Transition from hadronic to partonic interactions for a composite spin-1/2 model of a nucleon

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A simple model of a composite nucleon is developed in which a fermion and a boson, representing quark and diquark constituents of the nucleon, form a bound state owing to a contact interaction. Photon and pion couplings to the quark provide vertex functions for the photon and pion interactions with the composite nucleon. By a suitable choice of cutoff parameters of the model, realistic electromagnetic form factors are obtained. When a pseudoscalar pion-quark coupling is used, the pion-nucleon coupling is predominantly pseudoscalar. A virtual photopion amplitude is considered in which there are two types of contributions: hadronic contributions where the photon and pion interactions have an intervening propagator of the nucleon or its excited states, and contact-like contributions where the photon and pion interactions occur within a single vertex. At large Q, the contact-like contributions are dominant. The model nucleon exhibits scaling behavior in deep-inelastic scattering and the normalization of the parton distribution provides a rough normalization of the contact-like contributions. Calculations for the virtual photopion amplitude are performed using kinematics appropriate to the occurrence as a meson-exchange current in electron-deuteron scattering. The results show that the contact-like terms can dominate the meson-exchange current for Q > 1 GeV/c. There is a direct connection of the contact-like terms to the off-forward parton distributions of the model nucleon.

1. INTRODUCTION

At low energies and momentum transfers, nuclei are described in term of nucleons [1,2]. Interactions between the nucleons are modelled successfully by exchange of mesons [3-5], or more simply by potentials. When nuclei are probed at very high momentum transfer, e.g., in electron scattering, partons within the nucleons and mesons become the dominant scatterers [6,7]. Interactions between the partons are described by QCD. Between the high and low momentum transfer regimes, there is a transition region where a good description is lacking. The meson-exchange dynamics does not account in a satisfactory way for the composition of the nucleons and mesons. Therefore, it is of interest to study quark-based composite models of hadrons in order to get some insight on the limits of validity of a hadronic description. Electron scattering data for momentum transfer Q < 1 GeV/c often meet dual descriptions: models based on hadrons on one hand and models based on quark phenomenology on the other [8]. Moreover, the two kinds of description generally are not reconciled to one another in the sense that there is no smooth transition from one to the other as Q increases. Perturbative QCD descriptions are mainly qualitative and not properly normalized at low energy [9]. In the mesonic description, the mechanism of hard scattering from quarks that predominates in the perturbative QCD description is hidden or absent.

In this paper, we develop a simple model of a nucleon as a bound state of a fermion and a boson with the goal of gaining some insight into the transition region where, as Q increases, one passes from the dominance of hadronic processes to the dominance of scattering from the constituents of a nucleon. One may think of this model as having a quark and a spin-0 diquark bound together to make a nucleon and its excited states. The model is covariant and gauge invariant, but it lacks confinement. Excited states of the nucleon are a continuum of quark and diquark scattering states. Thus, it is mainly useful for processes where nucleon resonances do not play an important role. One such case is meson-exchange currents in nuclei.

An essential feature arises from composition: there are contact-like terms in second-order interactions. These are required by gauge invariance and they play a small but significant role at low energy, for example, in low-energy theorems [10-12]. For very large momentum transfer, the contact-like terms become dominant. They contain the leading-order mechanism for the external probe to scatter from the partons without any intervening hadronic state.
When a hadronic state exists between interactions, it produces form factors that fall rapidly with increasing Q, thus quenching the scattering. This is the fate of hadron-like terms in the second-order interactions, i.e., the terms that provide a hadronic interpretation at low momentum transfer.

In the limit that one of the interactions transfers a large momentum, the contact-like terms tend to the off-forward parton distributions for the composite nucleon model [12,14]. For the simple model that we consider, there is a clean separation of the hadron-like and contact-like contributions to second-order interactions. Interactions of the model nucleon with an electromagnetic probe have some realistic features. By introducing cutoff parameters, the nucleon’s charge and magnetic form factors can be described reasonably. At low momentum transfers, interactions of the model nucleon can be interpreted in terms of hadronic dynamics. For asymptotically large momentum transfer Q, at fixed r = Q^2/(2M^2), scaling obtains. We calculate the resulting parton distribution f(x).

In Sec. 2 we formulate the model in terms of a Lagrangian for a fermion and a boson interacting via a contact interaction. The model is not renormalizable: it is regulated by introducing subtraction terms of the Pauli-Villars type [15]. We consider only the simplest subset of contributions to the fermion-boson correlator. This produces a spin-1/2 propagator with a single bound state pole ("the nucleon") at mass M. Electromagnetic and pionic interactions are introduced in Sec. 3 as couplings to the fermion constituents ("the quark"). For simplicity, couplings to the boson ("the diquark") are omitted. For pseudoscalar coupling of the pion to the quark, the model produces mostly pseudovector coupling to the nucleon. It would be purely pseudovector if the mass of the quark were zero and there were no regulators of fermionic type.

In Sec. 4 we consider a virtual photopion amplitude involving second-order interactions with the composite nucleon. Two types of interaction occur: first, interactions with intervening propagation of a nucleon or its excited states and second, contact-like contributions where photon and pion interactions with the nucleon occurs within the same vertex. A standard analysis based upon elementary particles with form factors is compared with the composite nucleon analysis. In Sec. 5 we consider deep inelastic scattering from the nucleon for finite Q and as Q → ∞. In Sec. 6 we present calculations of the virtual photon production amplitude for a kinematical situation that arises in non-conservation contributions to electron-antielectron scattering. Calculations show that contact-like terms can become dominant for Q ≈ 1 GeV/c for some processes. Conclusions are presented in Sec. 7. A more complete description of the details of the calculations is given in four appendices.

II. COMPOSITE NUCLEON MODEL

A fermion and a boson interacting via a contact interaction can generate a composite spin-1/2 particle. For this purpose, the following Lagrangian is used [11].

\[ L = \bar{\psi}(x)(i\gamma^\mu - m)\psi(x) + \frac{1}{2\alpha}(\partial_\mu \phi(x)\partial^\mu \phi(x) - m^2\phi^2(x)) + g\phi(x)\bar{\psi}(x)\gamma^\mu\psi(x). \]  

(1)

where \( \psi(x) \) is the field for a fermion of mass m and \( \phi(x) \) is the field for a boson of mass \( \mu \). The fermion-boson interaction with coupling constant \( g \) is not renormalizable; finite results are obtained by introducing a Pauli-Villars regulator of mass \( \Lambda_1 \).

A bound state appears as a pole in the fermion-boson correlator,

\[ G(p) = i \int \frac{d^4q}{(2\pi)^4}\langle 0 | \left( \psi(\vec{x})\gamma^\mu(\phi(\vec{q})\phi(\vec{0})) \right) | 0 \rangle. \]  

(2)

Figure 1 shows the sequence of elementary bubble graphs that contribute to \( G(p) \) in a perturbative expansion. Because this sequence is sufficient to exhibit a bound state, contributions beyond those shown in Fig. 1 are not considered.

Summing the bubble graphs of Fig. 1 produces

\[ G(p) = \left( \frac{1}{1 - \Sigma(p)} \right). \]  

(3)

Here, \( \Sigma(p) \) is the contribution of a single fermion-boson loop,

\[ \Sigma(p, m, \mu, \Lambda_1) = ig \int \frac{d^4k}{(2\pi)^4} S(\vec{p} - \vec{k}, m) D(k, \mu, \Lambda_1). \]  

(4)

where the propagator for the fermion is \( S(p, m) = (\vec{p}^2 - m^2)^{-1} \). With a Pauli-Villars regulator of mass \( \Lambda_1 \), included, the propagator for the boson line is

\[ D(k, \mu, \Lambda_1) = \left( \frac{1}{k^2 - \mu^2 + i\eta} \right)^{-1} \left( \frac{1}{k^2 - \Lambda_1^2 + i\eta} \right)^{-1}. \]  

(5)

A generalization of the model that is suitable for describing a nucleon’s form factor is obtained by including additional regulator terms as follows.

\[ \Sigma(p) = ig \int \frac{d^4k}{(2\pi)^4}\left[ S(\vec{p} - \vec{k}, m) - aS(\vec{p} - \vec{k}, \mu) - (1 - \beta)D(k, \mu, \Lambda_1) \right] \times [D(k, \mu, \Lambda_1) + \beta D(k, \mu, \Lambda_2)]. \]  

(6)

where

\[ a = \frac{m^2 - m_0^2}{m_0^2 - m}, \]  

(7)

and

\[ \beta = \frac{\Lambda_1^2 - \Lambda_2^2}{\Lambda_1^2 - \Lambda}. \]  

(8)

The constants \( a \) and \( \beta \) are selected so that high loop momentum is cut off as \( k^2 \to 0 \). It is evident that the generalized form for \( \Sigma(p) \) is equal to a linear combination of the elementary bubble graph terms, \( \Sigma_a \), defined above.

\[ \Sigma(p) = \Sigma_a(p, m, \mu, \Lambda_1) + \beta \Sigma_b(p, m, \mu, \Lambda_2) - \alpha [\Sigma_a(p, m, \mu, \Lambda_1) + \beta \Sigma_b(p, m, \mu, \Lambda_2)] - (1 - \alpha) [\Sigma_a(p, m, \mu, \Lambda_1) + \beta \Sigma_b(p, m, \mu, \Lambda_2)]. \]  

(9)

When a bound state of mass \( M \) is present, the pole in the composite system propagator \( G(p) \) has the form

\[ G(p) = \frac{Z_0}{\vec{p} - M + i\eta}, \]  

(10)

where \( Z_0 \) is a wave-function renormalization factor. A renormalized propagator \( \tilde{G}(p) \) is obtained by dividing \( G(p) \) by \( Z_0 \) such that there is unit residue for the nucleon pole. The remainder \( R(p) \) is regular at \( \vec{p} = M \) and it represents excited state contributions. In the model considered, the excited state spectrum is a continuum of quark-diquark states. This is an unrealistic feature for a nucleon so the model should be used where the effects of resonances are not important.

The most general form for \( \Sigma(p) \) that is allowed by Lorentz invariance is

\[ \Sigma(p) = A(p^2) \rho + B(p^2). \]  

(11)

Presence of the bound state pole means that \( \Sigma(p) = 1 \) at \( \vec{p} = M \). This condition leads to

\[ Z_0^{-1} = \left( \frac{d^4p}{2\pi^4} \right) \rho \left( \frac{d^4p}{2\pi^4} \right) \rho + \left( \frac{d^4p}{2\pi^4} \right) B(\rho + \rho + \rho + \rho) \]  

(12)

where \( A_0 = A(M^2), A_0^* = dA(p^4)/dp^4|_{p=m}, \) and similarly for \( B_0^* \).

For later use, we introduce covariant projection operators,

\[ \frac{L^p(p)}{W_\rho + \rho/2W_\rho}. \]  

(13)

where \( \rho = + or - \), \( W_\rho = \sqrt{\rho^2} \) and \( L^p(p) + L^-(p) = 1 \). Projecting the propagator to the \( \rho = + \) and \( - \) subspaces in which \( \rho \) takes the values \( \pm W_\rho \), leads for the renormalized propagator, to

\[ \tilde{G}(p) = G^+(p) + G^-(p) \]  

(14)

where

\[ G(p) = \frac{Z_0^*}{\rho L(p)} - \rho \left( \frac{d^4p}{2\pi^4} \right) A\rho(p^4). \]  

(15)

To summarize this section, the composite model of a nucleon is formulated in a covariant way as a bound state of a spin-1/2 quark and a spin-0 diquark. Details of the calculation of \( A(p^2), B(p^2) \) and \( Z_0 \) are given in Appendix A.
III. PHOTON AND PION INTERACTIONS

Electromagnetic interactions are introduced via a fermion-photon coupling term in the lagrangian: \( \mathcal{L}_\gamma = \frac{e}{\sqrt{2}} \gamma^\mu A_\mu(x) \bar{\psi}(x) \), where \( e = \frac{\alpha}{\pi} + 1 \) is the charge operator for the quark. Photons couple to the baryon in order to keep the model simple. Consequently, the model proton is composed of a quark of charge \( e \) and a neutral dipole. The model neutrons are composed of a neutral quark and dipole and therefore have no electromagnetic interactions.

Inserting a photon into the propagator as indicated in Figure 2 produces the form
\[
\mathcal{G}(p) \equiv \mathcal{G}_\gamma(p, p, G(p)) \tag{16}
\]
where \( \mathcal{A}_\gamma \) describes the photon-nucleon vertex. One extracts the photon-nucleon (dressed) interaction as the residue of the two poles at \( p = M \) and \( p = M \), which leads to
\[
u(p, M) \equiv \mathcal{A}_\gamma(p, p, G(p)) \tag{17}
\]
where \( \nu(p, M) \) is the Dirac spinor for mass \( M \) and momentum \( p \) on the mass shell. The \( Z_2 \) factor and Dirac spinor factors arise from the parts of the initial- and final-state propagators that attach to the vertex \( \mathcal{A}_\gamma \). It is convenient to absorb the \( Z_2 \) factor into \( \mathcal{A}_\gamma \) to obtain a renormalized vertex \( \mathcal{A}_\gamma \). For momentum \( p \) and \( p \) that are either on-shell or off-shell, the renormalized vertex involves a fermion-boson loop insertion in the fermion propagator as follows,
\[
\mathcal{A}_\gamma(p, p, G(p)) = \mu_2 Z_2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{\nu}(p - k, m)\gamma_\mu \nu(p - k, m)D(k, \mu, A)}{k^2 - m^2 + i\epsilon} \tag{18}
\]
In the generalized model with additional Pauli-Villars regulators, the vertex is defined up to a sign, which is determined in Eq. (9). In each \( \Sigma_\gamma \), the fermion propagator \( \Sigma(k - m, m) \) for mass \( m \) is replaced by \( \Sigma(p - k, m)\gamma_\mu \nu(p - k, m) \). Gauge invariance requires that the vertex satisfy the following Ward-Takahashi identity [16,17],
\[
(\gamma_\mu - pk)\mathcal{A}_\gamma(p, p, p) = G^{-1}(p) - G^{-1}(p) = \Sigma(p) - \Sigma(p) \tag{19}
\]
This is satisfied when the photons couple to all fermion propagators in the same way. The 

In general, the vertex function can be decomposed in terms of charge and magnetic form factors \( F_2 \) and \( F_3 \). For the off-shell case, there is an additional form factor \( F_3 \). Moreover, all form factors depend upon \( \theta \) and \( \phi \). In order to have scalar form factors, it is not necessary to project with the operators \( L^+ \) and \( L^- \) to commute the \( \theta \) and \( \phi \) toward the propagators so that they may be replaced by \( \rho W_\nu \) or \( \rho ij \nu W_\nu \). This analysis is carried out in Appendix B. It produces
\[
\mathcal{A}_\gamma(p, p, p) = \sum_{j=1,2,3} \Delta_j \mathcal{A}_j(p, p, p) \tag{20}
\]
where
\[
\Delta_j = F_j^{(p)}(p, p, p) + i\mu_2 \phi F_j^{(p)}(p, p, p) \tag{21}
\]
and \( q = \theta, \phi \). Each scalar form factor is a different function depending on the values of \( \theta \) and \( \phi \), e.g., \( F_3 \) is different from \( F_3 \). We shall return to this point shortly.

For the off-shell situation, owing to time-reversal invariance, one only has \( F_3^{(p)}(p, q) \) and \( F_3^{(p)}(q, p) \), which are the usual charge and magnetic form factors of the proton. With three fermion masses and three boxon masses as parameters, the generalized model allows a reasonable fit to the proton's electromagnetic form factors. Figure 3 shows \( F_1^{(p)}(q^2) \) and \( F_2^{(p)}(q^2) \) in comparison with the dipole form \( F_{\text{dipole}} \) and \( F_{\text{dipole}} \) for each is used to characterize experimental form factors. The parameter values used are: \( m = 38, m_1 = 36, m_2 = 0, \mu = 72, \alpha_1 = 85, \alpha_2 = 50 \), all in GeV. The bound state is at \( M = 93826 \) GeV. The anomalous magnetic moment of the composite nucleus is \( k = 2M F_2^{(0)}(0) / 2.086 \), which may be compared with the standard model \( 1.79 \) GeV.

A parallel analysis may be made for couplings of an elementary pion to the quark by adding a pseudoscalar \( \tau \)-quark interaction \( \mathcal{L}_\pi = \phi^a \pi^a(x) \gamma_\mu \bar{\psi}(x) \) to the lagrangian. Figure 2 shows one pion insertion into the propagator. This produces
\[
G_\pi(p) = \gamma^0 \mathcal{A}_\pi(p, p, G_\pi(p)) \tag{22}
\]
where \( \mathcal{A}_\pi \) and \( \mathcal{A}_\pi \) are the pion-nucleon form factors. A renormalized form-factor function is calculated from a fermion-boson loop graph with a pseudoscalar insertion on the meson, as follows,
\[
\lambda_\pi(p, p, p) = i\mu_2 Z_2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{\nu}(p - k, m)\gamma_\mu \nu(p - k, m)D(k, \mu, A)}{k^2 - m^2 + i\epsilon} \tag{23}
\]
In the generalized model, the pion-nucleon form factor is in a sum of terms, one for each term in Eq. (9). In each \( \Sigma_\pi \), the fermion propagator \( \Sigma(p - k, m) \) for mass \( m \) is replaced by \( \Sigma(p - k, m)\gamma_\mu \nu(p - k, m) \).

Appendix B details
\[
\lambda_\pi(p, p, p) = \sum_{j=1,2,3} L_j(p, p, p)\gamma_\mu \nu(p, p, p) \tag{24}
\]
Figure 4 shows the resulting \( nN \) form factor \( F_2^{(p)}(q^2) \) for on-mass-shell nucleon momenta. It is quite similar to the magnetic form factor.

When one leg of the vertex function is off the mass shell, the form factors differ from the on-shell results. We wish to relate the off-shell effects to those appropriate to a hadronic vertex that is sandwiched between elementary Dirac vector and scalar factors. For this purpose, it is necessary to incorporate off-shell effects from the propagators into the off-shell.

In general, one encounters an off-shell vertex function sandwiched between propagators, as follows,
\[
\mathcal{G}(p) = \mathcal{G}(p, \mathcal{A}(p, \mathcal{G}(p)) \tag{25}
\]

The renormalized propagator of the composite system may be written as
\[
\mathcal{G}(p) = \frac{Z_2(p)}{Z_2(p) + Z_2(p)} \mathcal{G}(p) \tag{26}
\]
where \( Z_2(p) = (1 - W_\nu - W_\nu - W_\nu) \) and \( Z_2(p) \) are scalar functions. In the limit that \( W_\nu \to -M, Z_2 \to Z_2 \), and in the limit that \( W_\nu \to -M, Z_2 \to Z_2 \). For a point particle the factors \( Z_2(p) \) are unity, e.g., an elementary off-shell effects due to the propagator. A factor \( Z_2(p) \) is redistributed to the remaining \( Z_2(p) \) in the propagator in the propagators should be distributed to vertices functions preceding or following

Figure 4 shows the variation with off-shell momentum \( p \) for the \( F_3^{(p)}(M, q^2, p^2) \) form factor, with \( p^2 \) being the off-shell momentum. In each case, \( Z_2^{(p)} \) is included for the off-shell leg. Similar results are obtained for the \( F_2^{(p)} \) and \( F_3^{(p)} \) form factors. Roughly, when \( p^2 \) varies from 0 to 1 M, the form factor varies from 0.8 to 1.4 times the full momentum.

For couplings between \( g \) and \( p \), the form factors generally are off-shell because the momentum \( p \) of the off-mass state differs from \( W_\nu = -M \), where \( W_\nu \to \sqrt{p^2} \). Typically, up to couplings are evaluated near \( W_\nu = M \), and thus they should include a factor \( Z_2^{(p)} \) from the negative-energy propagator in order to be compared with experimental couplings.

The interaction between the \( F_2^{(p)} \) and \( F_2^{(p)} \) is different in each case. It is shown in Figures 5 and 6. In each case, the \( F_2^{(p)} \) form factor is shown as the ratio to the on-shell \( F_2^{(p)} \) form factor, and a factor \( Z_2^{(p)} \) is included. Although the pure pseudoscalar operator \( \gamma_\mu \) appears in each \( F_2^{(p)} \), the \( F_2^{(p)} \) form factor is different in each case. It is shown in Figures 5 and 6. The \( F_2^{(p)} \) form factor is shown as the ratio to the on-shell \( F_2^{(p)} \), and a factor \( Z_2^{(p)} \) is included. It is instructive to compare with an elementary vertex that contains a fraction of a pseudoscalar and a fraction of a pseudoscalar couplings as follows,
\[
M_{\gamma\mu}(p, p, p) = \lambda_{\gamma\mu} \gamma_\mu \gamma_\nu \tag{27}
\]
Expanding by use of the projection operators and specializing in on-mass-shell kinematics yields

$$A_{k}^{\text{ Born}}(p_{1}, p_{2}) = \sum_{f_{1}, f_{2}, r_{1}, r_{2}} L_{f_{1}}^{\text{Born}}(p_{1}) L_{f_{2}}^{\text{Born}}(p_{2}) \left( \frac{\sigma(f_{1} + p_{1})}{3} + \lambda \lambda_{i} + \frac{\sigma(f_{2} + p_{2})}{3} \right) L_{f_{1}}^{\text{Born}}(p_{2}).$$  \(28\)

On mass shell, the \(++\) vertex is \(\eta\) independent of the mixing parameter \(\lambda\). The \(+-\) vertex is proportional to \((1 - \lambda)\) and thus is suppressed for pseudovector coupling. A measure of the fraction of pseudovector coupling shows up in the ratio of \(+ +\) and \(+ -\) form factors. For the composite nucleon model, we define an equivalent pseudovector fraction in order to give a simple interpretation of the different \(+ +\) and \(+ -\) couplings as follows,

$$\lambda = 1 - \frac{F_{0}^{\text{Born}}(p_{1}, p_{2})^{2} + F_{1}^{\text{Born}}(p_{1}, p_{2})^{2}}{2F_{2}^{\text{Born}}(p_{1}, p_{2})^{2}} \right|_{W_{1} = W_{2} = M}. \tag{29}$$

Figure 8 shows this ratio for the model nucleons. In the low Q range, the composite model produces 75% pseudovector coupling of the pion starting from a pseudoscalar coupling to the quark. (If the factors \(\sqrt{2F_{i}^{\text{Born}}(p_{1}, p_{2})^{2}}\) were omitted, it would be 94% pseudovector.) At \(Q \approx 1 \text{ GeV/c}\), the vertex becomes closer to pseudoscalar.

To summarize this section, the model nucleon has realistic charge and magnetic form factors. The pion form factor is similar to the magnetic one and the \(S\) vertex is about 25% pseudovector and 75% pseudoscalar. Couplings between \(+ +\) and \(+ -\) states differ, which is a general feature of off-shell vertices.

**IV. SECOND-ORDER INTERACTIONS - THE VIRTUAL PHOTOPION AMPLITUDE**

Using the couplings discussed in the previous section, we consider a virtual photopion production process. This involves inserting a photon and a pion in all possible ways into the propagator and extracting the scattering amplitude as the residue of the poles in \(G(p_{1})\) and \(G(p_{2})\) as before. We also consider a standard hadronic treatment of the same process for comparison.

A. Composite nucleon analysis

For the process in which a nucleon with initial momentum \(p_{i}\) absorbs a photon of momentum \(q\) propagates with momentum \(p_{i} + q\) and subsequently emits a pion of momentum \(r\), ending up with momentum \(p_{2}\), where \(p_{2} + q = p_{1} + r\), the resulting amplitude is shown in Fig. 9 and is given by (omitting isospin factors)

$$V_{\text{s}}(p_{1}, p_{2}, q, p_{i}, p_{r}) = \sum_{f_{1}} \hat{u}(p_{2}) 2f^{r}_{f_{1} q} f_{q_{1} p_{1}}^{a} (p_{1}, p_{2} + q + r) 2f_{q_{1} p_{1}}^{d_{f_{1} q}} (p_{1} + q, p_{r}, u(p_{1})). \tag{30}$$

For the crossed process in which the nucleon first emits a pion of momentum \(r\) and subsequently absorbs a photon of momentum \(q\), ending up with the same momentum \(p_{1}\), the amplitude is (omitting isospin factors)

$$V_{\text{s}, q}(p_{1}, p_{2}, q, p_{i}, p_{r}) = \sum_{f_{1}} \hat{u}(p_{2}) 2f^{r}_{f_{1} q} f_{q_{1} p_{1}}^{a} (p_{1}, p_{2} + q + r) 2f_{q_{1} p_{1}}^{d_{f_{1} q}} (p_{1} + q, p_{r}, u(p_{1})). \tag{31}$$

These contributions to the photopion amplitude will be referred to as “Born” terms.

In the analysis, two factors of \(Z_{2}\) arise, one from the external wave functions and another from the pole term of the propagator. These factors are absorbed into the two vertex functions so that all quantities appearing in Eq. (30) and (31) are renormalized. Renormalized photon vertex function \(\tilde{F}_{a}^{\gamma\pi}\) is defined in Eq. (21) in terms of form factors \(F_{a}^{\gamma\pi}\), \(F_{a}^{\gamma\pi}\), and \(F_{a}^{\gamma\pi}\). Propagation has been split into separate factors for \(p = \pm\) and \(\rho = \pm\) states using covariant projection operators. Note that \(G^{+\pm}\) contains the nucleon pole term and the excited states, which in this case are quark and diquark scattering states. Similarly, \(G^{-}\) is the negative-energy propagation that occurs in Z-graphs. However, the standard Z-graph is based on noncovariant projection of the propagator and this causes some differences when nucleon momenta are not close to the mass shell. All of these elements arise also in a hadronic description.

The variation of \(V_{\text{s}, q}\) with momentum transfer is characterized roughly by \(F(q^{2})\) \(G^{\pm}(p + q) F(r^{2})\), where \(F(q^{2})\) is a typical form factor and \(G^{\pm}\) is the positive-energy propagator. At large \(q^{2}\) and \(r^{2}\), these contributions become small owing to the form factors involved. A similar estimate holds for \(V_{\text{s}}\). Excited states of the nucleon do little to alter

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**References:**

16, 17

Note the order of isospin factors is important as they do not commute. Gauge invariance implies conservation of the EM current, viz., \(q^{\mu} A_{\mu} = 0\) when the pion is on mass shell, i.e., \(r^{2} = m_{\pi}^{2}\). When the pion is off mass shell, there is in general a nonzero result proportional to \(G_{a}^{\pm}(r) = m^{2} - r^{2}\). The required form is realized in the photo-pion amplitude \(A_{\gamma}\) because of the following Ward-Takahashi identities [16,17].

$$q^{\mu} V_{\text{s}, q} = u(p_{2}) g_{\mu}^{\pi a} \tilde{A}_{a}(p_{1}, p_{2}, q) u(p_{1}) \tag{37}$$

$$q^{\mu} V_{\text{s}} = -\bar{u}(p_{2}) g_{\mu}^{\pi a} \tilde{A}_{a}(p_{1}, p_{2}, q) u(p_{1}) \tag{38}$$

$$q^{\mu} C_{\text{s}, q} = \bar{u}(p_{2}) g_{\mu}^{\pi a} \tilde{B}_{a}(p_{1}, p_{2}, q) u(p_{1}) \tag{39}$$

$$q^{\mu} C_{\text{s}} = \bar{u}(p_{2}) g_{\mu}^{\pi a} \tilde{B}_{a}(p_{1}, p_{2}, q) u(p_{1}) \tag{40}$$

These identities may be derived by use of Eqs. (19), (32) and (33). In the contact-like terms, one needs to use the elementary Ward-Takahashi identity for Dirac propagators.
\[ q^p S(p + \mathbf{q} \cdot m)_{\lambda} S(p \cdot m) = S(p \cdot m) - S(p + \mathbf{q} \cdot m) \]  
(41)

The pion-in-flight term is rewritten in terms of a commutator involving the nucleon's charge operator, \( \hat{\mathcal{F}} \), using the isospin identity \( T^3 \hat{\mathcal{F}} = [\hat{\mathcal{F}}, \hat{T}^3] \). Its contribution to the divergence of the amplitude then is

\[ q^p \hat{A}_{p}(p, r) = \left[ \hat{\mathcal{F}}(w\tilde{s}) \hat{P}_p, \hat{S}_p \right] \hat{A}_p(w\tilde{s}) \eta(p, \mathbf{r} \cdot q) \left[ G^p_r(\mathbf{r}) - G^p_r(\mathbf{r} - q) \right] \]

(42)

Contributions to \( q^p \hat{A}_p \) from the Born terms are cancelled exactly by the contributions from the contact-like terms that have the same isospin factors, and the remaining contributions from the contact terms are cancelled by the second term in the pion-in-flight contribution. This leaves only a term proportional to \( G^p_r(\mathbf{r}) \) that vanishes for on-shell pions. The full amplitude in gauge invariant and the presence of the contact-like terms is essential for this result.

The distinguishing feature of the contact-like terms is that no propagator for the composite system occurs between interactions. Thus, there is not a separate form factor for each interaction. However, the contact-like terms do depend upon the momentum transfer. They differ from a form factor mainly by the presence of an extra fermion propagator in the loop integrals of Eqs. (32) and (33). If the extra propagator lines were shrunk to a point, the contact-like terms would be related to form factors at momentum transfer \( q^p \). This suggests that the contact terms should behave like \( \text{Im}[\Gamma(p^2)(\mathbf{r}) \eta(p)] \), where \( \eta(p) \) accounts for the extra propagator. Our calculations show that \( \eta(p) \) is given roughly by \( \eta(p) \approx \frac{\text{Im} \Gamma(p^2)}{\text{Re} \Gamma(p^2)} \), where \( \mathbf{r} \) is a typical fermion mass. Comparing with \( V_{\lambda\sigma} \) and \( V_{\lambda\nu} \), the contact-like terms fall more slowly with increasing momentum transfer and ultimately they dominate the scattering.

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**D. Elementary particle with form factors analysis**

A standard treatment of meson-exchange currents in nuclear physics is to construct graphs corresponding to elementary particles and then to insert form factors at the vertices [20,21]. The form factors are obtained from on-shell matrix elements, e.g., from phenomenological fits to electron scattering data for a free proton target.

Treating the composite nucleon in this way, there are Born contributions \( V_{\lambda\nu} \) and \( V_{\lambda\sigma} \), which are evaluated using the \( F^p \) form factors, and the pion-in-flight term. We consider both pseudoscalar and pseudoscalar pion-nucleon coupling in the elementary particle amplitude, and there is an additional contact term \( \hat{A}_{p}(E_{\text{EM}}(p^2)) \) in the pseudovector case that results from gauge coupling to the pion field.

The elementary amplitude with pseudovector pion coupling is defined as

\[ \hat{A}_{p}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) = iF^{p}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) + \hat{\mathcal{F}} \hat{A}_{p}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) + \hat{A}_{p}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) + \hat{A}_{p}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}), \]

where

\[ V_{\lambda\nu}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) \equiv \left( \hat{\mathcal{F}} \left( \frac{1}{2M} \right) \hat{P}_p \hat{P}_p \right) \frac{\mathbf{r} \cdot q}{2M} \times \left[ \hat{F}^p_r(q) \right] \eta(p). \]

Similarly, the crossed contribution is

\[ V_{\lambda\nu}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) \equiv \hat{\mathcal{F}} \left( \frac{1}{2M} \right) \hat{P}_p \hat{P}_p \right) \frac{\mathbf{r} \cdot q}{2M} \times \left[ \hat{F}^p_r(q) \right] \eta(p). \]

In Eq. (44), a pseudovector vertex factor \( \left( \hat{\mathcal{F}} \left( \frac{1}{2M} \right) \hat{P}_p \hat{P}_p \right) \) has been evaluated by use of \( \hat{\mathcal{F}}(w\tilde{s}) \hat{P}_p \hat{P}_p = \hat{\mathcal{F}}(w\tilde{s}) \eta(p) \). Similarly, in Eq. (45), a pseudoscalar vertex factor \( \left( \hat{\mathcal{F}} \left( \frac{1}{2M} \right) \hat{P}_p \hat{P}_p \right) \) has been replaced by \( \eta(p) \) by use of the Dirac equation. When pseudoscalar pion coupling is used, these factors are omitted.

Note that the transition matrix elements to an intermediate negative-energy state in Eqs. (44) and (45) are based upon the same form factor as for the on-shell transition to an intermediate positive-energy state in the elementary amplitudes. However, when the pion vertex is pseudovector there is reduced coupling to the negative-energy states. In the corresponding Born amplitudes of Eqs. (30) and (31), transitions are based upon off-shell vertex functions that differ in general for the two transitions.

Hadronic contact terms are implied by the off-shell factors \( \left( \hat{\mathcal{F}} \left( \frac{1}{2M} \right) \hat{P}_p \hat{P}_p \right) \) in Eqs. (44) and (45). For example, in the first factor may be rewritten as \( \left( \hat{\mathcal{F}} \left( \frac{1}{2M} \right) \hat{P}_p \hat{P}_p \right) \) where the summand of the second part cancels the pseudovector. A corresponding rearrangement applies to \( \hat{A}_{p}^{\text{EM}}(p, \mathbf{r} \cdot q, \mathbf{p}) \). As a consequence, the pseudovector nucleon propagator between the photograph absorption and pion emission. However, the form factors at the pion and to such terms as hadronic contact terms. They are distinct.

The resulting hadronic contact terms for pseudovector pion coupling have parts in which the \( F^p \) term factor appears at the electromagnetic vertex. These contact terms exactly cancel with the \( \text{EM}(p^2) \) term. This leaves only interaction between neither the contact terms from the off-shell vertices nor the one from changing the derivative amplitudes. After the cancellations in the pseudovector elementary amplitude, the only surviving differences from the on-shell form factors and the conservation of four-momentum. Except when \( q \) happens to be equal to the difference between the use of two on-shell momenta, one cannot have an on-shell vertex. In the elementary Born amplitudes of Eqs. (44) and (45) on \( p^2 \) and \( p^2 \) are therefore evaluated with on shell initial and final momenta whose difference is the momentum transfer, i.e., \( p_\mu = (E_0, 0, 0, -Q^2) \) and \( p_\nu = (E_0, 0, 0, -Q^2) \), where \( E_0 \) is the free nucleon's four-momentum, even though these are not momenta, is often used when the on-shell dependence of the vertex function is unknown, but it does not have a sound theoretical basis for a composite particle.

A standard nonrelativistic analysis would be similar to the elementary analysis described above. Relativistic couplings would be used. Calculations based on such a definition of a nonrelativistic amplitude will be discussed in Sec. 6.

To summarize this section, the composite nucleon model has features which are similar to those of a hadronic string, which also has, at least in principle, vertex functions involving \( \mu \), and that are different functions for the different vertices in their production. The main feature that distinguishes the composite particle analysis from an elementary particle distribution.

V. DEEP INELASTIC SCATTERING

In order to obtain a rough normalization of the contact-like terms, we consider deep inelastic scattering from the composite nucleon. It is characterized by a hadronic tensor that takes a gauge-invariant form as follows [22,23].

\[ W^{\mu\nu}(x, \mathbf{Q}^2) = - \left( \frac{\mathcal{K}^{\mu\nu} - \mathcal{K}^{\mu\nu}}{q^2} \right) W^0 + \left( \frac{\mathcal{K}^{\mu\nu} - \mathcal{K}^{\mu\nu}}{q^2} \right) W^1. \]

(46)

Structure functions \( W_1^0 \) and \( W_2^0 \) depend on two scalar invariants: \( Q^2 = -q^2 \) and \( \nu = -p \cdot q/M \). In the limit \( Q^2 \to \infty \), with \( x \ll Q^2/(2M^2) \) held fixed, these functions become dependent only on \( x \) as follows [24].

\[ M W_1^0(x, Q^2) = \frac{1}{2} f(x), \]

(47)

and

\[ \nu W_2^0(x, Q^2) = \frac{1}{2} f(x + 1). \]

(48)

This scaling behavior in a consequence of scattering from point-like constituents of the nucleon, with \( q \to \) being the probability of scattering from a parton that carries a fraction \( x \) of the nucleon's momentum.

\[ \frac{1}{2} (0, 0, 0, 0, 0, 0) \text{ or } x \in \text{ the c.m. frame of the initial state}, \]

\[ x = (0, 0, 0, 0, 0, 0) \text{ or } x \in \text{ the c.m. frame of the final state}, \]

\[ 2M W_1^0(x, Q^2) = \frac{1}{2} f(x), \]

(49)
and in the asymptotic limit

\[ f(x) = \lim_{Q^2 \to \infty} 2M W_V(x, Q^2). \]

Batiz and Gross [25] have analyzed the scaling limit for a composite nucleon model that is essentially similar to the one used in this paper. Their analysis is for one space and one time dimension. They show that scaling in a general gauge involves a cancellation between a gauge-dependent part of the impulse approximation graph of Fig. 10 and a gauge-dependent part of the final-state interaction. This is related to the Ward identities of Eqs. (38) and (49) which imply that gauge invariance requires cutting both the Born and contact-like terms. Using the Landau prescription, Batiz and Gross split the impulse amplitude into a gauge-invariant part and a remainder. The gauge-invariant part of the impulse graph provides the scaling result. The gauge variant remainder cancels with part of the final-state interaction such that the resultant contribution vanishes at least as fast as 1/Q^2. In this section, we follow Batiz and Gross by using the Landau prescription for three space dimensions and one time dimension. The results differ because of integrals over the angles of final state particles and because the phase space in 3D differs from that in 1D.

The hadronic tensor based on the impulse graph of Fig. 10 is calculated in the c.m. frame of the final state,

\[ W^{\mu\nu} = \frac{1}{2} \sum_{\alpha} \left[ \frac{1}{4 \pi W^4} \int d\Phi_T T^{\alpha\tau} T^\tau_{\mu\nu} \right] \]

(51)

where W is the total energy of the final state that contains an on-shell quark of momentum p_1 = (E_1, p_1) and an on-shell boson of momentum (W - E_1, -p_1). Amplitude T^\alpha\tau describes the impulse approximation graph for scattering from the fermion constituent. Using the Landau prescription as in Ref. [25] to obtain a gauge-invariant current, this is

\[ T^\alpha\tau = \sqrt{2g_2^2M} a_1(p_1, p_2) \left[ \frac{2p_1 \cdot q}{p^2} - \frac{2p_1 \cdot q}{p^2} - \frac{q^2}{\mu^2} \right] u(p, M). \]

(52)

where \( a_1(p_1, p_2) \) is a Dirac spinor for the quark of mass m and \( u(p, M) \) is a Dirac spinor for the nucleon of mass M. The factor \( \sqrt{2g_2^2M} \) is the vertex function for the nucleon to fragment into a quark and a boson.

An equivalent form of the hadronic tensor, which we use, is

\[ W^{\mu\nu} = \frac{1}{4 \pi W^4} \int d\Phi_T \frac{1}{(p_1 - q)^2 - m^2 + i\epsilon} \]

(53)

where we define

\[ M^{\mu\nu} = \frac{1}{2} T \left( \left( \frac{2p_1 \cdot q}{p^2} - \frac{2p_1 \cdot q}{p^2} - \frac{q^2}{\mu^2} \right) \left(p + M\right) \right) \times \left( \frac{2p_1 \cdot q}{p^2} - \frac{2p_1 \cdot q}{p^2} - \frac{q^2}{\mu^2} \right) \left(p + M\right). \]

(54)

Specializing to M^{\alpha\tau}, we find

\[ M^{\alpha\tau} = 2(4p_1^2 + Q^2)(p_1 + M + m) + 4M \left( E_1 - p_1^2 - Q^2 \right), \]

(55)

where terms not involving \( \alpha \) components have arisen from use of \( \alpha^2 = -1 \). Expressing the vectors in the c.m. frame, where \( p_1 = (E_1, p_1, 0, 0), p_2 = (E_2, p_2, 0, 0), \) leads to the following expression

\[ M^{\alpha\tau} = 8|p_1|^2 \sin^2 \theta \cos^2 \phi \left[ (E_1 - p_1^2) E_1 - p_1^2 \right] \left( \cos^2 \theta + M + m \right) + 4M \left( E_1^2 - p_1^2 \right) \left( \cos^2 \theta + M + m \right) \]

(56)

Carrying out the elementary angle integrations produces

\[ W^{\mu\nu} = \frac{1}{2 \pi} \frac{1}{8 \pi W^4} \left( C_0 \frac{a}{a^2 - b^2} + C_1 \ln \left( \frac{a + b}{a - b} \right) + C_2 \right) \]

(57)

\[ a = -Q^2 + 2p_1^2, \quad b = 2p_1^2 \]

(58)

\[ C_0 = 4M^4 \left[ 1 + 2Q^2 E_1 + M m \right] \]

(59)

\[ C_1 = 4p_1^2 \left[ 1 - 2Q^2 E_1 + M m \right] \]

(60)

\[ C_2 = 8|p_1|^2 \left( \frac{4p_1^2}{p_1^2 + M m} \right) \frac{1}{2} \frac{1 - 3a^2}{b^2} - (2Mm)^2 \left( \frac{4p_1^2}{p_1^2 + M m} \right) \frac{1}{2} \]

(61)

\[ \frac{e^{2m}}{2}, \quad \frac{e^{2m}}{2} \]

(62)

and we have defined

\[ F_1(x) = m^2 \left( \frac{1}{x} + \rho + x^2 \right), \]

(63)

Furthermore, the phase-space factor approaches a constant,

\[ \frac{p_1^2}{W^4} = \frac{1}{2}. \]

(64)

In the expressions given above, the limiting form as \( Q^2 \to \infty \) with \( x = Q^2/(2M m) \) is fixed is indicated following the arrow. We have used a number of kinematical relations that can be found in the paper of Batiz and Gross, i.e.,

\[ \ln(a+b)/(a-b) \]

Note that \( \ln(a+b)/(a-b) \) arises in the structure function. Because \( a+b \to -F_1(x)/(1-x) \) is independent of Q^2, this is cancelled when the Pauli-Villars subtraction is scaling in 3+1 dimensions depends upon the subtraction, which was not the case in the 1+1 dimensional analysis of mass m, a boson of mass \( m \), and a Pauli-Villars subtraction of mass \( A_1 \).

\[ f(x; m, m, A_1) = \frac{\|p_1\|^2}{16 \pi^2} \left( \frac{1}{F_1(x)} - \frac{1}{F_1(z)} \right) \ln \left( \frac{F_1(x)}{F_1(z)} \right) \]

(65)

For the case in which additional subtractions are made, the parton distribution becomes a linear combination of terms as in Eq. (9), i.e.,

\[ f(x) = \sum \left[ \alpha f(x; m, \mu, A_1) + \beta f(x; m, \mu, A_2) + \gamma f(x; m, \mu, A_3) \right] \]

\[ \beta \left[ f(x; m, m_1, A_2) + \gamma f(x; m, m_2, A_3) \right] \]

\[ \beta \left[ f(x; m, m_1, A_2) + \gamma f(x; m, m_2, A_3) \right] \]

(66)

Figure 11 shows the inelastic structure function \( 2M W_V(x, Q^0) \) and its limit as \( Q^2 \to \infty, x(x), \) for the same parameters that allow a reasonable description of the nucleon form factors. For finite Q^2, the energy transfer \( x = Q^2/(2M m) \) also finite. It must be greater than the total mass of the constituent quark and quark for each combination that enters
Eq. (66) in order to avoid spurious threshold effects. This restricts \( x \) to be too close to 1. For finite \( Q \), we show \( W(x \rightarrow Q) \) only for \( x > \) greater than about 200 MeV above threshold. The solid line in Fig. 11 shows the asymptotic limit \( Q \rightarrow +\infty \), which is \( x(x) \). Already for \( Q > 2 \text{ GeV/cm} \), the inelastic structure function is close to its asymptotic limit for our quark-diquark model of a nucleus. Our result for \( x(x) \) is more peaked than that obtained previously by Mineo, Bent and Yazaki \(^{26} \) from consideration of a three-quark model, and both results lack sufficient strength near \( x = 1 \) in comparison with experimentally determined parton distributions. Reference \(^{26} \) considers what time one should expect for \( x(x) \) at low \( Q \). Using QCD evolution to relate high and low \( Q \), the nucleon’s parton distribution is found to be properly accounted for by multiplying the x(x) that we find, with a peak near \( x = m/M \), where \( m \) is the lightest fermion mass. However, our results at \( Q^2 = 4 \text{ GeV}^2 \), which is the same \( Q^2 \) as in the model used by the authors, are not unlike the x(x) that we find, with a peak near \( x = m/M \), where \( m \) is the lightest fermion mass. This is a deficiency of the model used. Our results for the parton distribution are influenced by the choice of subtractions that have been incorporated in order to obtain a good description of the nucleon’s form factors. We have been able to fit the parton distributions in our model by selecting parameters. One should be normalizing factor \( z E \), the parton distribution is normalized according to

\[
\int_{0}^{1} dx \frac{\alpha(x)}{x} = 1. \tag{67}
\]

See Ref. \(^{25} \) and Appendix A in this regard. However, only the fermion constituent mass enters and the momentum sum rule is

\[
\int_{0}^{1} dx \frac{\alpha(x)}{x} = 304. \tag{68}
\]

Thus, 70% of the momentum is carried by the diquark in this model.

VI. CALCULATIONS FOR MESON-EXCHANGE CURRENT AMPLITUDE

Virtual photoproduction amplitude \( A^0 \) that has been defined in Eq. (36) and discussed in Sec. 4 contributes to the electromagnetic current in electron-deuteron scattering. For this case, the emitted pion is absorbed on a second nucleon and, in general, there is no loop integration involving the pion momentum and the deuteron wave functions of initial and final states. For electron-deuteron scattering, the loop integration requires important contributions from the quasifermion kinematics indicated in Fig. 12. Each nucleon in the deuteron, only one of which is shown, has initial momentum \( p_1 = p - q \). When the plus sign is absorbed on the second nucleon, its final momentum also becomes \( p_1 = p - q \). The second nucleon, not shown, has initial momentum \( p_2 = p - q \). When the plus sign is absorbed on the second nucleon, its final momentum also becomes \( p_2 = p - q \). The second nucleon, not shown, has initial momentum \( p_2 = p - q \). This process begins and ends with the two nucleons at zero relative momentum. It is favorable because the deuteron wave function is largest at zero relative momentum. Pion-in-flight terms vanish for the selected kinematics, since \( p_2 - p_1 = 0 \). We are left with the contributions from the quasifermion kinematics with collinear momenta that we consider only three amplitudes that are significant. An incoherent, non-flip amplitude, \( A^{\text{inc}} \), occurs in the time-component of the photoproduction amplitude as follows,

\[
A^{\text{inc}} \propto s_1 s_2 \alpha_1 \alpha_2, \tag{69}
\]

where the isospin factor involves an anticonntrum. Two isospin flip amplitudes, \( b \) and \( c \), occur in the space-vertex parts of the photoproduction amplitude as follows,

\[
A^{b}_{s_1 s_2} \propto k \phi \frac{q_2}{q_1} s_1 s_2 \alpha_1 \alpha_2, \tag{70}
\]

\[
A^{c}_{s_1 s_2} \propto k \phi \frac{q_1}{q_2} s_1 s_2 \alpha_1 \alpha_2. \tag{71}
\]

where the isospin factors involve a commutator. Two isospin flip amplitudes, \( b \) and \( c \), occur in the space-vertex parts of the photoproduction amplitude as follows,

\[
A^{b}_{s_1 s_2} \propto k \phi \frac{q_2}{q_1} s_1 s_2 \alpha_1 \alpha_2, \tag{70}
\]

\[
A^{c}_{s_1 s_2} \propto k \phi \frac{q_1}{q_2} s_1 s_2 \alpha_1 \alpha_2. \tag{71}
\]

where the isospin factors involve a commutator.

Although we calculate only the photoproduction amplitude, its role as a meson-exchange current in electron-deuteron scattering is of interest. In that case, the isospin wave function of the pion for the pions are replaced by the isospin operators \( f_5 \) for the second nucleon. The isospin nonflip amplitude, \( a \), is the only one that contributes as a meson-exchange current amplitudes contribute. For brevity of the deuteron, both isospin nonflip and isospin-flip amplitudes contribute.

A. Isospin nonflip amplitude \( a \)

Figure 13 shows the absolute value of Born and contact-like contributions to amplitude \( a \) in comparison with simple estimates of these contributions suggested in Sec. 4. \( C_{1,2} \) and \( C_{3,4} \) are constants and \( s(\alpha) \) is the function used. The contact-like amplitude differs from the elementary one on the quark-diquark mass, and the Born contributions are a little less than the contact-like terms. A factor of this comparison is to show that the contact-like contributions are decreasing with \( Q \) faster than a form factor. There is a zero in the Born amplitude that is not reproduced in the estimate. However, the estimate is quite good.

In Fig. 14, we show \( |a| \) for the Born (long dash line) and full amplitudes (solid line) for the composite nucleon. Also shown (dashed) is the elementary amplitude \( a \) that is based upon pseudoscalar pion coupling. Finally, we show (dotted line) a nonrelativistic amplitude that is based upon pseudoscalar pion coupling and the standard positive-energy propagator: \( \Lambda^{(p)}(p^0 - \sqrt{m^2 + p^2}) \). Thus, the nonrelativistic amplitude differs from the elementary one by omission of the Z-graph part. Form factors used in the elementary particle and nonrelativistic analyses are based for all cases except that on-shell \( f \) and form factors are used. For small \( Q \), the Born contributions to two amplitudes that involve an intermediate propagator for a nucleon in the elementary nucleon-propagator form. In the vicinity of \( Q = 1.2 \text{ GeV/cm} \), two amplitudes, pass through zero. They are negative at high \( Q \) and their magnitude is 40% to 50% of the full amplitude at \( Q = 3 \text{ GeV/cm} \). The elementary amplitude based on pseudoscalar pion coupling has a zero near \( Q \approx 2.8 \text{ GeV/cm} \). It provides the best approximation to the results of the composite model because it is much smaller in magnitude for \( Q > 1.5 \text{ GeV/cm} \). The amplitude for the composite nucleon model is dominated at large \( Q \) by the contact-like terms, which do not change signs. Although the nonrelativistic amplitude tends at large \( Q \) to a magnitude similar to that of the Born amplitude, it has the opposite sign and is not a useful approximation to the full result.

Figure 15 shows the contact-like amplitude of the composite model, (solid line). The part of the Born amplitude that comes from excited states and Z-graphs is shown by the long dash line. It has been calculated by evaluating the Born amplitude based on the full propagator of the composite model. Next we show by the dash dot line the sum of parts of the composite nucleon amplitude that do not come from the Born terms, i.e., the sum of contact-like parts, excited-state parts and Z-graph parts. Thus, the dash-dot line shows the sum amplitude based on pseudoscalar pion coupling. The excited-states plus Z-graph part of the composite-nucleon Born at low \( Q \), but it decreases rapidly with \( Q \). Because a less point-like structure for the composite nucleon would be the Born contributions of the composite model at large \( Q \) is expected to hold more generally. The Z-dependence of the elementary amplitude is not shown.

Because the pion vector of the composite model is about 75% pseudovector, we consider next the same set of comparisons using an elementary amplitude in which the pion coupling is pseudovector. In this model, we also include elementary amplitude based on pseudoscalar pion coupling (dashed line) provides a poorer approximation to the composite nucleon result than the Born result. This shows that it has a zero near 1 GeV/c and has the wrong sign at large \( Q \). Figure 17 shows the difference between the pseudovector elementary amplitude and the nonrelativistic amplitude that uses pseudoscalar pion coupling by the dashed line. We find that the use of pseudoscalar pion coupling in the elementary amplitude provides a better approximation to the amplitude of the composite model. This is because it has a large Z-graph contribution that approximates the
rough normalization of the contact-like parts because the parton distribution is normalized (see Eq. 67)). However, our model of a composite nucleon as a bound state of a quark and diquark yields a parton distribution that is peaked near $x = m_{NN}$, the ratio of quark to nucleon mass, whereas the data suggest much less peaking and more strength at low $x$ values than the model gives.

Calculations for the virtual photoproduction amplitude are performed using kinematics appropriate to its occurrence as a meson-exchange current in electron-deuteron scattering. The results show that the contact-like terms dominate the meson-exchange current for $Q > 1$ GeV/c for the case of elastic electron-deuteron scattering. As $Q$ increases, the dominance of the contact-like terms over the Born terms of the composite nucleon can become very large, suggesting that hadronic processes become unimportant when this occurs. Our results indicate that contact-like terms still have substantial Q dependence when they become dominant.

For the inelastic electron deuteron scattering, both isospin-nonflip and isospin-flip parts of the photoproduction amplitude can contribute. Two of the three contributing amplitudes are dominated at large Q by contact-like terms and the other is not. A more complete calculation using deuteron wave functions is needed to understand the role of contact-like contributions in deuteron breakup.

Off-shell effects in the hadronic vertex functions are found to be significant in the composite model. They cause a significant suppression of Born contributions to the virtual photoproduction amplitude for $Q \geq 1$ GeV/c. This result is model-dependent, but it suggests that use of on-shell form factors could be a poor approximation for momenta that are significantly off the mass shell.

An elementary amplitude based upon pseudoscalar pion coupling fails to provide a useful approximation to the full result of the composite model for the isospin-nonflip amplitude. This can be improved somewhat by using pseudoscalar pion coupling in the elementary amplitude. The increased Z-graph contribution gives a better approximation to the contact-like terms of the composite nucleon, but not to the underlying physics.

Composition effects require contact-like terms in second-order interactions. They have a direct connection to off-forward parton distributions and can dominate the scattering at large Q as they contain the leading partonic scattering process. Hadronic form factors and off-shell effects tend to quench the Born scattering processes that involve intermediate hadronic states.

For the considered model nucleon, we find that scattering from the quark constituent can be significant at modest Q values such as $Q > 1$ to 2 GeV/c. Once partonic scattering becomes dominant, it is expected to remain dominant for higher Q. Where the transition to dominance of the partonic interactions actually takes place is a matter of great interest. The model calculation of this paper suggests that this is determined by the size of contact-like contributions, or equivalently, by the size of the off-forward parton distributions. It may occur in some processes at momentum transfer as low as 1 GeV/c and seems to be likely by 2 GeV/c for the considered isoscalar meson-exchange current.

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APPENDIX A: SELF-ENERGY AND LOOP INTEGRALS

Details that complete calculations but are omitted from the text are collected in this appendix. The fermion-boson self-energy graph, defined in Eq. (4), vertex functions defined in Eqs. (18) and (23) and the contact-like terms defined in Eqs. (32) and (33) require evaluations of Feynman integrals and subsequent reductions of the Dirac matrices to standard form. Integrations over loop momentum $k^l$ are performed by standard methods. Integrals of the type $\int d^4k/(2\pi)^4\, 1/(k^2 - m^2)^p$ for $p > 1$ are reduced using Feynman parameters $\alpha, \alpha, \ldots, \alpha$, into a single denominator function of the form $(k - \ell)^2 - m^2)^p$, where the shift vector $\ell^p$ and the function $F$ depend upon the external momenta and Feynman parameters. Numerator functions involve one power of the loop momentum, $k^l$ for each fermion propagator.

Two divergent k-integrations arise and these are evaluated by subtraction. The required formulas are:

By and large, divergent integrations are considered in which there are two types of contributions: hadronic contributions where the photon and pion interactions have an intervening propagator of the nucleon, or its excited states, and contact-like contributions where the photon and pion interactions occur within a single vertex. By suitable regularization of the two types of contribution is controlled by Ward-Takahashi identities at low momentum transfer. At high momentum transfer, scaling behavior is obtained for the composite nucleon already by $Q \approx 2$ GeV/c. This provides a
\[ \frac{g}{16\pi^2} \ln \left( \frac{F_\Delta}{F_{\Delta}} \right) + \frac{g}{16\pi^2} \left( \frac{1}{F_\Delta} - \frac{1}{F_{\Delta}} \right) \mu' \nu'. \]  

(A2)

In all other cases, the k-integrations can be performed before substitutions by using the formulas (for \( n \geq 3 \)),

\[ \langle x \rangle = \sum_{k = 1}^{n} \left( k^2 \right)^{-1} \frac{1}{\left( 1 + k^2 \right)^{n/2}} \left( \frac{1}{F_\Delta} - \frac{1}{F_{\Delta}} \right) \mu' \nu'. \]

(A3)

where

\[ n_{\mu'\nu'} = g^{\mu'\nu'} + g^{\mu'\nu'} + g^{\mu'\nu'}. \]

(A4)

Considering the self-energy of an elementary fermion-boson bubble graph, we have

\[ \Sigma_{\mu}(p, m, \mu, \Lambda) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{P + m}{P^2 - m^2 + \Gamma} \left( \frac{1}{k^2 - \alpha^2 + \eta} \right). \]

(A5)

Using the Feynman parameterization

\[ \frac{1}{\alpha^2} = \int_0^1 \frac{dx}{[x(1-x)|\alpha|^2]}, \]

(A6)

to combine denominators, we have in this case the shift vector \( t = \alpha p \) and denominator functions \( F_\varphi \) and \( F_{\Lambda} \), where the general form is

\[ F_\varphi = \alpha \Lambda^2 + (1 - \alpha) m^2 - \alpha(1 - \alpha) \beta \rho^2. \]

(A7)

Integrating over loop momentum produces the two scalar parts defined in Eq. (11), as follows,

\[ \{ A(p^2), B(p^2) \} = \frac{g}{16\pi^2} \int_0^1 \frac{dx}{[x(1-x)|\alpha|^2]} \ln \left( \frac{F_\varphi}{F_{\Lambda}} \right) \{ \alpha, m \}. \]

(A8)

Using these formulas and the condition \( MA(M^2) + MB(M^2) = 1 \), one may determine the coupling constant \( g \) such that there is a bound state of mass \( M \), where \( M < m + \mu \). The corresponding formulas for \( A'(p^2) \) and \( B'(p^2) \) are obtained by differentiating with respect to \( p^2 \),

\[ \{ A'(p^2), B'(p^2) \} = -\frac{g}{16\pi^2} \int_0^1 \frac{dx}{[x(1-x)|\alpha|^2]} \{ \alpha, m \}. \]

(A9)

Wave-function renormalization constant \( Z_1 \) has contributions from the elementary bubble graph that may be expressed, using Eqs. (12) and (A9), as follows,

\[ Z_1 = \frac{g}{16\pi^2} \int_0^1 \frac{dx}{[x(1-x)|\alpha|^2]} \ln \left( \frac{F_\varphi}{F_{\Lambda}} \right) (\alpha, m). \]

(A10)

and for \( Z_2 \), \( p^2 = M^2 \) in \( F_\varphi \) and \( F_{\Lambda} \), Using the identity,

\[ \int_0^1 \frac{dx}{[x(1-x)|\alpha|^2]} (1 - \alpha)m^2 + \alpha \ln \left( \frac{F_\varphi}{F_{\Lambda}} \right) = 0, \]

(A11)

an equivalent expression for \( Z_1 \) is,

\[ Z_1 = \frac{g}{16\pi^2} \int_0^1 \frac{dx}{[x(1-x)|\alpha|^2]} (\alpha, m) \ln \left( \frac{F_\varphi}{F_{\Lambda}} \right) - \ln \left( \frac{F_\varphi}{F_{\Lambda}} \right). \]

(A12)

The integral here is the same as for the contribution of the elementary bubble graph to the normalization of the parton distribution, showing that the factor \( Z_2 \) guarantees the normalization as in Eq. (67). Equation (A11) can be verified by integrating by parts the term involving \( 2\alpha - 1 \).

APPENDIX B: THREE-POINT FUNCTIONS

Three-point functions required for photon and pom vertices are defined by Eqs. (18) and (23). They are calculated numerically from formulas involving integrations over two Feynman parameters, \( \alpha_1 \) and \( \alpha_2 \). The denominator function is

\[ F_{\alpha} = \alpha_1 L + \alpha_2 (m^2 - p^2) + (1 - \alpha_1 - \alpha_2)(m^2 - p^2) + \mu^2, \]

and the shift vertex in

\[ \mu = \alpha_1 \rho_1 + (1 - \alpha_1 - \alpha_2) \rho_2. \]

(B1)

Moments of loop momentum need to be expanded in terms of the independent external momenta, which we choose to be \( p_1 \) and \( p_2 \). For this expansion, we define,

\[ <1> = C_0, \]

(B2)

\[ <k_1^2> = C_{11} \rho_1^2 + C_{12} \rho_1 \rho_2 + C_{13} \rho_1, \]

(B3)

\[ <k_2^2> = C_{21} \rho_1^2 + C_{22} \rho_2^2 + C_{23} \rho_2, \]

(B4)

where the coefficients are calculated from

\[ \{ C_0, C_{11}, C_{12}, C_{13}, C_{22}, C_{23}, C_{24}, C_{25}\} = \frac{-g}{16\pi^2} \int_0^1 \frac{dx_1}{F_{\alpha}} \int_0^{1-x_1} \frac{dx_2}{F_{\alpha}} \left( \frac{1}{F_\varphi} - \frac{1}{F_{\Lambda}} \right), \]

(A10)

and the final coefficient is

\[ C_{24} = \frac{g}{32\pi^2} \int_0^1 \frac{dx_1}{F_{\alpha}} \int_0^{1-x_1} \frac{dx_2}{F_{\alpha}} \left( \frac{1}{F_\varphi} - \frac{1}{F_{\Lambda}} \right). \]

(B7)

Finally, the Dirac matrices of numerators of two fermion propagators are simplified to standard forms with the assistance of projection operators \( L^2(p_1) \) and \( L^2(p_2) \). Once a factor \( \rho_1 \) is commuted, if necessary, to act on \( L^2(p_1) \), it becomes \( \rho_1 \tilde{W}_i \), where \( W_i = \sqrt{g} \). Similarly commuting \( \rho_2 \) as necessary to act on \( L^2(p_2) \), the final expressions for form factors are:

\[ F_{\alpha}^{\rho_1 \mu_i}(p_1, p_2) = (p_2 W_i + m)(p_1 W_i + m)C_0 + (p_2 W_i + m)\rho_1 \tilde{W}_i C_{11} + \rho_1 W_i \rho_1 \rho_2 W_i C_{12} - (1 - \alpha_1 - \alpha_2) C_{24} \]

(B8)

\[ F_{\alpha}^{\rho_2 \rho_i}(p_1, p_2) = -(p_2 W_i + m)C_{12} + (p_1 W_i + m)C_{23} + (p_2 W_i + m)\rho_2 W_i C_{23} - \rho_1 W_i \rho_2 W_i C_{24} + (1 - \alpha_1 - \alpha_2) C_{23} \]

(B9)

\[ F_{\alpha}^{\rho_1 \rho_2}(p_1, p_2) = (p_2 W_i + m)C_{12} + (p_1 W_i + m)C_{23} + (p_2 W_i + m)\rho_2 W_i C_{23} - \rho_1 W_i \rho_2 W_i C_{24} + (1 - \alpha_1 - \alpha_2) C_{23} \]

(B10)

\[ F_{\alpha}^{\rho_1 \rho_2}(p_1, p_2) = (p_2 W_i + m)(p_1 W_i + m)C_{24} - (p_2 W_i + m)\rho_1 W_i C_{24} - (1 - \alpha_1 - \alpha_2) C_{24} \]

(B11)

Dependences of the form factors on \( \rho_1, \rho_2 \) and off-shell momenta are made explicit in these formulas.
APPENDIX C: EXPANSION IN TERMS OF COVARIANTS AND MATRIX ELEMENTS OF BORN TERMS

It is convenient to expand amplitudes $V_{\alpha}^{sa}$ and $V_{\alpha}^{sa}$ in terms of kinematical covariants and associated scalar amplitudes. For the kinematical covariants, we use helicity matrix elements of a set of eight Dirac operators, as follows,

\begin{align}
\xi_{\alpha}^{\sigma} & = \xi_{\alpha}^{\sigma}(p_{\alpha}) \gamma_{\alpha}^{\sigma} u_{\alpha}(p_{\alpha}) \\
\xi_{s}^{\sigma} & = \xi_{s}^{\sigma}(p_{s}) \gamma_{\alpha}^{\sigma} u_{s}(p_{s}) \\
\xi_{t}^{\sigma} & = \xi_{t}^{\sigma}(p_{t}) \gamma_{\alpha}^{\sigma} u_{t}(p_{t}) \\
\xi_{4}^{\sigma} & = \gamma_{4} \xi_{4}^{\sigma}(p_{4}) \gamma_{\alpha}^{\sigma} u_{4}(p_{4}) \\
\xi_{5}^{\sigma} & = \gamma_{5} \xi_{5}^{\sigma}(p_{5}) \gamma_{\alpha}^{\sigma} u_{5}(p_{5}) \\
\xi_{6}^{\sigma} & = \xi_{6}^{\sigma}(p_{6}) \gamma_{\alpha}^{\sigma} u_{6}(p_{6}) \\
\xi_{7}^{\sigma} & = \xi_{7}^{\sigma}(p_{7}) \gamma_{\alpha}^{\sigma} u_{7}(p_{7}) \\
\xi_{8}^{\sigma} & = \xi_{8}^{\sigma}(p_{8}) \gamma_{\alpha}^{\sigma} u_{8}(p_{8})
\end{align}

where $\alpha$ and $\sigma$ denote the helicities of initial and final states.

The direct Born graph of Eq. (31) is expanded as follows,

\begin{align}
V_{\alpha}^{sa} & = \sum_{\sigma} V_{\alpha}^{sa} \xi_{\alpha}^{\sigma} (p_{\alpha}) \xi_{s}^{\sigma} (p_{s}) \xi_{t}^{\sigma} (p_{t}) \xi_{4}^{\sigma} (p_{4}) \xi_{5}^{\sigma} (p_{5}) \xi_{6}^{\sigma} (p_{6}) \xi_{7}^{\sigma} (p_{7}) \xi_{8}^{\sigma} (p_{8})
\end{align}

where the scalar coefficients are

\begin{align}
V_{D_{1}}^{sa} & = \sum_{\sigma} F_{s}^{\sigma} \frac{1}{D_{1}} \left[ (W_{\alpha}^{s} - \rho M) F_{t}^{\sigma} + \rho F_{t}^{\sigma} + (p_{t} - q_{t})^{2} \right] \\
V_{D_{2}}^{sa} & = \sum_{\sigma} F_{t}^{\sigma} \frac{1}{D_{2}} \left[ -2 \rho F_{t}^{\sigma} \right] \\
V_{D_{3}}^{sa} & = \sum_{\sigma} F_{s}^{\sigma} \frac{1}{D_{3}} \left[ \rho (F_{t}^{\sigma} - F_{t}^{\sigma}) \right] \\
V_{D_{4}}^{sa} & = \sum_{\sigma} F_{s}^{\sigma} \frac{1}{D_{4}} \left[ 2 \rho F_{t}^{\sigma +} \right] \\
V_{D_{5}}^{sa} & = \sum_{\sigma} F_{s}^{\sigma} \frac{1}{D_{5}} \left[ 2 \rho F_{t}^{\sigma} \right] \\
V_{D_{6}}^{sa} & = \sum_{\sigma} F_{s}^{\sigma} \frac{1}{D_{6}} \left[ (W_{\alpha}^{s} - \rho M) F_{t}^{\sigma} + \rho F_{t}^{\sigma} + (p_{t} - q_{t})^{2} \right] \\
V_{D_{7}}^{sa} & = \sum_{\sigma} F_{s}^{\sigma} \frac{1}{D_{7}} \left[ (W_{\alpha}^{s} + \rho M) F_{t}^{\sigma} + \rho F_{t}^{\sigma} + (p_{t} - q_{t})^{2} \right]
\end{align}

In the $V_{\alpha n}$ expressions, $D_{s}^{\alpha}(p) = 2W_{\alpha r}^{-1} F_{t}^{\alpha} \left[ 1 - B((p_{t} - q_{t})^{2} - \rho W_{\alpha s} A((p_{t} - q_{t})^{2})) \right]$. In the $V_{\alpha n}$ expressions, $D_{s}^{\alpha}(p) = 2W_{\alpha r}^{-1} F_{t}^{\alpha} \left[ 1 - B((p_{t} - q_{t})^{2} - \rho W_{\alpha s} A((p_{t} - q_{t})^{2})) \right]$.

APPENDIX D: CONTACT-LIKE TERMS

Contact-like terms involve three fermion propagators and Dirac matrices $\gamma_{\mu}$ and $\gamma_{5}$. For $C_{s,\alpha}$, we have

\begin{align}
C_{s,\alpha} & = \tilde{u}(p_{s}) \int \frac{dk}{(2\pi)^{3}} S(p_{t} - k; m) \gamma_{5} S(p_{t} + q - k; m) \gamma_{\alpha} \gamma_{5} S(p_{t} - k; m) \\
D(k; \mu, \Lambda_{1}) & \equiv u(p_{s})
\end{align}

We denote the numerator of this expression as

\begin{align}
N_{s,\alpha}(k) & = \tilde{u}(p_{s}) \left( \gamma_{5} (\not{p}_{t} - \not{q} + m) \gamma_{\alpha} (\not{p}_{t} - \not{k} + m) \gamma_{5} (\not{p}_{t} - \not{k} + m) \right) u(p_{s})
\end{align}
The Feynman parameterization used is

\[ \frac{1}{d_1 d_2 d_3 d_4} = 3! \int [da] \frac{1}{(a_1 a_2 + a_2 a_3 + a_3 a_4 + a_4 a_1)^4} \]  

(33)

where

\[ \int [da] = \int_0^1 da_1 \int_{-a_1}^{1-a_1} da_2 \int_{-a_2}^{1-a_2} da_3 \int_{-a_3}^{1-a_3} da_4. \]  

(34)

and

\[ a_4 = 1 - a_1 - a_2 - a_3. \]  

(35)

This leads to a shift vector

\[ \ell = a_2 p_f + a_3 (q_1 + q) + a_4 p_i, \]  

(36)

and a denominator function, for boson mass \( \Lambda \),

\[ F_k = a_1[m^2 - p_f^2] + a_2[m^2 - (p_1 + q)^2] + a_3[m^2 - p_i^2] + a_4 \Lambda^2 + \ell^2. \]  

(37)

The required integration is

\[ C_{5\nu}^a = \frac{6g^2}{16\pi^2} \int [da] \int \frac{dk}{k^2} \psi^\nu(k) \left( \frac{1}{(k - \ell)^2 - F_k^2} + \frac{1}{(k - \ell)^2} - \frac{1}{F_k^2} \right). \]  

(38)

From the general rules stated above, one sees that a \( k^\nu \) in the numerator in general is replaced by \( \ell^\nu \) after integration over \( k \), but there are additional contributions from combinations of \( k^\nu k^\nu \) that involve \( \ell^\nu \). Therefore we write

\[ k^\nu = \ell^\nu + (k - \ell)^\nu, \]  

and expand in powers of \( k - \ell \). Terms that are odd in \( k - \ell \) do not contribute because of symmetry. The parts that do contribute are

\[ N_{5\nu}^a(k) = N_{5\nu}^a(\ell) + \Delta N_{5\nu}^a(k - \ell), \]  

(39)

where

\[ \Delta N_{5\nu}^a(k) = 6(p_f) \left( \phi_f - f - i + m \right) \gamma_\nu \gamma_\rho \gamma_\sigma \phi_f - f - i + m \right) \gamma_\nu \gamma_\rho \gamma_\sigma \phi_f - f - i + m \right) \psi(p_i). \]  

(40)

Integration over \( k \) produces

\[ C_{5\nu}^a = \frac{g^2}{16\pi^2} \int [da] \left( N_{5\nu}^a(\ell) \left( \frac{1}{F_{5\nu}^2} - \frac{1}{F_{5\nu}^2} \right) - \frac{1}{2} (\Delta N_{5\nu}^a) \right) \psi(p_i). \]  

(41)

where

\[ (\Delta N_{5\nu}^a) = 6(p_f) \left( \phi_f - f - i + m \right) \gamma_\nu \gamma_\rho \gamma_\sigma \phi_f - f - i + m \right) \gamma_\nu \gamma_\rho \gamma_\sigma \phi_f - f - i + m \right) \psi(p_i). \]  

(42)

The resulting expressions are reduced and expanded in terms of scalar amplitudes times kinematical covariants defined in Appendix C as follows:

\[ C_{5\nu}^a = \sum_{n=1}^{8} C_{5n}^a n_n^a, \]  

(43)

where

\[ C_{5n}^a = \int [da] \left[ - \left( a_1 - MA \right) - MA(a_2 + a_3 + a_4) \right] \]  

(44)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(45)

\[ C_{5n}^a = \int [da] \left[ -(a_3 + A_3) \right] \]  

(46)

\[ C_{5n}^a = \int [da] \left[ -(a_3 + A_3) \right] \]  

(47)

\[ C_{5n}^a = \int [da] \left[ -A_4 - MA - MA(a_2 + a_3 + a_4) \right] \]  

(48)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(49)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(50)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(51)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(52)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(53)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(54)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(55)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(56)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(57)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(58)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(59)

\[ C_{5n}^a = \int [da] \left[ -(a_2 + a_3)(A_2 + A_3) \right] \]  

(60)
\[ A_{\alpha} = 2 M^2 (\alpha_2 + \alpha_3 + \alpha_4) - 2 p_\mu \cdot (p_\nu + q)(\alpha_2 + \alpha_1) + 2 q_\mu \cdot (p_\nu + q) \alpha_1 \]
\[ + 2 p_\nu \cdot q \alpha_1 \left( \frac{1}{F_{1,2}} - \frac{1}{F_{1,3}} \right) \]  

(D29)

Here \( p_{\text{int}} = p_\nu + q \) is the intermediate momentum, and \( \ell, F_{1,2} \), and \( F_{1,3} \), are expressed as appropriate for \( C_{\text{int}} \).

A very similar analysis is carried out for the crossed contact-like term, \( C_{\text{int}} \).

\[ C_{\text{int}} = \hat{u}(p_\nu) \int \frac{d^4 k}{(2\pi)^4} S(p_\nu - k - m) \gamma^\nu S(p_\nu - q - k - m) \gamma^5 \]
\[ \times S(p_\nu - k - m) D(k, \mu, \lambda_1) \hat{u}(p_\nu). \]  

(D30)

We denote the numerator of this expression as

\[ N_{\text{int}} = \hat{u}(p_\nu) \left[ (p_\nu - \ell - m) \gamma^\nu (p_\nu - \ell - \mu + m) \gamma^5 \right] \hat{u}(p_\nu). \]  

(D31)

Proceeding as before leads to a shift vector

\[ \ell = \alpha_4 p_\nu + \alpha_3 (p_\nu - q) + \alpha_2 p_\nu \]

and a denominator function, for boson mass \( A_1 \),

\[ F_{1,2} = \alpha_1 (m^2 - p_\mu^2) + \alpha_3 m^2 - (p_\nu - q)^2 + \alpha_2 m^2 - p_\mu^2 + \alpha_1 A_1^2 + \ell^2. \]  

(D33)

Numerators factors that produce nonzero results are expressed in a similar way as above,

\[ N_{\text{int}}(k) = N_{\text{int}}(\ell) + \Delta N_{\text{int}}(k - \ell), \]  

(D34)

where

\[ \Delta N_{\text{int}}(k) = \hat{u}(p_\nu) \left[ (p_\nu - \ell - m) \gamma^\nu \gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 \gamma^\nu (p_\nu - \ell - m) \gamma^\mu \right] \hat{u}(p_\nu). \]  

(D35)

Integration over \( k \) produces

\[ C_{\text{int}} = -\int \frac{d^4 k}{(2\pi)^4} \left\{ N_{\text{int}}(\ell) \left( \frac{1}{F_{1,2}} - \frac{1}{F_{1,3}} \right) - \frac{1}{2} \left( \Delta N_{\text{int}} \right) \left( \frac{1}{F_{1,2}} - \frac{1}{F_{1,3}} \right) \right\}, \]  

(D36)

where

\[ \left( \Delta N_{\text{int}} \right) = \hat{u}(p_\nu) \left[ (p_\nu - \ell - m) \gamma^\nu \gamma^5 \gamma^\mu \gamma^\rho \gamma^\sigma \hat{u}(p_\nu), \right. \]
\[ + \gamma^\nu \gamma^5 \gamma^\rho \gamma^\sigma (p_\nu - \ell - m) \gamma^\mu \gamma^\sigma \hat{u}(p_\nu) \]  

(D37)

The resulting expression is expanded in terms of scalar amplitudes times the kinematical covariants,

\[ C_{\text{int}} = \sum_{\alpha=1}^{8} C_{\alpha} a_\mu^\alpha, \]  

(D38)

where

\[ + \alpha_3 + \alpha_4) \]  

\[ C_{X1} = -C_{D1} \]  

(D39)

\[ C_{X2} = \int [da] \left[ -\alpha_2 (A_2 + \alpha_3 A_4) \right] \]  

(D40)

\[ C_{X3} = \int [da] \left[ -\alpha_2 (A_2 + A_3) \right] \]  

(D41)

\[ C_{X4} = -C_{D4} \]  

(D42)

\[ C_{X5} = \int [da] \left[ \alpha_2 (A_2 - A_3) + \alpha_2 (A_3 - A_4) \right] \]  

(D43)

\[ C_{X6} = \int [da] \left[ A_1 + \alpha_2 A_2 + A_4 \right] \]  

(D44)

\[ C_{X7} = \int [da] \left[ -A_1 - \alpha_2 (A_2 + A_4) - \alpha_2 (A_3 + A_4) \right] \]  

(D45)

\[ C_{X8} = C_{D8} \]  

(D46)

and where the functions \( A_1 \) to \( A_8 \) take the same form as before, except that \( p_{\text{int}} = p_\nu - q \), and the appropriate \( \ell, F_{1,2} \), and \( F_{1,3} \), for \( C_{\text{int}} \) must be used. Also, equations such as \( C_{X1} = -C_{D1} \) mean that \( C_{X1} \) takes the same form as \( -C_{D1} \), but of course must be evaluated with the appropriate \( \ell, \), and so on. Function \( A_8 \) takes a different form from \( A_4 \), as follows,

\[ A_8 = 2 M^2 (\alpha_2 + \alpha_3 + \alpha_4) + 2 p_\nu \cdot (p_\nu - q) \alpha_2 - 2 p_\nu \cdot (p_\nu - q) (\alpha_2 + \alpha_3) \]
\[ -2 p_\nu \cdot q \alpha_1 \left( \frac{1}{F_{1,2}} - \frac{1}{F_{1,3}} \right). \]  

(D47)

Calculations have been performed in two ways. One uses the expressions given above and the other uses expressions that have been developed by use of the symbolic manipulation program SCHOONSCHIP in order to reduce the Dirac matrices to the desired forms and FORMF to calculate the moments of the one-loop graphs [32]. Two independent computer codes were written and checked against one another to verify that the algebra and the numerics was done correctly.

FIG. 1. Bubble graphs that contribute to the composite particle propagator. Solid line represents the spin-1/2 quark and dashed line represents the spin-0 boson.
FIG. 2. Photon-quark and pion-quark insertions in the composite particle propagator.

FIG. 3. Electromagnetic and pion form factors for composite particle.
FIG. 4. Dependence of form factor $F^{++}_{1}(Q^2, M^2, p^2)$, including propagator factor as discussed in text, on off-shell variable $p^2$, where $p^2/M^2 = 1.2$ (dot line), 1.1 (dashed line), 1.0 (solid line), 0.9 (dot-dash line and long-dash line).

FIG. 5. Dependence of the ratio $F^{++}_{1}(Q^2, M^2, p^2)/F^{++}_{1}(Q^2)$ on off-shell variable $p^2$, where $p^2/M^2 = 1.2$ (dot line), 1.1 (dashed line), 1.0 (solid line), 0.9 (dot-dash line and long-dash line). Propagator factor is included as discussed in text.
FIG. 6. Dependence of ratio $F_2^{-}(Q^2, M^2, p^2) / F_2^{++}(Q^2)$ on off-mass-shell variable $p^2$, where $p^2/M^2 = 1.2$ (dot line), 1.1 (dashed line), 1.0 (solid line), 0.9 (dot-dash line) and 0.8 (long dash line). Propagator factor is included as discussed in text.

FIG. 7. Dependence of form factor $F_5^{+-}(Q^2, M^2, p^2) / F_5^{++}(Q^2)$ on off-mass-shell variable $p^2$, where $p^2/M^2 = 1.2$ (dot line), 1.1 (dashed line), 1.0 (solid line), 0.9 (dot-dash line) and 0.8 (long dash line). Propagator factor is included as discussed in text.
FIG. 8. Pseudovector fraction of pion-nucleus coupling.

FIG. 10. Impulse approximation for deep inelastic scattering.

FIG. 11. Structure function for inelastic scattering. $Q = 1.5$ GeV/c (dot line), $Q = 2.5$ GeV/c (dash line), $Q = \infty$ (solid line).
FIG. 12. Quasi-free kinematics for photopion amplitude in meson-exchange current. The pion (dashed line) is absorbed by a second nucleon, not shown, such that the each nucleon absorbs half the photon momentum.

FIG. 13. Born amplitude (solid line) is compared with product of dipole form factors and propagator (dotted line). Contact-like contribution (long dash line) is compared with estimate based on product of dipole form factor and factor $S(Q) = \kappa^2/(Q^2 + \kappa^2)$, with $\kappa = 0.2$ GeV. (dotted line).
FIG. 14. Isospin-nonflip a amplitude: full amplitude (solid line), elementary amplitude based on pseudoscalar pion coupling (dash line), nonrelativistic amplitude (dot line) and Born amplitude of composite model (long dash line).

FIG. 15. Contact-like contribution of composite model (solid line), excited states and Z-graph part of Born amplitude for composite model (long dash line), sum of contact-like, excited states and Z-graph parts (dot dash line) and Z-graph part of pseudoscalar elementary amplitude (dash line).
FIG. 16. Isospin-nonflip a amplitude: full amplitude (solid line), elementary amplitude based on pseudovector pion coupling (dash line), nonrelativistic amplitude (dot line) and Born amplitude of composite model (long-dash line).

FIG. 17. Contact-like contribution of composite model (solid line), excited states and 2-graph part of Born amplitude for composite model (long-dash line), sum of contact-like, excited states and 2-graph parts (dot-dash line) and difference between elementary amplitude based on pseudovector pion coupling and nonrelativistic amplitude (dash line).
FIG. 18. Isospin-flip b amplitude: full amplitude (solid line), elementary amplitude based on pseudoscalar pion coupling (dash line), nonrelativistic amplitude (dot line) and Born amplitude of composite model (long dash line).

FIG. 19. Isospin-flip c amplitude: full amplitude (solid line), elementary amplitude based on pseudoscalar pion coupling (dash line), nonrelativistic amplitude (dot line) and Born amplitude of composite model (long dash line).