Software Verification and Validation for Commercial Statistical Packages Utilized by the Statistical Consulting Section of SRTC

by

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Software Verification & Validation for Commercial Statistical Packages Utilized by the Statistical Consulting Section of SRTC (U)

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Prepared by
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ABSTRACT

The purpose of this report is to provide software verification and validation (v&v) for the statistical packages utilized by the Statistical Consulting Section (SCS) of the Savannah River Technology Center (SRTC). The need for this v&v stems from the requirements of the Quality Assurance (QA) programs that are frequently applicable to the work conducted by SCS.

This report covers the following software and computing platforms:

Computing Platform	Operating System	Software	Version
IBM Personal Computer 300PL or IBM Personal Computer 300XL (i.e., any Pentium II processor)	Windows NT Version 4.0	IMP®	3.2.2
		Microsoft Excel®	97 SR-1
		MIXSOFT™	2.3
		Statgraphics Plus®	4.0
Digital AlphaServer Model 4100 5/533	VMS-AXP	SAS®	6.12
	Open VMS V7		

The IBM Personal Computer 300PL and 300XL are both Pentium II based desktops. Therefore, the software verification and validation in this report is valid interchangeably between both platforms. As new computing platforms, statistical packages, or revisions to existing packages are introduced into the Statistical Consulting Section, the appropriate problems from this report are to be re-evaluated using these new tools, and this report is to be revised to address their verification and validation.

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INTRODUCTION

The mission of the Statistical Consulting Section (SCS) of the Savannah River Technology Center (SRTC) is to apply statistical thinking, methods, and computing in collaborative decision support, technology development, and continuous improvement at the Savannah River Site and to disseminate our knowledge and experience into the Federal Government complex via the Work For Others program. Computers and computer software are essential tools utilized by the SCS statisticians in pursuit of this mission. Many of these software programs are site-licensed and general purpose while some are special-purpose statistical packages.

As a general rule, memoranda, research reports, and technical reports prepared by members of SCS in response to customer requests for assistance are technically reviewed as part of the quality assurance (QA) for the section.¹ In SRTC, calculations are normally checked by alternate means (e.g., hand calculations) performed by an independent reviewer, but this is not always completely possible on modeling or other complicated calculations performed by some software programs. However, the technical review would certainly include an assessment of the appropriateness of the statistical approach and routines utilized in the document. In addition, validation and verification (v&v) of the software utilized for the analysis is frequently a requirement of the applicable QA program directing the investigation. This requirement is typically addressed uniquely in the document or in the supporting task plan, etc. The purpose of this report is to provide a central repository for the software verification and validation (v&v) for the statistical packages utilized by SCS. This document also demonstrates the v&v of some simple statistical software such as Excel spreadsheets. As new computing platforms, statistical packages, or revisions to existing packages are introduced into the Statistical Consulting Section, the appropriate problems from this report or new problems are to be evaluated using these new tools, and this report is to be revised to address their verification and validation.

The software considered in this report is commercial software (some of which is site-licensed). This study is to demonstrate that the vendor's software will perform correctly, as designed. The SRTC approach is to take problems with known solutions from peer reviewed publications and run them on the commercial software to demonstrate that the vendor's program does indeed perform as designed. The solutions of these problems are generated using software routines that are frequently utilized at SRTC. Running these routines using SRTC platforms and systems software and generating the appropriate answers to the "textbook" problems demonstrates the v&v of the software under consideration.

BACKGROUND

This report covers the software and computing platforms as identified in Table 1:

Table 1: SCS Computing Platforms and Associated Statistical Software Programs

Computing Platform	Operating System	Software	Version
IBM Personal Computer 300PL or IBM Personal Computer 300XL	Windows NT Version 4.0	IMP®	3.2.2
		Microsoft Excel®	97 SR-1
		MIXSOFT™	2.3
		Statgraphics Plus®	4.0
Digital AlphaServer Model 4100 5/533	VMS-AXP Open VMS V7	SAS®	6.12

¹ Such reviews may be a requirement of the applicable QA program directing a particular technical task.

The IBM Personal Computer 300PL and 300XL are both Pentium II based desktops. Therefore, the software v&v in this report is valid interchangeably between both platforms. JMP is a product of SAS Institute, Inc. [1]. The SAS system is a set of products. Those considered in this report include Base SAS [2], SAS/QC [3], SAS/STAT [4], and SAS/IML [5 and 6]. Microsoft Excel is a site-licensed product at the Savannah River Site. Mixsoft is a specialized software program for mixtures and other constrained-region problems [7]. Statgraphics contains numerous statistical routines and is a product of Manugistics, Inc.[8]. Other products (such as terminal emulation and virus protection packages) are also involved in the utilization of these platforms and software. These are not deemed important to the performance of the statistical programs and are not reviewed in this report.

The results from using each of the above packages to analyze the problems discussed below are organized by package as an appendix to this report.

DISCUSSION

In this section, problem types frequently encountered by members of SCS are identified. An example of each problem is selected from a well-established statistical textbook. The example is analyzed using a feature or features of the appropriate software described in the previous section. The results generated by the various software packages are compared to the information from the textbook and/or to each other for validation and verification. Little discussion is provided regarding the details of the problems, the underlying statistical theory, the statistical routines, or the statistical results. Information about the statistical packages, their capabilities, and details regarding their outputs can be found in their respective published documentation. These references, along with those cited as the sources of the problems, may be used to provide these details. The purpose of this report is show that the statistical packages, when used appropriately, provide reliable results.

Descriptive Statistics

The first area to be explored in this report is that of descriptive statistics, summary information about a set of data. Consider the set of data presented in Table 2, which is taken from Table 2.1 on page 40 of reference [9].

Table 2: Data on Lot Size and Number of Man-Hours

Production Run	Lot Size	Man-Hours
i	X_i	Y_i
1	30	73
2	20	50
3	60	128
4	80	170
5	40	87
6	50	108
7	60	135
8	30	69
9	70	148
10	60	132

From [9], the average of the lot size values, $\bar{X}$, is equal to 50 (see page 46), and several graphical depictions (including a Box Plot, Time Plot, and Stem-and-leaf Plot) of these lot size values are provided on page 114 of [9].

Using Excel Version 97 SR-1 on a Pentium II Processor Running Windows NT Version 4

The data from Table 2 were entered into Excel and the Excel Tools/Data Analysis/Descriptive Statistics pull-down menus were used to obtain descriptive statistics on the lot-size values that were cut and pasted into this report as Table A.1a in Appendix A. There are Excel functions that provide descriptive statistics as well. Table A.1b provides the results of applying some of these functions to the lot-size values.

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

The data from Table 2 were entered into JMP and the descriptive statistics capability of JMP (the Distribution-of-Y platform) was used to generate Exhibit B.1 in Appendix B for the lot size values. These results were determined by JMP, saved using JMP's journal feature, and imported (electronically) directly into this report. The average of the lot size values, 50, is included in the information presented by JMP. A Box Plot, a Stem-and-leaf plot, and a time plot (a plot by production run number) are also provided; these compare very favorably to the information on page 114 of [9].

Using SAS Version 6.12 on the AlphaServer Running Open VMS V7

The data from Table 2 were included in a SAS program that used PROC MEANS, PROC SUMMARY, and PROC UNIVARIATE to generate some descriptive statistics for the lot size values. The SAS program and results were downloaded to the PC and incorporated in this report. This information is provided in Exhibit C.1 of Appendix C.

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

The data from Table 2 were entered into Statgraphics and the numeric data one variable analysis routine of Statgraphics was used to generate Exhibit E.1 in Appendix E for the lot size values. These results were saved using Statgraphics' StatReporter feature, and imported (electronically) directly into this report. The average of the lot size values, 50, is included in the information presented by Statgraphics. A Scatter Plot, Box-and-Whisker Plot, Histogram, a Stem-and-leaf plot, and a Normal Probability Plot are also provided. The results compare very favorably to the information on page 114 of [9] and to the JMP output in Exhibit B.1.

Descriptive Statistics Summary Table

The critical descriptive information generated by the software packages reviewed above is summarized in Table 3.

Table 3: Summary of Descriptive Statistics for Lot-Size Values by Software Package

Source of Information/ Software Package	Mean	Standard Deviation	Standard Error
As described in [9] on page 46	50		
Excel Version 97 SR-1 on PC running Windows NT Version 4	50	19.4365	6.1464
JMP Version 3.2.2 on PC running Windows NT Version 4	50	19.4365	6.1464
SAS Version 6.12 on AlphaServer Running OpenVMS V7			
PROC MEAN, SUMMARY, and UNIVARIATE	50	19.4365	6.1464
Statgraphics Version 4.0 on PC running Windows NT Version 4.0	50	19.4365	6.1464

Table 3 summarizes what is revealed in the details of the exhibits covering this example: a consistent set of values for the descriptive statistics for these software packages across these computer platforms for the data of Table 2. Please note, however, that the output from the different packages often include different statistics.

Regression

The information presented in Table 2 also provides an opportunity for a look at various regression routines in fitting the simple linear model

$$Y = \beta_0 + \beta_1 X + \epsilon \quad (1)$$

where Y represents man-hours, X represents lot-size, the β 's represent the unknown coefficients that are to be estimated, and ϵ represents the error term (assumed to be independently, normally distributed with zero mean and constant variance over the Y's.)

From page 44 of [9], the estimate of β_0 , the y-intercept represented by b_0 , is determined to be 10.0 and the estimate of β_1 , the slope represented by b_1 , is determined to be 2.0.

Using Excel Version 97 SR-1 on a Pentium II Processor Running Windows NT Version 4

The data from Table 2 were entered into Excel and used to fit the model given in equation (1). Two methods were used to analyze these data with Excel. Tools/Data Analysis/Regression pull-down menus were used to fit the data to the model given by equation (1). The results were cut and pasted into this report as Table A.2 in Appendix A.

The matrix handling capabilities of Excel were also used to perform the least squares estimation of the regression parameters. The discussion of this approach to the data of Table 2 is provided in [9] on pages 207 and 208. The results from using Excel's matrix handling capabilities to analyze this problem were cut and pasted into this report as Table A.3 in Appendix A.

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

The data from Table 2 were entered into JMP and used to fit the model given in equation (1). Two methods were used to analyze these data with JMP. Exhibit B.2 in Appendix B provides the results from using the Fit Y By X platform to perform this analysis. Exhibit B.3 in Appendix B provides the results from using the Fit Model platform to perform the analysis. In both cases, the JMP results were journaled and imported into this report, and in both cases, the estimates for the slope and y-intercept are 2 and 10, respectively.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

The SAS set of procedures provides several ways of analyzing the data from Table 2. Exhibit C.2 in Appendix C provides the SAS/STAT program that utilizes PROC REG to perform the regression. Exhibit C.3 in Appendix C provides a SAS/IML program that estimates the β 's.

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

The data from Table 2 were entered into Statgraphics and used to fit the model given in equation (1). The Simple Regression method was used. The results are included in Exhibit E.2 in Appendix E. The StatReporter routine in Statgraphics was used to import the results into this report. The estimates for the slope and y-intercept are 2 and 10, respectively.

Regression Summary Table

The critical regression information generated by the software packages reviewed above is summarized in Table 4 along with the results from [9].

Table 4: Summary of Regression Statistics for Each Software Package

Source of Information/ Software Package	Estimate of Intercept	Estimate of Slope	R ²	Root Mean Square Error
As discussed in [9] on page 44	10	2		
Excel Version 97 SR-1 on a PC running Windows NT Version 4--- Regression	10	2	0.9956	2.7386
Excel Version 97 SR-1 on a PC running Windows NT Version 4--- Matrix handling capability	10	2		
JMP Version 3.2.2 on a PC running Windows NT Version 4 --- Fit Y by X	10	2	0.9956	2.7386
JMP Version 3.2.2 on a PC running Windows NT Version 4 --- Fit Model	10	2	0.9956	2.7386
SAS Version 6.12 on AlphaServer Running OpenVMS V7 PROC REG	10	2	0.9956	2.7386
SAS/IML Version 6.12 on Alphaserer Running OpenVMS V7	10	2		
Statgraphics Version 4.0 on a PC running Windows NT Version 4.0	10	2	0.9956	2.73861

Table 4 summarizes what is revealed in the exhibits covering this example: a consistent set of regression results from these software packages across these computer platforms for the data of Table 2.

ANOVA

Analysis of variance (ANOVA) models are versatile statistical tools for studying the relation between a dependent variable and one or more independent variables [9]. Several of these models are investigated in this section.

One-Way ANOVA

The example provided in Table 5 is from Table 14.1 on page 533 of [9]. In this table are recorded the number of cases sold by store for each of four package designs. An ANOVA is used to investigate for differences in sales across the four package designs.

**Table 5: Number of Cases Sold by Stores for Each of Four Package Designs---
Kenton Food Company Example**

Package Design	Cases Sold by Store		
	1	2	3
1	12	18	
2	14	12	13
3	19	17	21
4	24	30	

The discussion in [9] leads to the ANOVA results presented in Table 6 (this information appears as Table 14.4 on page 543 in [9]).

Table 6: ANOVA for Kenton Food Company Example

Source of Variation	SS	df	MS
Between designs	258	3	86
Error	46	6	7.67
Total	304	9	

Using Excel Version 97 SR-1 on a Pentium II Processor Running Windows NT Version 4

The data from Table 5 were entered into Excel, and Tools/Data Analysis/ANOVA: Single Factor pull-down menus were used to conduct the analysis of variance. The results were cut and pasted into this report as Table A.4 in Appendix A.

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

The data from Table 5 were entered into JMP, and the Fit Model platform was used to analyze these data. Exhibit B.4 in Appendix B provides the JMP results that were journaled and imported into this report.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

Two different tools available in the SAS system were used to analyze the data from Table 5. Exhibit C.4 in Appendix C provides the input and results of PROC ANOVA, and Exhibit C.5 in Appendix C provides this information for PROC GLM.

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

The data from Table 5 were entered into Statgraphics, and the One-Way ANOVA routine was used to analyze these data. Exhibit E.3 in Appendix E provides the Statgraphics results that were imported into this report using StatReporter.

One-Way ANOVA Summary Table

Some of the critical information from the ANOVA tables generated by the software packages reviewed above is summarized in Table 7 along with the results from [9].

Table 7: One-Way ANOVA Summary Statistics for Each Software Package

Source of Information/ Software Package	Sum of Squares Between	Sum of Squares For Error	Mean Squares for Designs	F Statistic for Differences
As discussed in [9] pages 543 and 548	258	46	86	11.2
Excel Version 97 SR-1 on Windows NT Version 4--- ANOVA: Single Factor	258	46	86	11.217
JMP Version 3.2.2 on a PC running Windows NT Version 4 --- Fit Model	258	46	86	11.217
SAS/STAT Version 6.12 on Alphaserer Running OpenVMS V7 PROC ANOVA	258	46	86	11.217
SAS/STAT Version 6.12 on Alphaserer Running OpenVMS V7 PROC GLM	258	46	86	11.217
Statgraphics Version 4.0 on a PC running Windows NT Version 4	258	46	86	11.22

Table 7 summarizes what is revealed in the exhibits covering this example: a consistent set of ANOVA results from these software packages across these computer platforms for the data of Table 3.

One-Way ANOVA with Random Factor

The example provided in Table 8 is from Table 17.3 on page 654 of [9]. In this table are recorded the ratings by five (randomly selected) personnel officers of Apex Enterprises for four randomly assigned (to each officer) candidates. An ANOVA is used to estimate the variation in ratings among all personnel officers of this company.

Table 8: Ratings by Personnel Officers of Apex Enterprises [9]

Officer	Candidate (j)			
(i)	1	2	3	4
A	76	64	85	75
B	58	75	81	66
C	49	63	62	46
D	74	71	85	90
E	66	74	81	79

The ANOVA for this problem is generated as in the previous section, but the interpretation of the information in the ANOVA under the conditions of a random factor lead to some additional calculations used to estimate the variance in ratings among the personnel officers. A discussion of the details of this estimation process is provided on page 660 of [9], leading to an estimate of 73.6 for this variance. Currently, Excel does not automatically generate this estimate as part of its ANOVA: Single Factor routine.

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

The data from Table 8 were entered into JMP, and the Fit Model platform was used (with a random factor designated in the fit) to analyze these data. Exhibit B.5 in Appendix B provides the JMP results that were journaled and imported into this report.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

PROC GLM and PROC VARCOMP demonstrate the capability of SAS to handle this type of problem for the data in Table 8. The inputs and results from each of these two procedures for solving this problem are provided in Exhibits C.6 and C.7 in Appendix C.

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

The data from Table 8 were entered into Statgraphics, and the ANOVA-Variance Components routine was used to analyze these data. Exhibit E.4 in Appendix E provides the Statgraphics results that were imported into this report using StatReporter

One-Way ANOVA (with a Random Factor) Summary Table

Some of the critical information from the ANOVA tables generated by the software packages reviewed above is summarized in Table 9 along with the results from [9]. Note that PROC GLM provides the equation for solving for the desired estimate. Using this equation along with the ANOVA information leads to an estimate of 73.6 for the rating variance.

**Table 9: One-Way ANOVA (Random Factor) Summary Statistics
for Each Software Package**

Source of Information/ Software Package	Sum of Squares Between	Sum of Squares For Error	Mean Squares for Offices	Estimate of Rating Variance
As discussed in [9] pages 655 and 660	1480	1134	370	73.6
JMP Version 3.2.2 on a PC running Windows NT Version 4 --- Fit Model (random)	1480	1134	370	73.6
SAS Version 6.12 on Alphaserer Running OpenVMS V7 PROC GLM	1480	1134	370	73.6
SAS Version 6.12 on Alphaserer Running OpenVMS V7 PROC VARCOMP	1480	1134	370	73.6
Statgraphics Version 4.0 on a PC running Windows NT Version 4.0	1480	1134	370	73.6

Table 9 summarizes what is revealed in the details of the exhibits covering this example: a consistent set of ANOVA results for these software packages across these computer platforms for the data of Table 8.

Two-Way ANOVA

The example provided in Table 10 is from Table 21.2 on page 787 of [9]. In this table automobile insurance premiums (in dollars) are provided for a city of small, medium, and large size in each of two regions (East and West) of the US. An ANOVA is used to investigate differences between the regions and among the cities. An assumption is made that there is no interaction between these two factors.

Table 10: Insurance Premiums [9]

Insurance Premiums in Dollars			
		Region	
		East	West
Size of City	Small	140	100
	Medium	210	180
	Large	220	200

The discussion in [9] leads to the ANOVA results presented in Table 11 (this information also appears in Table 21.2 on page 787 in [9]).

Table 11: ANOVA for Insurance Example

Source of Variation	SS	df	MS
Size of City	9,300	2	4,650
Region	1,350	1	1,350
Error	100	2	50
Total	10,750	5	

Using Excel Version 97 SR-1 on a Pentium II Processor Running Windows NT Version 4

The data from Table 10 were entered into Excel, and Tools/Data Analysis/ANOVA: Two Factors Without Replication pull-down menus were used to conduct the analysis of variance. The results were cut and pasted into this report as Table A.5 in Appendix A.

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

The data from Table 10 were entered into JMP, and the Fit Model platform was used to analyze these data. Exhibit B.6 in Appendix B provides the JMP results that were journaled and imported into this report.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

The data from Table 10 were analyzed using PROC ANOVA and PROC GLM of the SAS system. Exhibits C.8 and C.9 in Appendix C provide the inputs and results from using these two procedures to perform this analysis.

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

The data from Table 10 were entered into Statgraphics, and the Multi Factor ANOVA routine was used to analyze these data. Exhibit E.5 in Appendix E provides the Statgraphics results that were imported into this report using StatReporter

Two-Way ANOVA Summary Table

Some of the critical information from the ANOVA tables generated by the software packages reviewed above is summarized in Table 12 along with the results from [9].

Table 12: Two-Way ANOVA Summary Statistics for Each Software Package

Source of Information/ Software Package	Sum of Squares for City Size	Sum of Squares For Region	Sum of Squares for Error	F Statistic for Region
As discussed in [9] pages 787 and 788	9300	1350	100	27
Excel Version 97 SR-1 on a PC running Windows NT Version 4--- ANOVA: Two-Factors without replication	9300	1350	100	27
JMP Version 3.2.2 on a PC running Windows NT Version 4 --- Fit Model	9300	1350	100	27
SAS Version 6.12 on Alphaserer Running OpenVMS V7 PROC ANOVA	9300	1350	100	27
SAS Version 6.12 on Alphaserer Running OpenVMS V7 PROC GLM	9300	1350	100	27
Statgraphics Version 4.0 on a PC running Windows NT Version 4	9300	1350	100	27

Table 12 summarizes what is revealed in the exhibits covering this example: a consistent set of ANOVA results from these software packages across these computer platforms for the data of Table 10.

Two-Factor Nested ANOVA

A nested two-factor model differs from the previous two-factor (crossed) model in that the levels of the second factor are unique to each level of the first factor. An example of this situation is provided in Table 13 (this example is provided as Table 26.1 on page 971 of [9]).

**Table 13: Sample Data for Nested Two-Factor Study
(Training School Example from [1])**

Factor A (School)	Factor B (Instructor)	
	1	2
Atlanta	25	14
	29	11
Chicago	11	22
	6	18
San Francisco	17	5
	20	2

The discussion in [9] leads to the ANOVA results presented in Table 14 (this information appears in Table 26.5 on page 981 in [9]).

Table 14: ANOVA for Training School Example

Source of Variation	SS	df	MS
Schools	156.5	2	78.25
Instructors within Schools	567.5	3	189.17
Error	42.0	6	7.00
Total	766.0	11	

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

JMP does provide the desired estimate. The data from Table 13 were entered into JMP, and the Fit Model platform was used to analyze these data. Exhibit B.7 in Appendix B provides the JMP results that were journaled and imported into this report.

If both of these factors were random instead of fixed for the data in Table 13, the questions of interest would be different (what variation in scores is due to school? and what variation in scores is due to instructor?) and the test statistics to answer these questions would be different (this is discussed on page 984 of [9]). JMP handles this type of problem in its Fit Model platform. Using this approach leads to the results presented in Exhibit B.8 in Appendix B.

From the discussion of page 985 of [9], the test statistics for this random effects problem are given by

$$\text{Test for schools : } F = \frac{MSA}{MSB(A)} = \frac{78.25}{189.17} = 0.414$$

and

$$\text{Test for instructors : } F = \frac{MSB(A)}{MSE} = \frac{189.17}{7} = 27.0$$

From Exhibit B.8, the test statistic for schools is 0.414 and for instructors is 27.0.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

The SAS system's PROC ANOVA was used to analyze the data in Table 13 and the results are presented in Exhibit C.10. Exhibit C.11 provides the results from the use of PROC GLM to analyze these same data.

If both factors are assumed to be random, there are still at least two-ways to analyze these data with SAS: PROC GLM and PROC VARCOMP. Exhibits C.12 and C.13 provide the inputs and results for these two procedures. Note that PROC VARCOMP does not compute the F statistic for schools, but the procedure does estimate this variance (-27.7) as does the JMP procedure, by following the equation on page 985 of [9]. (A negative estimate indicates that this variance is not statistically significant for these data.)

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

The data from Table 13 were entered into Statgraphics, and the Special, Advanced Regression, General Linear Model platform was used to analyze these data. Exhibit E.6 in Appendix E provides the Statgraphics results that were imported into this report using StatReporter. No option is provided for allowing both factors to be random. However, the Statgraphics results using Variance Components are presented in Exhibit E.7.

Two-Factor Nested ANOVA Summary Table

Some of the critical information from the ANOVA tables generated by the software packages reviewed above is summarized in Table 15 along with the results from [9].

Table 15: Two-Way Nested ANOVA Summary Statistics for Each Software Package

Source of Information/ Software Package	Sum of Squares for School	Sum of Squares For Instructor	Sum of Squares for Error	F Statistic for Schools
As discussed in [9] pages 981 - 984	156.5	567.5	42	11.2
JMP Version 3.2.2 on Windows NT Version 4 --- Fit Model	156.5	567.5	42	11.2
SAS/STAT Version 6.12 on Alphaserer Running OpenVMS V7 PROC ANOVA	156.5	567.5	42	11.2
SAS/STAT Version 6.12 on Alphaserer Running OpenVMS V7 PROC GLM	156.5	567.5	42	11.2
Statgraphics for Windows Version 4.0	156.5	567.5	42	11.18

Table 15 summarizes what is revealed in the exhibits covering this example: a consistent set of ANOVA results from these software packages across these computer platforms for the data of Table 13.

Some of the critical information from the ANOVA tables generated by the software packages reviewed above for the situation in which the two factors of Table 13 are random is summarized in Table 16 along with the results from [9].

**Table 16: Two-Way Nested and Random ANOVA Summary Statistics
for Each Software Package**

Source of Information/ Software Package	Sum of Squares for School	Sum of Squares For Instructor	Sum of Squares for Error	F Statistic for Schools
As discussed in [9] pages 981 - 984	156.5	567.5	42	0.414
JMP Version 3.2.2 on Windows NT Version 4 --- Fit Model	156.5	567.5	42	0.414
SAS/STAT Version 6.12 on Alphaserer Running OpenVMS V7 PROC GLM	156.5	567.5	42	0.414
SAS/STAT Version 6.12 on Alphaserer Running OpenVMS V7 PROC VARCOMP	156.5	567.5	42	
Statgraphics for Windows Version 4.0	156.5	567.5	42	-

Table 16 summarizes what is revealed in the exhibits covering this example: a consistent set of ANOVA results from these software packages across these computer platforms for the data of Table 13 with both factors random.

Experimental Designs

Another major area of interest is that of experimental design. Two important types of problems in this area, which are addressed in this section, are fractional factorial experiments and mixture experiments. Several packages are utilized by SCS in planning these types of experiments.

Fractional Factorial

An excellent aid in the planning of these types of experiments is provided in Table 9A.1 on pages 182 and 183 of [10]. A portion of this table covering 6-factors experiments is provided in Table 17.

**Table 17: Selected Fractional Factorial Experiments
of the Complete Factorial Experiment for a 6-Factor Study**

Number of Factors	Number of Test Runs	Fraction	Resolution	Defining Equations	Added Factors
6	8	1/8	III	I = ABD I = ACE I = BCF	4=12 5=13 6=23
	16	1/4	IV	I=ABCE	5=123

The features of this quarter fraction design for this 6-factor study of interest

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

Using the Design platform of JMP, candidate designs involving 6 factors can be explored. One option presented is a 16-run experiment (a quarter fraction of the complete factorial experiment). Exhibit B.9 in Appendix B provides the results of selecting this option from the list of JMP candidates.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

PROC FACTEX in SAS/QC can be used to generate such designs. The input and results for this SAS procedure are provided in Exhibit C.14 of Appendix C.

Using Mixsoft Version 2.3 on a Pentium II Processor Running Windows NT Version 4

This is a specialized software program that aids in experimental designs. Exhibit D.1 in Appendix D provides the results of using this program to select a fractional factorial experiment consisting of 16 trials for a 6-factor problem.

Using Statgraphics Version 4.0 on a Pentium II Processor Running Windows NT Version 4

Using the Experimental Design platform of Statgraphics a quarter fraction of the complete factorial experiment for a 6-factor problem was selected. The results appear in Exhibit E.8 in the Appendix.

Fractional Factorial Summary

Four different packages were used to generate this fractional factorial experiment, and the results are identical except that the columns for the Mixsoft results are in a different order (these can be rearranged to match results from the other packages exactly).

Mixture

Mixture experiments have been of critical importance in the support of DWPF and in other studies of the vitrification of legacy materials. In mixture experiments, the factors are ingredients of a mixture, and their levels are not independent. Extreme vertices designs are used to support these types of problems. For a full discussion, see Chapter 9 of reference [11]. An example from this reference will be used to illustrate the capabilities of the software utilized by SCS in support of mixture experiments. This example is discussed in Section 9.3.2 and involves a mixture of three components with each component being bounded as given in the equation (2)

$$0.20 \leq x_1 \leq 0.60 \qquad 0.10 \leq x_2 \leq 0.60 \qquad 0.10 \leq x_3 \leq 0.50 \qquad (2)$$

where the three components are represented by the x_i 's. The discussion in [4] on pages 353 through 358 identifies 6 extreme vertices for the region defined by equation (2). These extremes are given in Table 18 [10].

Table 18: Extreme Vertices for Region Defined by Equation (2)

Count	x_1	x_2	x_3	Sum
1	0.6	0.3	0.1	1
2	0.3	0.6	0.1	1
3	0.2	0.6	0.2	1
4	0.2	0.3	0.5	1
5	0.4	0.1	0.5	1
6	0.6	0.1	0.3	1

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

Using the Design platform of JMP, the region defined by equation (2) was entered and the Extreme Vertices design option invoked. Exhibit B.10 in Appendix B provides the results of selecting this option under the JMP software.

Using Mixsoft Version 2.3 on a Pentium II Processor Running Windows NT Version 4

Once again, Mixsoft is specialized software; one of its capabilities is mixture experimental design. Exhibit D.2 in Appendix D provides the inputs and outputs generated by this program to select generate the extreme vertices for the problem defined by equation (2).

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

Exhibit C.15 in Appendix C provides the input and results from using the mixture design capabilities provided in SAS/QC to generate the extreme vertices for the region defined by equation (2).

Using Statgraphics, Version 4.0 on a Pentium II Processor Running Windows NT Version 4

Using the Experimental Design platform of Statgraphics, the region defined by equation (2) was entered and the Extreme Vertices design option selected. Exhibit E.9 in the Appendix provides the Statgraphics results.

Mixture Summary

Four different packages were used to generate the set of extreme vertices for the mixture experiment described by equation (2), and the results are identical to those of Table 18 across all four packages.

Optimal Designs

Selecting an optimal design from a set of candidate points is frequently a necessity during the planning of an experiment. Computer-aided design of experiments routines utilize one or more of design optimality criteria to choose such a set of points (the design) from a candidate list of points. Almost all of these computer-aided design routines are model dependent. Once a model is chosen and a list of candidate design points is specified, a particular design (of a designated size) that minimizes or maximizes a particular criterion is selected from the candidate points. One of the more frequently selected criteria for choosing a design is

D-optimality, which seeks to minimize the determinant of $(X'X)^{-1}$ where each row of the matrix X is a design point, i.e., a set of explanatory variables: $x_1, x_2, \dots, x_p$.

This is a model-dependent criterion, and a design that is optimal for one model form, for example a first-degree model such as (3), will not necessarily be optimal for another model such as a second-

degree model. The example to be considered as part of this report is the use of this criterion to select 8 design points from those listed in Table 19.

Table 19: Face-Centered Cube Design

Pattern	x_1	x_2	x_3	Comment
+00	1	0	0	Axial
++	1	1	-1	FF
---	-1	-1	-1	FF
000	0	0	0	Center-FF
00-	0	0	-1	Axial
000	0	0	0	Center-FF
-00	-1	0	0	Axial
00+	0	0	1	Axial
+++	1	1	1	FF
+-+	1	-1	1	FF
000	0	0	0	Center-FF
--+	-1	-1	1	FF
-+-	-1	1	-1	FF
0-0	0	-1	0	Axial
+--	1	-1	-1	FF
0+0	0	1	0	Axial
000	0	0	0	Center-Ax
000	0	0	0	Center-Ax
000	0	0	0	Center-FF
---+	-1	1	1	FF

The design points provided in this table represent a "face-centered cube design," similar to that discussed in [10]. The optimal design for a linear model in x_1 , x_2 , and x_3 with an intercept term is given by the shaded rows of Table 19. These points make up the fractional factorial part of the face-centered cube design.

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

A feature of the JMP software package is its D-Optimal Design routine to choose a set of points (the design) from a candidate list of points [1]. The data from Table 19 were entered into JMP and the D-optimal routine evoked to select the best set of 8 points from the set of 20 points. The results from this process are provided in Exhibit B.11 in Appendix B. Values of the D-Optimality criteria, including D-efficiency, are provided as part of the output from this routine. The JMP spreadsheet resulting from the process is also provided in Exhibit B.11, and it shows the rows selected as "optimal" for a linear model.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

Exhibit C.16 in Appendix C provides the results from using PROC OPTEX in SAS to select an optimal design.

Using Statgraphics Version 4.0 on a Pentium II Processor Running Windows NT Version 4

Exhibit E.10 in Appendix E provides the results from using the D-Optimal Design routine in Statgraphics.

Optimal Design Summary

Three different packages were used to generate a set of eight design points from those in Table 19 that would be D-optimal for a linear model. The two data sets resulting from the JMP and SAS procedures were identical. Statgraphics had a mirror image design point (+--) for one of the eight points selected by JMP (-++). However, the design efficiencies are identical. The Statgraphics design was input into JMP and the efficiency statistics reproduced.

Control Charts

As a final area of interest in this review of software, a problem in statistical process control is explored. The construction of $\bar{x}$ -bar and s charts will be illustrated using an example from [12] (these data are provided in Table 5.1 on page 83 of [12]). These data (along with summary information) are presented in Table 20.

Table 18: Data in Subgroups Obtained at Regular Intervals
(Example 5.1, Table 5.1 of [12])

Subgroup	x1	x2	x3	x4	Average	Std Dev
1	72	84	79	49	71.00	15.47
2	56	87	33	42	54.50	23.64
3	55	73	22	60	52.50	21.70
4	44	80	54	74	63.00	16.85
5	97	26	48	58	57.25	29.68
6	83	89	91	62	81.25	13.28
7	47	66	53	58	56.00	8.04
8	88	50	84	69	72.75	17.23
9	57	47	41	46	47.75	6.70
10	13	10	30	32	21.25	11.35
11	26	39	52	48	41.25	11.53
12	46	27	63	34	42.50	15.76
13	49	62	78	87	69.00	16.87
14	71	63	82	55	67.75	11.53
15	71	58	69	70	67.00	6.06
16	67	69	70	94	75.00	12.73
17	55	63	72	49	59.75	9.98
18	49	51	55	76	57.75	12.42
19	72	80	61	59	68.00	9.83
20	61	74	62	57	63.50	7.33

$\bar{x}$ -Bar and s Charts

The values were entered into Excel, and the summary statistics were computed using the AVERAGE and STDEV functions of Excel. These are the values that appear in the last two columns of Table 20, and they agree with the information in [12]. The data were also entered into JMP, and the sample means and standard deviations were computed. These summary statistics were computed using JMP's "grouping" feature. Table B.1 in Appendix B provides these values (which agree with Table 20).

Using JMP Version 3.2.2 on a Pentium II Processor Running Windows NT Version 4

Using the Graph/Control Charts platform of JMP, the $\bar{x}$ -bar and s charts for the data of Table 20 were generated after the values were entered into JMP. These charts appear as Exhibit B.12 in Appendix B.

Using SAS Version 6.12 on AlphaServer Running Open VMS V7

Exhibit C.17 in Appendix C provides the inputs to and results from using PROC SHEWHART in SAS to generate these control charts.

Using Statgraphics Version 4.0 on a Pentium II Processor Running Windows NT Version 4

Using the Special/Quality Control/Variables Control Charts/ $\bar{x}$ -bar and s option of Statgraphics, the $\bar{x}$ -bar and s charts for the data in Table 20 were generated using Statgraphics. These charts appear as Exhibit E.11 in Appendix E.

Control Chart Summary Table

Some of the critical information from the control charts generated by the software packages reviewed above is provided in Table 21 along with the results from [12].

Table 21: Control Chart Summary Statistics for Each Software Package

Source of Information/ Software Package	Center-line for X-bar Chart	Upper Control limit for x-bar Chart	Center- line for s Chart	Upper Control limit for s Chart
As discussed in [12] pages 83-96	59.4	82.1	13.9	31.5
JMP Version 3.2.2 on a PC running Windows NT Version 4 --- Fit Model	59.4	82.1	13.9	31.5
SAS/QC Version 6.12 on AlphaServer Running OpenVMS V7 PROC SHEWHART	59.4	82.1	13.9	31.5
Statgraphics Version 4.0 on a PC running for Windows Version 4.0	59.4	82.1	13.9	31.5

Table 21 summarizes what is revealed in the exhibits covering this example: a consistent set of control charts from these software packages across these computer platforms for the data of Table 20.

CONCLUDING COMMENTS

The statistical analyses completed in this study provide an important verification and validation of the statistical software and computer platforms utilized by the members of SCS. The IBM Personal Computer 300PL and 300XL are both Pentium II based desktops. Therefore, the software v&v in this report is valid interchangeably between both platforms.

Problems frequently encountered by members of SCS are identified; an example of each problem is selected from a well-established statistical textbook; the example is analyzed using a feature or features of the software (as appropriate) described in the previous section; and the results generated by the various software packages are compared to the information from the textbook and/or to each other for validation and verification. Little discussion is provided regarding the details of the problems, the underlying statistical theory, the statistical routines, or the statistical results. Information about the statistical packages, their capabilities, and details regarding their outputs can be found in their respective published documentation. These references along with those cited as the sources of the problems may be used to provide these details.

This report has shown that these statistical packages, when used appropriately, provide reliable results over a broad range of problem types. This effort is not intended to diminish the importance of the technical review process. As seen in the discussion above, selecting the appropriate statistical approach and model for the problem at hand and the appropriate feature of the available software for its solution are important issues. An important part of the technical review process is to confirm the appropriateness of these decisions.

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APPENDICES

Appendix A: Excel Results

- Table A.1a: Excel Descriptive Statistics for Lot-Size Values in Table 2
- Table A.1b: Excel Functions for Lot-Size Values in Table 2
- Table A.2: Excel Regression of Information in Table 2
- Table A.3: Excel Matrix Handling Capabilities Used to Conduct Regression Analysis for Information in Table 2
- Table A.4: Excel ANOVA
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Appendix B: JMP Results

- Table B.1: JMP Sample Statistics for Data from Table 20
- Exhibit B.1: JMP Output for Descriptive Statistics of Lot Size Information in Table 2
- Exhibit B.2: JMP Output for Regression Model for Information in Table 2 Using Fit Y by X
- Exhibit B.3: JMP Output for Regression Model for Information in Table 2 Using Fit Model
- Exhibit B.4: JMP Output for ANOVA of Information in Table 5 Using Fit Model
- Exhibit B.5: JMP Output for ANOVA of Information in Table 8 Using Fit Model with Random Factor
- Exhibit B.6: JMP Output for ANOVA of Information in Table 8 Using Fit Model with Two Factors
- Exhibit B.7: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with A Nested Factor
- Exhibit B.8: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with Random Factors
- Exhibit B.9: JMP Output for a Fractional Factorial Experiment using the Design Experiment Feature
- Exhibit B.10: JMP Output for Mixture Problem Defined by Equation (2)
- Exhibit B.11: JMP's D-Optimality Results
- Exhibit B.12: JMP Results for x-Bar and s Charts for Data in Table 20

Appendix C: SAS Results

- Exhibit C.1: SAS Input for and Results from Descriptive Statistics for Lot Size Values in Table 2
- Exhibit C.2: SAS Input for and Results from Proc Reg for Table 2 Data
- Exhibit C.3: SAS Input for and Results from SAS/IML Program for Regression of Table 2 Data
- Exhibit C.4: SAS Input for and Results from SAS/STAT Proc ANOVA for Analyzing of Table 5 Data
- Exhibit C.5: SAS Input for and Results from SAS/STAT Proc GLM for Analyzing of Table 5 Data
- Exhibit C.6: SAS Input for and Results from SAS/STAT Proc GLM for Analyzing of Table 8 Data
- Exhibit C.7: SAS Input for and Results from SAS/STAT Proc VARCOMP for Analyzing of Table 8 Data
- Exhibit C.8: SAS Input for and Results from SAS/STAT Proc ANOVA for Analyzing of Table 8 Data
- Exhibit C.9: SAS Input for and Results from SAS/STAT Proc GLM for Analyzing of Table 8 Data
- Exhibit C.10: SAS Input for and Results from SAS/STAT Proc ANOVA for Analyzing of Table 13 Data
- Exhibit C.11: SAS Input for and Results from SAS/STAT Proc GLM for Analyzing of Table 13 Data
- Exhibit C.12: SAS Input for and Results from SAS/STAT Proc GLM for Analyzing of Table 13 Data
- Exhibit C.13: SAS Input for and Results from SAS/STAT Proc VARCOMP for Analyzing of Table 13 Data
- Exhibit C.14: SAS Input for and Results from SAS/QC Proc Factex for Experimental Design
- Exhibit C.15: SAS Input and Output for Mixture Problem Defined by Equation (2)
- Exhibit C.16: SAS Input for and Results from SAS/QC Proc Optex for D-Optimality
- Exhibit C.17: SAS Input and Output for Control Chart Example from Table 20

Appendix D: Mixsoft Results

Exhibit D.1: Mixsoft Output for a Fractional Factorial Experiment using the Design Experiment Feature

Exhibit D.2: Mixsoft Input and Output for Mixture Problem Defined by Equation (2)

Appendix E: Statgraphics Results

Exhibit E.1: Statgraphics Output for Statistics of Lot Size Information in Table 2

Exhibit E.2: Statgraphics Output for Table 2 Data Using Simple Regression

Exhibit E.3: Statgraphics Output for Table 5 Data Using One-Way ANOVA

Exhibit E.4: Statgraphics Output for Table 8 Data using Variance Components Analysis

Exhibit E.5: Statgraphics Output for Table 10 Data using Two Factor ANOVA

Exhibit E.6: Statgraphics Output for Table 13 Data using a Nested

Exhibit E.7: Statgraphics Output for Table 13 Data using Variance Component Analysis

Exhibit E.8: Statgraphics Output for Fractional Factorial Design

Exhibit E.9: Statgraphics Output for Extreme Vertices

Exhibit E.10: Statgraphics D-Optimality Results

Exhibit E.11: Statgraphics X-bar and S Charts - X

Appendix A: Excel Results

Table A.1a: Excel Descriptive Statistics for Lot-Size Values in Table 2

Lot Size (X_i)	
Mean	50
Standard Error	6.146362972
Median	55
Mode	60
Standard Deviation	19.43650632
Sample Variance	377.7777778
Kurtosis	-1.066608997
Skewness	-0.113491711
Range	60
Minimum	20
Maximum	80
Sum	500
Count	10
Largest(1)	80
Smallest(1)	20
Confidence Level(95.0%)	13.90404962

Table A.1b: Excel Functions for Lot-Size Values in Table 2

EXCEL Function	Value	Description
Count	10	Number of data
Average	50	Average of data
Sum	500	Sum of data
Minimum	20	Minimum of data
Maximum	80	Maximum of data
Median	55	Median of data
Mode	60	Mode of data
DEVSQ	3400	Sum of Squares of Deviations about the Mean
Std Dev	19.43650632	Standard deviation of data
Skew	-0.11349171	Skewness of data
Kurt	-1.06660900	Kurtosis of data
Geomean	46.12054471	Geometric mean of data
Harmean	41.93709436	Harmonic mean of data

Appendix A: Excel Results

Table A.2: Excel Regression of Information in Table 2

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.997801							
R Square	0.995608							
Adjusted R Square	0.995059							
Standard Error	2.738613							
Observations	10							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	13600	13600	1813.333	1.02E-10			
Residual	8	60	7.5					
Total	9	13660						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	10	2.502939	3.995302	0.003976	4.228208	15.77179	4.228208	15.77179
Lot Size (Xi)	2	0.046967	42.58325	1.02E-10	1.891694	2.108306	1.891694	2.108306

Appendix A: Excel Results

Table A.3: Excel Matrix Handling Capabilities Used to Conduct Regression Analysis for Information in Table 2

Matrix Approach:											
Y =		73	X =		1	30					
		50			1	20					
		128			1	60					
		170			1	80					
		87			1	40					
		108			1	50					
		135			1	60					
		69			1	30					
		148			1	70					
		132			1	60					
Estimates of Beta's		=	INVERSE[(X'X)]X'Y		=	10	=	estimate of intercept			
						2		estimate of slope			
X' =		1	1	1	1	1	1	1	1	1	
		30	20	60	80	40	50	60	30	70	60
X'X=		10	500	X'Y=		1100					
		500	28400			61800					
INVERSE(X'X)		0.835294	-0.01471								
		-0.01471	0.000294								

Appendix A: Excel Results

Table A.4: Excel ANOVA

Anova: Single Factor

SUMMARY

<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
1	2	30	15	18
2	3	39	13	1
3	3	57	19	4
4	2	54	27	18

ANOVA

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	258	3	86	11.21739	0.007135	4.757055
Within Groups	46	6	7.666667			
Total	304	9				

Appendix A: Excel Results

Table A.5: Excel ANOVA: Two-Factor Without Replication

Anova: Two-Factor Without Replication						
SUMMARY	Count	Sum	Average	Variance		
Small	2	240	120	800		
Medium	2	390	195	450		
Large	2	420	210	200		
East	3	570	190	1900		
West	3	480	160	2800		
ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Rows	9300	2	4650	93	0.010638	19.00003
Columns	1350	1	1350	27	0.035099	18.51276
Error	100	2	50			
Total	10750	5				

Appendix B: JMP Results

Table B.1: JMP Sample Statistics for Data from Table 20

Subgroup	N Rows	Mean(x)	Std Dev(x)
1	4	71	15.4704
2	4	54.5	23.64318
3	4	52.5	21.70253
4	4	63	16.8523
5	4	57.25	29.68024
6	4	81.25	13.27592
7	4	56	8.041559
8	4	72.75	17.23127
9	4	47.75	6.70199
10	4	21.25	11.35415
11	4	41.25	11.52895
12	4	42.5	15.7586
13	4	69	16.87207
14	4	67.75	11.52895
15	4	67	6.055301
16	4	75	12.72792
17	4	59.75	9.979145
18	4	57.75	12.41974
19	4	68	9.831921
20	4	63.5	7.325754

Appendix B: JMP Results

Exhibit B.1: JMP Output for Descriptive Statistics of Lot Size Information in Table 2

Quantiles

maximum	100.0%	80.000
	99.5%	80.000
	97.5%	80.000
	90.0%	79.000
	75.0%	62.500
quartile	50.0%	55.000
quartile	25.0%	30.000
minimum	10.0%	21.000
	2.5%	20.000
	0.5%	20.000
	0.0%	20.000

Note: There are several ways to determine estimates of quantiles. JMP computes them as follows [1]

"To compute the Pth quantile of N nonmissing values in a column, arrange the N values in ascending order and call these column values $y_1, y_2, \dots, y_N$. Compute the rank number for the Pth quantile as $\frac{P}{100}(N+1)$

If the result is an integer, the Pth quantile is that rank's corresponding value. If the result is not an integer, the Pth quantile is found by interpolation. Denote the integer portion of the computed rank number as I and the fractional portion as f. The Pth quantile, denoted q_P , is computed $q_P = (1-f)y_I + (f)y_{I+1}$

If $I=N$, then y_N is taken as the quantile."

Moments

Mean	50.00000
Std Dev	19.43651
Std Error Mean	6.14636
Upper 95% Mean	63.90416
Lower 95% Mean	36.09584
N	10.00000
Sum Weights	10.00000

Appendix B: JMP Results

Exhibit B.1: JMP Output for Descriptive Statistics of Lot Size Information in Table 2
(continued)

Stem and Leaf

Stem	Leaf	Count
8	0	1
7	0	1
6	000	3
5	0	1
4	0	1
3	00	2
2	0	1

Multiply Stem.Leaf by 10

Appendix B: JMP Results

Exhibit B.2: JMP Output for Regression Model for Information in Table 2 Using Fit Y by X

Linear Fit

Man-Hours (Yi) = 10 + 2 Lot Size (Xi)

Summary of Fit

RSquare	0.995608
RSquare Adj	0.995059
Root Mean Square Error	2.738613
Mean of Response	110
Observations (or Sum Wgts)	10

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	13600.000	13600.0	1813.333
Error	8	60.000	7.5	Prob>F
C Total	9	13660.000		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	10	2.502939	4.00	0.0040
Lot Size (Xi)	2	0.046967	42.58	<.0001

Appendix B: JMP Results

Exhibit B.3: JMP Output for Regression Model for Information in Table 2 Using Fit Model

Response: Man-Hours (Yi)

Summary of Fit

RSquare	0.995608
RSquare Adj	0.995059
Root Mean Square Error	2.738613
Mean of Response	110
Observations (or Sum Wgts)	10

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	5	27.333333	5.4667	0.5020
Pure Error	3	32.666667	10.8889	Prob>F
Total Error	8	60.000000		0.7662
Max RSq				
0.9976				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	10	2.502939	4.00	0.0040
Lot Size (Xi)	2	0.046967	42.58	<.0001

Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
Lot Size (Xi)	1	1	13600.000	1813.333	<.0001

Whole-Model Test

Appendix B: JMP Results

Exhibit B.3: JMP Output for Regression Model for Information in Table 2 Using Fit Model
(continued)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	13600.000	13600.0	1813.333
Error	8	60.000	7.5	Prob>F
C Total	9	13660.000		<.0001

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
13600.000	1813.333	1	<.0001

Appendix B: JMP Results

Exhibit B.4: JMP Output for ANOVA of Information in Table 5 Using Fit Model

Response: Case Sold
Summary of Fit

RSquare	0.848684
RSquare Adj	0.773026
Root Mean Square Error	2.768875
Mean of Response	18
Observations (or Sum Wgts)	10

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	18.5	0.89365	20.70	<.0001
Package [1-4]	-3.5	1.64781	-2.12	0.0778
Package [2-4]	-5.5	1.440968	-3.82	0.0088
Package [3-4]	0.5	1.440968	0.35	0.7404

Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
Package Design	3	3	258.00000	11.2174	0.0071

Whole-Model Test

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	258.00000	86.0000	11.2174
Error	6	46.00000	7.6667	
C Total	9	304.00000		

Prob>F
0.0071

Appendix B: JMP Results

Exhibit B.4: JMP Output for ANOVA of Information in Table 5 Using Fit Model
(continued)

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
258.00000	11.2174	3	0.0071

Least Squares Means

Level	Least Sq Mean	Std Error	Mean
1	15.00000000	1.957890021	15.0000
2	13.00000000	1.598610508	13.0000
3	19.00000000	1.598610508	19.0000
4	27.00000000	1.957890021	27.0000

Appendix B: JMP Results

Exhibit B.5: JMP Output for ANOVA of Information in Table 8 Using Fit Model with Random Factor
(continued)**Response: Rating
Summary of Fit**

RSquare	0.566182
RSquare Adj	0.450497
Root Mean Square Error	8.694826
Mean of Response	71
Observations (or Sum Wgts)	20

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	71	1.944222	36.52	<.0001
Officer [A-E]	4	3.888444	1.03	0.3199
Officer [B-E]	-1	3.888444	-0.26	0.8005
Officer [C-E]	-16	3.888444	-4.11	0.0009
Officer [D-E]	9	3.888444	2.31	0.0352

Expected Mean Squares

The Mean Square per row by the Variance Component per column

EMS	Intercept	Officer (i)
Intercept	0	0
Officer (i)	0	4

plus 1.0 times Residual Error Variance

Variance Component Estimates

Component	Var Comp Est
Officer (i)	73.6
Residual	75.6

These estimates based on equating Mean Squares to
Expected Value.**Test Denominator Synthesis**

Source	MS Den	DF Den	Denom MS Synthesis
Officer (i)	75.6	15	Residual

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob>F
Officer (i)	1480	370	4	4.8942	0.0100

Appendix B: JMP Results

Exhibit B.5: JMP Output for ANOVA of Information in Table 8 Using Fit Model with Random Factor
(continued)

Whole-Model Test

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	1480.0000	370.000	4.8942
Error	15	1134.0000	75.600	Prob>F
C Total	19	2614.0000		0.0100

Officer (i)

Appendix B: JMP Results

Exhibit B.5: JMP Output for ANOVA of Information in Table 8 Using Fit Model with Random Factor
(continued)

Effect Test				
	Sum of Squares	F Ratio	DF	Prob>F
	1480.0000	4.8942	4	0.0100
Denominator MS Synthesis:		Residual		
Least Squares Means				
Level	Least Sq Mean	Std Error	Mean	
A	75.00000000	4.347413024	75.0000	
B	70.00000000	4.347413024	70.0000	
C	55.00000000	4.347413024	55.0000	
D	80.00000000	4.347413024	80.0000	
E	75.00000000	4.347413024	75.0000	

Warning: Std Err calculated with respect to Synthetic Denominator.

Appendix B: JMP Results

Exhibit B.6: JMP Output for ANOVA of Information in Table 10 Using Fit Model with Two Factors

Response: Premium (\$)**Summary of Fit**

RSquare	0.990698
RSquare Adj	0.976744
Root Mean Square Error	7.071068
Mean of Response	175
Observations (or Sum Wgts)	6

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	175	2.886751	60.62	0.0003
Size of [Large-Small]	35	4.082483	8.57	0.0133
Size of [Medium-Small]	20	4.082483	4.90	0.0392
Region[East-West]	15	2.886751	5.20	0.0351

Effect Test

Source	Nparam	DF	Sum of Squares	F Ratio	Prob>F
Size of City	2	2	9300.0000	93.0000	0.0106
Region	1	1	1350.0000	27.0000	0.0351

Whole-Model Test

Appendix B: JMP Results

Exhibit B.6: JMP Output for ANOVA of Information in Table 10 Using Fit Model with Two Factors
(continued)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	10650.000	3550.00	71.0000
Error	2	100.000	50.00	Prob>F
C Total	5	10750.000		0.0139

Size of City

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
9300.0000	93.0000	2	0.0106

Least Squares Means

Level	Least Sq Mean	Std Error	Mean
Large	210.0000000	5.000000000	210.000
Medium	195.0000000	5.000000000	195.000
Small	120.0000000	5.000000000	120.000

Appendix B: JMP Results

Exhibit B.6: JMP Output for ANOVA of Information in Table 10 Using Fit Model with Two Factors
(continued)

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
1350.0000	27.0000	1	0.0351

Least Squares Means

Level	Least Sq Mean	Std Error	Mean
East	190.0000000	4.082482905	190.000
West	160.0000000	4.082482905	160.000

Appendix B: JMP Results

Exhibit B.7: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with A Nested Factor

Response: Class Learning Scores (coded)
Summary of Fit

RSquare	0.94517
RSquare Adj	0.899478
Root Mean Square Error	2.645751
Mean of Response	15
Observations (or Sum Wgts)	12

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	15	0.763763	19.64	<.0001
School[Atlanta-San Fra]	4.75	1.080123	4.40	0.0046
School[Chicago-San Fra]	-0.75	1.080123	-0.69	0.5134
School[Atlanta]:Instruct[1-2]	7.25	1.322876	5.48	0.0015
School[Chicago]:Instruct[1-2]	-5.75	1.322876	-4.35	0.0048
School[San Fra]:Instruct[1-2]	7.5	1.322876	5.67	0.0013

Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
School	2	2	156.50000	11.1786	0.0095
Instructor[School]	3	3	567.50000	27.0238	0.0007

Whole-Model Test

Appendix B: JMP Results

Exhibit B.7: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with A Nested Factor
(continued)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	724.00000	144.800	20.6857
Error	6	42.00000	7.000	Prob>F
C Total	11	766.00000		0.0010

School

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
156.50000	11.1786	2	0.0095

Least Squares Means

Level	Least Sq Mean	Std Error	Mean
Atlanta	19.75000000	1.322875656	19.7500
Chicago	14.25000000	1.322875656	14.2500
San Francisco	11.00000000	1.322875656	11.0000

Appendix B: JMP Results

Exhibit B.7: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with A Nested Factor
(continued)

Effect Test			
Sum of Squares	F Ratio	DF	Prob>F
567.50000	27.0238	3	0.0007

Least Squares Means		
Level	Least Sq Mean	Std Error
[Atlanta]1	27.00000000	1.870828693
[Atlanta]2	12.50000000	1.870828693
[Chicago]1	8.50000000	1.870828693
[Chicago]2	20.00000000	1.870828693
[San Francisco]1	18.50000000	1.870828693
[San Francisco]2	3.50000000	1.870828693

Appendix B: JMP Results

Exhibit B.8: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with Random Factors
(continued)**Response: Class Learning Scores (coded)**
Summary of Fit

RSquare	0.94517
RSquare Adj	0.899478
Root Mean Square Error	2.645751
Mean of Response	15
Observations (or Sum Wgts)	12

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	15	0.763763	19.64	<.0001
School[Atlanta-San Fra]	4.75	1.080123	4.40	0.0046
School[Chicago-San Fra]	-0.75	1.080123	-0.69	0.5134
School[Atlanta]:Instruct[1-2]	7.25	1.322876	5.48	0.0015
School[Chicago]:Instruct[1-2]	-5.75	1.322876	-4.35	0.0048
School[San Fra]:Instruct[1-2]	7.5	1.322876	5.67	0.0013

Expected Mean Squares

The Mean Square per row by the Variance Component per column

EMS	Intercept	School	Instructor[School]
Intercept	0	0	0
School	0	4	2
Instructor[School]	0	0	2

plus 1.0 times Residual Error Variance

Variance Component Estimates

Component	Var Comp Est
School	-27.7292
Instructor[School]	91.08333
Residual	7

These estimates based on equating Mean Squares to Expected Value.

Test Denominator Synthesis

Source	MS Den	DF Den	Denom MS
Synthesis			
School	189.167	3	Instructor[School]
Instructor[School]	7	6	Residual

Appendix B: JMP Results

Exhibit B.8: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with Random Factors
(continued)

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob>F
School	156.5	78.25	2	0.4137	0.6940
Instructor[School]	567.5	189.167	3	27.0238	0.0007

Whole-Model Test

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	724.00000	144.800	20.6857
Error	6	42.00000	7.000	Prob>F
C Total	11	766.00000		0.0010

Appendix B: JMP Results

Exhibit B.8: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with Random Factors
(continued)

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
156.50000	0.4137	2	0.6940

Denominator MS Synthesis: Instructor[School]

Least Squares Means

Level	Least Sq Mean	Std Error	Mean
Atlanta	19.75000000	6.876893679	19.7500
Chicago	14.25000000	6.876893679	14.2500
San Francisco	11.00000000	6.876893679	11.0000

Warning: Std Err calculated with respect to Synthetic Denominator.

Appendix B: JMP Results

Exhibit B.8: JMP Output for Nested ANOVA of Information in Table 13 Using Fit Model with Random Factors
(continued)

Effect Test

Sum of Squares	F Ratio	DF	Prob>F
567.50000	27.0238	3	0.0007

Denominator MS Synthesis: Residual

Least Squares Means

Level	Least Sq Mean	Std Error
[Atlanta]1	27.00000000	1.870828693
[Atlanta]2	12.50000000	1.870828693
[Chicago]1	8.50000000	1.870828693
[Chicago]2	20.00000000	1.870828693
[San Francisco]1	18.50000000	1.870828693
[San Francisco]2	3.50000000	1.870828693

Warning: Std Err calculated with respect to Synthetic Denominator.

Appendix B: JMP Results

Exhibit B.9: JMP Output for a Fractional Factorial Experiment using the Design Experiment Feature

Pattern	X1	X2	X3	X4	X5	X6
-----	-1	-1	-1	-1	-1	-1
---+++	-1	-1	-1	1	1	1
--++++	-1	-1	1	-1	1	1
-++--	-1	-1	1	1	-1	-1
-+---+	-1	1	-1	-1	1	-1
-+-++	-1	1	-1	1	-1	1
-++--	-1	1	1	-1	-1	1
-++++-	-1	1	1	1	1	-1
+----+	1	-1	-1	-1	-1	1
+---++	1	-1	-1	1	1	-1
+--+-	1	-1	1	-1	1	-1
+---+	1	-1	1	1	-1	1
++---	1	1	-1	-1	1	1
+++-	1	1	-1	1	-1	-1
+++--	1	1	1	-1	-1	-1
+++++	1	1	1	1	1	1

Fractional Factorial Structure

Factor Confounding Rules

X5 = X2*X3*X4

X6 = X1*X3*X4

Aliasing Structure

X1 = X2*X5*X6 = X3*X4*X6 = X1*X2*X3*X4*X5

X2 = X1*X5*X6 = X3*X4*X5 = X1*X2*X3*X4*X6

X3 = X1*X4*X6 = X2*X4*X5 = X1*X2*X3*X5*X6

X4 = X1*X3*X6 = X2*X3*X5 = X1*X2*X4*X5*X6

X5 = X1*X2*X6 = X2*X3*X4 = X1*X3*X4*X5*X6

X6 = X1*X2*X5 = X1*X3*X4 = X2*X3*X4*X5*X6

X1*X2 = X5*X6 = X1*X3*X4*X5 = X2*X3*X4*X6

X1*X3 = X4*X6 = X1*X2*X4*X5 = X2*X3*X5*X6

X1*X4 = X3*X6 = X1*X2*X3*X5 = X2*X4*X5*X6

X1*X5 = X2*X6 = X1*X2*X3*X4 = X3*X4*X5*X6

X1*X6 = X2*X5 = X3*X4 = X1*X2*X3*X4*X5*X6

X2*X3 = X4*X5 = X1*X2*X4*X6 = X1*X3*X5*X6

X2*X4 = X3*X5 = X1*X2*X3*X6 = X1*X4*X5*X6

X1*X2*X3 = X1*X4*X5 = X2*X4*X6 = X3*X5*X6

X1*X2*X4 = X1*X3*X5 = X2*X3*X6 = X4*X5*X6

Appendix B: JMP Results

Exhibit B.10: JMP Output for Mixture Problem Defined by Equation (2)

X_1	X_2	X_3	Dimen
0.4	0.1	0.5	0
0.6	0.1	0.3	0
0.6	0.3	0.1	0
0.2	0.3	0.5	0
0.3	0.6	0.1	0
0.2	0.6	0.2	0
0.2	0.45	0.35	1
0.6	0.2	0.2	1
0.5	0.1	0.4	1
0.25	0.6	0.15	1
0.45	0.45	0.1	1
0.3	0.2	0.5	1
0.383333	0.333333	0.283333	2

JMP is capable of evaluating more than just the extreme vertices of this region. A value of 0 for Dimen column indicates that the row corresponds to an extreme vertex of the mixture region, a value of 1 indicates an edge of the region, and finally, a 2 value indicates the centroid of the region. This centroid is computed as part of the discussion in [1], and the value reported there (on page 358) is (0.384,0.333,0.283) the same value as shown in the table above.

Appendix B: JMP Results

Exhibit B.11: JMP's D-Optimality Results

Optimal Design Controls

N Desired 8
N Random 3
K Value 3
Trips 1

N 8 ----- Ready -----
Trips 1

Best Design

D-efficiency 100
A-efficiency 100
G-efficiency 100
AvgPredSE 0.5590
N 8.0000

		Correlations			
Corr	Intercept	X1	X2	X3	
Intercept	1:0000	0.0000	0.0000	0.0000	
X1	0.0000	1.0000	0.0000	0.0000	
X2	0.0000	0.0000	1.0000	0.0000	
X3	0.0000	0.0000	0.0000	1.0000	

Pattern	X1	X2	X3	Comment	OptCount	OptStdPred
+00	1	0	0	Axial	0	0.5
++	1	1	-1	FF	1	0.707107
--	-1	-1	-1	FF	1	0.707107
000	0	0	0	Center-FF	0	0.353553
00-	0	0	-1	Axial	0	0.5
000	0	0	0	Center-FF	0	0.353553
-00	-1	0	0	Axial	0	0.5
00+	0	0	1	Axial	0	0.5
+++	1	1	1	FF	1	0.707107
+-	1	-1	1	FF	1	0.707107
000	0	0	0	Center-FF	0	0.353553
-+	-1	-1	1	FF	1	0.707107
-+-	-1	1	-1	FF	1	0.707107
0-0	0	-1	0	Axial	0	0.5
+-	1	-1	-1	FF	1	0.707107
0+0	0	1	0	Axial	0	0.5
000	0	0	0	Center-Ax	0	0.353553
000	0	0	0	Center-Ax	0	0.353553
000	0	0	0	Center-FF	0	0.353553
++	-1	1	1	FF	1	0.707107

Appendix B: JMP Results

Exhibit B.12: JMP Results for x-Bar and s Charts for Data in Table 18

Appendix B: JMP Results

Exhibit B.12: JMP Results for x-Bar and s Charts for Data in Table 18 (continued)

Appendix C: SAS Results

Exhibit C.1: SAS Input for and Results from Descriptive Statistics for Lot Size Values in Table 2

```

data example1;
  infile cards;
  input prod_run lot_size man_hrs;
cards;
1 30 73
2 20 50
3 60 128
4 80 170
5 40 87
6 50 108
7 60 135
8 30 69
9 70 148
10 60 132
;
proc summary data=example1 noprint;
  var lot_size;
  output out=outex1 n=s_size min=s_min max=s_max
    mean=s_mean std=s_std stderr=s_stderr;
proc print data=outex1;
run;
proc means data=example1 noprint;
  var lot_size;
  output out=outex2 n=s_size min=s_min max=s_max
    mean=s_mean std=s_std stderr=s_stderr;
proc print data=outex2;
run;
proc univariate data=example1 noprint;
  var lot_size;
  output out=outex3 n=s_size min=s_min max=s_max
    mean=s_mean std=s_std stdmean=s_stderr;
proc print data=outex3;
run;

```

OBS	_TYPE_	_FREQ_	S_SIZE	S_MIN	S_MAX	S_MEAN	S_STD	S_STDERR
1	0	10	10	20	80	50	19.4365	6.14636

OBS	_TYPE_	_FREQ_	S_SIZE	S_MIN	S_MAX	S_MEAN	S_STD	S_STDERR
1	0	10	10	20	80	50	19.4365	6.14636

OBS	S_SIZE	S_MEAN	S_STD	S_STDERR	S_MAX	S_MIN
1	10	50	19.4365	6.14636	80	20

Appendix C: SAS Results

Exhibit C.2: SAS Input for and Results from PROC REG for Table 2 Data

```
options ls=80 ps=66;
data example1;
  infile cards;
  input prod_run lot_size man_hrs;
cards;
1 30 73
2 20 50
3 60 128
4 80 170
5 40 87
6 50 108
7 60 135
8 30 69
9 70 148
10 60 132
;
proc reg data=example1;
  model man_hrs = lot_size;
  output out=outex1a;
  proc print data=outex1a;
  run;
```

Model: MODEL1

Dependent Variable: MAN_HRS

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	13600.00000	13600.00000	1813.333	0.0001
Error	8	60.00000	7.50000		
C Total	9	13660.00000			
Root MSE	2.73861	R-square	0.9956		
Dep Mean	110.00000	Adj R-sq	0.9951		
C.V.	2.48965				

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	10.000000	2.50293945	3.995	0.0040
LOT_SIZE	1	2.000000	0.04696682	42.583	0.0001

Appendix C: SAS Results

Exhibit C.3: SAS Input for and Results from SAS/IML Program for Regression of Table 2 Data

```
proc iml;  
  reset noname;  
  x={1 30,1 20,1 60,1 80,1 40,1 50,1 60,1 30,1 70,1 60};  
  y={73, 50, 128, 170, 87, 108, 135, 69, 148, 132};  
  betahat=INV(x`*x)*(x`*y);  
  reset name;  
  print betahat;  
  quit;  
  run;
```

```
BETAHAT  
      10  
       2
```


Appendix C: SAS Results

Exhibit C.4: SAS Input for and Results from SAS/STAT PROC ANOVA for Analyzing of Table 5 Data

```
data example3;
  infile cards;
  input design store n_cases;
cards;
1 1 12
1 2 18
2 1 14
2 2 12
2 3 13
3 1 19
3 2 17
3 3 21
4 1 24
4 2 30
;

proc anova data=example3;
  class design;
  model n_cases = design;
run;
```

Analysis of Variance Procedure

Class Level Information

Class	Levels	Values
DESIGN	4	1 2 3 4

Number of observations in data set = 10

Analysis of Variance Procedure

Dependent Variable: N_CASES

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	258.00000000	86.00000000	11.22	0.0071
Error	6	46.00000000	7.66666667		
Corrected Total	9	304.00000000			

R-Square	C.V.	Root MSE	N_CASES Mean
0.848684	15.38264	2.7688746	18.000000

Source	DF	Anova SS	Mean Square	F Value	Pr > F
DESIGN	3	258.00000000	86.00000000	11.22	0.0071

Appendix C: SAS Results

Exhibit C.5: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 5 Data

```
data example3;
  infile cards;
  input design store n_cases;
cards;
1 1 12
1 2 18
2 1 14
2 2 12
2 3 13
3 1 19
3 2 17
3 3 21
4 1 24
4 2 30
;
proc glm data=example3;
  class design;
  model n_cases = design;
  output out=outex3a;
proc print data=outex3a;
run;
```

General Linear Models Procedure

Class Level Information

Class	Levels	Values
DESIGN	4	1 2 3 4

Number of observations in data set = 10

General Linear Models Procedure

Dependent Variable: N_CASES

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	258.00000000	86.00000000	11.22	0.0071
Error	6	46.00000000	7.66666667		
Corrected Total	9	304.00000000			

R-Square	C.V.	Root MSE	N_CASES Mean
0.848684	15.38264	2.7688746	18.000000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
DESIGN	3	258.00000000	86.00000000	11.22	0.0071

Source	DF	Type III SS	Mean Square	F Value	Pr > F
DESIGN	3	258.00000000	86.00000000	11.22	0.0071

Appendix C: SAS Results

Exhibit C.6: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 8 Data

```

data example4;
    infile cards;
    input officer $1 cand yscore;
cards;
A 1 76
A 2 64
A 3 85
A 4 75
B 1 58
B 2 75
B 3 81
B 4 66
C 1 49
C 2 63
C 3 62
C 4 46
D 1 74
D 2 71
D 3 85
D 4 90
E 1 66
E 2 74
E 3 81
E 4 79
;

proc glm data=example4;
    class officer;
    model yscore = officer;
    random officer/test;
    output out=outex4a;
proc print data=outex4a;
run;

```

General Linear Models Procedure

Class Level Information

Class	Levels	Values
OFFICER	5	A B C D E

Number of observations in data set = 20

General Linear Models Procedure

Dependent Variable: YSCORE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1480.000000	370.0000000	4.89	0.0100
Error	15	1134.000000	75.6000000		
Corrected Total	19	2614.000000			

Appendix C: SAS Results

Exhibit C.6: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 8 Data
(Continued)

		R-Square	C.V.	Root MSE	YSCORE Mean	
		0.566182	12.24623	8.6948260	71.000000	
Source	DF	Type I SS	Mean Square	F Value	Pr > F	
OFFICER	4	1480.000000	370.000000	4.89	0.0100	
Source	DF	Type III SS	Mean Square	F Value	Pr > F	
OFFICER	4	1480.000000	370.000000	4.89	0.0100	

General Linear Models Procedure

Source Type III Expected Mean Square
 OFFICER Var(Error) + 4 Var(OFFICER)

General Linear Models Procedure
Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: YSCORE

Source: OFFICER
 Error: MS(Error)

		Denominator	Denominator	F Value	Pr > F
DF	Type III MS	DF	MS		
4	370	15	75.6	4.8942	0.0100

Appendix C: SAS Results

Exhibit C.7: SAS Input for and Results from SAS/STAT PROC VARCOMP for Analyzing of Table 8 Data

```

data example4;
  infile cards;
  input officer $1 cand yscore;
cards;
A 1 76
A 2 64
A 3 85
A 4 75
B 1 58
B 2 75
B 3 81
B 4 66
C 1 49
C 2 63
C 3 62
C 4 46
D 1 74
D 2 71
D 3 85
D 4 90
E 1 66
E 2 74
E 3 81
E 4 79
;
proc varcomp data=example4 method=typel;
  class officer;
  model yscore = officer;
run;

```

Variance Components Estimation Procedure

Class Level Information

Class	Levels	Values
OFFICER	5	A B C D E

Number of observations in data set = 20

Variance Components Estimation Procedure

Dependent Variable: YSCORE

Source	DF	Type I SS	Type I MS
OFFICER	4	1480.00000000	370.00000000
Error	15	1134.00000000	75.60000000
Corrected Total	19	2614.00000000	

Source

Expected Mean Square

OFFICER Var(Error) + 4 Var(OFFICER)

Error Var(Error)

Variance Component	Estimate
Var(OFFICER)	73.60000000
Var(Error)	75.60000000

Appendix C: SAS Results

Exhibit C.8: SAS Input for and Results from SAS/STAT PROC ANOVA for Analyzing of Table 10 Data

```
data example5;
  infile cards;
  input c_size $7. region $5. premium;
cards;
small east 140
small west 100
medium east 210
medium west 180
large east 220
large west 200
;
proc anova data=example5;
  class c_size region;
  model premium = c_size region;
run;
```

Analysis of Variance Procedure

Class Level Information

Class	Levels	Values
C_SIZE	3	large medium small
REGION	2	east west

Number of observations in data set = 6

Analysis of Variance Procedure

Dependent Variable: PREMIUM

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	10650.000000	3550.000000	71.00	0.0139
Error	2	100.000000	50.000000		
Corrected Total	5	10750.000000			

R-Square	C.V.	Root MSE	PREMIUM Mean
0.990698	4.040610	7.0710678	175.00000

Source	DF	Anova SS	Mean Square	F Value	Pr > F
C_SIZE	2	9300.000000	4650.000000	93.00	0.0106
REGION	1	1350.000000	1350.000000	27.00	0.0351

Appendix C: SAS Results

Exhibit C.9: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 10 Data

```

data example5;
  infile cards;
  input c_size $7. region $5. premium;
cards;
small east 140
small west 100
medium east 210
medium west 180
large east 220
large west 200
;
proc glm data=example5;
  class c_size region;
  model premium = c_size region;
  output out=outex5a;
proc print data=outex5a;
run;

```

General Linear Models Procedure

Class Level Information

Class	Levels	Values
C_SIZE	3	large medium small
REGION	2	east west

Number of observations in data set = 6

General Linear Models Procedure

Dependent Variable: PREMIUM

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	10650.000000	3550.000000	71.00	0.0139
Error	2	100.000000	50.000000		
Corrected Total	5	10750.000000			

R-Square	C.V.	Root MSE	PREMIUM Mean
0.990698	4.040610	7.0710678	175.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
C_SIZE	2	9300.0000000	4650.0000000	93.00	0.0106
REGION	1	1350.0000000	1350.0000000	27.00	0.0351

Source	DF	Type III SS	Mean Square	F Value	Pr > F
C_SIZE	2	9300.0000000	4650.0000000	93.00	0.0106
REGION	1	1350.0000000	1350.0000000	27.00	0.0351

Appendix C: SAS Results

Exhibit C.10: SAS Input for and Results from SAS/STAT PROC ANOVA for Analyzing of Table 13 Data

```
data example6;
  infile cards;
  input school $14. instruct rating;
cards;
Atlanta      1 25
Atlanta      1 29
Atlanta      2 14
Atlanta      2 11
Chicago      1 11
Chicago      1  6
Chicago      2 22
Chicago      2 18
San Francisco 1 17
San Francisco 1 20
San Francisco 2  5
San Francisco 2  2
;
proc anova data=example6;
  class school instruct;
  model rating = school instruct(school);
run;
```

Analysis of Variance Procedure

Class Level Information

Class	Levels	Values
SCHOOL	3	Atlanta Chicago San Francisco
INSTRUCT	2	1 2

Number of observations in data set = 12

Analysis of Variance Procedure

Dependent Variable: RATING

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	724.00000000	144.80000000	20.69	0.0010
Error	6	42.00000000	7.00000000		
Corrected Total	11	766.00000000			

R-Square	C.V.	Root MSE	RATING Mean
0.945170	17.63834	2.6457513	15.000000

Source	DF	Anova SS	Mean Square	F Value	Pr > F
SCHOOL	2	156.50000000	78.25000000	11.18	0.0095
INSTRUCT(SCHOOL)	3	567.50000000	189.16666667	27.02	0.0007

Appendix C: SAS Results

Exhibit C.11: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 13 Data

```
data example6;
  infile cards;
  input school $14. instruct rating;
cards;
Atlanta      1 25
Atlanta      1 29
Atlanta      2 14
Atlanta      2 11
Chicago      1 11
Chicago      1 6
Chicago      2 22
Chicago      2 18
San Francisco 1 17
San Francisco 1 20
San Francisco 2 5
San Francisco 2 2
;
proc glm data=example6;
  class school instruct;
  model rating = school instruct(school);
  output out=outex6a;
proc print data=outex6a;
run;
```

General Linear Models Procedure

Class Level Information

Class	Levels	Values
SCHOOL	3	Atlanta Chicago San Francisco
INSTRUCT	2	1 2

Number of observations in data set = 12

General Linear Models Procedure

Dependent Variable: RATING

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	724.00000000	144.80000000	20.69	0.0010
Error	6	42.00000000	7.00000000		
Corrected Total	11	766.00000000			

R-Square	C.V.	Root MSE	RATING Mean
0.945170	17.63834	2.6457513	15.000000

Appendix C: SAS Results

Exhibit C.11: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 13 Data
(continued)

Source	DF	Type I SS	Mean Square	F Value	Pr > F
SCHOOL	2	156.50000000	78.25000000	11.18	0.0095
INSTRUCT(SCHOOL)	3	567.50000000	189.16666667	27.02	0.0007
Source	DF	Type III SS	Mean Square	F Value	Pr > F
SCHOOL	2	156.50000000	78.25000000	11.18	0.0095
INSTRUCT(SCHOOL)	3	567.50000000	189.16666667	27.02	0.0007

Appendix C: SAS Results

Exhibit C.12: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 13 Data

```
data example6;
  infile cards;
  input school $14. instruct rating;
cards;
Atlanta      1 25
Atlanta      1 29
Atlanta      2 14
Atlanta      2 11
Chicago      1 11
Chicago      1  6
Chicago      2 22
Chicago      2 18
San Francisco 1 17
San Francisco 1 20
San Francisco 2  5
San Francisco 2  2
;
proc glm data=example6;
  class school instruct;
  model rating = school instruct(school);
  random school instruct(school)/test;
  output out=outex6b;
proc print data=outex6b;
run;
```

General Linear Models Procedure

Class Level Information

Class	Levels	Values
SCHOOL	3	Atlanta Chicago San Francisco
INSTRUCT	2	1 2

Number of observations in data set = 12

General Linear Models Procedure

Dependent Variable: RATING

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	724.00000000	144.80000000	20.69	0.0010
Error	6	42.00000000	7.00000000		
Corrected Total	11	766.00000000			

R-Square	C.V.	Root MSE	RATING Mean
0.945170	17.63834	2.6457513	15.000000

Appendix C: SAS Results

Exhibit C.12: SAS Input for and Results from SAS/STAT PROC GLM for Analyzing of Table 13 Data
(continued)

Source	DF	Type I SS	Mean Square	F Value	Pr > F
SCHOOL	2	156.50000000	78.25000000	11.18	0.0095
INSTRUCT(SCHOOL)	3	567.50000000	189.16666667	27.02	0.0007
Source	DF	Type III SS	Mean Square	F Value	Pr > F
SCHOOL	2	156.50000000	78.25000000	11.18	0.0095
INSTRUCT(SCHOOL)	3	567.50000000	189.16666667	27.02	0.0007

General Linear Models Procedure

Source	Type III Expected Mean Square
SCHOOL	Var(Error) + 2 Var(INSTRUCT(SCHOOL)) + 4 Var(SCHOOL)
INSTRUCT(SCHOOL)	Var(Error) + 2 Var(INSTRUCT(SCHOOL))

General Linear Models Procedure
Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: RATING

Source: SCHOOL		Denominator		Denominator			
Error: MS(INSTRUCT(SCHOOL))			DF	MS	F Value	Pr > F	
DF	Type III MS		3	189.16666667	0.4137	0.6940	
2	78.25						
Source: INSTRUCT(SCHOOL)		Denominator		Denominator			
Error: MS(Error)			DF	MS	F Value	Pr > F	
DF	Type III MS		6	7	27.0238	0.0007	
3	189.16666667						

Appendix C: SAS Results

Exhibit C.13: SAS Input for and Results from SAS/STAT PROC VARCOMP for Analyzing of Table 13 Data

```
data example6;
  infile cards;
  input school $14. instruct rating;
cards;
Atlanta      1 25
Atlanta      1 29
Atlanta      2 14
Atlanta      2 11
Chicago      1 11
Chicago      1  6
Chicago      2 22
Chicago      2 18
San Francisco 1 17
San Francisco 1 20
San Francisco 2  5
San Francisco 2  2
;
proc varcomp data=example6 method=type1;
  class school instruct;
  model rating = school instruct(school);
run;
```

Variance Components Estimation Procedure

Class Level Information

Class	Levels	Values
SCHOOL	3	Atlanta Chicago San Francisco
INSTRUCT	2	1 2

Number of observations in data set = 12

Variance Components Estimation Procedure

Dependent Variable: RATING

Source	DF	Type I SS	Type I MS
SCHOOL	2	156.50000000	78.25000000
INSTRUCT(SCHOOL)	3	567.50000000	189.16666667
Error	6	42.00000000	7.00000000
Corrected Total	11	766.00000000	

Source	Expected Mean Square
SCHOOL	$\text{Var}(\text{Error}) + 2 \text{ Var}(\text{INSTRUCT}(\text{SCHOOL})) + 4 \text{ Var}(\text{SCHOOL})$
INSTRUCT(SCHOOL)	$\text{Var}(\text{Error}) + 2 \text{ Var}(\text{INSTRUCT}(\text{SCHOOL}))$
Error	$\text{Var}(\text{Error})$

Variance Component	Estimate
Var(SCHOOL)	-27.72916667
Var(INSTRUCT(SCHOOL))	91.08333333
Var(Error)	7.00000000

Appendix C: SAS Results

Exhibit C.14: SAS Input for and Results from SAS/QC PROC FACTEX for Experimental Design

```
proc factex;  
  factors x1 x2 x3 x4 x5 x6;  
  model res=4;  
  size fraction=4;  
  output out=outex7;  
proc print data=outex7;  
run;
```

OBS	X1	X2	X3	X4	X5	X6
1	-1	-1	-1	-1	-1	-1
2	-1	-1	-1	1	1	-1
3	-1	-1	1	-1	1	1
4	-1	-1	1	1	-1	-1
5	-1	1	-1	-1	1	-1
6	-1	1	-1	1	-1	1
7	-1	1	1	-1	-1	1
8	-1	1	1	1	1	-1
9	1	-1	-1	-1	-1	1
10	1	-1	-1	1	1	-1
11	1	-1	1	-1	1	-1
12	1	-1	1	1	-1	1
13	1	1	-1	-1	1	1
14	1	1	-1	1	-1	-1
15	1	1	1	-1	-1	-1
16	1	1	1	1	1	1

Appendix C: SAS Results

Exhibit C.15: SAS Input and Output for Mixture Problem Defined by Equation (2)

```

libname new 'sys$scratch:.';
"
  libname textbk01 xport 'sys$scratch:textbk01' cc=none;
"
"
%adxgen;
"
%adxmix;
"
%adxinit
"
%adxxvert(ev,x1 .2-.6/ x2 .1-.6/ x3 .1-.5,1);
"
  proc sort data=ev
                                out=new.ev_pts;
    by x1 x2 x3;
  proc copy in=new out=textbk01 memtype=data;
    select ev_pts;
run;

```

Extreme Vertices (Dimen=0) and Centroid (Dimen=2)

x_1	x_2	x_3	DIMEN
0.2	0.3	0.5	0
0.2	0.6	0.2	0
0.3	0.6	0.1	0
0.383333	0.333333	0.283333	2
0.4	0.1	0.5	0
0.6	0.1	0.3	0
0.6	0.3	0.1	0

The output from this SAS run was "FTP'd" to the IBM PC using WS_FTP32 Version 3.00 by Ipswitch, Inc., 1996, as a SAS transport file. The file was then imported into JMP and copy and pasted into this document.

Appendix C: SAS Results

Exhibit C.16: SAS Input for and Results from SAS/QC PROC OPTEX for D-Optimality

```
data example2;
  infile cards;
  input x1 x2 x3;
cards;
1 0 0
1 1 -1
-1 -1 -1
0 0 0
0 0 -1
0 0 0
-1 0 0
0 0 1
1 1 1
1 -1 1
0 0 0
-1 -1 1
-1 1 -1
0 -1 0
1 -1 -1
0 1 0
0 0 0
0 0 0
0 0 0
-1 1 1
;
proc optex data=example2;
  examine var;
  generate criterion=d n=8;
  model x1 x2 x3;
  output out=outex2;
proc print data=outex2;
run;
```

Design Number	D-efficiency	A-efficiency	G-efficiency	Prediction Standard Error
1	100.0000	100.0000	100.0000	0.5590
2	100.0000	100.0000	100.0000	0.5590
3	100.0000	100.0000	100.0000	0.5590
4	100.0000	100.0000	100.0000	0.5590
5	100.0000	100.0000	100.0000	0.5590
6	100.0000	100.0000	100.0000	0.5590
7	100.0000	100.0000	100.0000	0.5590
8	100.0000	100.0000	100.0000	0.5590
9	100.0000	100.0000	100.0000	0.5590
10	100.0000	100.0000	100.0000	0.5590

Examining Design Number 1

Log determinant of the information matrix = 8.3178E+00
Maximum prediction variance over candidates = 0.5000
Average prediction variance over candidates = 0.3125
Average variance of coefficients = 0.1250

D-Efficiency = 100.0
A-Efficiency = 100.0

Appendix C: SAS Results

Exhibit C.16: SAS Input for and Results from SAS/QC PROC OPTEX for D-Optimality
(Continued)

Variance Matrix				
	INTERCEPT	X1	X2	X3
INTERCEPT	0.125	0.000	0.000	0.000
X1	0.000	0.125	0.000	0.000
X2	0.000	0.000	0.125	0.000
X3	0.000	0.000	0.000	0.125

	OBS	X1	X2	X3
	1	-1	-1	-1
	2	-1	-1	1
	3	-1	1	-1
	4	-1	1	1
	5	1	-1	-1
	6	1	-1	1
	7	1	1	-1
	8	1	1	1

Appendix C: SAS Results

Exhibit C.17: SAS Input and Output for Control Chart Example from Table 18

```
data example8;
"
    infile cards;
"
    input sg x1-x4;
"
cards;
"
1 72 84 79 49
2 56 87 33 42
3 55 73 22 60
4 44 80 54 74
5 97 26 48 58
6 83 89 91 62
7 47 66 53 58
8 88 50 84 69
9 57 47 41 46
10 13 10 30 32
11 26 39 52 48
12 46 27 63 34
13 49 62 78 87
14 71 63 82 55
15 71 58 69 70
16 67 69 70 94
17 55 63 72 49
18 49 51 55 76
19 72 80 61 59
20 61 74 62 57
;
data example9;
    set example8;
    x=x1; keep sg x;
    output;
    x=x2; keep sg x;
    output;
    x=x3; keep sg x;
    output;
    x=x4; keep sg x;
    output;
run;
proc shewhart data=example9;
    xchart x*sg /
        type=estimate sigmas=3 stddeviations limitn=4;
    schart x*sg;
run;
```

Appendix C: SAS Results

Exhibit C.17: SAS Input and Output for Control Chart Example from Table 18
(Continued)

3 Sigma Limits

For n=4:

Subgroup Sizes: n=4

Appendix C: SAS Results

Exhibit C.17: SAS Input and Output for Control Chart Example from Table 18
(Continued)

Subgroup Sizes: n=4

Appendix D: Mixsoft Results

Exhibit D.1: Mixsoft Output for a Fractional Factorial Experiment using the Design Experiment Feature

MIXSOFT VERSION 2.3, MARCH 1998
 TWOLEV VERSION 2.3, MARCH 1998
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 ALL RIGHTS RESERVED

6 VARIABLES

CODED LEVELS OF -1 AND +1 USED FOR DESIGN VARIABLES.

FRACTIONAL FACTORIAL DESIGN, 16 POINTS

Run	A	B	C	D	E	F
1	-1	-1	-1	-1	-1	-1
2	1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	1
4	1	1	-1	-1	-1	-1
5	-1	-1	1	-1	1	-1
6	1	-1	1	-1	-1	1
7	-1	1	1	-1	-1	1
8	1	1	1	-1	1	-1
9	-1	-1	-1	1	-1	1
10	1	-1	-1	1	1	-1
11	-1	1	-1	1	1	-1
12	1	1	-1	1	-1	1
13	-1	-1	1	1	1	1
14	1	-1	1	1	-1	-1
15	-1	1	1	1	-1	-1
16	1	1	1	1	1	1

Appendix D: Mixsoft Results

Exhibit D.2: Mixsoft Input and Output for Mixture Problem Defined by Equation (2)

MIXSOFT VERSION 2.3, MARCH 1998
VERT VERSION 2.3, MARCH 1998
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3 COMPONENTS

COMPONENT	LOWER BOUNDS	UPPER BOUNDS
1	0.200000E+00	0.600000E+00
2	0.100000E+00	0.600000E+00
3	0.100000E+00	0.500000E+00

TOLERANCE VALUE = 0.1000E-05

COMPONENT VALUE TOLERANCE VECTOR

TOLV(1) = 0.5000E-04
TOLV(2) = 0.5000E-04
TOLV(3) = 0.5000E-04

THE CONSTRAINT REGION HAS 6 VERTICES.

ALL VERTICES: 6 OBTAINED

1	0.2000E+00	0.6000E+00	0.2000E+00
2	0.3000E+00	0.6000E+00	0.1000E+00
3	0.6000E+00	0.3000E+00	0.1000E+00
4	0.2000E+00	0.3000E+00	0.5000E+00
5	0.6000E+00	0.1000E+00	0.3000E+00
6	0.4000E+00	0.1000E+00	0.5000E+00

Appendix E: Statgraphics Results

Exhibit E.1: Statgraphics Output for Statistics of Lot Size Information in Table 2

Analysis Summary

Data variable: X

10 values ranging from 20.0 to 80.0

Summary Statistics for X

Count = 10
Average = 50.0
Median = 55.0
Mode = 60.0
Geometric mean = 46.1205
Variance = 377.778
Standard deviation = 19.4365
Standard error = 6.14636
Minimum = 20.0
Maximum = 80.0
Range = 60.0
Lower quartile = 30.0
Upper quartile = 60.0
Interquartile range = 30.0
Skewness = -0.113492
Std. skewness = -0.146517
Kurtosis = -1.06661
Std. kurtosis = -0.688493
Coeff. of variation = 38.873%
Sum = 500.0

Appendix E: Statgraphics Results

Exhibit E.1: Statgraphics Output for Statistics of Lot Size Information in Table 2
(continued)

Box-and-Whisker Plot

Percentiles for X

0.5%	= 20.0
2.5%	= 20.0
10.0%	= 25.0
25.0%	= 30.0
50.0%	= 55.0
75.0%	= 60.0
90.0%	= 75.0
97.5%	= 80.0
99.5%	= 80.0

Note: There are several ways to determine estimates of quantiles. Statgraphics computes these as outlined in Hayes [13]. The Pth quantile is estimated as $100(i - 0.5)/N$ where i is the rank. Linear interpolation is used for other quantiles.

Moments

Histogram for X

Appendix E: Statgraphics Results

Exhibit E.1: Statgraphics Output for Statistics of Lot Size Information in Table 2
(continued)

Stem-and-Leaf Display for X: unit = 1.0 1/2 represents 12.0

```

1  2|0
3  3|00
4  4|0
5  5|0
5  6|000
2  7|0
1  8|0

```

Confidence Intervals for X

95.0% confidence interval for mean: 50.0 +/- 13.9041 [36.0959,63.9041]

95.0% confidence interval for standard deviation: [13.3691,35.4835]

Appendix E: Statgraphics Results

Exhibit E.2: Statgraphics Output for Table 2 Data Using Simple Regression

Regression Analysis - Linear model: $Y = a + b \cdot X$

Dependent variable: Y

Independent variable: X

Parameter	Estimate	Standard Error	T Statistic	P-Value
Intercept	10.0	2.50294	3.9953	0.0040
Slope	2.0	0.0469668	42.5833	0.0000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	13600.0	1	13600.0	1813.33	0.0000
Residual	60.0	8	7.5		
Total (Corr.)	13660.0	9			

Correlation Coefficient = 0.997801

R-squared = 99.5608 percent

Standard Error of Est. = 2.73861

Analysis of Variance with Lack-of-Fit

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	13600.0	1	13600.0	1813.33	0.0000
Residual	60.0	8	7.5		
Lack-of-Fit	27.3333	5	5.46667	0.50	0.7662
Pure Error	32.6667	3	10.8889		
Total (Corr.)	13660.0	9			

Appendix E: Statgraphics Results

Exhibit E.3: Statgraphics Output for Table 5 Data Using One-Way ANOVA

Dependent variable: Sales

Factor: Design

Number of observations: 10

Number of levels: 4

Summary Statistics for Sales

Design	Count	Average	Variance
1	2	15.0	18.0
2	3	13.0	1.0
3	3	19.0	4.0
4	2	27.0	18.0
Total	10	18.0	33.7778

Design	Standard deviation	Minimum	Maximum
1	4.24264	12.0	18.0
2	1.0	12.0	14.0
3	2.0	17.0	21.0
4	4.24264	24.0	30.0
Total	5.81187	12.0	30.0

Design	Range	Std. skewness	Std. kurtosis
1	6.0		
2	2.0	0.0	
3	4.0	0.0	
4	6.0		
Total	18.0	1.23305	0.358593

Appendix E: Statgraphics Results

Exhibit E.3: Statgraphics Output for Table 5 Data Using One-Way ANOVA
(continued)

ANOVA Table for Sales by Design

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Between groups	258.0	3	86.0	11.22	0.0071
Within groups	46.0	6	7.66667		
Total (Corr.)	304.0	9

Appendix E: Statgraphics Results

Exhibit E.3: Statgraphics Output for Table 5 Data Using One-Way ANOVA
(continued)

**Table of Means for Sales by Design
with 95.0 percent LSD intervals**

Design	Count	Std. error		Lower limit	Upper limit
		Mean	(pooled s)		
1	2	15.0	1.95789	11.6124	18.3876
2	3	13.0	1.59861	10.234	15.766
3	3	19.0	1.59861	16.234	21.766
4	2	27.0	1.95789	23.6124	30.3876
Total	10	18.0			

Appendix E: Statgraphics Results

Exhibit E.4: Statgraphics Output for Table 8 Data using Variance Components Analysis

Dependent variable: Y
Factors: Officer

Number of complete cases: 20

Analysis of Variance for Y

Source	Sum of Squares	Df	Mean Square	Var. Comp.	Percent
TOTAL (CORRECTED)	2614.0	19			
Officer	1480.0	4	370.0	73.6	49.33
ERROR	1134.0	15	75.6	75.6	50.67

Level	Count	Mean	Standard Deviation
GRAND MEAN	20	71.0	11.7294
Officer			
A	4	75.0	8.60233
B	4	70.0	10.0995
C	4	55.0	8.75595
D	4	80.0	8.98146
E	4	75.0	6.68331

Appendix E: Statgraphics Results

Exhibit E.5: Statgraphics Output for Table 10 Data using Two Factor ANOVA

Multifactor ANOVA - Y

Dependent variable: Y

Factors:

Size

Region

Number of complete cases: 6

Analysis of Variance for Y

Type III Sums of Squares

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
MAIN EFFECTS					
A:Size	9300.0	2	4650.0	93.00	0.0106
B:Region	1350.0	1	1350.0	27.00	0.0351
RESIDUAL	100.0	2	50.0		
TOTAL (CORRECTED)	10750.0	5			

All F-ratios are based on the residual mean square error.

Appendix E: Statgraphics Results

Exhibit E.5: Statgraphics Output for Table 10 Data using Two Factor ANOVA
(continued)

Means and 95.0 Percent LSD Intervals

Table of Least Squares Means for Y with 95.0 Percent Confidence Intervals

Level	Count	Mean	Std. Error	Lower Limit	Upper Limit
GRAND MEAN	6	175.0			
Size					
Large	2	210.0	5.0	188.487	231.513
Medium	2	195.0	5.0	173.487	216.513
Small	2	120.0	5.0	98.4867	141.513
Region					
East	3	190.0	4.08248	172.434	207.566
West	3	160.0	4.08248	142.434	177.566

Multiple Range Tests for Y by Size

Method: 95.0 percent LSD			
Size	Count	LS Mean	Homogeneous Groups
Small	2	120.0	X
Medium	2	195.0	X
Large	2	210.0	X
Contrast			
		Difference	+/- Limits
Large - Medium		15.0	30.4243
Large - Small		*90.0	30.4243
Medium - Small		*75.0	30.4243

* denotes a statistically significant difference.

Appendix E: Statgraphics Results

Exhibit E.6: Statgraphics Output for Table 13 Data using a Nested Model

General Linear Models

Number of dependent variables: 1
Number of categorical factors: 2
Number of quantitative factors: 0

Analysis of Variance for Y

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	724.0	5	144.8	20.69	0.0010
Residual	42.0	6	7.0		
Total (Corr.)	766.0	11			

Type III Sums of Squares

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
School	156.5	2	78.25	11.18	0.0095
Instructor(School)	567.5	3	189.167	27.02	0.0007
Residual	42.0	6	7.0		
Total (corrected)	766.0	11			

All F-ratios are based on the residual mean square error.

R-Squared = 94.517 percent
R-Squared (adjusted for d.f.) = 89.9478 percent
Standard Error of Est. = 2.64575
Mean absolute error = 1.83333
Durbin-Watson statistic = 1.89881

Appendix E: Statgraphics Results

Exhibit E.6: Statgraphics Output for Table 13 Data using a Nested Model
(continued)

95.0% confidence intervals for coefficient estimates (Y)

Parameter	Estimate	Standard Error	Lower Limit	Upper Limit	V.I.F.
CONSTANT	15.0	0.763763	13.1311	16.8689	
School	4.75	1.08012	2.10703	7.39297	1.33333
School	-0.75	1.08012	-3.39297	1.89297	1.33333
Instructor(School)	7.25	1.32288	4.01303	10.487	1.0
Instructor(School)	-5.75	1.32288	-8.98697	-2.51303	1.0
Instructor(School)	7.5	1.32288	4.26303	10.737	1.0

Appendix E: Statgraphics Results

Exhibit E.6: Statgraphics Output for Table 13 Data using a Nested Model
(continued)

**Table of Least Squares Means for Y
with 95.0 Percent Confidence Intervals**

Level	Count	Mean	Std. Error	Lower Limit	Upper Limit
GRAND MEAN	12	15.0	0.763763	13.1311	16.8689
School					
Atlanta	4	19.75	1.32288	16.513	22.987
Chicago	4	14.25	1.32288	11.013	17.487
SanFancisco	4	11.0	1.32288	7.76303	14.237
Instructor within School					
1 Atlanta	2	27.0	1.87083	22.4222	31.5778
1 Chicago	2	8.5	1.87083	3.92224	13.0778
1 SanFancisc	2	18.5	1.87083	13.9222	23.0778
2 Atlanta	2	12.5	1.87083	7.92224	17.0778
2 Chicago	2	20.0	1.87083	15.4222	24.5778
2 SanFancisc	2	3.5	1.87083	-1.07776	8.07776

Multiple Comparisons for Y by School

Method: 95.0 percent LSD			
School	Count	LS Mean	Homogeneous Groups
SanFancisco	4	11.0	X
Chicago	4	14.25	X
Atlanta	4	19.75	X
Contrast			
		Difference	+/- Limits
Atlanta - Chicago		*5.5	4.57776
Atlanta - SanFancisco		*8.75	4.57776
Chicago - SanFancisco		3.25	4.57776

* denotes a statistically significant difference.

Appendix E: Statgraphics Results

Exhibit E.7: Statgraphics Output for Table 13 Data using Variance Component Analysis

Dependent variable: Y

Factors:

School

Instructor

Number of complete cases: 12

Analysis of Variance for Y

Source	Sum of Squares	Df	Mean Square	Var. Comp.	Percent
TOTAL (CORRECTED)	766.0	11			
School	156.5	2	78.25	0.0	0.00
Instructor	567.5	3	189.167	91.0833	92.86
ERROR	42.0	6	7.0	7.0	7.14

Variance Component Plot

Y

Level	Count	Mean	Standard Deviation
GRAND MEAN	12	15.0	8.34484
School			
Atlanta	4	19.75	8.61684
Chicago	4	14.25	7.13559
SanFrancisco	4	11.0	8.83176

Appendix E: Statgraphics Results

Exhibit E.7: Statgraphics Output for Table 13 Data using Variance Component Analysis
(continued)

Instructor			
1	2	27.0	2.82843
2	2	12.5	2.12132
1	2	8.5	3.53553
2	2	20.0	2.82843
1	2	18.5	2.12132
2	2	3.5	2.12132

Appendix E: Statgraphics Results

Exhibit E.8: Statgraphics Output for Fractional Factorial Design

Screening Design Attributes

Design Summary

Design class: Screening
 Design name: Quarter fraction 2^{6-2}

Base Design

Number of experimental factors: 6 Number of blocks: 1
 Number of responses: 1
 Number of runs: 16 Error degrees of freedom: 2
 Randomized: No

Factors	Low	High	Continuous
X1	-1.0	1.0	Yes
X4	-1.0	1.0	Yes
X3	-1.0	1.0	Yes
X2	-1.0	1.0	Yes
X6	-1.0	1.0	Yes
X5	-1.0	1.0	Yes

Run	X1	X2	X3	X4	X5	X6
1	-1	-1	-1	-1	-1	-1
2	-1	-1	-1	1	1	1
3	-1	-1	1	-1	1	1
4	-1	-1	1	1	-1	-1
5	-1	1	-1	-1	1	-1
6	-1	1	-1	1	-1	1
7	-1	1	1	-1	-1	1
8	-1	1	1	1	1	-1
9	1	-1	-1	-1	-1	1
10	1	-1	-1	1	1	-1
11	1	-1	1	-1	1	-1
12	1	-1	1	1	-1	1
13	1	1	-1	-1	1	1
14	1	1	-1	1	-1	-1
15	1	1	1	-1	-1	-1
16	1	1	1	1	1	1

Appendix E: Statgraphics Results

Exhibit E.8: Statgraphics Output for Fractional Factorial Design
(continued)

Alias Structure

Contrast Estimates

1	A
2	B
3	C
4	D
5	E
6	F
7	AB+CE
8	AC+BE
9	AD+EF
10	AE+BC+DF
11	AF+DE
12	BD+CF
13	BF+CD

Appendix E: Statgraphics Results

Exhibit E.9: Statgraphics Output for Extreme Vertices

Design Summary

Design class: Mixture
Design name: Extreme vertices

Base Design

Number of components: 3
Number of responses: 1
Number of runs: 6
Model type: Linear
Randomized: Yes

Components	Low	High	Units
X1	0.2	0.6	
X2	0.1	0.6	
X3	0.1	0.5	

Mixture total = 1.0

Run	X1	X2	X3
1	0.6	0.3	0.1
2	0.6	0.1	0.3
3	0.3	0.6	0.1
4	0.2	0.6	0.2
5	0.4	0.1	0.5
6	0.2	0.3	0.5

Appendix E: Statgraphics Results

Exhibit E.10: Statgraphics D-Optimality Results

Optimize Experiment

Selection criterion: D-optimality
Desired number of runs: 8
Selection method: Forward
Model order: 1

Number of runs already completed: 0
Additional candidate runs: 20

D-optimal Design

Design has been reduced to 8 runs.
D-efficiency = 100.0%
A-efficiency = 100.0%
G-efficiency = 100.0%

Select	Condition	X1	X2	X3
*	1	-1	-1	1
	2	0	0	0
*	3	1	-1	1
	4	0	1	0
*	5	1	1	1
	6	0	0	0
	7	0	0	0
*	8	1	-1	-1
*	9	-1	1	1
	10	0	0	-1
*	11	-1	1	-1
	12	-1	0	0
	13	0	0	1
	14	0	0	0
	15	0	0	0
	16	0	-1	0
*	17	-1	-1	-1
	18	0	0	0
	19	1	0	0
*	20	1	1	-1

* indicates a run selected to achieve D-optimality

Appendix E: Statgraphics Results

Exhibit E.11: Statgraphics X-bar and S Charts - X

Initial Study for X

Number of subgroups = 20
Average subgroup size = 4.0
0 subgroups excluded

X-bar Chart

UCL: +3.0 sigma = 82.0667
Centerline = 59.4375
LCL: -3.0 sigma = 36.8083

S Chart

UCL: +3.0 sigma = 31.496
Centerline = 13.8991
LCL: -3.0 sigma = 0.0

Estimates

Process mean = 59.4375
Process sigma = 15.0861
Mean sigma = 13.8991

Appendix E: Statgraphics Results

Exhibit E.11: Statgraphics X-bar and S Charts - X
(continued)

Subgroup Reports

All Subgroups

X = Excluded

* = Beyond Limits

Subgroup	Size	X-bar	S
1	4	71.0	15.4704
2	4	54.5	23.6432
3	4	52.5	21.7025
4	4	63.0	16.8523
5	4	57.25	29.6802
6	4	81.25	13.2759
7	4	56.0	8.04156
8	4	72.75	17.2313
9	4	47.75	6.70199
10	4	* 21.25	11.3541
11	4	41.25	11.5289
12	4	42.5	15.7586
13	4	69.0	16.8721
14	4	67.75	11.5289
15	4	67.0	6.0553
16	4	75.0	12.7279
17	4	59.75	9.97914
18	4	57.75	12.4197
19	4	68.0	9.83192
20	4	63.5	7.32575

S Chart for X

UCL = 31.50

CTR = 13.90

LCL = 0.00

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