Josephson Coupling and Plasma Resonance in Vortex Crystal

L.N. Bulaevskii\textsuperscript{a}, A. E. Koshelev\textsuperscript{b},

\textsuperscript{a}Los Alamos National Laboratory, Los Alamos, NM 87545

\textsuperscript{b}Materials Science Division, Argonne National Laboratory, Argonne, IL 60439

To be published in the proceedings of the International Conference on Materials and Mechanisms of Superconductivity and High Temperature Superconductors VI, Physica C, Houston, Texas, February, 20-25, 2000,

This work was supported by the NSF Office of Science and Technology Centers, grant No. 91-20000, and by the U.S. Department of Energy, BES-Materials Sciences, Under Contract No. W-31-109-ENG-38. Work in Los Alamos was supported by the U.S. Department of Energy.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
Josephson coupling and plasma resonance in vortex crystal

A. E. Koshelev* and L. N. Bulaevskii**

*Materials Science Division, Argonne National Laboratory, Argonne, IL 60439
**Los Alamos National Laboratory, Los Alamos, NM 87545

We consider the magnetic field dependence of the plasma resonance frequency in vortex crystal state. We found that low magnetic field induces a small correction to the plasma frequency proportional to the field. The slope of this linear field dependence is directly related to the average distance between the pancake vortices in the neighboring layers, wandering length. This length is determined by both Josephson and magnetic couplings between layers. At higher fields the Josephson coupling is suppressed collectively and is determined by elastic energy of the vortex lattice. Analyzing experimental data, we found that (i) the wandering length becomes comparable with the London penetration depth near \( T_c \), (ii) at small melting fields \( (< 20 \, \text{G}) \) the wandering length does not change much at the melting transition demonstrating existence of the line liquid phase in this field range, and (iii) the self consistent theory of pancake fluctuations describes very well the field dependence of the Josephson plasma resonance frequency up to the melting point.

1. Introduction

Josephson coupling characterizes ability of layered superconductors to carry supercurrents across the layers. In very anisotropic superconductors this coupling is suppressed by magnetic field applied along the c-axis. Thermal fluctuations and uncorrelated pinning lead to misalignment of pancake vortices induced by the magnetic field [1] (see Fig. 1). Misalignment results in nonzero phase difference and in the suppression of Josephson interlayer coupling. This suppression is quantitatively characterized by the "local coherence factor" \( C \equiv \langle \cos \varphi_{n,n+1}(r) \rangle \), where \( \varphi_{n,n+1}(r) \) is the gauge-invariant phase difference between layers \( n \) and \( n+1 \), \( \langle \ldots \rangle \) means average over thermal disorder and pinning. Josephson plasma resonance (JPR) measurements in highly anisotropic layered superconductors [2] probe directly the interlayer Josephson coupling and the effect of pancake vortices on this coupling, because the squared JPR frequency, \( \omega_p^2 \), is almost always proportional to the average interlayer Josephson energy [3],

\[
\omega_p^2 \approx \omega_0^2 C \propto J_0 C, \tag{1}
\]

where \( \omega_0(T) = c/\sqrt{\lambda_c(T)} \) is the zero field plasma frequency, \( \lambda_c(T) \) is the c-component of the London penetration depth, \( \epsilon_0 \) is dielectric constant, and \( J_0 \) is the Josephson critical current.

![Figure 1. Meandering of pancakes along the vortex line in layered superconductors and schematic grayscale plot of \( \cos \phi_{n,n+1} \) near two misaligned pancakes.](image)

The JPR measurements performed in the liquid vortex phase at relatively high magnetic fields, \( B > B_J = \Phi_0/\lambda_3^2 \) revealed that the plasma frequency drops approximately as \( 1/\sqrt{B} \) [2]. Here \( \lambda_J = \gamma s \) is the Josephson length, \( \gamma \) is the anisotropy ratio and \( s \) is the interlayer distance. The above dependence is characteristic for the...
pancake liquid weakly correlated along the c axis [4]. In this phase many pancake vortices contribute to the suppression of the phase difference at a given point. In contrast, in the vortex solid pancake vortices form aligned stacks. JPR measurements in Bi$_2$Sr$_2$CaCu$_2$O$_{8-x}$ (Bi-2212) crystals [5,6] have shown that in the fields above 20 Oe the interlayer phase coherence changes drastically at the transition line, implying the decoupling nature of the first-order melting transition in agreement with theoretical expectations (see, e.g., Ref. [7]).

In this paper we consider Josephson coupling and JPR in the vortex lattice. In this state suppression of coupling is caused by thermal fluctuations of pancake vortices near the equilibrium crystal positions. This problem has been considered before using different approximations [1] However quantitative calculation suitable for comparison with experiment [5,6] has never been done. At small fields, when vortices act independently, $\omega_p^2$ decreases linear with $B$. The linear dependence was observed experimentally in Refs. [5,6] in solid state in Bi-2212 crystals. In fields below 20 Oe near $T_c$ this linear dependence extends to the liquid state providing evidence for a line structure of the vortices in the liquid at low fields. The regime of independent vortices has been considered in Ref. [8]. In this paper we extend our consideration to higher fields up to the melting field.

2. Low fields. Isolated vortex lines

Consider magnetic fields $B \ll \Phi_0/4\pi \lambda_{ab}^2 B_j$. At these fields regions of suppressed coupling are localized near the vortex lines (pancake stacks) and do not overlap (single vortex regime)[8]. The field-induced change in $C$ in this regime is given by $\delta C \equiv 1 - C = BI/\Phi_0$, where $I = \int d^2r \left(1 - \cos \left(\varphi_{n,n+1}(r)\right)\right)$, and $\varphi_{n,n+1}(r)$ is the phase difference induced by fluctuation displacements $u_n$ in a single line. The same integral determines the tilt stiffness due to the Josephson coupling and take

$$\varphi_{n,n+1}(r) = \arctan \frac{u_{n+1} - u_n}{x_{n+1} - x_n}.$$ 

At $r > R$ we can take $\varphi_{n,n+1}$ in linear with respect to $u_n$ approximation,

$$\varphi_{n,n+1}(r, k_z) \approx -\int \frac{dk_z}{2\pi} k_z\left[u(k_z) \times \nabla\right] K_0 \left(\frac{k_z r}{\gamma}\right),$$

where $u(k_z) = \sum_{n} \exp(-i k_z n) u_n$, $k_z \equiv (2/s) \sin(k_z/2)$, and $K_0(z)$ is the modified Bessel function. At intermediate distances $r_w \ll r \ll \lambda_J$ both expressions give the same simple result $\varphi_{n,n+1}(r) \approx \left[r \times u_{n-1,n+1}/r^2\right]$. Using above asymptotics of $\varphi_{n,n+1}$ we obtain

$$I = \frac{\pi}{2} \int \frac{dk_z}{2\pi} \left(1 - \cos(k_z)\right) \left|u(k_z)\right|^2 \times \ln \left(\frac{3.72 \lambda_J^3}{u_{n+1,n}^2 (1 - \cos(k_z))}\right).$$

Weak dependence on $u_{n,n+1}$ under the logarithm leads to the nonharmonic tilt energy [9] In the following we will treat this nonharmonicity using the self consistent harmonic approximation (SCHA), which results in the substitution $\ln(A/\omega_{p,(n+1)}^2) \to \ln(0.24 A/r_w^2)$. Approximate evaluation of the above integral gives a simple practical relation connecting the field-induced correction of the plasma frequency $\omega_p(B, T)$ with $r_w$ for the case $r_w < \lambda_J < \alpha$:

$$\frac{\omega_p^2(T) - \omega_p^2(B, T)}{\omega_0^2(T)} \approx \frac{\pi B \alpha}{2 \Phi_0} \ln \left(\frac{0.8 \lambda_J}{r_w}\right).$$

This relation allows one to extract $r_w^2$ from the plasma resonance measurements.

We now calculate $r_w^2$ when wandering of the vortex lines is caused by thermal fluctuations. In the single vortex regime $r_w^2$ is determined by the wandering energy consisting of the Josephson and magnetic contributions,

$$F_w \approx \frac{1}{2\pi} \int \frac{dk_z}{2\pi} \left[\varepsilon_1 k_z^2 + w_M\right] \left|u(k_z)\right|^2,$$

where $\varepsilon_1 \approx (\varepsilon_0/\gamma^2) \ln \left(1.33 \gamma/(r_w k_z)\right)$ is the line tension due to the Josephson coupling, $w_M \approx$
(\varepsilon_0/\lambda_{ab}^2) \ln(1.5\lambda_{ab}/r_w) is the effective cage potential, which appears due to nonlocal magnetic interactions between pancake vortices in different layers (it describes the magnetic tilt stiffness at wave vectors $k_s > 1/r_w$)[10,7], and $\varepsilon_0 \equiv \Phi_0^2/(4\pi \lambda_{ab}^2)$. Assuming Gaussian fluctuations we obtain

$$r_w^2 = \frac{8T}{\omega_{w0}^2} \frac{1}{1 + \zeta + \sqrt{1 + \zeta}}$$

where the parameter $\zeta(T) \approx 4\lambda_{ab}^2(T)/\lambda_J^2$ describes the relative roles of the Josephson and magnetic interactions. Substituting this result into Eq. (2) we obtain

$$\frac{\omega_0^2 - \omega_w^2}{\omega_0^2} \approx \frac{4\pi \lambda_{ab}^2 BT}{\varepsilon_0 \Phi_0} \frac{1}{1 + \zeta + \sqrt{1 + \zeta}}$$

This result of the single vortex regime is valid in both solid and liquid vortex states for $B < B_J$, because in this field range wandering of lines at short scales does not change much at the melting point. Eq. (5) describes fairly well suppression of plasma frequencies at small fields [8]. Using data of Ref. [5] for underdoped Bi-2212 with $T_c \approx 84.45$ K we estimate using Eqs. (2) $r_w \approx 1$µm at 77 K. This estimate is in good agreement with the theoretical calculation (4) and it is comparable with both $\lambda_{ab}$ and $\lambda_J$ at this temperature.

3. High fields, $B > \Phi_0/\lambda_J^2$, $\Phi_0/\lambda_{ab}^2$

Two competing effects start to influence pancake fluctuations and field dependence of average Josephson energy with increase of the field. First, vortex interactions diminish pancake fluctuations. On the other hand, collective suppression of the Josephson energy decreases tilt stiffness and enhances pancake fluctuations. Using general relations connecting phase perturbations with elastic lattice deformations (see, e.g. in Ref. [1]) we obtain

$$\delta C = \frac{(\varphi_{n.0}^2+n^2)}{2} \approx \frac{(2\pi s n_w)^2}{2}$$

expression (6) for $\delta C$ can be naturally split into the collective contribution $\delta C_{coll}$, corresponding to $Q = 0$ term in the $Q$-summation,

$$\delta C_{coll} \approx \frac{2\pi s n_w}{2} \int d^2qdk_z \left( \frac{k_z^2}{(2\pi)^3} \right) \left( \frac{k_z^2}{q^2} \right)$$

and the local contribution $\delta C_{loc}$ coming from $Q > 0$ terms. At high fields $B > \Phi_0/\lambda_J$ we obtain
approximate expression for $\delta C_{\text{loc}}$, which resembles the single-vortex result (2)

$$\delta C_{\text{loc}} \approx \frac{\pi n_0 a^2}{2} \ln \frac{0.58a}{r_w}.$$  \hspace{1cm} (9)

In general, relative role of the collective term in $\delta C$ grows with field. We use above expressions to calculate the field dependence of $C$ for comparison with JPR data.

Recently detailed measurements of field dependence of JPR frequency in the vortex crystal state of Bi-2212 have been done by M. Gaifullin et al. using frequency scan.[6] To compare our calculations with JPR data we need to know $\lambda_{ab}(T)$ and $\gamma = \lambda_c/\lambda_{ab}$. $\lambda_c$ is extracted directly from JPR frequency at $B = 0$, $\lambda_c(T) = c/\sqrt{\omega_0(T)}$ taking $c_0 = 11$ and $\gamma$ is chosen as a fitting parameter. Figure 2 shows comparison between computed dependence $C(B)$ and measured relative JPR frequency $(\omega_p(B)/\omega_p(0))^2$ for three values of temperature. We also show obtained values of $\lambda_{ab}$ and $\gamma$. We obtain $\gamma$, that slightly grows with temperature (from 460 at 40 K to 510 at 60 K). Such enhancement of the effective anisotropy factor is expected due to the phase fluctuations.

In conclusion, we have calculated the field dependence of the JPR frequency in the vortex crystal. In the single vortex regime at low magnetic fields the JPR provides a direct probe for meandering of individual lines. Theory of pancake fluctuations gives a very good description of the field dependence of the plasma frequency up to the melting field. The authors thank M. Gaifullin, Y. Matsuda, T. Tamegai, and T. Shibuchi for providing their experimental data prior publication.

REFERENCES

8. L. N. Bulaevskii, et al. preprint, cond-mat/9907462