THE INTERACTIONS OF HEAVY QUARKONIUM: 
SOME RECENT RESULTS

D. Kharzeev

RIKEN-BNL Research Center and Physics Department, 
Brookhaven National Laboratory, 
Upton, New York 11973-5000, USA

Abstract

Some recent developments in the theory of heavy quarkonium interactions are presented. First, we consider quarkonium–quarkonium scattering at very low energies – an analog of the Van der Waals interaction. These long-range forces have very surprising properties in QCD – in particular, as a consequence of scale anomaly, the strength of the interaction at large distances appears independent of the coupling constant and is entirely determined by the non–perturbative vacuum energy density. Second, we argue that the scale anomaly can play a dominant role also in high–energy scattering, and discuss the possible origin of the “soft” Pomeron.

1 Introduction

Heavy quarkonium has a very special place in the hadron world. Its size is small enough to make perturbation theory meaningful, and the description of $J/\psi$ properties and decays was among the first successful applications of perturbative QCD. It is not sufficiently small though to make non-perturbative effects totally negligible; however, they can be analyzed in a systematic way, providing a unique information about the strength of soft gluon fields in the QCD vacuum. Quarkonium is also a unique probe of the gluon field produced in heavy ion collisions [1].

Of particular interest is the problem of heavy quarkonium interactions with other hadrons. It underlines a number of important, and still poorly understood, theoretical issues, and is of great practical significance, e.g., for the
phenomenology of quarkonium suppression in nuclear collisions. In this talk I will describe two recent results – one on the scattering of heavy quarkonia at very low energies, another on high–energy scattering. The common idea behind these two examples is to explore the influence of the QCD vacuum on hadron scattering. The presentation will be very schematic, and I refer the interested reader to the original papers [2] and [3] for all details.

2 The long–range forces of QCD

2.1 Perturbation theory

Let us begin with a somewhat academic problem – the scattering of two heavy quarkonium states at very low energies. The Wilson operator product expansion allows one to write down the scattering amplitude (in the Born approximation) of two small color dipoles in the following form[4]:

\[
V(R) = -i \int dt \langle 0 | T \left( \sum_i c_i O_i(0) \right) \left( \sum_j c_j O_j(x) \right) | 0 \rangle,
\]

where \( x = (t, R) \), \( O_i(x) \) is the set of local gauge-invariant operators expressible in terms of gluon fields, and \( c_i \) are the coefficients which reflect the structure of the color dipole. At small (compared to the binding energy of the dipole) energies, the leading operator in (1) is the square of the chromo-electric field \( (1/2)g^2E^2 \) [5, 4]. Keeping only this leading operator, we can rewrite (1) in a simple form

\[
V(R) = -i \left( \tilde{d}_2 \frac{a_0^2}{\epsilon_0} \right)^2 \int dt \langle 0 | T \left( \frac{1}{2} \frac{g^2E^2(0)}{E_0} \right) \frac{1}{2} \frac{g^2E^2(t, R)}{E_0} | 0 \rangle,
\]

where \( \tilde{d}_2 \) is the corresponding Wilson coefficient defined by

\[
\tilde{d}_2 \frac{a_0^2}{\epsilon_0} = \frac{1}{3N} \langle \phi | r^i \frac{1}{H_a + \epsilon} r^i | \phi \rangle,
\]

where we have explicitly factored out the dependence on the quarkonium Bohr radius \( a_0 \) and the Rydberg energy \( \epsilon_0 \); \( N \) is the number of colors, and \( | \phi \rangle \) is the quarkonium wave function, which is Coulomb in the heavy quark limit. In [6] quarkonium states.

\footnote{The Wilson coefficients \( \tilde{d}_2 \), evaluated in the large \( N \) limit, are available for \( S \) [4] and \( P \) quarkonium states.}
physical terms, the structure of (2) is transparent: it describes elastic scattering of two dipoles which act on each other by chromo-electric dipole fields; color neutrality permits only the square of dipole interaction. It is convenient to express $g^2E^2$ in terms of the gluon field strength tensor [7]:

$$g^2E^2 = -\frac{1}{4}g^2G_{\alpha\beta}G^{\alpha\beta} + g^2(-G_{\alpha\alpha}G_0 + \frac{1}{4}g_{00}G_{\alpha\beta}G^{\alpha\beta}) = \frac{8\pi^2}{b} \theta_\mu + g^2\theta_0^G.$$ (4)

where

$$\theta_\mu = \frac{\beta(g)}{2g}G^{\alpha\beta}_{\alpha\beta} = -\frac{bg^2}{32\pi^2}G^{\alpha\beta}_{\alpha\beta}.$$ (5)

Note that as a consequence of scale anomaly [8], $\theta_\mu$ is the trace of the energy-momentum tensor of QCD in the chiral limit of vanishing light quark masses.

Let us now introduce the spectral representation for the correlator of the trace of energy-momentum tensor:

$$\langle 0|T\theta_\mu^\mu(0)\theta_\mu^\mu(x)|0\rangle = \int d\sigma^2 \rho_\theta(\sigma^2)\Delta_F(x;\sigma^2),$$ (6)

where $\rho_\theta(\sigma^2)$ is the spectral density and $\Delta_F(x;\sigma^2)$ is the Feynman propagator of a scalar field. Using the representation (6) in (2), we get

$$V_\theta(R) = -\left(\frac{a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 \int d\sigma^2 \rho_\theta(\sigma^2) \frac{1}{4\pi R} e^{-\sigma R}.$$ (7)

The potential (7) is simply a superposition of Yukawa potentials corresponding to the exchange of scalar quanta of mass $\sigma$.

Our analysis so far has been completely general; the dynamics enters through the spectral density. In perturbation theory, for $SU(N)$, one has

$$\rho_\theta^p(q^2) = \left(\frac{bg^2}{32\pi^2}\right)^2 \frac{N^2 - 1}{4\pi^2} q^4.$$ (8)

Substituting (8) into (7) and performing the integration over invariant mass $\sigma^2$, we get, for $N = 3$

$$V_\theta(R) = -g^4\left(\frac{a_0^2}{\epsilon_0}\right)^2 \frac{15}{8\pi^3} \frac{1}{R^7}. $$ (9)
Figure 1: Contributions to the scattering amplitude from (a) two gluon exchange and (b) correlated two pion exchange.

The $\propto R^{-7}$ dependence of the potential (9) is a classical result known from atomic physics [9]; as is apparent in our derivation (note the time integration in (7)), the extra $R^{-1}$ as compared to the Van der Waals potential $\propto R^{-6}$ is the consequence of the fact that the dipoles we consider fluctuate in time, and the characteristic fluctuation time $\tau \sim \epsilon_0^{-1}$, is small compared to the spatial separation of the "onia" : $\tau \ll R$.

Let us note finally that the second term in (4) gives the contribution of the same order in $g$; this contribution is due to the tensor $2^{++}$ state of two gluons and can be evaluated in a completely analogous way. Adding this contribution to (9), changes the factor of 15 in (9) to 23, and we reproduce the result of ref. [4], which shows the equivalence of the spectral representation method used here and the functional method of ref. [4].

2.2 Beyond the perturbation theory:
scale anomaly and the role of pions

At large distances, the perturbative description breaks down, because, as can be clearly seen from (7), the potential becomes determined by the spectral density at small $q^2$, where the transverse momenta of the gluons become small. At small invariant masses, we have therefore to saturate the physical spectral density by the lightest state allowed in the scalar channel – two pions. Since, according to (5), $\theta_\alpha$ is gluonic operator, this requires the knowledge of the
coupling of gluons to pions. This looks like a hopeless non-perturbative problem, but it can nevertheless be rigorously solved, as it was shown in ref. [10] (see also [7]). The idea of [10] is the following: at small pion momenta, the energy-momentum tensor can be accurately computed using the low-energy chiral Lagrangian:

$$\theta_\mu = -\partial_\mu \pi^a \partial^\mu \pi^a + 2m_\pi^2 \pi^a \pi^a + \cdots$$  \hspace{1cm} (10)

Using this expression, in the chiral limit of vanishing pion mass one gets an elegant result [10]

$$\langle 0 | \frac{\beta(g)}{2g} G^{\alpha \beta a} G_{a \alpha b} | \pi^+ \pi^- \rangle = q^2.$$ \hspace{1cm} (11)

Now that we know the contribution to the pion-pair spectral density can be easily computed by performing the simple phase space integration, with the result

$$\rho_0^{\pi \pi}(q^2) = \frac{3}{32\pi^2} q^4,$$ \hspace{1cm} (12)

which leads to the following long-distance potential:

$$V^{\pi \pi}(R) = -\left(\frac{a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 \frac{3}{2} \left(2m_\pi\right)^4 \frac{m_\pi^{1/2}}{(4\pi R)^{5/2}} e^{-2m_\pi R} \quad \text{as } R \to \infty.$$ \hspace{1cm} (13)

Note that, unlike the perturbative result which is manifestly $\sim g^4$, the amplitude (13) is $\sim g^0$ – this “anomalously” strong interaction is the consequence of scale anomaly.²

While the shape of the potential in general depends on the spectral density, which is fixed theoretically only at relatively small invariant mass, the overall strength of the non-perturbative interactions is fixed by low energy theorems and is determined by the energy density of QCD vacuum. Indeed, in the heavy quark limit, one can derive the following sum rule [2]:

$$\int d^3R \left( V(R) - V^{pt}(R) \right) = \left(\frac{a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right) \frac{m_\pi^{1/2}}{16 |\epsilon_{vac}|},$$ \hspace{1cm} (14)

which expresses the overall strength of the interaction between two color dipoles in terms of the energy density of the non-perturbative QCD vacuum.

²Of course, in the heavy quark limit the amplitude (13) will nevertheless vanish, since $a_0 \to 0$ and $\epsilon_0 \to \infty$. 
3 High–energy scattering: scale anomaly and the “soft” Pomeron

Our second example is the scattering of quarkonia at high energies. In the framework of perturbation theory, a systematic approach to high energy scattering was developed by Balitsky, Fadin, Kuraev and Lipatov [11], who demonstrated that the “leading log” terms in the scattering amplitude of type \((g^2 \ln s)^n\) (where \(g\) is the strong coupling) can be re-summed, giving rise to the so-called “hard” Pomeron. Diagrammatically, BFKL equation describes the \(t\)-channel exchange of “gluonic ladder” (see Fig.2a) – a concept familiar from the old-fashioned multi-peripheral model [12].

The starting point of the approach proposed in [3] is the following: among the higher order, \(O(\alpha_s^3)\) \((\alpha_s = g^2/4\pi)\), corrections to the BFKL kernel we isolate a particular class of diagrams which include the propagation of two gluons in the scalar color singlet channel \(J^{PC} = 0^{++}\) (see Fig. 2-b). We show then that, as a consequence of scale anomaly, these, apparently \(O(\alpha_s^2)\), contributions become the dominant ones, \(O(\alpha_s^0)\).

One way of understanding the disappearance of the coupling constant in the spectral density of the \(g^2 G^2\) operator is to assume that the non-perturbative QCD vacuum is dominated by the semi-classical fluctuations of the gluon field. Since the strength of the classical gluon field is inversely proportional to the coupling, \(G \sim 1/g\), the quark zero modes, and the spectral density of their pionic excitations, appear independent of the coupling constant.

The explicit calculation using the methods of [13] yields the power-like behavior of the total cross section [3]:

\[
\sigma_{tot} = \sum_{n=0}^{\infty} \sigma_n = \sigma_{BORN} \Delta ,
\]

where \(\sigma_{BORN}\) is the cross section due to two gluon exchange, and the non-perturbative contribution to the intercept \(\Delta\) is

\[
\Delta = \frac{\pi^2}{2} \times \left(\frac{8\pi}{b}\right)^2 \times \frac{18}{32\pi^2} \int \frac{dM^2}{M^6} \left( \rho_{phys}^{qg}(M^2) - \rho_{QCD}^{qg}(M^2) \right) .
\]
Using the chiral formula (12) for $\rho_{\psi}^{phys}$ for $M^2 < M_0^2$, we obtain [3]

$$\Delta = \frac{1}{48} \ln \frac{M_0^2}{4m_\pi^2}. \quad (17)$$

The precise value of the matching scale $M_0^2$ as extracted from the low-energy theorems depends somewhat on detailed form of the spectral density, and can vary within the range of $M_0^2 = 4 \div 6$ GeV$^2$ [14], [2]. Fortunately, the dependence of Eq. (17) on $M_0$ is only logarithmic, and varying it in this range leads to

$$\Delta = 0.08 \div 0.1, \quad (18)$$

in surprising agreement with the phenomenological intercept of the “soft” Pomeron, $\Delta \simeq 0.08$.

At present, the language used in the description of hadron interactions at low and high energies is very different. Yet, as the two examples discussed above imply, both limits may appear to be determined by the same fundamental object – the QCD vacuum.
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