DETERMINING THE TRANSFER FUNCTION FOR UNSTEADY PRESSURE MEASUREMENTS USING A METHOD OF CHARACTERISTICS SOLUTION*

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ABSTRACT

An inverse Fourier transform method for removing lag from pressure measurements has been used by various researchers, given an experimentally derived transfer function to characterize the pressure plumbing. This paper presents a Method of Characteristics (MOC) solution technique for predicting the transfer function and thus easily determining its sensitivity to various plumbing parameters. The MOC solution has been used in the pipeline industry for some time for application to transient flow in pipelines, but it also lends itself well to this application. For highly nonsteady pressures frequency-dependent friction can cause significant distortion of the traveling waves. This is accounted for in the formulation. A simple bench experiment and proof-of-principle test provide evidence to establish the range of validity of the method.

INTRODUCTION

When possible, measurement of unsteady pressures is best accomplished with in-situ, high-frequency response, flush-diaphragm pressure transducers. However, in many situations, either the geometry of the experiment or the medium in which the experiment is conducted may preclude such an optimum solution^{1,2}. In the experiment which engendered the development of the method which this paper describes, both of these constraints were active. The differential pressures were being measured across thin, ribbon-like structures that had no space for in-situ transducers, and the experiment was conducted in a water tow tank. Were thin, surface-mounted transducers to have been used, it would have been necessary to coat them to protect them from the water. For the very low pressures being measured (~ 0.20 psi full-scale), the errors associated with the "bridging" of the diaphragm by the coating material would have been significant. Further, the heat transfer characteristics of the water (as compared to air) would have increased the transducers' temperature sensitivity and the subsequent errors associated with that sensitivity. Therefore, since it was necessary to locate the transducers

on the tow tank carriage, long capillary-like tubes were necessary to connect them to the pressure taps on the towed model. To avoid the errors associated with air-water interfaces, the tubes (and variable reluctance transducers) were filled with deaerated water and purged of any entrapped air bubbles. It should be admitted that prior to the test, the first author (who had spent his entire professional life working with air as a test medium) had naively assumed that measurement lag would not be a problem due both to water's "incompressibility" and to the very high acoustic speed in water coupled with the very low velocities (~ 4 ft/s) expected in the tow tank experiment. Subsequent examination of the test data proved this assumption not only wrong but *dreadfully* wrong. In a flurry of Monday-morning quarterbacking, it was found that the acoustic speed of water is significantly reduced (from ~ 4915 ft/s to ~ 200 ft/s) in the plastic tubing used for a majority of the capillary. Further, the high-sensitivity variable reluctance transducers required significant diaphragm movement as a function of pressure and, therefore, a significant flow of water to effect such movement. The very small diameter of the tubing coupled with its long length introduced large amounts of frictional resistance to that flow.

A method of characteristics (MOC) solution, outlined in *Fluid Transients in Systems*, a textbook³ on transient flow analysis, was adapted to the transducer plumbing system to derive the response at the transducer to an input signal at the measurement orifice. It was found that when a signal proportional to the dynamic pressure (obtained from the measured carriage velocity which experienced no lag) was input to the MOC method, the resulting response was in every way similar to the actual pressure traces obtained during the test. Concurrent with this observation exposure to a paper by Sims-Williams and Dominy⁴ suggested the possibility of deriving an inverse function that could be used to reconstruct the pressure at the orifice, given the measured pressure and the MOC-derived transfer function associated with the pressure tubing. They had used a method first described by Irwin, Cooper, and Girard⁵, in which an experimentally obtained transfer function was

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used to correct signal distortion caused by pressure tubing. Another method used to correct the distortion effects of tubing systems on pressure measurements relies on an experimental determination of the transfer function to establish a natural frequency and damping ratio for use in a second-order linear filter approximation to the system response⁶.

This paper describes the application of the MOC solution to obtain the necessary transfer function and an experiment that was used to validate the method. The formulation accounts for the strong effects of elasticity of the tubing walls on the acoustic speed in water. *It must be emphasized that we do not claim the development of this methodology — merely the application of it to the pressure lag problem endemic to instrumentation in aerospace simulation facilities and particularly acute in water facilities.*

NOMENCLATURE

a	acoustic speed
A	internal cross-sectional area of tube
B	tubing characteristic impedance
C^+	refers to conditions along + characteristic line — i.e., $a = +dx/dt$
C^-	refers to conditions along — characteristic line — i.e., $a = -dx/dt$
D	inside diameter of tube
e	tubing wall thickness
E	modulus of elasticity of tube wall material
f	Darcy-Weisbach friction factor
K	bulk compression modulus of fluid medium
p	pressure
p^+	pressure immediately downstream of node point
p^-	pressure immediately upstream of node point
p_f	frictional pressure drop per unit length
Δp_i	pressure drop across node point i
Q	volumetric flow rate
R	tubing resistance coefficient
Re	Reynolds number
t	time
u	velocity
V	volume of transducer at pressure, p
W	weighting function
x	distance along tubing
η	Poisson's ratio for tube wall material
λ_1, λ_2	multipliers
ν	kinematic viscosity
ξ	non-dimensional time
ρ	density

Subscripts

i	index on location <i>along</i> tubing
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j	index on time
P	point at which velocity and pressure are being calculated
A	point upstream of P along + characteristic (see calculation grid)
B	point downstream of P along — characteristic (see calculation grid)

METHOD OF CHARACTERISTICS FORMULATION

In a pressure tubing system the flow can be reasonably assumed to be not only laminar but also basically one-dimensional in character. In the Method of Characteristics formulation, simplifying assumptions are made that transform the partial differential equations governing this flow into total differential equations. These equations may be integrated to yield finite difference algebraic equations which are then solved numerically.

For the pressure tubing system used in this analysis, the flow is governed by the continuity and momentum equations written in terms of two dependent variables, velocity and pressure, and two independent variables, distance along the tubing and time. Following the derivation outlined by Zielke⁷:

Continuity Equation

The continuity equation for one-dimensional unsteady flow may be written³

$$\frac{\partial u}{\partial x} + \frac{u}{\rho a^2} \frac{\partial p}{\partial x} + \frac{1}{\rho a^2} \frac{\partial p}{\partial t} = 0, \quad (1)$$

but for $(u/a^2) \ll 1$, the equation reduces to

$$\rho a^2 \frac{\partial u}{\partial x} + \frac{\partial p}{\partial t} = 0. \quad (2)$$

Momentum Equation

The one-dimensional, unsteady momentum equation can be written³

$$\frac{\partial p}{\partial x} + \rho u \frac{\partial u}{\partial x} + \rho \frac{\partial u}{\partial t} + p_f = 0, \quad (3)$$

in which p_f is the pressure drop per unit length due to fluid friction. The inertial term, $\rho u(\partial u/\partial x)$, can be considered negligible for the low-speed flows and the closed tubing system with which we are working, so we are left with

$$\frac{\partial p}{\partial x} + \rho \frac{\partial u}{\partial t} + p_f = 0. \quad (4)$$

Characteristics Equations

We can now form two new equations from Equations (2) and (4) by linearly combining them using multipliers, λ_1 and λ_2 .

$$\frac{\partial p}{\partial x} + \rho \frac{\partial u}{\partial t} + p_f + \lambda_1 \left[\rho a^2 \frac{\partial u}{\partial x} + \frac{\partial p}{\partial t} \right] = 0 \quad (5)$$

$$\frac{\partial p}{\partial x} + \rho \frac{\partial u}{\partial t} + p_f + \lambda_2 \left[\rho a^2 \frac{\partial u}{\partial x} + \frac{\partial p}{\partial t} \right] = 0 \quad (6)$$

Now by setting $\lambda_1 = 1/a$ and $\lambda_2 = -1/a$ and rearranging,

$$\left[a \frac{\partial p}{\partial x} + \frac{\partial p}{\partial t} \right] + \rho a \left[a \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} \right] + a p_f = 0 \quad (7)$$

$$\left[-a \frac{\partial p}{\partial x} + \frac{\partial p}{\partial t} \right] + \rho a \left[-a \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} \right] - a p_f = 0 \quad (8)$$

By setting $a = dx/dt$ and $-dx/dt$ in Equations (7) and (8), respectively, we obtain the following two pairs of equations which we have identified as the C^+ and C^- equations. The partial differential equations have been converted to total differential Equations (9) and (11), each of which is valid and can be integrated only along its appropriate characteristic line, Equation (10) or (12).

$$C^+: \begin{cases} \frac{dp}{dt} + \rho a \frac{du}{dt} + a p_f = 0 & (9) \\ \frac{dx}{dt} = +a & (10) \end{cases}$$

$$C^-: \begin{cases} \frac{dp}{dt} - \rho a \frac{du}{dt} - a p_f = 0 & (11) \\ \frac{dx}{dt} = -a & (12) \end{cases}$$

Finite Difference Formulation

We divide the pressure tubing into an even number of reaches N , separated by $N-1$ node points. The length of all reaches should be set such that the transit time of an acoustic wave over any reach is equal. The acoustic speed in a fluid contained in elastic tubing anchored at its upstream end is given by Wylie³:

$$a = \sqrt{\frac{K/\rho}{1 + [(K/E)(D/e)]c_1}}, \quad (13)$$

where for thick-walled ($D/e < 25$) tubing,

$$c_1 = \frac{2e}{D}(1+\eta) + \frac{D}{D+e}(1-\eta/2). \quad (14)$$

Figure 1 shows a typical grid for solving the pressure tubing problem.

Figure 1. Characteristics grid

The characteristic equations are integrated along the appropriate characteristic lines, C^+ and C^- , to give the algebraic finite difference equations. Thus, the two dependent variables, u and p , are solved for at Point P using the two equations relating conditions at Point P to known conditions at points A and B. It should be noted that at this point, the pressure has been defined to have two possible values at any computation point, depending upon whether the point was approached from upstream along the C^+ characteristic (in which case it carries a - superscript) or from downstream along the C^- characteristic (in which case it carries a + superscript). An additional equation is added at each point to define the pressure drop across the point. This allows introduction of known pressure losses, Δp_P , associated with sudden expansions, contractions, orifices, etc. By introducing the tubing cross-sectional area, A , and writing the equations in terms of flow rate, Q , instead of velocity to assure conservation of mass at discontinuities in tubing size, we obtain the following difference equations:

$$p_P^- - p_A^+ + \frac{\rho a}{A}(Q_P - Q_A) + a\Delta t p_{fA} = 0 \quad (15)$$

$$p_P^+ - p_B^- - \frac{\rho a}{A}(Q_P - Q_B) - a\Delta t p_{fB} = 0 \quad (16)$$

$$p_P^- - p_P^+ = \Delta p_P. \quad (17)$$

If the flow were steady, the pressure loss due to fluid friction would be modeled by the familiar

$$p_{f, steady} = \frac{f}{DA^2} \frac{\rho Q^2}{2}, \quad (18)$$

and since for laminar flow

$$f = \frac{64}{Re} = \frac{64A\nu}{QD} \quad (19)$$

$$p_{f, steady} = \frac{32\rho\nu}{AD^2} Q. \quad (20)$$

However, for the unsteady flow associated with the problem we are modeling, the fluid friction becomes frequency dependent. In such flows the fluid in the center of the tube is dominated by inertial effects, whereas that at the walls responds primarily to viscous forces. The velocity distribution is no longer parabolic, and in the extreme the fluid near the wall may even be moving in the opposite direction from the overall mass average. In 1966 Zielke studied this type of flow as the subject of a Ph.D. thesis working under Professor Wylie. His derivation of the effects of this unsteadiness on the MOC solution is outlined in Reference 7 and is too complex to duplicate here. The results, however, are not extremely complex and simply reduce to splitting the friction pressure loss into two components, steady and unsteady. The steady loss is equal to that given by Equation 20, and the unsteady loss is calculated with a convolution integral that factors in the past history of the flow.

$$p_{f, unsteady} = \frac{16\rho\nu}{AD^2} \int_0^t \frac{\partial Q}{\partial \tau}(\tau) W(t-\tau) d\tau, \quad (21)$$

where the weighting function is given by

$$W(\xi) = e^{-26.3744\xi} + e^{-70.8493\xi} + e^{-135.0193\xi} + e^{-218.9216\xi} + e^{-322.5544\xi} \quad \xi > 0.02 \quad (22a)$$

$$W(\xi) = 0.282095\xi^{-1/2} - 1.25 + 1.057855\xi^{1/2} + 0.9375\xi + 0.396696\xi^{3/2} - 0.351563\xi^2 \quad \xi < 0.02 \quad (22b)$$

where the dimensionless time, ξ , is defined by

$$\xi = \frac{4\nu t}{D^2}. \quad (23)$$

Algebraic Solution

After starting with known initial conditions at time zero, the solution proceeds by determining p and Q at alternate grid points at time Δt , proceeding to time $2\Delta t$, and solving for the interior intervening grid points from the previously determined p 's and Q 's and the boundary points directly from the known boundary conditions. This is continued until the desired time span has been calculated. Thus, by properly minding one's p 's and Q 's, a complete solution is obtained.

The simultaneous solution for p and Q at any interior point is obtained after defining two new parameters: the tubing's "characteristic impedance," B_i , and its "resistance coefficient," R_i , over reach, i .

$$B_i = \frac{\rho a_i}{A_i} \quad (24)$$

$$R_i = \frac{32\rho\nu a_i \Delta t}{A_i D_i^2} \quad (25)$$

After rearranging Equations 15 and 16, we obtain

$$C^+ : p_P^- = C_A - B_{i-1} Q_P \quad (26)$$

$$C^- : p_P^+ = C_B + B_i Q_P, \quad (27)$$

where

$$C_A = p_A^+ + (B_{i-1} - R_{i-1}) Q_A - \frac{R_{i-1}}{2} \sum_{j=1,3,\dots}^{k-1} (Q_{i,k-j+1} - Q_{i,k-j-1}) W(j\Delta t) \quad (28)$$

$$C_B = p_B^- - (B_i - R_i) Q_B + \frac{R_i}{2} \sum_{j=1,3,\dots}^{k-1} (Q_{i,k-j+1} - Q_{i,k-j-1}) W(j\Delta t). \quad (29)$$

Equations 17, 26, and 27 can be solved to give:

$$p_P^- = \frac{C_A B_i + (C_B + \Delta p_P) B_{i-1}}{B_i + B_{i-1}} \quad (30)$$

$$p_P^+ = \frac{(C_A - \Delta p_P) B_i + C_B B_{i-1}}{B_i + B_{i-1}} \quad (31)$$

$$Q_P = \frac{C_A - C_B - \Delta p_P}{B_i + B_{i-1}} \quad (32)$$

Boundary Conditions

To solve the finite difference equations, boundary conditions need to be supplied at both ends of the tubing. For the pressure response problem, the pressure at the upstream boundary simply becomes the orifice pressure as a function of time, whereas the flow rate can be obtained directly from Equation 27. For a system terminating at the pressure transducer – i.e., a gage pressure or absolute pressure transducer – a zero flow rate boundary condition (with corresponding pressure obtained from Equation 26) would be appropriate at the downstream end of the tubing *if the transducer diaphragm exhibited negligible movement over the pressure range of interest*. However, if the transducer diaphragm deforms noticeably, a relationship between the volumetric displacement caused by this movement and the exhibited pressure must be obtained. A pressure boundary condition is then obtained as a time integral of the volumetric flow rate entering that boundary.

For the tow tank test since differential pressure transducers were used, the entire plumbing system stretched from the orifice supplying pressure to one side of the diaphragm to the orifice supplying pressure to the reference side. The tubing on both sides of the diaphragm were mirror images of each other. It was assumed that the differential pressure was obtained by all of the pressure acting on one orifice while the other was held at zero reference pressure. (This assumption was validated numerically by applying half of the pressure at one end and a negative half at the other end with identical results.) Thus the boundary condition supplying the transfer function was at a node point T in the center of the tubing. At this point Δp_P was obtained by integrating the flow rate at that point, T:

$$\begin{aligned} \Delta p_T &= \left(\frac{dp}{dV} \right) \int_0^t Q_T dt \\ &= \left(\frac{dp}{dV} \right) \sum_{j=1,3,\dots}^k Q_{T,j} (2\Delta t), \end{aligned} \quad (33)$$

where (dp/dV) is the experimentally derived slope of pressure versus transducer volumetric displacement.

CALCULATION OF TRANSFER FUNCTION

The transfer function was calculated by applying a sinusoidal pressure at the upstream end of the tubing; calculating the response, Δp_T , at the transducer, as

outlined above; and determining a least squares fit of the output data (constrained to be a sine wave with frequency equal to the input frequency). The gain and phase at that particular frequency were then determined by the relationship between the input and output sine waves. Figure 2 shows a typical plot of such a calculation.

Figure 2. Transducer Response to 0.5 Hz Sine Wave at Orifice

EXPERIMENTAL VALIDATION OF TRANSFER FUNCTION

An experiment was conducted to validate the transfer function calculated by the MOC method (see Figure 3). In this experiment a reference pressure transducer was "short-coupled" to a tank of water whose pressure was also measured by a response transducer coupled to the tank through 6 ft. of 0.038 in. ID x 0.062 in. OD stainless steel tubing connected to 14 ft. of 0.050 in. ID x 0.090 in. OD Tygon tubing. This tubing connection was identical to that used in the calculation (and in the water tow tank experiment mentioned above). The pressure tubing on the reference side of the differential transducer was a mirror image and was connected to a reference tank kept at constant pressure. An audio loudspeaker pressurized the tank with a sinusoidal pressure of known frequency. The amplitude ratio and phase relationship between the response transducer output and the reference transducer output defined the system transfer function as a function of frequency. Figure 4 shows a comparison between the transfer function calculated using the method outlined above with that obtained experimentally. In the MOC calculation, 200 terms (adding more terms does not significantly affect the answer) were included to represent velocity history for determining the unsteady fluid friction term, $\Delta p_{f \text{ unsteady}}$. Error bars represent a two-sigma scatter in the experimental data (most of the error bars cannot be seen in the figure because they are hidden by the size of the data points.)

Figure 3. Apparatus for Experimental Validation of MOC-Calculated Transfer Function

(a)

(b)

Figure 4. Comparison of Experimentally Determined System Transfer Function with MOC Calculations

Relatively good agreement is obtained for the gain, whereas poor agreement is shown for the phase for frequencies above ~2 Hz. It is assumed that the gain shows better agreement since the factors influencing

pressure drop (tube diameter and laminar friction modeling) were relatively well known, whereas the primary influence on phase – namely the extensibility of the tubing walls – was not. The value used for Young's modulus for the Tygon tubing was obtained from the manufacturer but was certainly open to doubt. In fact, during the experimental phase it was found that the phase repeatability was strongly influenced by the temperature of the Tygon tubing. Fortunately the velocity profile for the tow tank was not dominated by "high" frequencies, so those errors were not critical to the use of the method in this case. Were time correlation of higher frequency data important, it would be required that the phase relationship be better determined. It should also be noted that the actual time lag does not increase with frequency to a large degree, since it is proportional to the phase divided by the frequency.

PROOF OF PRINCIPLE TEST

A second experiment was conducted in which the speaker was removed and the tank was used as a means of applying a time-varying pressure signal that was typical of the signals measured in the water tow tank. Water was poured into the tank, agitated at varying rates, and then evacuated. The pressure signal measured by the response transducer was then corrected using the calculated transfer function, and the result reproduced the signal measured by the reference transducer to a very high degree (see Figure 5). We have included an expanded plot of that comparison for the time span from 12 to 18 seconds when the agitation points out the inadequacy of the phase modeling in recapturing a time-accurate reconstructed pressure. (It should be noted that the phase lag problem was not evident when the experimentally-derived transfer function was used.)

Figure 5. Proof of Principle Pressure Reconstruction

The inability to reconstruct a steady-state pressure of ~ 0.191 psi (the plot shows a reconstruction of ~ 0.193 psi) is not due to the reconstruction but rather to the experimental error in measurement – i.e., the response and reference transducers are not expected to agree to a precision greater than $\pm 1\%$. A complete error analysis can be found in Reference 1.

A bit needs to be said here about the method used for correcting the response pressure signal by use of the transfer function. In a manner similar to that outlined in Reference 5, first, we represent the pressure at the upstream end of the tubing, p_{ref} , by a Fourier series,

$$p_{ref}(t) = \sum_{n=-\infty}^{\infty} A_n e^{in\omega t} \quad (34)$$

where A_n are complex Fourier coefficients. Similarly the response pressure, p_{resp} , can be represented as,

$$p_{resp}(t) = \sum_{n=-\infty}^{\infty} B_n e^{in\omega t} \quad (35)$$

The system transfer function relates these two in the following way:

$$B_n = T_n A_n \quad (36)$$

so the desired coefficients A_n can be calculated from

$$A_n = B_n / T_n \quad (37)$$

Finally, the desired corrected (or reconstructed) orifice pressure may be obtained by computing the inverse Fourier transform of $A_n(\omega)$. This is best achieved by the use of Fast Fourier Transform (FFT) techniques which require $(2)^n$ terms, where n is an integer. This was achieved in our case by taking exactly $(2)^n$ points of time data and calculating the FFT of that data to obtain $(2)^n$ B_n 's. We fit cubic splines to both the gain and phase terms of the transform, interpolated to obtain the gain and phase at the required n frequencies, recast them into complex form, performed the term-by-term complex divide and then took the inverse FFT of the result.

However, there was one intervening step that was necessary to obtain meaningful results. Since the gain dropped to such a low value (~ 0.001 @ 20 Hz), division results in amplification of the noise to the degree that it obscures the real data. As a result, the A_n coefficients of the reconstructed reference pressure Fourier transform were zeroed for frequencies greater than 15 Hz or less than -15 Hz prior to the inverse FFT process.

MOC METHOD APPLICATIONS

A Method of Characteristics solution similar to that used for calculating transient pipe flows has been successfully applied to the pressure tubing response problem. While the application derived here was specifically validated for very low speed flows with water as the pressure conduit medium, the method should be extensible to the wind tunnel problem so long as the assumptions made in simplifications leading to Equations (2) and (4) are not violated. While experimental determination of dp/dV (used in Equation 33) was a simple process with water, it is assumed that a simple equation of state argument could give reasonable values for large volume transducers being used with a medium such as air, which has a much lower bulk modulus than water.

Once derived and programmed, the method can give very accurate sensitivity coefficients for determining the effects of varying certain tubing parameters or installing restrictors in the tubing.

We feel that this general approach could also have applications to other instrumentation lag problems where the reasons for the lag are not only known but can be modeled. Solution of the model simulation (either analytically or numerically) could be used with an inverse Fourier transform technique to resolve at least part of the lag problem.

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