Lattice Design for a 50 on 50-GeV Muon Collider

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Lattice Design for a 50 on 50-GeV Muon Collider *

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Abstract

Two modes are being considered for a 50 on 50-GeV muon collider: one being a high-luminosity ring with broad momentum acceptance (dp/p of ~ 0.12%, rms) and the other lower luminosity with narrow momentum acceptance (dp/p of ~ 0.003%, rms). To reach the design luminosities, the value of beta at collision in the two rings must be 4 cm and 14 cm, respectively. In addition, the bunch length must be held comparable to the value of the collision beta to avoid luminosity dilution due to the hour-glass effect. To assist the rf system in preventing the bunch from spreading in time, the constraint of isochronicity is also imposed on the lattice. Finally, the circumference must be kept as small as possible to minimize luminosity degradation due to muon decay. Two lattice designs will be presented which meet all of these conditions. Furthermore, the lattice designs have been successfully merged into one physical ring with mutual components; the only difference being a short chicane required to match dispersion and floor coordinates from one lattice to the other.

1 INTRODUCTION

After one $\mu^+$ bunch and one $\mu^-$ bunch have been accelerated to collision energy, the two bunches are injected into the collider ring, which is a fixed-field storage ring. Collider ring lattices have been developed for two of the collision energies: 100 GeV[1] and 3 TeV[2] in the center of mass. In addition, a preliminary interaction region for a 500 GeV lattice has been designed.

Three operational modes are proposed[1] for the 100-GeV collider, each requiring a different machine optics. The following sections discuss a 100-GeV collider lattice for two of the modes, the broad momentum (dp/p of 0.12%, rms) application and the monochromatic mode (dp/p of 0.003%), as well as comparison with a 3-TeV collider lattice.

2 COLLIDER LATTICES

Design criteria Stringent criteria have been imposed on the collider lattice designs in order to attain the specified luminosities. The first and most difficult criterion to satisfy is provision of an Interaction Region (IR) with extremely low $\beta^*$ values at the collision point consistent with acceptable dynamic aperture. The required $\beta^*$ values for the 100-GeV collider are 4 cm for the broad momentum-width case and 14 cm for the narrow-width case. For the 3 TeV machine, $\beta^*$ is only 3 mm. These $\beta^*$ values were tailored to match the longitudinal bunch lengths in order to avoid luminosity dilution from the hour-glass effect. Achieving this requirement in the 3 TeV lattice is complicated by the high peak beta-function values in the final-focus quadrupoles requiring 8-10 cm radial apertures. The correspondingly weakened gradients combined with the ultra-high energy make for a long final-focus structure. (In contrast, the lower energy and larger $\beta^*$ values in the 100 GeV collider lead to an efficient, compact final-focus telescope.) Compounding the problem, particularly for the 3-TeV design, is the need to protect superconducting coils from the decay products of the muons. Placing a tungsten shield between the vacuum chamber and the coils can increase the radial aperture in the 3-TeV quadrupole by as much as 6 cm, lowering still further available gradients. Final-focus designs must also include collimators and background sweep dipoles, and other provisions for protecting the magnets and detectors from muon-decay electrons. Effective schemes have been incorporated into the current lattices.

Another difficult constraint imposed on the lattice is that of isochronicity. A high degree of isochronicity is required in order to maintain the short bunch structure without excessive rf voltage. In the lattices presented here, control over the momentum compaction is achieved through appropriate design of the arcs.

A final criterion especially important in the lower-energy colliders, is that the ring circumference be as small as feasible in order to minimize luminosity degradation through decay of the muons. To achieve small circumference requires high fields in the bending magnets as well as a compact, high dipole packing-fraction design. To meet the small circumference demand, 8 T poletip fields have been assumed for all superconducting magnets, with the exception of the 3-TeV final-focus quadrupoles, whose poletips are assumed to be as high as 12 T. In addition, design studies for still higher field dipoles are in progress. rf system The rf requirements depend on the momentum compaction of the lattice and on the parameters of the muon bunch. For the case of very low momentum spread, synchrotron motion is negligible and the rf system is used solely to correct an energy spread generated through the impedance of the machine. For the cases of higher momentum spreads, there are two approaches. One is to make the momentum compaction zero to high order through lattice design. Then the synchrotron motion can be eliminated, and the rf is again only needed to compensate the induced energy spread correction. Alternatively, if some momentum compaction is retained, then a more powerful rf system...
is needed to maintain the specified short bunches. In either case, rf quadrupoles will be required to generate BNS [3, 4] damping of the transverse head-tail instability.

3-TeV lattice The 3-TeV ring has a roughly racetrack design with two circular arcs separated by an experimental insertion on one side, and a utility insertion for injection, extraction, and beam scraping on the other. The experimental insertion includes the interaction region (IR) followed by a local chromatic correction section (CCS) and a matching section. The chromatic correction section is optimized to correct the ring’s linear chromaticity, which is almost completely generated by the low beta quadrupoles in the IR. In designs of $e^+e^-$ colliders, it has been found that local chromatic correction of the final focus is essential, as was found to be the case here. The 3-TeV IR and CCS are displayed in Fig. 1.

100-GeV lattices In contrast, the 100-GeV ring geometry is highly compact and more complicated than a racetrack, but the lattice has regions with the same functions as those of the 3-TeV ring.

The need for different collision modes in the 100-GeV machine led to an Interaction Region design with two optics modes: one with broad momentum acceptance ($\Delta p/p$ of 0.12%, rms) and a collision $\beta$ of 4 cm, and the other basically monochromatic ($\Delta p/p$ of 0.003%, rms) and a larger collision $\beta$ of 14 cm. This lattice design, shown in Fig. 2 and Fig. 3, has a total circumference of about 350 m with arc modules accounting for only about a quarter of the ring circumference.

The low beta function values at the IP are mainly produced by three strong superconducting quadrupoles in the Final Focus Telescope (FFT) with pole-tip fields of 8 T. Because of significant, large-angle backgrounds from muon decay, a background-sweep dipole is included in the final-focus telescope and placed near the IP to protect the detector and the low-$\beta$ quadrupoles [5]. It was found that this sweep dipole, 2.5 m long with an 8 T field, provides sufficient background suppression. The first quadrupole is located 5 m away from the interaction point, and the beta functions reach a maximum value of 1.5 km in the final focus telescope, when the maxima of the beta functions in both planes are equalized. For this maximum beta value, the quadrupole apertures must be at least 11 cm in radius to accommodate $5\sigma$ of a 90 $\pi$ mm mrad, 50-GeV muon beam (normalized rms emittance) plus a 2 to 3 cm thick tungsten liner [6]. The natural chromaticity of this interaction region is about $-60$. 

Local chromatic correction of the muon collider interaction region is required to achieve broad momentum acceptance. The CC contains two pairs of sextupoles, one pair for each transverse plane, all located at locations with high dispersion. The sextupoles of each pair are located at positions of equal, high beta value in the plane (horizontal or vertical) whose chromaticity is to be corrected, and very low beta waist in the other plane. Moreover, the two sextupoles of each pair are separated by a betatron phase advance of near $\pi$, and each sextupole has a phase separation of $(2n + 1)\pi/2$ from the IP, where $n$ is an integer. The result of this arrangement is that the geometric aberrations of each sextupole are canceled by its companion while the chromaticity corrections add.

The sextupoles of each pair are centered about a minimum in the opposite plane ($\beta_{\text{min}} < 1$ m), which provides chromatic correction with minimal cross correlation between the planes. A further advantage to locating the opposite plane’s minimum at the center of the sextupole, is that this point is $\pi/2$ away from, or “out of phase” with, the source of chromatic effects in the final focus quadrupoles; i.e. the plane not being chromatically corrected is treated like the IP in terms of phase to eliminate a second order chromatic aberration generated by an “opposite-plane” sex-

![Figure 2: 4 cm $\beta^*$ Mode showing half of the IR, local chromatic correction, and one of three arc modules.](image)
In this lattice example, the CC was optimized to be as short as possible. The $\beta_{\text{max}}$ is only 100 m and the $\beta_{\text{min}} = 0.7$ m, giving a $\beta_{\text{ratio}}$ between planes of about 150, so the dynamic aperture is not compromised by a large amplitude-dependent tuneshift.

This large beta ratio, combined with the opposite-plane phasing, allows the sextupoles for the opposite planes to be interleaved, without significantly increasing the nonlinearity of the lattice. In fact, interleaving improved lattice performance compared to that of a non-interleaved correction scheme, due to a shortening of the chromatic correction section, which lowers its chromaticity contribution. The use of somewhat shallower beta-minima with less variation in beta through the sextupoles was also applied to soften the chromatic aberrations, although this caused a slight violation of the exact $\pi$ phase advance separation between sextupole partners. The retention of an exact $\pi$ phase advance difference between sextupoles was found to be less important to the dynamic aperture than elimination of minima with $\beta_{\text{min}} < 0.5$ m.

The total momentum compaction contributions of the IR, CC, and matching sections is about 0.04. The total length of these parts is 173 m, while that of the momentum-compaction-correcting arc is 93 m. From these numbers, it follows that this arc must and does have a negative momentum compaction of about $-0.09$ in order to offset the positive contributions from the rest of the ring.

The arc module is shown as a part of Fig. 2. It has the small beta functions characteristic of FODO cells, yet a large, almost separate, variability in the momentum compaction of the module which is a characteristic associated with the flexible momentum compaction module [7].

A very preliminary calculation of the dynamic aperture without optimization of the lattice nor inclusion of errors and end effects is given in Fig. 4. One would expect that simply turning off the chromatic correction sextupoles in the 4 cm $\beta^*$ mode would result in a linear lattice with a large transverse aperture. With only linear elements, the 4 cm $\beta^*$ optics showed to be strongly nonlinear with limited on-momentum dynamic acceptance.

A normal form analysis using COSY INFINITY showed that the tune-shift-with-amplitude was large, which was the source of the strong nonlinearity in the seemingly linear lattice. Numerical studies showed a similar dynamic aperture and tune-shift-with-amplitude terms. This ruled out the possibility that the dynamic aperture was limited by the low beta points in the local chromatic correction section and points to the IR as the source of the nonlinearity. Further analytical study using perturbation theory showed that the first-order contribution to the tune shift with amplitude is proportional to $\gamma_{\pi,\pi}$ and $\gamma_{\pi,\pi}$, which are large in this IR. These terms come from the nonlinear terms of $p_x/p_0$ and $p_y/p_0$, which, to the first order, equal the angular divergence of a particle. It was therefore concluded and later shown that the dynamic aperture of the more relaxed $\beta^*$ of 14 cm would not have the same strong nonlinearities due to the reduced angular terms. In fact, the tune shift with amplitude was less by an order of magnitude; hence the large transverse acceptance shown in Fig. 4 (dashed line).

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3 REFERENCES