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A Field-Theory Approach to Modeling Helical FCG's

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Abstract

Often helical flux-compressor generator (FCG) design codes are essentially circuit codes which utilize known equations for parameterizing circuit elements such as armature and stator inductance. We present an analytical model that is based more on first principals. The stator inductance is calculated using a definition of inductance in terms of the magnetic vector-potential. The calculation accounts for winding-pitch, bifurcations, and works for any ratio of length to diameter. The currents on the armature are calculated self-consistently and are not assumed to simply 'mirror' the stator currents. Resistive losses and magnetic diffusion losses are calculated less rigorously but we are working on better methods. Details of the model and comparison with experiment will be presented.

Introduction

Modeling of helical flux compression generators is not a new enterprise, many papers on this subject can be found in most editions of the Megagauss conference proceedings [1-6]. We present a model and computer code that calculates the generator inductance from first principals using the magnetic vector potential. The calculation does not assume the stator pitch angle is small, or that azimuthal currents which flow on the armature surface 'mirror' those flowing on the stator. The armature currents are calculated self-consistently. The stator and armature can be of arbitrary aspect ratio and the stator can have an arbitrary number of sections and parallel wires. Any spacing between wires is allowed and accounted for.

The model in its present state is linear. One consequence of this assumption is that heating effects are not calculated. Losses due to resistance and magnetic diffusion are included but are not calculated with the same rigor as the generator inductance.

In this paper we discuss the most important features of the model and then compare the computer code results with experiments using two very different helical flux-compressors.

Stator Self-Inductance

By using Stokes Theorem, stator self-inductance can be expressed in terms of the magnetic vector potential, A .

$$L = \frac{\Phi}{I} = \int \frac{\mathbf{B} \cdot d\mathbf{S}}{I} = \oint \frac{\mathbf{A} \cdot d\mathbf{l}}{I} \quad (1)$$

The magnetic vector potential at any point in space from a system of wires with known current densities is given by,

$$\mathbf{A}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{\mu_0}{4\pi} \int_{Vol} \frac{\mathbf{J}(\mathbf{x}', \mathbf{y}', \mathbf{z}') d\mathbf{x}' d\mathbf{y}' d\mathbf{z}'}{|\mathbf{r} - \mathbf{r}'|} \quad (2)$$

Consider n parallel wires, with pitch angle θ , on a cylinder of radius a , carrying a total current I . From eq. (2) the vector potential from this set of wires, at any point in space, is

$$A_x(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{\mu_0}{4\pi} \sum_j^n \int_{\phi_{j \text{ init}}}^{\phi_{j \text{ final}}} \frac{-I \cos\theta \sin\phi_j a (1 + \tan^2\theta)^{1/2} d\phi_j}{n \sqrt{(x - a \cos\phi_j)^2 + (y - a \sin\phi_j)^2 + (z - (z_{0i} + a \tan\theta (\phi_j - \phi_{j \text{ init}})))^2}}$$

$$A_y(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{\mu_0}{4\pi} \sum_j^n \int_{\phi_{j \text{ init}}}^{\phi_{j \text{ final}}} \frac{I \cos\theta \cos\phi_j a (1 + \tan^2\theta)^{1/2} d\phi_j}{n \sqrt{(x - a \cos\phi_j)^2 + (y - a \sin\phi_j)^2 + (z - (z_{0i} + a \tan\theta (\phi_j - \phi_{j \text{ init}})))^2}} \quad (3)$$

$$A_z(x, y, z) = \frac{\mu_0}{4\pi} \sum_j^n \int_{\phi_{j \text{ init}}}^{\phi_{j \text{ final}}} \frac{I \sin \theta \ a (1 + \tan^2 \theta)^{1/2} d\phi_j}{n \sqrt{(x - a \cos \phi_j)^2 + (y - a \sin \phi_j)^2 + (z - (z_{0i} + a \tan \theta (\phi_j - \phi_{j \text{ init}})))^2}}$$

The integration variable ϕ monotonically increases with the number of wire turns. Z_{0i} is the initial z -position of the n wires. All wires in each section begin at the same z -position but have different initial values of ϕ , usually evenly spaced from $m\pi$ to $(m+1)\pi$. If the stator has bifurcations and thus, multiple sections with different pitch angles, then there is an additional summation over sections.

The unit vector $d\mathbf{l}$ that points along a wire which lies on a cylinder of radius a , with pitch angle θ is given in either cylindrical or cartesian coordinates by,

$$d\mathbf{l} = a \tan \theta \ d\phi \ \hat{z} + a \ d\phi \ \hat{\phi} = a \ d\phi (\tan \theta \ \hat{z} - \sin \phi \ \hat{x} + \cos \phi \ \hat{y}) \quad (4)$$

With expressions for the components of $\mathbf{A}$ at any point in space, and the unit vector $d\mathbf{l}$ which defines the direction of a stator wire, we can use eq. (1) to calculate the stator self-inductance.

$$\oint_{\text{stator}} \mathbf{A} \circ d\mathbf{l} = \oint (-A_x(\mathbf{r}_{\text{wire}}) \sin \phi + A_y(\mathbf{r}_{\text{wire}}) \cos \phi + A_z(\mathbf{r}_{\text{wire}}) \tan \theta) a \ d\phi \quad (5)$$

The vector $\mathbf{r}_{\text{wire}}$ depends on the integration variable ϕ as follows.

$$\mathbf{r}_{\text{wire}} = (a - \epsilon) \cos \phi \ \hat{x} + (a - \epsilon) \sin \phi \ \hat{y} + (a - \epsilon) \tan \theta (\phi - \phi_{\text{init}}) \ \hat{z} \quad (6)$$

The line integral is performed around the stator, on the inside surface of the wire, $(a - \epsilon)$. The exact value of ϵ is unimportant because the stator self-inductance depends only weakly on the exact current distribution in the wire. (We usually let $\epsilon = \Delta - \delta$ where Δ and δ are the wire radius and a nominal skin depth, respectively.) If there are multiple parallel wires in a section then the line integral follows one of those wires, usually the center one. Note that eq. (5) is actually a double integral because each component of the vector potential, at every point along the stator, involves an integral over all current carrying conductors that make up the stator. The implicit double-integral provides the usual N^2 dependence of inductance on wire turns.

As the armature expands and stator turns are shorted out the stator self-inductance is recalculated as described above using a time-dependent initial starting position for the remaining turns. The numerical code that calculates the stator-self inductance as described above gives excellent agreement with analytic formulas for less complicated geometries.

Armature Mutual Inductance

The presence of the armature decreases the generator inductance. Azimuthal currents on the armature surface exclude flux from the armature interior. We calculate these currents self-consistently, without making the usual assumption that as a function of z the armature line-current density is equal and opposite to the stator line-current density. The expanding armature is broken into a series of rings each with its own radius and current. Once the armature shorts to the stator its effective length decreases, but the number of rings used to model what's left of the armature is kept constant, thus maintaining spatial resolution at the end of the generator run when the armature is quite short.

The flux linking each armature ring is made up of 3 parts; that due to the stator current, that due to the current of all the other rings, and that due to the current in the ring itself. Stator flux linking a ring is calculated by integrating the stator vector-potential around the ring (see eq. (1)). This line integral is done at a fixed z (the middle of the ring) so only involves the x and y components of the stator vector-potential given in eq. 3. Flux linking a ring due to the other rings is equivalent to a calculation of mutual inductance. Reference [7] describes how to calculate mutual inductance between coaxial circular filaments. We use the same procedure but include a second integration in z to include the effect of finite ring width. The self-flux of a ring is obtained in a similar fashion but the z -location of the 2 rings coincide. After all 3 of these calculations are performed we write the flux linking the i^{th} ring as,

$$\Phi_i = \Phi_i^{\text{stator}} + M_{i,j} \text{Jring}_j \quad (7)$$

Here M_{ij} is the mutual inductance matrix and Jring is the vector that represents the unknown values of the ring currents. Note that because we are dealing with rings, and not filaments, M_{ij} and Jring_j have units of

H-m and A/m, respectively. We now make the assertion that the armature currents exclude all the flux from the interior of each ring. After setting the LHS of eq. (7) to zero and inverting the matrix M_{ij} we can solve for the ring currents,

$$J_{ring_j} = -M_{i,j}^{-1} \Phi_i^{stator} \quad (8)$$

Using eq. (2) we can now calculate the vector potential at any point in space due to the armature,

$$A_{arm_x}(x,y,z) = \sum_i^{N_{rings}} \frac{\mu_o}{4\pi} \int_{z_{start_i}}^{z_{end_i}} \left(\int_0^{2\pi} \frac{-J_{ring_i} \sin\phi b_i d\phi}{\sqrt{(x-b_i \cos\phi)^2 + (y-b_i \sin\phi)^2 + (z-z_{ring_i})^2}} \right) dz \quad (9)$$

A similar expression can be found for the y-component. The vector b_i represents the radius of the rings and varies in time according to the expansion angle of the armature and detonation velocity of the high-explosive [8]. The z-component of the armature vector potential is zero because the armature currents that contribute to flux compression are purely azimuthal. Finally, we calculate the mutual inductance between the armature and stator by doing the following line integral around the stator,

$$M = \frac{\oint_{stator} A_{arm}(x,y,z) \circ dl}{I} \quad (10)$$

Again, we have checked the results of this approach against known solutions for simple cases of coaxial solenoids and find excellent agreement.

Stator Resistive Losses

Ohmic losses in the stator are calculated in a manner similar to that found in references [4 and 9]. A time-dependent effective 'skin depth' (δ) is calculated based on a characteristic time constant τ ,

$$\tau(t) = \frac{I}{\partial I / \partial t}, \quad \delta(t) = \sqrt{\frac{\tau(t)}{\sigma \pi \mu_o}} \quad \text{where } \sigma \text{ is the wire conductivity.} \quad (11)$$

$$R_{stator}(t) = \sum_i^{\# \text{ of Sections}} \int_{\phi_i^{init}(t)}^{\phi_i^{final}(t)} \frac{a(1 + \tan^2 \theta_i)^{1/2} d\phi}{n_i \pi r_w \delta_i(t) \sigma} \quad (12)$$

Here r_w is the wire radius, θ_i is the pitch angle of section i , and n_i is the number of wires in parallel for section i .

Armature Resistive Losses

Armature resistive losses take into account both the circuit current I which flows in the axial direction, and the inductive current that flows in the azimuthal direction around each armature ring.

$$R_{armature}(t) = \sum_i^{\# \text{ of Rings}} (1 + [\frac{J_{ring_i}(t)}{I/2\pi b_i(t)}]^2) \frac{\Delta z_i(t)}{\sigma 2\pi b_i(t) \delta(t)} \quad (13)$$

Ring length and radius are both time-dependent and are given by Δz and b , respectively.

Magnetic Diffusion Losses

Magnetic flux loss is not due solely to resistive losses. At any time t , inside the conductors that compose the armature and stator, there exists magnetic energy (not available for flux compression) that is roughly equivalent to the integrated power, or energy, that has been lost from joule heating. Only a solution to the magnetic diffusion equation, using the problem geometry, can give the exact relationship between lost magnetic energy due to flux diffusion, and lost magnetic energy due to resistive heating. However, it has been shown that for a wide range of geometries and time-dependent current profiles, flux loss due to these two mechanisms are nearly equal [10]. At the present time we estimate diffusion losses by assuming they are equal to the armature and stator resistive losses. We are currently working on a model that explicitly calculates flux loss due to both joule heating and magnetic diffusion. This model solves the magnetic diffusion equation using an approximate geometry and will also serve to introduce non-linear effects due to conductor heating.

The Circuit Equations

As with many previous flux compressor codes a circuit model is solved to predict the generator performance. The most important terms that go into the circuit model have already been discussed. The most general circuit we use for a FCG is shown below.

Figure 1 – Circuit schematic for a helical flux compressor generator.

Many of the helical FCGs built at Los Alamos involve the use of an air-core transformer [11]. The primary, secondary, and mutual inductance are measured and used as circuit parameters in the model. The following two coupled-circuit equations are solved to obtain the results.

$$\int_0^t \frac{I_p}{C} dt + (L_p + L_g(t) + L_{stray}) \dot{I}_p + I_p (R_g(t) + R_{stray} + R_p) + I_p \dot{L}_g(t) + M \dot{I}_s = 0 \quad (14)$$

$$M \dot{I}_p + (L_s + L_{load}) \dot{I}_s + I_s R_{load} = 0 \quad (15)$$

Here I_p and I_s refer to the primary and secondary currents, respectively.

Comparison with Experiments

The MARK 9 generator [12] provides a benchmark because results and all the important parameters needed to model the device have been published. The stator ID is 35.6 cm and the armature OD is 17.3 cm. The overall length is 112 cm with 4 sections having 5, 10, 20, and 40 parallel wires, respectively. Other specifications can be found in the reference. Figures 2 and 3 show a comparison of the experimental data (taken from the graphs in the reference) and the simulation.

Figures 2 and 3—A comparison of experiment and simulation of I and dI/dt for the Mark 9 generator with a 56 nH load.

The simulation uses all the parameters as listed in the original publication but includes an additional $1\text{ m}\Omega$ of load resistance. Without this additional resistance the agreement with experiment is still excellent until the last 15% of the generator run where the simulation diverges from the experimental data and peaks at 39 MA and 1×10^{12} A/sec. Though the agreement between experiment and theory as shown in figures 2 and 3 appears excellent, the sensitivity of the simulation to a small amount of stray resistance means that this result cannot be considered a validation of the model.

More recently using our computer model we have designed and tested a small generator. This generator has a stator radius of a 6.3 cm, an armature radius of 2.8 cm, and is 24 cm long. The stator has 4 sections with 1,2,4, and 8 parallel wires, respectively. This generator drives a 75 nH, 28 m Ω load. Comparisons between the model and experiment are shown in figures 4 and 5.

Figures 4 and 5 – A comparison of theory and experiment for the LANL small generator.

Only measured load parameters are used for the simulation. Simulation results are insensitive to any stray resistance due to the relatively high (28 m Ω) load resistance. For this reason, in contrast to the Mark 9 device, the small generator makes for a better test of the model.

In fact, we assert the following general statement. A good test of any generator model is to have it predict the performance of any generator driving a higher impedance load than would normally be optimum, thus making less important any anomalous losses that are either unknown or difficult to measure. It is possible, or even likely, to have good agreement between theory and experiment with a high impedance load and poor agreement with a low (or optimum) impedance load. A conclusion one can draw from such a circumstance is that major effects included in the model are correct but smaller effects, that play an important role with a low-impedance load, either are not included or are not calculated accurately enough.

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