Analysis of the Rotopod: An All Revolute Parallel Manipulator

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Abstract

This paper introduces a new configuration of parallel manipulator call the Rotopod which is constructed from all revolute type joints. The Rotopod consists of two platforms connected by six legs and exhibits six Cartesian degrees-of-freedom. The Rotopod is initially compared with other all revolute joint parallel manipulators to show its similarities and differences. The inverse kinematics for this mechanism are developed and used to analyze the accessible workspace of the mechanism. Optimization is performed to determine the Rotopod design configurations which maximum the accessible workspace based on desirable functional constraints.

1 Introduction

This paper presents the kinematic and workspace analysis of the Rotopod, an all revolute joint, six degree-of-freedom (DOF) parallel manipulator developed at Sandia National Laboratories by Bieg [1] (See Figure 1). The Rotopod is under developed for several target applications. One area is machining applications, taking advantage of the stiffness properties of parallel manipulators in general, and specific dexterity properties of the Rotopod. Another area is micro-level applications such as micro-surgery and micro-assembly, which take advantage of the precise positioning accuracy of parallel manipulators.

The Rotopod consists of two platforms connected by six legs and exhibits six Cartesian DOF’s. Each leg is driven by one actuator at the base joint. The design of the Rotopod takes advantage of the design trend to move the actuators to the base of the manipulator such that they are not part of the moving mass of the mechanism. As with all parallel mechanism designs, the workspace is limited compared to serial link manipulators. Therefore, the focus of our current design efforts has been to maximize the accessible workspace of the Rotopod.

The paper is organized as follows. Section 2 presents related background on parallel manipulators and shows how the Rotopod is unique compared to currently known all revolute joint mechanisms. Section 3 describes the Rotopod configuration in detail. Section 4 presents the inverse kinematic analysis, and these results are applied in Section 5 for workspace analysis. Section 6 presents the results of optimization of the accessible workspace. Finally, Section 7 presents the conclusions and future work.

2 Background

Parallel manipulators have been studied by many researches in the past years. Many have focused on the Stewart platform design with linear actuators [2,3], while others have focused on manipulators which combine linear and revolute joints [4]. Several mechanism configurations have been developed which consist of all revolute type joints, the class to which the Rotopod belongs.

Table 1 shows a comparison of some of the known all revolute parallel mechanism designs. A similar table was presented by Cleary [5] related to the SMARTee device, and we have updated this table to include the Rotopod and other mechanisms which we have become familiar. Using the nomenclature of Cleary, a pitch joint is defined as a joint that rotates about an axis that is tangential to the base circle of the manipulator. The base circle is defined as being centered at the base of the mechanism, with a radius equal to the distance from this center to the first joint axis. A roll joint is defined as a joint that rotates about the corresponding link’s axis of elongation. A yaw joint rotates about an axis colinear with a base radial line.

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Table 1: 6 DOF Parallel Manipulators with All Revolute Type Joints

<table>
<thead>
<tr>
<th>Mechanism Name (if applicable)</th>
<th>Reference</th>
<th>Number of Legs</th>
<th>Structure of First 3 Joints</th>
<th>Leg Joint DOF’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funabashi</td>
<td>Funabashi [6]</td>
<td>6</td>
<td>pitch yaw pitch</td>
<td>1-1-1-3</td>
</tr>
<tr>
<td>RSI</td>
<td>RSI [8]</td>
<td>3</td>
<td>pitch pitch-yaw</td>
<td>1-2-3</td>
</tr>
<tr>
<td>SMARTee</td>
<td>Brooks [5,9]</td>
<td>3</td>
<td>pitch-roll pitch</td>
<td>2-1-3</td>
</tr>
<tr>
<td>Rotopod</td>
<td>Bieg [1]</td>
<td>6</td>
<td>pitch-yaw pitch</td>
<td>2-1-3</td>
</tr>
<tr>
<td>Tesar</td>
<td>Tesar [10]</td>
<td>3</td>
<td>yaw-pitch pitch</td>
<td>2-1-3</td>
</tr>
</tbody>
</table>

The table shows that the Rotopod has a unique kinematic arrangement. The Rotopod’s leg DOF’s of 2-1-3 is similar to both the SMARTee and Tesar’s mechanism, but differ in the number of legs (three for both SMARTee and Tesar, six for the Rotopod) and the axis orientation for the first two joints. The mechanism also shares similarities to Funabashi’s mechanism in the structure of the first three joints (pitch yaw pitch), but Funabashi includes a physical link between the first and second joint of the leg structure, whereas the Rotopod has zero link length and thus the first joint is a universal joint with two DOF’s.

3 Mechanism Description

The Rotopod mechanism is shown in Figure 1. It consists of a fixed base and a moving top platform which are connected with six legs. Each leg consists of a two DOF universal joint at the base, a link of length $L_1$, a single DOF rotational joint at the elbow, a second link of length $L_2$, and a three DOF spherical joint at the moving top platform. The two DOF universal joint at the base is oriented such that the first rotation produces an orientation change about a fixed axis tangent to the base circle (pitch), but the axis is not constrained to lie in the plane of the base circle. This first rotation is actuated by the motor. The second rotation of the universal joint produces an orientation change about an axis perpendicular to both the axis of the first rotation and the axis along the link’s length. The axis of the single DOF elbow joint produces a pitch motion.

Figure 2 shows the nomenclature used to describe the Rotopod. A fixed base frame is attached at the center of the base circle with $z_b$ normal to the plane of the base circle, and $x_b$ and $y_b$ in the plane of the base circle with $x_b$ oriented symmetrically between joints 1 and 6. A moving platform frame is attached to the center of the platform circle with $z_p$ normal to the plane of the platform circle, and $x_p$ and $y_p$ in the plane of the platform circle with $x_p$ oriented symmetrically between joints 1 and 6. The base circle has a radius $r_b$ from the base frame origin to the center of the universal joint. The platform circle has a radius $r_p$ from the platform frame origin to the center of the spherical joint. The angle $\alpha$ describes the separation of legs 1-6, 2-3 and 4-5 on the base circle, and legs 1-2, 3-4 and 5-6 on the platform circle. The angle $\beta_i$ describes the absolute orientation of the base universal joint $i$ from $x_b$ about $z_b$. Similarly, the angle $\lambda_i$ describes the orientation of the platform spherical joint on leg $i$ from $x_p$ about $z_p$. $\theta_{1i}$ and $\theta_{2i}$ specify the angles of the universal joint about the joint axes $Z_{1i}$ and $Z_{2i}$ respectively, and $\theta_{3i}$ the angle of the elbow joint about axis $Z_{3i}$. Since the axis of joint 1 is not constrained to lie in the $x_b$,$y_b$ plane, the angle $\phi$ describes the rotation of this axis out of the plane about a radial axis from the base origin to the center of the joint. The angle $\rho$ describes the orientation of the first link with respect to the base circle in the initial assembled configuration of the mechanism establishing the direction of the fixed reference frame for leg $i$.

4 Inverse Kinematics

The inverse kinematics of a parallel manipulator involves solving for the values of the driven joints of the legs given the position and orientation of the moving top platform relative to the fixed base. For the Rotopod, this involves solving for the first angle, $\theta_{1i}$, of the universal
joint for each leg. However, the solution approach described below also yields the angles $\theta_2$ and $\theta_3$, for each leg which can be useful in computing the location of the extended elbow.

The inverse kinematics of the Rotopod can be solved in closed-form. The solution approach involves writing two expressions for the vector $d_i$ shown in Figure 2 which is the relationship for an individual leg between the centers of the universal joint at the base and the spherical joint at the moving platform. The first is an expression for $d_i$ in terms of the vectors which describe how the legs are attached to the base and moving platforms, $p_{bi}$ and $p_{pi}$ respectively, and the relationship between the base and top platform, $p_{plat}$. The second expression for $d_i$ is written in terms of the joint angles and link lengths of each leg, treating it as a serial link manipulator. These two expressions for $d_i$ can then be equated and solved for the angles of each leg, $\theta_1$, $\theta_2$, and $\theta_3$.

4.1 Compute $d_i$ through the base and platform

From Figure 2, it can be observed that

$$d_i = p_{plat} + p_{pi} - p_{bi} \quad (1)$$

where each of the vectors is expressed with respect to the base frame. The vector $p_{plat}$ is given for the inverse kinematics problem. The components of $p_{plat}$ are $[ppat_b, ppat_pla, pplat]$. The vector $p_{bi}$ specifies the location of the center of the universal joint for each leg. The parameter $a$ specifies the separation of each pair of legs as shown in Figure 2, and the parameter $p_i$ specifies the angle of leg $i$ from the base $x$ axis in terms of $\alpha$. The symmetry axes between each leg pair are 120° apart. The components of $p_{bi}$ are $[r_i\cos(\beta_i), r_i\sin(\beta_i), 0]$.

The vector $p_{pi}$ specifies the location of the center of the spherical joint of each leg with respect to the platform origin, but expressed with respect to the base frame. This vector can be computed from $p_{bi}$ with the equation

$$p_{pi} = \text{base} R \text{ plat} p_{bi} \quad (2)$$

The term $\text{base plat} R$ is a 3x3 matrix which expresses the orientation of the platform relative to the base. This matrix is given for the inverse kinematics problem and can be written in a variety of forms (e.g. Euler angles). The following equations will use the elements of the matrix, $R_{ij}$. The components of $\text{plat} p_{pi}$ are $[r_i\cos(\lambda_i), r_i\sin(\lambda_i), 0]$ where $\lambda_i$ represents the angle of leg $i$ from the platform $x$ axis and is expressed in terms of the leg separation parameter $\alpha$. Note that $\alpha$ is the same parameter used for the base leg separation.

Expanding equation (2) and substituting into equation (1) yields the components of $d_i$.

$$d_{ix} = R_{11}r_ip\cos(\lambda_i) + R_{12}r_ip\sin(\lambda_i) + p_{plat} - r_i\cos(\beta_i)$$
$$d_{iy} = R_{21}r_ip\cos(\lambda_i) + R_{22}r_ip\sin(\lambda_i) + p_{plat} - r_i\sin(\beta_i)$$
$$d_{iz} = R_{31}r_ip\cos(\lambda_i) + R_{32}r_ip\sin(\lambda_i) + p_{plat}$$

$d_i$ as stated is expressed in terms of the base frame, but needs to be expressed in terms of the local leg frame in order to be equated with the formulation of $d_i$ in the next section. Therefore,

$$d_i = \left[ \begin{array}{c} \text{base} \\ \text{leg} \end{array} R \right]^T \text{base} d_i \quad (4)$$

The orientation of the local leg frame is determined from three rotations of the base frame. The first is a rotation about the $z_b$ by angle $\beta_i$. The remaining two rotations are specified with respect to the initial assembled configuration of the Rotopod as shown in Figure 2. The angle $\phi$ specifies the rotation of the plane defined by the leg links from a vertical plane and establishes the fixed angle of the motor driving the first joint with respect to the plane of the base. The angle $\rho$ specifies the rotation of the first leg link from the plane of the base. Given these three angles, the local leg frame can be established from the base frame with the following three rotations:

1. Rotate about $z_b$ by $\beta_i$
2. Rotate about $x'$ by $-\left(90° + k\phi \right)$ where $k = +1$ for legs 1,3,5 and $k=-1$ for legs 2,4,6
3. Rotate about $z'$ by $\psi = \rho - 180°$

Therefore,

$$\left[ \begin{array}{c} \text{base} \\ \text{leg} \end{array} R \right] = \text{Rot}(z_b, \beta_i)\text{Rot}(x', -\left(90° + k\phi \right))\text{Rot}(z', \rho - 180°)$$

and

$$\left[ \begin{array}{c} \text{base} \\ \text{leg} \end{array} R \right]^T =$$

$$\begin{bmatrix}
-c\beta_i c\phi - k s\beta_i s\phi s\rho & - s\beta_i c\rho + k c\beta_i s\phi s\rho & c\phi s\rho \\
-c\beta_i s\phi & s\beta_i s\phi c\rho & s\beta_i s\phi c\rho + k c\beta_i s\phi c\rho & c\phi c\rho \\
-s\beta_i c\phi & -s\beta_i c\phi & c\beta_i c\phi & -k s\phi
\end{bmatrix}$$

(6)

where for example $c\beta_i = \cos(\beta_i)$ and $s\phi = \sin(\phi)$.

4.2 Compute $d_i$ through the leg structure

As shown in Figure 2, an expression for $d_i$ can also be developed through the links and joints of the leg structure of the Rotopod. The approach is to treat the leg structure as a serial link manipulator and to develop the leg
kinematics following the approach of Craig[12]. Table 2 shows the Denavit-Hartenberg (DH) parameters for the Rotopod leg structure shown in Figure 2.

Table 2: DH Parameters for the Rotopod

<table>
<thead>
<tr>
<th>Link</th>
<th>( \alpha_k )</th>
<th>( \alpha_{k-1} )</th>
<th>( a_k )</th>
<th>( d_k )</th>
<th>( \theta_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>-90(^\circ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>90(^\circ)</td>
<td>( \ell_1 )</td>
<td>0</td>
<td>0</td>
<td>( \theta_3 )</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>( \ell_2 )</td>
<td>0</td>
<td>0</td>
<td>( \theta_4 )</td>
</tr>
</tbody>
</table>

Using these parameters, the 4x4 matrix \( \mathbf{T}_4 \) can be written which expresses the position and orientation of a reference frame located at the center of the spherical joint relative to the leg frame. The first three elements of the fourth column of this matrix correspond to the elements of the vector \( \mathbf{d}_4 \) expressed in the leg frame. These elements are

\[
\begin{align*}
\mathbf{d}_{iX} &= (L_1 + L_2 c \theta_3) c \theta_1 c \theta_2 - L_2 s \theta_1 s \theta_3 \\
\mathbf{d}_{iY} &= (L_1 + L_2 c \theta_3) s \theta_1 c \theta_2 + L_2 s \theta_1 s \theta_3 \\
\mathbf{d}_{iZ} &= -(L_1 + L_2 c \theta_3) s \theta_2 
\end{align*}
\]  

(7)

By equating the elements of equations (4) and (7), the individual joint angles can be determined. The expression for \( \theta_3 \) is determined by squaring and adding each of the equations in (7).

\[
\theta_3 = \cos^{-1}\left( \frac{r^2 - L_2^2 - L_2^2}{2L_1 L_2} \right) 
\]

(8)

where 

\[
r^2 = \mathbf{d}_{iX}^2 + \mathbf{d}_{iY}^2 + \mathbf{d}_{iZ}^2 
\]

(9)

The expression for \( \theta_2 \) is determined from the equation for \( \mathbf{d}_{iX} \) in (7)

\[
\theta_2 = \sin^{-1}\left( \frac{\mathbf{d}_{iX}}{L_2 c \theta_1 + L_1} \right) 
\]

(10)

Finally, \( \theta_1 \) is determined from \( \mathbf{d}_{iX} \) in (7).

\[
\theta_1 = 2 \tan^{-1} \left( \frac{b \pm \sqrt{b^2 + a^2 - c^2}}{a + c} \right) 
\]

(11)

where

\[
\begin{align*}
a &= (L_2 c \theta_3 + L_1) c \theta_2 \\
b &= L_2 s \theta_3 \\
c &= \mathbf{d}_{iX}
\end{align*}
\]

5 Workspace Analysis

The inverse kinematic equations developed in the previous section can be used to determine the accessible workspace of the Rotopod. The accessible workspace is defined as the 3D volume that can be reached by the center point of the moving platform. Maximizing accessible workspace for parallel manipulators was one of the key motivations for developing the Rotopod design. One feature of this design is that the top platform can be lowered to nearly touch the base if the leg link dimensions meet certain constraints. This capability should lead to increased workspace volumes compared to parallel mechanism designs where the leg geometry does not permit such collapsing of the mechanism.

Several approaches have been developed for analyzing the accessible workspace of parallel mechanisms [13, 14]. Our approach utilizes a searching technique for locating the workspace boundary on planes parallel to the \( x_y_z \) plane at discrete \( z \) locations. Potential platform locations are determined to be inside or outside the workspace by calculating the inverse kinematics and then testing if each of the joint angles \( (\theta_{1i}, \theta_{2i}, \theta_{3i}) \) are real. The search increment is refined automatically such that the boundary locations are determined within a desired tolerance.

Once a valid assembled configuration of the mechanism is specified, the algorithm proceeds as follows:

1. Determine the maximum extension, \( z_{\text{max}} \), of the platform.
2. Determine the minimum extension, \( z_{\text{min}} \), of the platform.
3. Divide the range \( z_{\text{max}} - z_{\text{min}} \) into equal increments.
4. For each \( z \) plane, search along equally spaced radial lines to determine the \( x-y \) coordinates of the boundary.

For algorithm efficiency, the initial search for a boundary limit proceeds at a large search increment (e.g. 0.1 units) until the first point outside the boundary is located. The algorithm then automatically reduces the increment and continues searching about this point to locate the boundary to the desired accuracy (e.g. 0.001 units). Also, for step 4, a new search along a radial line is initiated with the value from the last radial search. All parameters in the search, such as the number of \( z \) increments in step (3), or the separation of the radial lines in (4), can be specified to determine the final resolution of the workspace map.

The workspace algorithm also computes a contained volume for the resulting workspace. First, a circular area is computed for each workspace layer in \( z \) which is based on the average radius from the radial line searches. That area is then multiplied by the \( z \) increment to obtain a
volume. The volumes for each of the individual layers is then summed to result in the total workspace volume. Note that the top most layer for the workspace has zero area because it is a point, and therefore has zero volume. Thus, there is not an extra layer projected into a volume as might initially be expected.

Figure 3 shows the workspace of a Rotopod with the following dimensions: \( r_b = 1.0, r_p = 0.5, l_1 = 1.0, l_2 = 1.0, \alpha = 60^\circ \).

*Figure 3: Rotopod Workspace*

The dimensions of this Rotopod are one that a designer might choose as an initial configuration. A value of \( \alpha = 60^\circ \) indicates that the legs are equally spaced. This configuration has the capability kinematically to lower the top platform to touch the base. The \( z \) range of the top platform is \( Z_{\min} = 0.0 \) and \( Z_{\max} = 1.94 \). The workspace volume is 2.9. The workspace was calculated with 25 \( z \) planes, and 10° radial increments per plane. The workspace boundaries were located to within 0.0001.

6 Workspace Optimization

Optimization has been performed to maximize the Rotopod's accessible workspace. The objective function to maximize was the accessible workspace volume as discussed in section 5. The design variables during optimization were \( \alpha, r_b, r_p, \) and \( l_1/l_2 \), the ratio of the two leg link lengths. The optimization was subject to the constraint \( l_1 + l_2 = 2.0 \) in order to bound the possible workspace volume. Use of the ratio \( l_1/l_2 \) allowed the constraint to be implied within the objective function, permitting the use of an unconstrained optimization function. The optimization was performed using the MATLAB optimization toolbox.

The optimization resulted in the following values for the design variables

\[
\begin{align*}
\alpha &= 58.3^\circ \\
r_b &= 0.44 \\
r_p &= 0.47 \\
l_1/l_2 &= 1.58
\end{align*}
\]

and the following operational parameters for the Rotopod workspace volume = 10.2

\[
Z_{\max} = 1.99 \\
Z_{\min} = 0.45
\]

The workspace for this Rotopod design is shown in Figure 4. The workspace maximization expands the upper "bowl" portion of the workspace and minimizes the smaller diameter "stem" as compared to the workspace shown in Figure 3. It is interesting to note that the optimization result in a \( Z_{\min} = 0.45 \), meaning that for this optimized design the top platform does not lower completely to touch the base as is possible with some designs.

*Figure 4: Maximum Accessible Workspace*

Another observation about the optimized design is the resulting design parameter \( \alpha = 58.3^\circ \). This means that the leg separation is nearly symmetrical around the base and platform, and that the platform attachment point for any given leg is almost directly above (in base \( z \)) the base attachment point when the platform frame is directly above the base frame. This results in a mechanism for which it is difficult to drive platform orientation changes about the \( z \) axis because the leg configuration can't easily generate the required torques in this direction. As a result, we have also studied optimized Rotopod designs which constrain the angle \( \alpha \) to be less than 30°. We have found that the larger values of \( \alpha \) in this range produce larger accessible workspaces. Figure 5 shows the workspace for a Rotopod that constrains the value of \( \alpha \) to 30°. The resulting design parameters are

\[
\begin{align*}
l_1/l_2 &= 1.48 \\
r_b &= 0.12 \\
r_p &= 0.11
\end{align*}
\]
and the resulting operational parameters are

workspace volume = 8.45
\[ Z_{\text{max}} = 1.99 \]
\[ Z_{\text{min}} = 0.38 \]

The resulting accessible workspace for this design is about 17% smaller than the optimized design with \( \alpha = 58^\circ \). Note that more of a “stem” structure is present in the workspace for this design as compared to the optimized design. Also note that the base and platform radiiues are much smaller for \( \alpha = 30^\circ \) than the design with \( \alpha = 58^\circ \). These are the tradeoffs for preserving the capability to drive platform orientation changes about \( z \).

Our current Rotopod designs are based on \( \alpha = 30^\circ \).

7 Conclusions and Future Work

The design of the Rotopod has been optimized for maximum accessible workspace. We have shown that with no constraints on the design variable \( \alpha \), the workspace shape assumes nearly a full “bowl” shape with no “stem” on the lower portion. However, we are concerned with the resulting value of \( \alpha = 58.3^\circ \) and the restriction this imposes on the ability to drive platform rotations about the \( z \) axis. Therefore, we have restricted \( \alpha \) to 30° to retain this degree of freedom, which results in a Rotopod design with 17% smaller workspace. We are currently fabricating an actuated device with these dimensions to further study the Rotopod design concept.

Other researches have shown that maximizing the accessible workspace of a parallel manipulator does not necessarily produce the best quality of workspace with respect to such measures as dexterity [15]. We will continue our workspace optimization studies to include such workspace quality measures as dexterity and manipulability [16].

Acknowledgment

We would like to acknowledge the contributions of Don Plymale at Sandia in our studies of parallel manipulators.

References

[8] RSI, British Columbia, Canada

Figure 5: Maximum Accessible Workspace for \( \alpha = 30^\circ \)