Large Acceptance Muon Storage Rings for Neutrino Production: 
Lattice Design

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Large Acceptance Muon Storage Rings for Neutrino Production: Lattice Design *

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Abstract
The possibility of achieving the high muon fluxes suggested in recent work on muon colliders has revived interest in the idea of using muon storage rings for neutrino production. Through proper design of the lattice, a significant fraction of the stored muons can be converted into an intense, low-divergence beam of neutrinos. This work examines the incorporation of a long, high-beta straight section for production of neutrino beams into a lattice which is otherwise optimized for transverse and longitudinal admittance. (The ring must be able to accept a very large emittance and large momentum spread muon beam.)

1 INTRODUCTION
A muon storage ring is also a source of neutrinos from muon decay. By recirculating the intense muon source beam, a muon storage ring is capable of efficient and intense neutrino production when the muons decay in long field-free regions (or straight sections). This paper discusses two 10-GeV muon storage rings designed to optimize neutrino-beam production.

2 DESIGN CONSIDERATIONS
There are three principal considerations when optimizing a muon storage ring for neutrino beam production. The overriding consideration is to design a ring with an exceptionally large acceptance, both transversely and longitudinally, thereby reducing as much as possible the beam cooling required. This condition also presumes large-emittance muon beams can be effectively accelerated. The next two are specific to neutrino production. One is the conversion of a significant fraction of the stored muons into a directed beam of neutrinos; that is, the production straight must occupy a substantial portion of the ring circumference. Another is that the properties of the secondary neutrinos include the dynamics of the muons in the production straight. This implies that the neutrino beam will be colinear (at least to within the limits set by the decay kinematics) if the divergence of the parent muon beam is less than or comparable to the muon decay angle (which is 10 mr at 10 GeV). Since the muon beam divergence in the decay straight can be controlled with a straightforward high-beta insert, the lattice parameters are not overly constrained. This allows the base lattice to be designed consistent with unusually large transverse and momentum admittances. (The intent here is that the greater the acceptance achieved in the storage ring, the fewer muon cooling stages will be required upstream with a considerable savings in complexity and expense.) The challenge, then, in the design of the storage ring is to extend the exceptionally large transverse and longitudinal emittances characteristic of the arc design to include the high-beta insertion. For reference, the muon beam entering the initial cooling stage has a normalized transverse emittance of 15,000 π mm-mr and a momentum spread of about ±5%.

3 BASIC LATTICE DESIGN
With long opposing straights, the natural layout of the storage ring is racetrack. Strong-focussing FODO cells are the best choice for the arcs because of their potential for large momentum acceptance (larger than ±5%) as compared with more complicated focussing structures. At 10-GeV, large-bore superconducting quadrupoles are recommended in order to maintain a strong gradient over the large aperture required to accomodate both the large transverse admittance and the large displacement of off-momentum orbits. The high-beta insert, on the other hand, is a weaker-focussing structure and must be carefully designed and matched to the arc to transmit the large range in momenta in addition to creating a parallel beam for neutrino production.

Table 1: Parameters of the large momentum acceptance arc cells for a 10-GeV muon storage ring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial</th>
<th>Later</th>
</tr>
</thead>
<tbody>
<tr>
<td>intermagnet spacing (m)</td>
<td>0.1451</td>
<td>0.2</td>
</tr>
<tr>
<td>dipole length (m)</td>
<td>3.422</td>
<td>0.645</td>
</tr>
<tr>
<td>dipole bend (rad)</td>
<td>π/18</td>
<td>0.174</td>
</tr>
<tr>
<td>dipole field (T)</td>
<td>1.7</td>
<td>9</td>
</tr>
<tr>
<td>quadrupole length (m)</td>
<td>0.4</td>
<td>0.387</td>
</tr>
<tr>
<td>arc quadrupole strength (m²)</td>
<td>0.75</td>
<td>1.8</td>
</tr>
<tr>
<td>arc quadrupole poletip field (T)</td>
<td>&gt; 5</td>
<td>7.8</td>
</tr>
<tr>
<td>arc quadrupole radius (cm)</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>cell phase advance (deg)</td>
<td>≈ 74</td>
<td>≈ 90</td>
</tr>
<tr>
<td>horiz. sextupole strength (T)</td>
<td>0</td>
<td>2.3</td>
</tr>
<tr>
<td>vert. sextupole strength (T)</td>
<td>0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

The initial design is simply constructed using two identical high-beta straights matched directly to the connecting arcs. Each arc is composed of six full FODO cells plus an extra half cell with cell properties given as the first example in Table 1. Two cells at each end of the arcs have reduced deflections to suppress dispersion in the two long straights. The arcs and dispersion suppression modules account for 44% of the ring which is 394 m in circumference. The high-beta straight for neutrino production was formed using an antisymmetric doublet quadrupole structure and repre-

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Figure 1: Initial design: The neutrino production region flanked by matching sections and dispersion-suppression cells

represents about 16% of the total ring circumference. The matching section between the arc and the high-beta straight accounts for 12% of the total circumference and its increased divergence contributes to the backgrounds associated with neutrino detection. Fig. 1 shows the beta and dispersion functions for the high-beta region, flanking matching sections, and dispersion suppression cells. The rest of the circumference is occupied by a second long straight which is opposite to and presently identical to the production straight. This region will eventually be redesigned as a utility section to include injection. The periodicity of this ring is one.

The admittance of the initial ring design was calculated using a \( \beta_{\max} \) in the high-beta quadrupoles of 200 m (from Fig. 1), a quadrupole radial aperture of 20 cm, and assuming a \( \pm 3 \sigma \) rms beam size. These parameters give a normalized rms emittance of 2111 mm-mr; given that the upstream acceleration is not the limiting aperture. Errors have not been included in calculating this ideal aperture.

The momentum dependence of the initial lattice was studied and tracked using MAD[2]. The tune of the ring is \( \nu_x = 5.46 \) and \( \nu_y = 5.65 \). Tracking studies indicate that this fractional tune is near optimal and also that the momentum aperture is best when the x and y plane fractional tunes are close. (Although symmetric inserts were studied, it was more difficult to make the fractional tunes between the two planes close—at least in the present lattice version—and the dynamic aperture was smaller when tracked.) Although a closed orbit exists for a \( dp/p \) of -6.7% to 5.8%, the orbit excursions were large at the low momenta and the betas too high at the high momenta. If an orbit cutoff of \( \pm 15 \) cm and a \( \beta_{\max} \) of 200 m is enforced, then the acceptable momentum range of the initial lattice design appears to be -5.3% to 5%.

4 LATTICE IMPROVEMENTS

Noting that the neutrino production straight is only 16% of the ring circumference, one of the first modifications to the design was to match the short FODO cells of the arc into much longer FODO cells to build up the high-beta production region. Use of a periodic unit allows almost complete flexibility in the length of the region. Additionally, peak beta values were reduced by a factor of two. The insertion was kept antisymmetric so that the periodicity of the ring remained one. The improvements in this later lattice allows for a production straight which can be easily varied from \( \approx 22\% \) of the total circumference for a 428 m ring to \( \approx 40\% \) for a 717 m circumference ring. In the initial design the matching sections are almost equivalent in length to the production straight (75% of), but in the later work, they fall to 30% of the length of the production straight in the 428 m ring and 10% in the 717 m ring.

A significant problem with the initial design is that the tune changes by \( \nu_x = 5.82, \nu_y = 6.01 \) to \( \nu_x = 5.11, \nu_y = 5.28 \) over the accepted momentum range. Such a large tune spread unavoidably covers an integer or half-integer resonance; consequently substantial beam loss is expected. Therefore, another improvement in the later design was to insert sextupole correctors at the center of each quadrupole in the arcs and cancel most of the linear chromaticity. (In the calculation, the quadrupoles were divided in two and short, 0.2 m sextupole magnets inserted; hence their large poletip fields. It should be possible instead to insert sextupole correction coils along the length of the quadrupole at a reduced field.)

Not only were sextupoles correctors added, the arc cell was also redesigned to keep the off-momentum excursions under \( \pm 8 \) cm and arc quadrupole apertures modest. Reducing off-momentum orbit excursions means shorter cells with stronger quadrupoles and shorter, stronger dipoles. The amount of bend per arc cell was left virtually unchanged so the number of cells per arc remains the same (7.5). Additionally, at the central momentum of 10 GeV, the arc quadrupole strengths were tuned to give to a near \( \pi/2 \) phase advance per cell (keeping the focussing and defocussing quadrupoles equal strength). The new parameters of the arc FODO cells are included in Table 1 for comparison. It should be noted that the 9 T fields in the dipoles can be reduced to 6–7 T without compromising the basic lattice design or its properties; this requires adding 2 to 4 cells per arc.
Figure 2: Later design: Lattice functions of the high-beta FODO cells for neutrino production flanked by matching sections, dispersion supression cells, and three arc modules.

5 LATTICE PERFORMANCE

The optimal dynamic aperture for both transverse and longitudinal was found to be near a fractional tune of \((2n + 1)\pi/2\). In the latest lattice shown Fig. 2, the fractional tune of the horizontal plane was set near 0.25 and the vertical plane near 0.75.) This choice keeps off-momentum particles from approaching either integer or half-integer resonances using the weakest sextupole strengths. It also minimizes second-order chromatic effects. With this fractional tune, the normalized rms emittance accepted by the high-beta quadrupoles—assuming a 20 cm quadrupole radius, a \(\beta_{\text{max}}\) of 100 m and a 3\(\sigma\) distribution—is about 4200 mm-mm. The transverse acceptance is limited by the high-beta quadrupole aperture and, fortunately, not by the nonlinear tuneshifts generated by the chromatic correction sextupoles.

With the sextupoles cancelling most of the linear chromaticity, the total tune swing is reduced from \(\delta\nu_x = 0.71, \delta\nu_y = 0.73\) to \(\delta\nu_x = 0.12, \delta\nu_y = 0.16\). Further reduction of this tune swing is possible, but does not appear necessary and stronger sextupoles decrease the currently large on-momentum aperture.

6 RESULTS

At the rms emittance quoted (2111 mm-mm), the rms angle of the muon beam in the 63.3 m-long production region (between the quadrupole doublets) is less than 1 mr, or less than 1/10 the natural opening angle of the neutrinos. It is therefore an insignificant contribution to the final divergence of the neutrino beam. However, the matching sections outside of the doublet contribute between 1 and 3 mr over 33.2 m, rising to 2-5 m over the 14 meters occupied by the high-beta quadrupole doublet.

In the later lattice, the rms angle of divergence, on average, in the production straight (assuming 4200 mm mm) is about 1.3 mr both horizontally and vertically, despite the increase in the transverse acceptance. There is a short burst of highly divergent beam, 14-17 mr, over the short distance (~1 m) comprising the high-beta quadrupole doublet. However, the resulting background neutrinos should dissipate rapidly before the intended target. The divergence in the remaining 12-13 m in each matching section (representing over 90% of the length of this section), drops to 3-5 mr, which is similar to the previous design. Overall, the contribution of the matching sections to the neutrino beam is reduced by a factor of 2.5-7 from the initial design.

7 DISCUSSION AND SUMMARY

In the initial design, all quadrupoles must have large, 20-25 cm radial apertures to accomodate the large displacements (~15 cm) of off-momentum orbits in the arcs and the 200 m high-beta peaks in the insert. This includes both arc and high-beta quadrupoles. In the arcs of the second design, the quadrupole apertures are smaller by about a factor of two due to the reduced orbit swing of off-momentum orbits. If needed, the strength and cell length can be adjusted to lower the poletip fields in both quadrupole and dipole components. Since the high-beta peak in the insert is lower by a factor of two, the transverse dynamic aperture of the second lattice is correspondingly larger.

To summarize, ideal lattices have been designed which successfully include a long, low-divergence, straight section for neutrino production without compromising an exceptional dynamic aperture in both transverse and momentum space. The production straights nominally occupy 20-40% of the ring circumference indicating a large conversion of stored muons into useable neutrinos. Although a large dynamic aperture has been obtained, clearly, a realistic set of errors must be introduced into the tracking to determine to what extent this ideal aperture deteriorates.

Thanks to S. Ohnuma for useful suggestions on sextupoles

8 REFERENCES