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**Research and Development to Complete Construction
of a Setup for Determination of the Longitudinal
Profile and Duration of Ultrashort Bunches**

E.D. Gazazian, M.I. Ivanian, and E.M. Laziev

March 1995

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**"Research & Development to Complete Construction of a Setup
for Determination of the Longitudinal Profile & Duration of
Ultrashort Bunches"**

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March, 1995

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**RESEARCH AND DEVELOPMENT
TO COMPLETE CONSTRUCTION OF A SETUP
FOR DETERMINATION OF THE LONGITUDINAL PROFILE
AND DURATION OF ULTRASHORT BUNCHES**

FINAL REPORT

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ABSTRACT

Here are presented the results of our investigations on the construction of a setup for determination of the longitudinal charge distribution function and duration of ultrashort electron bunches via the measurement of the time-profile of the bunch-generated optical transition radiation (OTR) burst.

In the Theoretical Part of the Final Report are obtained conditions that determine the adequacy between the time-profile of OTR flash and bunch longitudinal charge distribution function with account of finiteness of the bunch cross-section and bandwidth of the receiver. The competing processes (bremsstrahlung, luminescence) as well as the boundary roughness effect are considered.

The Experimental Part presents the justification for the choice of the target - an OTR-emitter, a description of the measurement scheme, and the main characteristics of the units the setup consists of. An account is given of the OTR simulator and of the setup calibration method. Demands are formulated both to the setup as a whole and to its units. Experimental results are given that confirm correctness of the chosen method.

In the Appendix we consider alternative methods of solution of the problem posed.

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INTRODUCTION

The present Report closes the Second stage of work aimed at construction of a setup for the measurement of the charge distribution longitudinal function in the bunches and duration of these bunches. So long as we deal with ultrashort (≈ 1 ps) bunches, solution of this problem is not trivial at all.

There exist the well-known direct methods of measurement with the use of a streak-camera. The best specimens of streak-cameras have a time resolution of ≈ 1 ps and cannot directly be applied for our aims. There also exist the indirect methods which suggest the use of bunch radiation (the transition, Cherenkov, synchrotron radiations), the Compton scattering on moving bunches, etc.

To solve our problem we suggest to use the optical transition radiation (OTR).

Our choice is due to a number of reasons.

It is known that when a single point-like charge traverses the boundary of two media (say, vacuum - perfect conductor), then the TR flash will have a δ -shaped time dependence and infinitely wide uniform spectrum. (This statement is valid if we neglect dispersion in both media.).

The number of emitted quanta will be proportional to squared value of charge.

If the same boundary is crossed by a charged particle bunch with finite longitudinal and transverse dimensions, then the TR burst duration will be equal to the time interval within which the bunch traverses the boundary. The number of emitted quanta at every given instant of time will be proportional to squared number of charges contained in the bunch cross-section which traverses the boundary at the same moment of time. The radiation spectrum will be determined both by duration (extension) of the bunch and the charge distribution in it (i.e. by the bunch form-factor). Thus, the TR flash shape and spectrum will contain information both on the duration of the bunch and the charge distribution function in it.

The choice of TR optical range is not accidental. It is known that in this wavelength region the methods of measurement of ultrashort light pulses (USP) are developed and successfully used.

These methods, too, can be divided into direct and indirect ones.

When using the direct methods the streak-cameras are successful. However all the above-cited remarks concerning the time resolution are valid for this case, too.

The indirect methods are based chiefly on the measurement of high-order correlation functions. However these methods can only be used to measure the coherent radiation.

The OTR coherent part corresponds to that spectrum region where the linear dimensions of the bunch are much smaller than the radiation wavelength. It can be readily shown that in this case the OTR spectrum will be indifferent relative to the bunch form-factor. This means that the OTR measurements in this spectral region will not contain information on the bunch charge distribution function.

To solve this problem we suggested to use the following scheme (see Fig. In. 1).

The incoherent OTR light pulse in the wavelength range $\lambda_0 \pm \Delta\lambda$ arrives at a special device (expander) which executes time stretching of the initial pulse by a definite law. After that the broadened pulse arrives at a linear optical amplifier and then at the streak-camera. The photochronogram obtained at the streak-camera is transferred via the CCD TV-camera to the PC where the initial OTR pulse shape is recovered.

The present Report consists of two parts: theoretical and experimental.

In the theoretical part attention is primarily given to determination of the receiver bandwidth.

As the main criterion there serves adequacy of the charge distribution function mapping by the time shape (profile) of the OTR burst intensity. The choice of the detection band is determined by the bunch form-factor, the availability of roughness on the bunch boundary, etc. All these problems are studied in detail in Sections 1 and 2. In Section 3 we investigated

the influence of competing processes and ensured that in virtue of some peculiarities of TR (polarization, angular and spatial distributions, energy dependence) these effects can be neglected.

The second part of the Report presents experimental results. Here information is given on the manufactured targets (OTR emitters) and the controlled-band optical filter based on the commercial monochromator. A description is given of the OTR simulator based on a special laser manufactured by us for this purpose and a two-passage disperser. The setup calibration system is described and results of measurements of the pulse from the OTR simulator are given.

In the Appendix we consider the alternative methods of solution of the problem posed.

The authors are thankful to the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the US Department of Energy for supporting this work.

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The authors express their special gratitude to Prof. A. Sessler for very helpful discussions, for permanently kind attention and patience with which he endured the major and minor trouble and complications connected with this work.

THE BLOCK DIAGRAM

THEORETICAL PART

(Sections 1 - 3)

1. DETERMINATION OF THE BUNCH CHARGE DISTRIBUTION LONGITUDINAL FUNCTION BY THE TIME PROFILE OF THE OTR BURST

Abstract

Here we generalized the earlier obtained theoretical results [1,2] that determine the adequacy between the time dependence (envelope) of the OTR burst intensity and the bunch charge distribution function. It is assumed that the bunch has finite transverse dimensions and different charge distribution in different (along the bunch) cross-sections. The obtained results allow to moderate the demands to the choice of the bandwidth of the OTR receiver.

Introduction

When a single point-like charge moving at a relativistic velocity v crosses the boundary of two media, say, a vacuum-ideal conductor boundary, then the total number of emitted TR photons will be proportional to charge squared. The time dependence of the TR intensity will be δ -shaped, and the radiation spectrum will be infinite if we neglect the frequency dispersion of each of the media.

If the same boundary is traversed by a charged bunch with finite longitudinal and transverse dimensions, then the duration of the TR flash will be equal to the time interval during which the bunch traverses the boundary. The number of emitted photons in every given instant of time will be proportional to the square of charge in the cross-section which at the same time moment traverses the boundary. In other words, the shape of the TR burst will contain information on charge distribution in the bunch and its duration. Similarly the spectral TR distribution will be determined by the bunch form-factor. Under the form-factor we will imply both the geometrical dimensions of the bunch and the charge distribution in it.

In [2] it was proved that the adequacy of the longitudinal distribution function reflection by the time shape of the TR burst intensity will take place in the case when the bunch transverse dimensions are much smaller than its length. Ibidem were obtained conditions that determine a bandwidth of the receiver for the TR flash detection.

Below we give new results which allow to determine a bandwidth of detection and correspondence of the time dependence of the TR burst to the longitudinal charge distribution function for bunches with finite transverse dimensions and arbitrary charge distribution in the bunch cross-sections.

The Bunch TR Field

The results given in [2] were obtained in the assumption that the three-dimensional bunch charge distribution function $Q(\vec{r})$ can be presented as a product

$$Q(\vec{r}) = Z(\tilde{z})R(r, \tilde{\varphi}) \quad (1.1)$$

where $Z(\tilde{z})$ and $R(r, \tilde{\varphi})$ respectively are the longitudinal and transverse mutually independent charge distribution functions; $r, \tilde{\varphi}, \tilde{z}$, - are coordinates of the cylindrical coordinate system connected with the conventional axis of the bunch (Fig. 1. 1).

Such an interpretation implies that the charges in all the cross-sections are distributed by the same law. Although this assumption is a generally-adopted one (see, e.g. [1]), nevertheless it is evident that in this case the range of considered bunches is substantially limited. Below we will prove that this restriction is unnecessary.

Under the longitudinal function we will imply the probability density of the charge location in the given point of the bunch cross-section along its axis:

$$Z(\tilde{z}) = \int_0^{\infty} \int_0^{2\pi} Q(\tilde{r}) r dr d\tilde{\varphi} \quad (1.2)$$

We will consider the general case for oblique traverse through the boundary. A trajectory of a bunch (axis $\tilde{z}$) moving towards the boundary at a velocity v makes an angle ψ with the z axis normal to the boundary.

The OTR is detected in the "far zone" in the frequency band $\omega_0 - \Omega \leq \omega \leq \omega_0 + \Omega$ determined by the receiver. Assuming that the boundary is perfectly planar we can write the OTR field in vacuum as a product of the amplitude function $\bar{U}(\bar{R})$ for a single particle into the bunch form-factor $F(L)$, where $L = \frac{v}{c} R - vt$, c is the light velocity in vacuum, t is the time:

$$\bar{E}(\bar{R}, t) = N \bar{U}(\bar{R}) F(L) \quad (1.3)$$

where N is the number of particles in the bunch.

As is known [3], the amplitude function $\bar{U}(\bar{R})$ in the oblique entrance case can be decomposed into two components: parallel (l) and normal (n) to the radiation plane, i.e.

$$\bar{U}(\bar{R}) = \bar{U}^l(\bar{R}) + \bar{U}^n(\bar{R}). \quad (1.4)$$

At perfectly conducting planar boundary

$$\begin{aligned} U^l(\bar{R}) &= -\frac{ev_z v}{\pi \delta c^2 R} \frac{\sin \theta - \frac{v_x}{c} \cos \varphi}{\delta}, \\ U^n(\bar{R}) &= \frac{ev_z v}{\pi c^2 R} \frac{\frac{v_x}{c} \cos \theta \sin \varphi}{\delta}, \\ \delta &= (1 - \frac{v_x}{c} \sin \theta \cos \varphi)^2 - \frac{v_z^2}{c^2} \cos^2 \theta, \end{aligned} \quad (1.5)$$

where is used a spherical coordinate system (R, θ, φ) connected with the radiation centre and axis z normal to the boundary (Fig. 1.1).

The time dependence of the TR field is determined by the bunch form-factor $L = \frac{v}{c} R - vt$

$$F(L) = \int_0^{\infty} \int_0^{2\pi} W(L', r, \tilde{\varphi}) r dr d\tilde{\varphi}, \quad L' = L - \mu, \quad (1.6)$$

where

$$W(L', r, \tilde{\varphi}) = \int_{-\infty}^{\infty} Q(\tilde{r}) \frac{\sin \frac{\Omega}{v} (L' - \tilde{z})}{L' - \tilde{z}} \exp \left[i \frac{\omega_0}{v} (L' - \tilde{z}) \right] d\tilde{z} \quad (1.7)$$

$$\mu = \frac{r}{\cos \varphi} \left[\left(\frac{v}{c} \sin \theta \cos \varphi - \sin \psi \right) \cos \tilde{\varphi} + \frac{v}{c} \sin \theta \sin \varphi \sin \tilde{\varphi} \cos \psi \right]. \quad (1.8)$$

As it follows from (1.8), parameter μ is determined by the difference in the path of beams between the initial points of radiation in the cross-section of the bunch traversing the boundary, i.e. parameter μ is stipulated by finite transverse dimensions of the bunch (see Fig. 1.1 and the Appendix to this section).

Radiation Field Detection

The OTR detection is performed in the optical region of spectrum; however the time profile of the OTR burst is ultimately a determined quantity. Hence requirements to the receiver must be formulated proceeding from adequacy of the OTR flash time profile to the bunch longitudinal charge distribution function. Here we will study in detail the problem of determination of the receiver's (detector's) bandwidth.

In [2] we assumed that the signal detection takes place in the band $\omega_0 - \Omega \leq \omega \leq \omega_0 + \Omega$; note that we required the fulfillment of narrow-bandness condition, i.e. $\Omega/\omega \ll 1$. Below we will show that this condition is not strictly necessary.

We present function $W(L', r, \tilde{\varphi})$ as two summands:

$$W(L', r, \tilde{\varphi}) = A + iB, \quad (1.9)$$

where

$$A = \int_{-\infty}^{\infty} Q(\tilde{r}) \frac{\sin \frac{\Omega}{v} (L' - \tilde{z})}{L' - \tilde{z}} d\tilde{z}, \quad (1.10)$$

$$B = \int_{-\infty}^{\infty} Q(\tilde{r}) \frac{\sin \frac{\omega_0}{2v} (L' - \tilde{z})}{(L' - \tilde{z})/2} \sin \frac{\Omega(L' - \tilde{z})}{v} \exp \left[i \frac{\omega_0(L' - \tilde{z})}{2v} \right] d\tilde{z}. \quad (1.11)$$

The first term (1.10) which does not contain central frequency ω_0 of the detection band may conventionally be called a "low-frequency" component of the signal, and the second term - "high-frequency" component. We pay attention at the circumstance that the second term tends to zero if:

- a. $\Omega \ll \omega_0, \omega_0 \rightarrow \infty$;
- b. $\Omega \rightarrow \infty, \omega_0 = \text{const}$;
- c. $\Omega \rightarrow \infty, \omega_0 \rightarrow \infty$.

In the last two cases the first term in (1.10) will be expressed directly through the three-dimensional distribution function:

$$A = Q(r, \tilde{\varphi}, L') \quad (1.12)$$

which coincides with the result obtained in [1] at infinitely wide detection band.

It follows from stated above that to determine the detector's bandwidth we must proceed from two conditions:

- smallness of parameter μ in expression (1.10);
- undistorted recovery of the OTR burst envelope.

As was shown in [2], the first condition will be fulfilled if the following inequality is satisfied:

$$\frac{\Omega}{v} a \sin \theta' (1 + \lg \psi) \ll 1 \quad (1.13)$$

where θ' is the angle between the direction of "geometrooptical reflection" of the bunch trajectory and the direction of observation, a is the maximum transverse dimension of the

bunch. This condition is a consequence of difference in the path of the rays that form the OTR field at finite transverse dimensions of the bunch. This condition contains no demands to the central frequency and limits the receiver bandwidth from above.

The second condition limits the bandwidth from below and is a result of the well-known "relation of uncertainties" in the signal theory:

$$\Omega\tau \geq 2 \quad \text{or} \quad \frac{\Omega}{v}\tilde{d} \geq 2 \quad (1.14)$$

where $\tilde{d}$ is the bunch effective length which at oblique entrance is somewhat larger than its real length d : $\tilde{d} = d + 4atg\psi$.

Now we will demonstrate the above-cited conditions (1.13) and (1.14) on the examples.

First we note that in the ultrarelativistic case the radiation is concentrated mostly in the direction of angles $\theta' \approx 1/\gamma$, where γ is the Lorentz factor. At $\psi = 45^\circ$, $a = 10^{-4}$ m, $d = 10^{-7}$ m these inequalities give:

$$\Omega v \ll 0.5\gamma \cdot 10^7 \text{ m}^{-1} \quad \text{and} \quad \Omega v \gg 2 \cdot 10^4 \text{ m}^{-1}. \quad (1.15)$$

Thus at $\gamma > 200$ and wavelength $\lambda \approx 500$ nm corresponding to frequency ω_0 , and the receiver's bandwidth in the range from 4 nm to 40 nm the adequacy of recovery of the bunch longitudinal profile to the OTR burst intensity envelope is provided in the assumption that the dispersive properties of the metal do not substantially distort the field pattern.

Under fulfillment of conditions (1.15) the TR field envelope in the frequency range $\omega_0 - \Omega \leq \omega \leq \omega_0 + \Omega$ is to be written in the form:

$$\vec{E}(\vec{R}, t) = \vec{U}(\vec{R})\tilde{Z}(L), \quad (1.16)$$

where

$$\tilde{Z}(L) = \int_{-\infty}^{\infty} Z(z) \frac{\sin \frac{\Omega(L-z)}{v}}{L-z} dz. \quad (1.17)$$

So long as the receiver measures the bunch OTR flash instantaneous power $I(\vec{R}, t)$ that arrives at some surface with angular dimensions $\theta < \theta' < \theta_0$, then

$$I(\vec{R}, t) = \frac{cR^2 \tilde{Z}^2(L)}{2} \int_0^{\theta_0} \{ |U'(\vec{R})|^2 + |U''(\vec{R})|^2 \} \sin\theta d\theta' \quad (1.18)$$

and the measured OTR signal power will be proportional to squared envelope of the longitudinal bunch charge distribution function.

Conclusions

The representation of the OTR field in the frequency band in the form of the "high-frequency" and "low-frequency" terms makes it possible to substantially moderate the demands to the signal "quasi-monochromaticity". This means that using a receiver with a wider band we thereby enlarge the number of detected photons and can reduce demands to the receiver's sensibility. In that case the restrictions on the bandwidth are imposed proceeding from the observed for the given metal values of dependence of its optical constants on frequency in the given band. For example, for silver (Ag) such a dependence can be neglected in the interval 500 ± 20 nm. The representation of the charge distribution in the form of (1.1)

allows to remove the problem of beam broadening which was considered in [2].

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Appendix to Section 1.

Account of Phase Shifts in Description of TR Field of Bunch

The phase factor of the time Fourier-component of the bunch TR field in vacuum can be presented in the form

$$\Phi = \exp \left[i \frac{\omega}{v} \left(\frac{v}{c} R' - vt' \right) \right], \quad (\text{Ap. 1})$$

where R' is the distance from the point at which the particle intersects the boundary surface to the point of observation, t' is the instant of time of the particle radiation arrival at the point of observation.

The bunch particle distribution function introduced in Section 1 describes the bunch particle distribution density in the coordinate system connected with the bunch and does not depend on its spatial orientation. We assume that all particles in the bunch move at constant velocity in parallel rectilinear trajectories. In that case the trajectory of one of the particles can be chosen as an arbitrary axis of the bunch and to describe the particles in the bunch we can introduce a Cartesian $(\tilde{x}, \tilde{y}, \tilde{z})$ or cylindrical $(r, \tilde{\varphi}, \tilde{z})$ coordinate system connected with the chosen axis ($\tilde{x} = \tilde{y} = 0$ or $r = 0$).

The orientation of the chosen axis coincides with the direction of velocity $\vec{v}$ and the bunch axis makes an angle φ with the normal to the boundary surface. The coordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$ will unambiguously be determined if the axis $\tilde{y}$ is directed parallel to the boundary surface.

The observation point is described in the Cartesian system (x, y, z) or in connected with it spherical coordinate system (R, θ, φ) (see Fig. 1.1). Axes x and y make a plane of boundary surface. Axis y is parallel to axis $\tilde{y}$, axis z is perpendicular to boundary surface and its positive direction coincides with the direction of motion of the velocity $\vec{v}$ projection on this axis. If the centre of the introduced system coincides with the radiation point of the bunch central particle ($r = 0$), then the coordinate systems (x, y, z) and $(\tilde{x}, \tilde{y}, \tilde{z})$ turn into each other at rotation around axis y (or $\tilde{y}$) by an angle of ψ . The unit vectors directed along the axes of the introduced Cartesian systems are connected by relations

$$\tilde{x} = \tilde{x} \cos \psi + \tilde{z} \sin \psi, \quad \tilde{y} = \tilde{y}, \quad \tilde{z} = \tilde{z} \cos \psi - \tilde{x} \sin \psi. \quad (\text{Ap.2})$$

The location of the particle being in the given cross-section of the bunch $\tilde{z} = \text{const}$ is described by the two-dimensional radius-vector $\tilde{r}$:

$$\tilde{r} = r(\tilde{x} \cos \tilde{\varphi} + \tilde{y} \sin \tilde{\varphi}). \quad (\text{Ap.3})$$

The point of boundary intersection by the same particle is described by the radius-vector $\vec{\rho}$ with coordinates (x, y) or (ρ, φ') (angle φ' is counted off from the axis x):

$$\vec{\rho} = \rho(\tilde{x} \cos \varphi' + \tilde{y} \sin \varphi'). \quad (\text{Ap.4})$$

The obtained expression can using (Ap.2) be written in the form of expansion in unit vectors of the Cartesian system connected with the bunch:

$$\vec{\rho} = \rho(\tilde{x} \cos \varphi' \cos \psi + \tilde{y} \sin \varphi' + \tilde{z} \cos \varphi' \sin \psi). \quad (\text{Ap.5})$$

It is evident (see Fig. 1.1) that the time lag (or advance) of the given particle of the bunch relative to its central particle located in the same cross-section is proportional to the projection ρ_z of vector $\vec{\rho}$ on axis $\tilde{z}$:

$$\Delta t = \frac{\rho_z}{v} = \frac{\rho}{v} \cos \varphi' \sin \psi. \quad (\text{Ap.6})$$

With account of $\rho_z = r_{\tilde{z}}$ and $\rho_y = r_{\tilde{y}}$ from (Ap.3) and (Ap.5) we obtain a relationship between coordinates (ρ, φ') of the point on the boundary and coordinates $(r, \tilde{\varphi})$ of the corresponding point in the cross-section of the bunch:

$$\rho \cos \varphi' = \frac{r \cos \tilde{\varphi}}{\cos \psi}, \quad \rho \sin \varphi' = r \sin \tilde{\varphi}. \quad (\text{Ap.7})$$

Substituting the expression for $\rho \cos \varphi'$ into (Ap.6) we have

$$\Delta t = \frac{r}{v} \tan \psi \cos \tilde{\varphi}. \quad (\text{Ap.8})$$

From (Ap.8) it follows, in particular, that at $\psi = 0$ (normal entrance) the time difference turns to zero - in this case all particles of the given cross-section of the bunch traverse the boundary simultaneously.

So long as the bunch has finite transverse dimensions, there takes place path-length difference between the optical paths of the rays corresponding to the initial radiation points for different particles. For the particle with coordinates $(r, \tilde{\varphi})$ this difference can be written relative to the central particle (at $R \gg \rho$) in the form:

$$\Delta R = \rho \cos(\hat{R}\vec{\rho}) = \rho \frac{\vec{R}\vec{\rho}}{R\rho}, \quad (\text{Ap.9})$$

where $\vec{R}$ is the observation point radius-vector coming from the radiation point on the boundary surface corresponding to the bunch central particle:

$$\vec{R} = R(\tilde{x} \sin \theta \cos \varphi + \tilde{y} \sin \theta \sin \varphi + \tilde{z} \cos \theta). \quad (\text{Ap.10})$$

Using relations (Ap.10) and (Ap.4) we obtain

$$\Delta R = \rho(\sin \theta \cos \varphi \cos \varphi' + \sin \theta \sin \varphi \sin \varphi') \quad (\text{Ap.11})$$

or using relations (Ap.7):

$$\Delta R = \frac{r \sin \theta}{\cos \psi} (\cos \varphi \cos \tilde{\varphi} + \sin \varphi \sin \tilde{\varphi} \cos \psi). \quad (\text{Ap.12})$$

Thus, the phase difference in the time Fourier-component of the TR field of the bunch particle with coordinates $(r, \tilde{\varphi})$ and its central particle is produced by the trajectory increment $v\Delta t$ owing to the bunch oblique entrance and the path-length difference of the rays coming from the boundary, ΔR .

Hence for the planar boundary the phase parameters R' and t' of the arbitrary particle in the bunch can be expressed through parameters R, t of the central particle and their increment $\Delta R, \Delta t$ which are functions of the particle coordinates $r, \tilde{\varphi}$:

$$R' = R + \Delta R, \quad t' = t - \Delta t. \quad (\text{Ap.13})$$

The phase factor (Ap.1) is now presented as

$$\Phi = \exp \left[i \frac{\omega(L - \mu)}{v} \right], \quad (\text{Ap.14})$$

where (see (1.8))

$$L = \frac{v}{c} R - v t, \quad (\text{Ap.15})$$

$$\mu = \frac{v}{c} \Delta R - v \Delta t = \frac{v r \sin \theta}{c \cos \psi} (\cos \varphi \cos \tilde{\varphi} + \sin \varphi \sin \tilde{\varphi} \cos \psi) - r \cos \tilde{\varphi} \operatorname{tg} \psi$$

In the direction of geometrooptical reflection of the bunch axis ($r=0$, $\varphi = \tilde{\varphi} = 0$) the parameter μ takes the value

$$\mu \rightarrow (v/c - 1) \operatorname{tg} \psi. \quad (\text{Ap.16})$$

i.e. turns to zero in the ultrarelativistic case ($\gamma \gg 1$).

(The Coordinate Systems)

Fig. 1.1.
($\mathcal{A}$ is the observation point)

2. THE INFLUENCE OF BOUNDARY ROUGHNESS ON THE TIME PROFILE OF THE BUNCH OPTICAL TRANSITION RADIATION BURST

Abstract

Here we considered the influence of the boundary inhomogeneities on adequacy of the mapping of the bunch charge longitudinal distribution by the shape of the TR burst. It is shown that at TR detection in a sufficiently narrow optical frequency band the distorting effect of radiated fluctuation field can be neglected.

Introduction

It is experimentally revealed [1] that if TR generates on a boundary with poorly treated surface, then the polarization degree of the observed radiation noticeably reduces and its spectral and angular distributions change. In this connection there arises a question: to what extent may the presence of inhomogeneities affect the adequacy of reproduction of the bunch charge longitudinal distribution by the time dependence of TR flash.

Earlier we considered the OTR of a point-like charge traversing the boundary with small and sloping roughness [2]. Ibidem were obtained conditions under which it is possible to obtain an undistorted pattern of OTR field space-time distribution.

Since that roughness has an impact on the time dependence of the bunch OTR flash, then our task is to determine the distorting action of the fluctuation fields.

Single Charge TR in the Presence of Roughness on the Boundary

In [2] we obtained expressions for a first approximation of perturbation theory for a single point-like charge OTR field due to the presence of small and sloping roughness on the boundary. With account of that approximation the time Fourier component of the OTR field in the "far zone" can be presented as

$$\begin{aligned} e'(\bar{R}, \omega) &= e'_0(\bar{R}, \omega) + e'_1(\bar{R}, \omega) \\ e''(\bar{R}, \omega) &= e''_0(\bar{R}, \omega) + e''_1(\bar{R}, \omega) \end{aligned} \quad (2.1)$$

where the superscripts refer to the components parallel ($\parallel$) and normal ($\perp$) to the radiation plane, and subscripts correspond to the zero (unperturbed) and the first (due to roughness) terms of expansion in the perturbation theory approximation.

We consider the general case of oblique entrance. The radius- vector $\bar{R}$ of the observation point comes from the radiation centre on the plane $z=0$ that approximates rough surface of the boundary $z = \xi(x, y)$. The same point relates to the spherical coordinate system $(R = |\bar{R}|, \theta, \varphi)$ (see. Fig. 1.1).

For the case of perfectly conducting flat boundary with roughness the zero-approximation corresponding to exact solution for ideal plane has the form

$$\begin{aligned} e'_0(\bar{R}, \omega) &= \frac{ev_z v}{\pi c^2 R} A'_0(\theta, \varphi) \exp(i \frac{\omega L}{v}), \quad A'_0(\theta, \varphi) = -(\sin \theta - \frac{v_x}{c} \cos \varphi) / \delta \\ e''_0(\bar{R}, \omega) &= \frac{ev_z v}{\pi c^2 R} A''_0(\theta, \varphi) \exp(i \frac{\omega L}{v}), \quad A''_0(\theta, \varphi) = \frac{v_x}{c} \cos \theta \sin \varphi / \delta, \end{aligned} \quad (2.2)$$

where $\delta = (1 - \frac{v_x}{c} \sin \theta \cos \varphi)^2 - \frac{v_z^2}{c^2} \cos^2 \theta$, $L = \frac{v}{c} R - vt$.

The roughness-induced fluctuation field, which is the first approximation is obtained in the form

$$\begin{aligned} e'_1(\bar{R}, \omega) &= \frac{e}{\pi v_z R} A'_1(\theta, \varphi, \omega) \exp(i \frac{\omega L}{v}), \\ e''_1(\bar{R}, \omega) &= \frac{e}{\pi v_z R} A''_1(\theta, \varphi, \omega) \exp(i \frac{\omega L}{v}), \end{aligned} \quad (2.3a)$$

$$\begin{aligned} A'_1(\theta, \varphi, \omega) &= X(k_{0x}, k_{0y}, \omega) \cos \varphi + Y(k_{0x}, k_{0y}, \omega) \sin \varphi, \\ A''_1(\theta, \varphi, \omega) &= -X(k_{0x}, k_{0y}, \omega) \sin \varphi + Y(k_{0x}, k_{0y}, \omega) \cos \varphi, \end{aligned} \quad (2.3b)$$

where

$$k_{0x} = \frac{\omega}{c} \sin \theta \cos \varphi, \quad k_{0y} = \frac{\omega}{c} \sin \theta \sin \varphi, \quad (2.4)$$

$$X(k_{0x}, k_{0y}, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_x \xi(k_{0x} - k_x, k_{0y} - k_y) dk_x dk_y, \quad (2.5a)$$

$$Y(k_{0x}, k_{0y}, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_y \xi(k_{0x} - k_x, k_{0y} - k_y) dk_x dk_y, \quad (2.5b)$$

$$P_x = i \frac{\omega}{c \delta} \left\{ \frac{\omega}{c} [(k_{0x} - k_x)g + f] + k_{0x} \tilde{q} \right\}, \quad (2.5c)$$

$$P_y = i \frac{\omega}{c \delta} \left\{ \frac{\omega}{c} (k_{0y} - k_y)g + k_{0y} \tilde{q} \right\}. \quad (2.5d)$$

Above we introduced notations:

$$g = \frac{v_z}{c} - \frac{v_x}{c} \frac{k_x}{\chi}, \quad f = \frac{v_x}{c} \left(\frac{\omega - k_x v_x}{v_z} \right), \quad \tilde{q} = \frac{k_x^2 + k_y^2}{\chi} - \frac{\omega - k_x v_x}{v_z}, \quad (2.6)$$

$$\tilde{\delta} = k_x^2 + k_y^2 + \left(\frac{\omega - k_x v_x}{v_z} \right)^2 - \frac{\omega^2}{c^2}, \quad \chi = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}$$

and ξ is a Fourier-image of roughness distribution function and has the form

$$\tilde{\xi}(k_x, k_y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \xi(x, y) \exp[-i(k_x x + k_y y)] dx dy \quad (2.7)$$

The expressions (2.5) as distinct from the corresponding expressions given in [2] do not contain derivatives of roughness distribution function in explicit form. They enter in (2.5) implicitly, which is confirmed by combinations of the type of $(k_{0(x,y)} - k_{(x,y)})$ and $\tilde{\xi}(k_{0(x,y)} - k_{(x,y)})$.

The asymmetry of the expressions of X and Y takes place even in the absence of roughness too and is explained by the fact that we consider the case of oblique incidence of the particle on the boundary.

From comparison of the zero (2.2) and the first (2.3) terms of expansion of the time Fourier-components of OTR field we can conclude on the difference in the nature of their frequency dependence - the unperturbed terms contain frequency in the exponent's index, the fluctuation fields described by the first expansion terms also possess extraexponential dependence. Precisely this circumstance leads to the distortion of time dependence of point-like charge OTR field in the presence of roughness.

Condition of Narrow Bandness of Detected Point-like Charge OTR Flash Signal

The far-zone fluctuation field due to roughness on the boundary in the detection band $\omega_0 - \Omega \leq \omega \leq \omega_0 + \Omega$ can be presented as an integral

$$e_1^{n,t}(\vec{R}, t) = \int_{\omega_0 - \Omega}^{\omega_0 + \Omega} \bar{e}_1^{n,t}(\vec{R}, \omega) d\omega. \quad (2.8)$$

If the bandwidth satisfies the monochromaticity conditions ($\omega_0 \rightarrow \infty$, $\Omega/\omega_0 \ll 1$), then the integrand in (2.8) is presented as a fast-oscillating exponent and an exponential factor slowly varying compared with it. The signal narrow bandness means small variation of a pre-exponential factor in the whole range of integration. This allows to replace the pre-exponential factor by its value at $\omega = \omega_0$ and place it outside the integral sign. The fluctuation field in that case is written as

$$\bar{e}_1^{n,t}(\vec{R}, t) = \frac{ev}{\pi v_z R} A_1^{n,t}(\theta, \varphi, \omega_0) \frac{\sin \frac{\Omega L}{v}}{L} \exp(i \frac{\omega_0 L}{v}). \quad (2.9)$$

It follows from (2.9) that the time dependence of the OTR field of a particle on the boundary with small and sloping inhomogeneities in a narrow frequency band will be identical to the time dependence of unperturbed OTR field of the same particle. This means that at detection of the OTR field in a relatively narrow frequency band the time profile of the OTR burst is not distorted.

Above we considered the behavior of the first term of expansion of the single charge OTR field in the approximation by perturbation theory which takes into account the influence of roughness on the OTR flash time dependence. The subsequent terms of OTR field expansion in the same approximation have the same structure - an integral within detected frequency interval which contains a slowly varying pre-exponential factor and a fast-oscillating exponent. (We will not present these expressions here because of their awkwardness.). Thus we can state that the adequacy of reproduction of charge distribution by the time dependence of the OTR flash shape is preserved for the considered type of roughness at signal detection in a narrow frequency band. The roughness parameters noticeably affect the amplitude distribution of the radiation field and its polarization characteristics.

Linear Bunch OTR Field

The results obtained for the point-like charge can be extended to the linear bunch case. Under the linear bunch we will imply a section of charged "filament" lying on the $\tilde{z}$ axis and moving along this axis at a velocity v . This approximation satisfactorily describes the bunches whose transverse dimensions are much smaller than their longitudinal ones.

In the linear bunch case all the particles traverse the boundary at the same point. This means that the time profile may undergo perturbation determined by the roughness distribution function $z = \xi(x, y)$ relative to that point. Although all the points lying on the boundary take part in formation of the fluctuation field, this field will be identical for all particles of the bunch. Hence, as follows from the above-considered point-like charge case, the time profile of the bunch OTR flash will not be distorted if the condition of signal detection narrow-bandness is fulfilled.

The linear bunch OTR field; with account of the first term in the perturbation theory approximation, can be written in the form

$$E'^n(\bar{R}, t) = N \frac{ev_z v}{\pi c^2 R} A'^n(\theta, \varphi, \omega_0) \hat{Z}(L) \quad (2.10)$$

where the amplitude factor

$$A'^n(\theta, \varphi, \omega_0) = A_0'^n(\theta, \varphi) + \frac{c^2}{v^2} A_1'^n(\theta, \varphi, \omega_0) \quad (2.11)$$

determines spatial distribution of the bunch OTR field, and

$$\hat{Z}(L) = \int_{-\infty}^{\infty} Z(z) \frac{\sin \frac{\Omega(L-z)}{v}}{L-z} \exp \left[i \frac{\omega_0(L-z)}{v} \right] dz \quad (2.12)$$

maps the bunch charge longitudinal distribution function.

The function $\hat{Z}(z)$, as shown above, at $\omega_0 \rightarrow \infty$ can be presented in the form

$$\hat{Z}(L) \rightarrow \tilde{Z}(L) = \int_{-\infty}^{\infty} Z(z) \frac{\sin \frac{\Omega(L-z)}{v}}{L-z} dz. \quad (2.13)$$

Bunch with Finite Transverse Dimensions

The pattern is considerably complicated for the bunch with finite transverse dimensions. It is perfectly clear that the charges located in different points of the given cross-section will traverse the boundary at different points of the boundary plane. We also note that these particles will traverse the boundary at different moments of time (see Fig. 1.1) because the bunch crosses the boundary plane at the angle ψ .

A particle with coordinates $r, \tilde{\varphi}$ in the bunch cross-section traverses the boundary at a point $\bar{\rho}$ with coordinates

$$\rho_x = r \cos \tilde{\varphi} \cos \psi, \quad \rho_y = r \sin \tilde{\varphi}. \quad (2.14)$$

With respect to the above-said, the amplitudes $A_1'^n$ will depend on the coordinates (2.14)

$$A_1'^n(\theta, \varphi, \omega_0, \bar{\rho}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D'^n(x, y) \xi(x + \rho_x, y + \rho_y) \exp[i(k_{0,x}x + k_{0,y}y)] dy dx, \quad (2.15)$$

where

$$D'^n(\bar{R}, t) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \begin{array}{l} P_x \cos \varphi + P_y \sin \varphi \\ -P_x \sin \varphi + P_y \cos \varphi \end{array} \right\} \exp[i(k_x x + k_y y)] dk_x dk_y. \quad (2.16)$$

The fluctuation OTR field of the bunch can be written as

$$E_1^{(n)}(\bar{R}, t) = \frac{Nev}{\pi v_z R} \int_{-\infty}^{\infty} \int_0^{2\pi} \int_0^{\varphi} A_1^{(n)}(\theta, \varphi, \omega_0, \bar{\rho}) Q(\bar{r}) \times \\ \times \frac{\sin \frac{\Omega(L - \tilde{z} - \tilde{\mu})}{v}}{L - \tilde{z} - \tilde{\mu}} \exp \left[i \frac{\omega_0(L - \tilde{z} - \tilde{\mu})}{v} \right] r dr d\tilde{\varphi} d\tilde{z}, \quad (2.17)$$

where integration is carried out over the bunch volume, $Q(\bar{r})$ is the bunch charge distribution function, $\bar{r} = \{\tilde{\varphi}, \tilde{z}, r\}$ are the bunch particle coordinates and

$$\tilde{\mu} = \mu + \frac{v}{c} \xi(\rho_x, \rho_y) \cos \theta. \quad (2.18)$$

Parameter μ characterizes phase shift and difference in the path of rays for particles in the bunch with finite transverse dimensions (1.8).

As follows from (2.17) and (2.18), the structure of the fluctuation OTR field of the bunch differs from the zero approximation by the presence of an additional term in the phase index of exponent $(v\xi \cos \theta/c)$ and by the dependence of coordinates of point $\bar{\rho}$ on the boundary surface.

We can, just like previously, make use of the condition of field "high-frequency" in (2.17) and express it through a "low-frequency" component

$$E_1^{(n)}(\bar{R}, t) = \frac{Nev}{\pi v_z R} \int_{-\infty}^{\infty} \int_0^{2\pi} \int_0^{\varphi} A_1^{(n)}(\theta, \varphi, \omega_0, \bar{\rho}) Q(\bar{r}) \frac{\sin \frac{\Omega(L - \tilde{z} - \mu)}{v}}{L - \tilde{z} - \tilde{\mu}} r dr d\tilde{\varphi} d\tilde{z}. \quad (2.19)$$

From (2.19) it follows that adequate mapping of longitudinal charge distribution function $Z(z)$ is provided if parameters $\tilde{\mu}$ and $\bar{\rho}$ are negligibly small or their variations along the entire cross-section of the bunch are insignificant.

The condition for parameter $\tilde{\mu}$ has the form

$$\frac{\Omega}{v} \tilde{\mu} \ll 1 \quad \text{at} \quad 0 \leq r \leq a, \quad (2.20)$$

where a is a maximum size of the bunch cross-section.

Using the estimates given in [2] for (3.20) we can write

$$\Omega[a \sin \theta' (1 + g \psi) + h \cos \theta] / v \ll 1, \quad (2.21)$$

where θ' is an angle between the directions of the observation point and "geometroptical reflection" of the bunch trajectory, θ is an angle between the z axis and observation point direction (see Fig. 1.1).

The condition (2.21) written for the rough boundary case differs from the similar condition for the perfectly smooth surface by the additional term $h \cos \theta$. As follows from (2.21), the presence of roughness as if enlarges the transverse size of the bunch. For the directions close to the cone of OTR field concentration this condition takes the form

$$\Omega[a(1+tg\psi)/\gamma+h\cos\psi]/v \ll 1. \quad (2.22)$$

At $\gamma \gg a(1+tg\psi)/h\cos\psi$ the roughness height becomes determinative. If $a = 100 \text{ nm}$, $h = 30 \text{ nm}$, $\psi = 45^\circ$ and $\gamma \gg 10$, then the crucial role in inequality (2.22) will be played by h .

For high values of γ and angles ψ not close to 90° the inequality (2.22) can be written in the form

$$\Omega h \cos\psi/v \ll 1 \quad (2.23)$$

and respectively at $\psi = 45^\circ$ and $\lambda = 500 \text{ nm}$ the bandwidth must satisfy the inequality $\Delta\lambda \ll 100 \text{ nm}$.

This condition is considerably less rigid than the demand of quasi-monochromaticity which was introduced in [2].

If we restrict ourselves to the approximation (1.1) for the bunch charge distribution function, then the condition (2.23) with conditions (1.13) and (1.14) prove to be sufficient for adequate mapping of the bunch longitudinal charge distribution function by the OTR time profile in the presence of roughness. In this case the fluctuation component of the OTR field can be written in the form

$$E_1^{i,n}(\vec{R}, t) = N \frac{ev}{\pi v_z R} S(\theta, \varphi) \tilde{Z}(L), \quad (2.24)$$

where

$$S(\theta, \varphi) = \int_0^{2\pi} \int_0^\pi R(r, \tilde{\varphi}) A_1^{i,n}(\theta, \varphi, \omega_0, \vec{\rho}) r dr d\tilde{\varphi},$$

and the function $Z(L)$ maps the time dependence of the bunch charge distribution provided that the distribution in every cross-section along the bunch is the same.

The use of the way of determination (1.2) for the longitudinal distribution function leads to an additional demand - small variation in the value of roughness in the region of bunch cross-section. This demand can be satisfied if, in particular, the average period of roughness l very much exceeds the dimensions of the bunch cross-section ($a \ll l$). In this case the OTR field will also be written in the form of (2.24) where now

$$S(\theta, \varphi) = \cos\psi A_1^{i,n}(\theta, \varphi, \omega_0, 0),$$

and the function $Z(L)$ has the sense of (1.2).

References

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- [2] E.D. Gazazian, M.I. Ivanian, E.M. Laziev & R.A. Avakian, A.E. Mailian, D.L. Oganessian, A.O. Vardanian, "R&D to Complete Construction of a Setup for Determination of the Longitudinal Profile and Duration of Ultrashort Electron Bunches (Intermediate Report)", LBL - , (1994).

3. THE COMPETING PROCESSES

The competing processes that occur on the boundary of two media when a charged bunch traverses the boundary are considered. It is shown that in virtue of some features of transition radiation (polarization, dependence on the energy of a particle incident on the boundary) these effects can be neglected.

In the general case the transition radiation is accompanied by the competing processes: bremsstrahlung and different kinds of luminescence.

Analyzing these processes and comparing the glow induced by them with the OTR glow we can unambiguously claim that the polarization, directivity and energy dependence of TR allow to reliably get rid of the background of competing processes.

The above statement is corroborated by a whole number of experimental studies which are reviewed in [1,2]. Below we will refer chiefly to the results given in those reviews.

In the experiment performed by P. Goldsmith and J. V. Jelley (see [1,2] and references therein) they observed the glow of the surface of Au, Ag and Al at their bombardment with the protons of 1 to 5 MeV energy. It was found that the detected glow was partially polarized, and the polarization degree was from 8% for gold to 20% for aluminum. The presence of polarization was used to identify the TR from other competing processes. The fact that the glow yield in that experiment increased proportionally with proton energy confirms that precisely TR was observed.

A more detailed study was performed by S. Mikhalyak who observed the glow occurring at irradiation of massive silver, aluminum and nickel targets with 60 keV electrons. The polarized component in his experiments achieved 90%. It was established that the target with a poorly treated surface produces unpolarized glow of pretty high intensity. The above-mentioned degree of polarization was attained after special chemical treatment and mechanical polishing. The experiment showed that the radiated energy grows linearly with electron energy.

In his studies W. Steinmann investigated the glow that occurred at bombardment of silver films with a 25 keV electron beam. To detect and analyze the radiation spectrum he used a quartz spectrograph with 10 nm resolution. A radiation was revealed which was characterized by a rather sharp peak on a wavelength of 330 nm.

In the experiments of R. W. Brown, P. Wessel and E. P. Trounson they used 50 nm thick silver films and 22 keV electrons incident normally on the film. This experiment also revealed a radiation with the above-cited spectral characteristics. The radiation angular distribution had a peak at 40° (the angle was counted off from the normal to the film directed toward the beam) falling practically down to zero at 0° and 90°.

The spectral distribution turned out to be essentially dependent on the angle of observation. At 30° the spectrum shape fits the one obtained in W. Steinmann's study, the radiation maximum drops at a wavelength of 340 nm.

All the experiments studied both the polarization and dependence of the radiated energy on the incident electron energy. The radiation, as a rule, always turned out practically polarized, and the polarization sign coincided with the one expected in TR theory. The polarization degree increased with electron energy, and in the region above 50 keV they observed practically 100% polarization. The deviation of linear dependence of glow yield on electron energy was observed mostly for low energies (to 30 keV) too.

All these allow to conclude that at low energies the role of bremsstrahlung and luminescence occurring in the surface film of the metal noticeably increases.

Indeed, the photon absorption coefficient for metals is high, and the glow induced by bremsstrahlung and luminescence will only emit from a thin surface layer of the metal. So long as the TR yield grows in proportion to incident particle energy γ , while the yield of luminescence and bremsstrahlung is inversely proportional to γ , then with growing electron energy the role of TR becomes predominant.

The studies of H. Boersch, C. Radloff and G. Sauerbrey were aimed to find out the dependence of the glow brightness on the target temperature and on the degree of the vacuum of the volume wherein the target located. They studied the glow of targets made of tungsten, tantalum, molybdenum, platinum, nickel, copper, aluminum, silver, cesium which were hit by an electron beam with the energy from 6 to 30 *keV*. The obtained results testify that the glow intensity does not change with vacuum variation from 10^{-5} to 10^{-9} Torr and does not depend on the target temperature.

As follows from the above brief review, the distinguishing peculiarities of TR (polarization, directivity, spectral distribution, energy dependence) allow one at high energies of the particle incident on the boundary to unambiguously get rid of the background caused by competing processes. Here we note that the polarized luminescence excitation [3] is impossible in our case.

References

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EXPERIMENTAL PART

(Sections 4 - 6)

4. Choice of Target Material and Thermal Conditions

In our previous Report [1] we considered in detail the physics of processes leading to ionization loss in the matter when it is traversed by relativistic electrons. In [1] we gave the expression (3.2) for ionization loss which is valid for electrons of energy higher than some hundreds *GeV*. Assuming that all ionization losses ultimately result in target heating, we can obtain from this formula that the heating of a target of about $40\ \mu\text{m}$ thickness by an electron bunch with 10^{12} electrons will make up a few shares of a degree. (Almost the same values are also obtained for electron energies of some tens *MeV* .). Note that the extent of heating for the whole class of metals (*Au, Ag, Al, Cu*) turns out nearly the same. Thus, the choice of target material is not critical from the viewpoint of heating.

5. OTR Emitter and Vacuum Chamber

By the present time we have manufactured a few targets of *Cu, Al* and *Ag* with systems of mounting in a vacuum tank and radiation exit to a receiver (detector). In Fig. 5.1 we present a photograph of the tank and one of the targets.

The controlled-band commercial monochromator is used as a light filter.

OTR EMITTER AND VACUUM TANK

Fig. 5. 1

6. The Measuring Device and Calibration

The main task at this stage of work was the development of the prototype - simulator of TR of about 1 ps duration which at the same time must play the role of the setup calibrator. By means of the mentioned simulator we investigated the processes of determination of the light pulse envelope, time broadening (expansion) of space-limited ultrashort pulses in linear dispersive medium of the expander (Fig. 6.1).

Realization of these problems was executed using the following units:

- a laser generating pulses lower than 1.5 ps ;
- a calibrator which comprises a cross-correlator allowing to determine the time profile and spectrum width of a laser pulse;
- a "picoross" that executes conversion of the laser coherent radiation into incoherent one (a TR flash simulator);
- expander, which represents a linear dispersive medium using which the light pulse time broadening is realized;
- a streak-camera with a resolution of $\approx 2\text{ ps}$;
- a system of information readout and its processing.

Below are given the main characteristics for each of the units.

Laser

As a radiation source generating picosecond coherent pulses there serves a specially developed by us picosecond laser on concentrated neodymium-doped phosphate glass with passive mode synchronization and inertia feedback.

The laser parameters are:

- wavelength $1.06\text{ }\mu\text{m}$;
- half-height pulse duration about 1 ps ;
- spectrum width 0.7 nm ;
- energy - $300\text{ }\mu\text{J}$.

Picoross

As was already shown in [1], the TR light pulse simulation is performed by transmitting the laser ultrashort pulse (USP) through the dye wherein stimulated Raman scattering (SRS) takes place. For pulses of a few picosecond duration the SRS is observed very seldom as an isolated process. As a rule, the SRS is accompanied by strong self-focusing which enhances the conversion efficiency, in which case the light beam decays into multiple "filaments". In dyes the SRS process can proceed simultaneously on several frequencies in every filament, irrespective of the others, and the superposition of beats of various SRS components ultimately results in a rather irregular structure. To this also contributes stimulated four-photon parametric scattering leading to the time expansion of the laser light pulse spectrum. As a result, the pulse at the dye's output represents incoherent superposition of a great number of pulses with modulated phases and different frequencies, while the time envelope of the pulse contains a large number of irregular spikes of different intensity and duration estimated as $\approx 0.1\text{ ps}$.

In our case we used a dye of "Rodamin-6G" type whose spectral width is not more than 15 nm , with a maximum intensity at a wavelength 580 nm .

Calibration System

Calibration of the laser is performed using a cross-correlator (Fig. 6.2) which is described below. The USP from the laser arrives through the telescope at the disperser wherein the initial USP is broadened. The broadened pulse $f(t)$ can be written in the form

$$f(t) = \int_{-\infty}^{\infty} f_0(\tau) \exp \left[i \frac{(t-\tau)^2}{2\beta_0^2 kz} \right] d\tau, \quad (6.1)$$

where $f_0(t)$ is the initial pulse, $k = 2\pi/\lambda$, z is the distance between the compressor's gratings, $\beta_0 = -2\pi mc/\omega_0^2 d \cos \theta_0$, m is the order of diffraction, ω_0 is the "carrier" frequency, d is the diffraction grating constant.

The broadened pulse splits into two parts, one of which propagates without change, and the other arrives at the amplitude transparent (diaphragm) by means of which the pulse $F(t)$ of a given shape and given spectrum is formed (Fig. 6.2)

$$F(t) = \text{sinc}(l/2\beta_0 z) \exp(i\omega t) \quad (6.2)$$

where l is the diaphragm slot width.

Then the two pulses $f(t)$ and $F(t)$ come to a nonlinear crystal LiJO_3 at the phase synchronism angle α_0 where noncollinear second-harmonic (SH) generation takes place [3].

The SH radiation from the nonlinear crystal output arrives at a properly oriented spectrograph (diffraction grating with a dispersion parameter $\beta = -2\pi cm/4\omega_0^2 d \cos \theta_0$) which is at a distance L from the nonlinear crystal. The transverse radiation energy distribution on the spectrograph's output corresponds to the dynamic spectrogram of broadened pulse $f(t)$ (see Fig. 6.3).

The SH energy distribution along the x axis of the dynamic spectrogram corresponding to the time scale will have the form

$$W(x, 0) = \int_{-\infty}^{\infty} \text{sinc} \left(\frac{l\xi}{2\beta_0 z} \right) |f(\xi - x(T_0 - T))|^2 \xi, \quad (6.3)$$

where $T_0 + T = L(2\sin\alpha + \cos\alpha \tan\psi)/u$, ψ is the slope angle between $f(t)$ and $F(t)$, u is the group velocity, s is the distance from the spectrograph to the CCD TV-camera.

Formula (6.3) expresses cross-correlation of the pulse of a given shape with envelope of the broadened pulse. By measuring the cross-correlation function we can restore the broadened pulse envelope.

The SH energy distribution along the y axis of the dynamic spectrogram corresponding to the frequency scale has the form

$$W(0, y) \approx \left| F \left(\frac{y}{s\beta} - \omega_0 \right) \right|^2, \quad (6.4)$$

where $F(\omega)$ coincides with the broadened pulse spectrum:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(i\omega t) dt.$$

With account of the broadened pulse narrow-bandness, from (6.4) it follows that the inverse dispersion along the y axis is

$$\frac{|\Delta y|}{\delta \lambda} = \frac{2\pi s \beta c}{\lambda_0^2} 10^{-7} \frac{mm}{A}. \quad (6.5)$$

Here it is evident that the equation of the line going through identical values of energy distribution (6.3) falling at different wavelengths (6.4) corresponds to the dependence $\lambda(t)$, which allows to determine the phase of $f(t)$. This means that the shape of the initial USP is determined from the expression (6.1) with respect to the knowledge of phase and shape of the broadened pulse.

Recording of dynamic spectrogram was performed using the CCD TV-camera the signal from which through appropriate interfaces arrived at PC for processing.

Thus, at the cross-correlator output we can determine both the envelope of the broadened pulse and its spectrum corresponding to the pulse energy distribution along two mutually perpendicular axes in (x, y) plane. The condition of narrow-bandness of the broadened pulse also allows to determine the phase spectrum of $f(t)$.

Broadening of Spatially Limited TR USP in the Expander

Spatial limitedness of real TR light pulses introduces new phenomena to the process of their propagation and conversion by optical systems. As was shown in [1], under the action of TR USP on the input of linear dispersive medium the intensity of the output broadened pulse is the result of convolution of the function of the light pulse envelope with the modulus of the disperser pulse characteristic. At the same time, when the light pulse passes through the disperser (the diffraction grating, prism) we observe a spatial shift of light pulse spectral components. This takes place when the disperser's angular dispersion is equivalent to dispersion of group velocity. This means that both the modulus of pulse characteristic of the linear dispersive medium and the intensity of the output broadened pulse are functions of the spatial coordinates. This circumstance leads to undesirable dependence of results of measurements of the time profile of the broadened light pulse on the spatial coordinate. In particular, when the distance traveled by the light pulse in the grating's disperser is much less than the confocal parameter (σ^2/λ) , where σ is the light beam width, the broadened pulse spectrum at disperser's output is

$$E(x, y, \omega) = E_0(\omega) \exp(ik\beta_0^2 \omega^2 z / 2) \prod(\omega - \omega_0) \exp\left[-\frac{(x - \beta_0 \omega z)^2 + y^2}{\sigma^2}\right], \quad (6.6)$$

where $E_0(\omega)$ is the spectrum of input noisy USP, θ_0 is the diffraction angle, m is the diffraction order, $\beta_0 = -2\pi cm/\omega_0^2 d \cos \theta_0$, d is the grating constant, ω_0 is the "carrier" frequency, $\prod(\omega - \omega_0)$ is the grating transmission band.

As follows from (6.6), to each coordinate x corresponds its spectral component ω . Hence, by placing at disperser's output a transparent with transmission function $S[x(\omega)]$ we can execute spatial filtration of the broadened pulse spectrum. To remove the spatial shift of spectral components, the broadened pulse (6.6) should be transmitted through the second disperser similar to the first one, which will result in doubling of dispersive broadening of the pulse.

Thus, with account of stated above we will obtain at the output of the two-passage disperser a broadened pulse

$$E_{\Sigma}(x, \omega, t) = \int_{-\infty}^{\infty} E_0(\omega) \prod(\omega - \omega_0) S[x(\omega)] \exp(ix\beta_0^2 \omega^2 z) \times \\ \times \exp\left[-\frac{x^2 + y^2}{\sigma^2}\right] \exp(-i\omega t) d\omega \quad (6.7)$$

According to [1], the intensity of the output broadened pulse will be

$$|E_z(x, y, t)|^2 = \exp\left[-2\frac{x^2 + y^2}{\sigma^2}\right] \int |E_0(\tau)|^2 |h(t - \tau)|^2 d\tau, \quad (6.8)$$

where

$$h(t - \tau) = \int_{-\infty}^{\infty} \prod (\omega - \omega_0) S[x(\omega)] \exp i[(k\beta_0^2 \omega^2 z) - \omega t] d\omega \quad (6.9)$$

is a pulse characteristic of the disperser.

As one of the principal conclusions of the performed investigation we can state the following: to remove the undesired dependence of the result of measurement of the broadened pulse time profile as well as to obtain spectral filtration of the broadened pulse, it is necessary to use a two-passage disperser.

Streak-Camera. Detection and Processing

(Description and main characteristics are given in [1])

Experimental Results

The measurements were carried out by the methods described in [1,2] (Fig.6.1 and Fig.6.2).

The laser light pulse by means of a beam-splitting mirror was divided in a 1:4 ratio, and its minor part was directed to the cross-correlator which executed the light pulse calibration. Under the calibration here we imply the determination of the time shape of the light pulse and its spectral width. The major part of the pulse fell at the picoross wherein the conversion of the coherent radiation into incoherent one took place with simultaneous wavelength transposition. Maximum intensity at the picoross output fell at a wavelength of 585 nm and the spectrum width measured by the monochromator made up approximately 10.0 nm.

From the picoross output the light pulse simulating the OTR pulse arrived at the two-passage disperser wherein the pulse broadening with simultaneous spectrum filtration to 5 nm was realized. Then the broadened pulse traveled to the streak-camera. The obtained at the streak-camera's output photochronogram via the CCD TV-camera was transferred to a PC where the processing of obtained results was performed.

Fig. 6.3 presents a dynamic spectrogram obtained in the measurement of spectral-time characteristics of the USP of the laser based on concentrated neodymium phosphate glass. Measurements were carried out using the above-described cross-correlator. The obtained results are recorded on a three-dimensional spectrogram, where both the time profile and the spectrum of the laser USP are presented. The measurement results show that the USP half-width duration is in the range 1.2 - 1.4 ps, and the spectrum width is $\Delta\lambda = 0.64 - 0.67$ nm.

Fig. 6.4 gives the results of convolution of the USP spectrum with the apparatus function of the spectrograph and presents the time profile of the laser USP. One can see from Fig. 6.3 that the pulse duration of the driver laser does not exceed 1.5 ps.

Fig. 6.5 presents results of photometry of the photochronogram of the broadened pulse of the OTR USP simulator from the streak-camera.

Fig. 6.6 shows the time profile of the recovered pulse. The obtained results testify that the broadening of USP which simulates the OTR flash is a factor of 8 at the recovered USP duration about 2 ps.

We performed preliminary study of the linear light amplifier and obtained that the own noise level corresponds to $5 \times 10^5 - 10^6$ photons in the broadened pulse.

In view of that we consider a possibility to exclude the amplifier from the scheme and to use another method.

To do that, we will give to the input window of the streak-camera a supporting, high-stability continuous laser light along with the broadened OTR pulse.

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THE BLOCK DIAGRAM

Fig. 6.1

CROSS-CORRELATOR

Fig. 6. 2.

DYNAMIC SPECTROGRAM

$$\frac{dV}{dt} = 0.703 \text{ mm/mm}$$

$$\frac{dV}{dx} = 0.381 \text{ ps/mm}$$

Fig. 6.3

Fig. 6.4

PHOTOCHRONOGRAM OF BROADENED PULSE

Fig. 6. 5

RECOVERED PULSE SHAPE

Fig. 6. 6

CONCLUSION

The results given in the Final Report unambiguously confirm the possibility to determine the bunch charge longitudinal distribution by the time profile of the OTR burst.

One of the most important questions is the problem of the receiver bandwidth optimization. This choice is conditioned by a number of factors. Of these the primal is the adequacy of mapping of the bunch charge longitudinal distribution by the time profile of OTR flash. The option of the receiver bandwidth is also affected by the bunch transverse dimensions and the presence of roughness on the boundary. It follows from the performed analysis that the bunch charge longitudinal distribution of a unity picosecond duration can be determined by the proposed method at the receiver's bandwidth of the order of 5 - 50 nm in a wavelength range nearly 500 nm.

From the consideration of the competing processes the OTR is accompanied by, we concluded that in the high-energy region ($\gamma > 100$) the background produced by these processes (all kinds of luminescence, bremsstrahlung) is insignificant, so these processes can be neglected.

Similarly, at high γ we can neglect the beam thermal losses in targets of a few micron thickness. The OTR characteristics are practically independent of material of the target which serves as the OTR emitter.

The setup prototype constructed in the process of work allowed to confirm experimentally the possibility to determine the time profile of incoherent optical pulse generated by the OTR simulator with duration from 1 ps to 15 ps.

A simulator of the OTR flash is constructed with the use of which the whole process of measurements was tested. A 1.5 ps pulse was by a factor of 8 broadened and shape shape was measured. The comparison of the recovered pulse shape with its original gives a good fit and corroborates correctness of the chosen methods.

An essential result of the performed study is the circumstance that they demonstrated inexpediency of use of the linear light amplifier and the possibility to exclude this element of the setup. We consider it more rational in the given case to apply the method of blending of the OTR pulse with the radiation of a supporting continuously operating high-stability laser.

The constructed prototype of the setup can already now be used for measurements on the beam.

APPENDIX 2
(Alternative Methods)

The Recovery of the Most Probable Longitudinal Profile and Duration of Bunches Using Transition Radiation Spectrum^{1,2}

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Abstract

The method is based on the relationship between the spectrum of the transition radiation (TR) excited by a train of electron bunches traversing a set of waveguides and the charge distribution function in the bunch. The measured spectrum is compared with the corresponding standard distribution spectra, and a conclusion on the most probable charge distribution function is made.

1. INTRODUCTION

The well-known TR can be used to determine the longitudinal profile and duration of the electron bunches [1-4].

Below we'll consider the method of determination of the charge distribution along the bunch. The method is based on the measurement of TR energy spectral distribution in the set of rectangular waveguides. Their dimensions are chosen such that the k -th waveguide cannot excite lower harmonics of bunches repetition rate where k is a harmonic's number. The experimentally measured dependence of radiation energy distribution over harmonics k is compared with the TR spectrum for standard distributions of charge along the bunch. According to chosen criteria a conclusion is made on the most probable shape of the longitudinal profile and duration of the bunch.

2. TR ENERGY LOSS IN A WAVEGUIDE

It is known that charge q while flying at a velocity v through a waveguide or cavity leaves part of its energy in the form of a radiation field. In the waveguide (cavity) structures two radiation mechanisms may be of practical significance: the TR and the Cerenkov radiation. Further we'll consider the radiators wherein the Cerenkov radiation conditions are not satisfied.

Let a charge q moving parallel to the y axis traverses an infinite rectangular waveguide in a direction parallel to wall b and normal to wider wall a .

The particle entrance coordinates are: $x_0 = a/2$, $y_0 = 0$, z_0 and escape coordinates: $x_1 = x_0$, $y_1 = b$, $z_1 = z_0$.

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In such a waveguide all the E_{mn} and H_{mn} waves with the odd indexes m will be excited. It is shown in [2] that energy loss for the H_{mn} wave radiation will be

$$W_{mn} = \frac{8q^2 \epsilon_m \pi^2 n^2}{a^3 b c^2 v^2 \lambda_{mn}^2} \operatorname{Re} \int_{-\infty}^{\infty} \frac{\sin^2 \left(\frac{\pi n}{b} - \frac{\omega}{v} \right) \frac{b}{2}}{\left[\left(\frac{\pi n}{b} \right)^2 - \left(\frac{\omega}{v} \right)^2 \right]^2} \omega^3 d\omega \quad (1)$$

where ϵ_m for $m \neq 0$ ($\epsilon_m = 1$), m and n are integers that determine the wave mode in the waveguide,

$$\lambda_{mn} = \pi \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^{1/2} \quad \text{and} \quad \gamma_{mn} = \left(\frac{\omega^2}{c^2} - \lambda_{mn}^2 \right)^{1/2}$$

As far as the waveguide is assumed to be empty, the Cerenkov radiation cannot arise in it. Then H_{mn} will be excited only by TR at the entrance and escape of the charge.

3. TR SPECTRUM IN A WAVEGUIDE

If the current carried by accelerated charge is written in the form of some time function $i_e(t)$, then the power radiated in the waveguide at frequency ω can be written as

$$P_\omega = I_\omega^2 R_{rad} \quad (2)$$

where I_ω is determined from Fourier expansion of function $i_e(t)$ and R_{rad} is "radiation impedance" of the waveguide [5].

It is quite evident that if $i_e(t) = i_e(t+T)$, i.e. represents a periodic train of bunches with a period T , then radiation will take place only at frequencies multiple of ω_b .

It is well known that if the complex spectrum (both of amplitudes and phases) is determined, then the time function shape can be recovered.

Function $i_e(t)$ describes charge distribution in a bunch; hence by measuring I_ω we can, in principle, solve the formulated problem. However the suggested method can be used to measure the amplitude spectrum only, so we can speak only about the most probable charge distribution function in a bunch and its length τ .

The value of R_{rad} can readily be found from (1). For H_{m0} it will be [5]

$$R_{rad} = \frac{4\pi\beta^2}{ab\omega} \sin \frac{\omega b}{2} \sum_{m=1}^M \frac{1}{\gamma_{m0}} \quad (3)$$

Now we consider a few standard charge distribution functions, using Fourier transformation.

a. Uniform distribution.

$$i(t) = \begin{cases} 0 & \dots \text{at} \dots t \leq -\tau/2 \\ I_0 & \dots \text{at} \dots -\tau/2 \leq t \leq \tau/2 \\ 0 & \dots \text{at} \dots t \geq \tau/2 \end{cases} \quad I(\omega\tau) = I_0\tau \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} \quad (4)$$

b. Triangular distribution.

$$i(t) = \begin{cases} 0 & \dots \text{at} \dots t \leq -\tau/2 \\ I_0(1 + \frac{2t}{\tau}) & \dots \text{at} \dots -\tau/2 \leq t \leq 0 \\ I_0(1 - \frac{2t}{\tau}) & \dots \text{at} \dots 0 \leq t \leq \tau/2 \\ 0 & \dots \text{at} \dots t \geq \tau/2 \end{cases} \quad I(\omega\tau) = \frac{I_0\tau}{2} \left(\frac{\sin \frac{\omega\tau}{4}}{\frac{\omega\tau}{4}} \right)^2 \quad (5)$$

c. Cosinusoidal distribution.

$$i(t) = \begin{cases} 0 & \dots \text{at} \dots t \leq -\tau/2 \\ I_0 & \dots \text{at} \dots -\tau/2 \leq t \leq \tau/2 \\ 0 & \dots \text{at} \dots t \geq \tau/2 \end{cases} \quad I(\omega\tau) = \frac{2I_0\tau}{\pi} \frac{\cos \frac{\omega\tau}{2}}{1 - \left(\frac{2}{\pi} \frac{\omega\tau}{2} \right)^2} \quad (6)$$

d. Normal distribution.

$$i(t) = I_0 \exp\left(-\left(\frac{t}{\tau_{eq}}\right)^2\right) \quad I(\omega\tau) = I_0\tau \exp\left(-\left(\frac{\omega\tau_{eq}}{2}\right)^2\right) \quad (7)$$

From (4)-(7) it follows, that if $\omega\tau \ll 1$, $I(\omega\tau) \approx \text{const}$ and frequency-independent. Hence: if we want to determine waveform of $i(t)$, we must measure the spectral function $I(\omega\tau)$ at high frequencies.

Any of the above-considered spectral functions can be expanded into a Taylor series

$$I(\omega\tau) \approx 1 - \alpha_1(\omega\tau)^2 + \alpha_2(\omega\tau)^4 - \dots \quad (8)$$

If we now put $\omega = k\omega_0$, then (8) can be rewritten in the form

$$I(k) \approx 1 - \alpha_1(k\tau)^2 + \alpha_2(k\tau)^4 \quad (9)$$

Now assume that we measure the spectral function $I(k)$ at three frequencies corresponding to harmonics k_1, k_2, k_3 . Restricting ourselves to three terms of expansion (9) we'll obtain a system

$$\begin{aligned} I_1 &\approx 1 - \alpha_1(k_1\tau)^2 + \alpha_2(k_1\tau)^4 \\ I_2 &\approx 1 - \alpha_1(k_2\tau)^2 + \alpha_2(k_2\tau)^4 \\ I_3 &\approx 1 - \alpha_1(k_3\tau)^2 + \alpha_2(k_3\tau)^4 \end{aligned} \quad (10)$$

Here it should be recalled that the coefficients α contain information on spectral function shape, and τ is the bunch duration

The system (10) has the following solutions

$$\begin{aligned} \alpha_1\tau^2 &= \frac{I_1(k_3^4 - k_2^4) - I_2(k_3^4 - k_1^4) + I_3(k_2^4 - k_1^4)}{(k_3^2 - k_2^2)(k_3^2 - k_1^2)(k_2^2 - k_1^2)} \\ \alpha_2\tau^4 &= \frac{I_1(k_3^2 - k_2^2) - I_2(k_3^2 - k_1^2) + I_3(k_2^2 - k_1^2)}{(k_3^2 - k_1^2)(k_3^2 - k_1^2)(k_2^2 - k_1^2)} \end{aligned} \quad (11)$$

The parameter α_1^2/α_2 or (α_2/α_1^2) is independent of τ and determined only by the shape of the spectral function $|I(\omega)|$ or, what is the same, $|I_k|$

In order to enhance the accuracy of determination of the spectral function, it is necessary to carry out measurements by triads of values of $I_1(k_1), I_2(k_2), I_3(k_3)$ on different parts of spectral function. So long as k_1, k_2, k_3 are integer (numbers of harmonics bunch running frequency ω_0), then expressions (11) can be simplified demanding that

$$k_3^2 - k_2^2 = k_2^2 - k_1^2 \quad (12)$$

and Eq.(12) can be solved in integers. The solution should be looked for in the interval of values of k from 5 to 60 in order to be able to use the industrial equipment. At the accelerating field frequency ≈ 3 GHz this corresponds to frequencies from 15 GHz to 180 GHz.

Then we'll obtain three triads of values of k :

1. $k_{11}=7, k_{12}=13, k_{13}=17$;
2. $k_{21}=17, k_{22}=25, k_{23}=31$;
3. $k_{31}=31, k_{32}=41, k_{33}=49$

on which we'll just carry out measurements of the spectral function (Fig.1)

Experimentally obtained spectral function $I(k)$, its first- and second-order derivatives are compared with the standard spectral functions. By chosen criteria the most probable charge distribution function is determined.

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Photometry Method to Define the Charged Particle Longitudinal Distribution Function in the Bunch^{1,2}

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Abstract

In this paper we propose the method of reconstruction of the longitudinal profile of the charged particle bunch by photometry of the film blackening due to optical transition radiation (OTR) generated by those bunches. Three techniques are considered to determine the modules of Fourier spectrum of the bunch particle distribution function envelope. The mathematical processing of the photometry results consists in solution of the Fredholm first-kind integral equation

1. INTRODUCTION

The present work shows the possibility to determine a Fourier transformation image moduls of longitudinal function of charge distribution in an electron bunch using the integral energy characteristic function of the bunch - angular distribution of energy flux density of OTR. The latter is emitted by vacuum-medium boundary and detected in a narrow frequency band of the spectrum optical region during the whole time of the bunch passage through the boundary.

The mentioned characteristic can be determined by results of photometry of the photofilm exposed to OTR effect. For simplicity we consider the normal entrance case. Generalization for the oblique entrance case causes no principal difficulties [1]

2. OTR FROM THE DISPERSIVE MEDIUM BOUNDARY

A linear bunch is moving along the positive direction of the z axis in vacuum towards the planar vacuum-medium boundary (normal to the z axis) at a velocity v . The photofilm's flat surface parallel to the boundary is located at a distance z_0 from it. The point on the film's surface is characterized by a radial ρ and polar φ coordinates of the cylindrical coordinate system connected with the z axis.

At $z_0 \gg l$ (l - OTR formation zone length) for the normal entrance case the OTR field is axially symmetric, linearly polarized and for film-detected frequency band can be presented as [1]:

$$E_\rho = Q(\rho, t) \cos \theta; E_z = Q(\rho, t) \sin \theta; E_\varphi = 0; \quad (1)$$

where

$$Q(\rho, t) = \int_{\omega_0 - \Omega}^{\omega_0 + \Omega} A(\rho, \omega) F\left(\frac{\omega}{v}\right) e^{i\frac{\omega}{v}L} d\omega \quad (2)$$

$$A(\rho, \omega) = \frac{1}{z_0} U(\theta, \omega), F\left(\frac{\omega}{v}\right) = \int_{-\infty}^{\infty} Z(x) \exp(i\frac{\omega}{v}x) dx \quad (3)$$

$$\theta = \arctg \rho z_0; L = \varphi(\rho) - vt; \varphi(\rho) = \beta(z_0^2 + \rho^2)^{1/2};$$

Here $U(\theta, \omega)$ is OTR frequency-angular characteristic of a single particle [2], $Z(x)$ is bunch particle distribution function, t - time, c - velocity of light in vacuum, $\beta = v/c$.

The film blackening produced as a result of OTR is defined by the expression [3]

$$I(\rho) \approx \int_{-\infty}^{\infty} |\vec{E}|^2 dt = \cos^2 \theta \int_{\omega_0 - \Omega}^{\omega_0 + \Omega} |A(\rho, \omega)|^2 \left| F\left(\frac{\omega}{v}\right) \right|^2 d\frac{\omega}{v}. \quad (4)$$

being a linear integral transformation [4] of squared Fourier-image modulus of distribution function. The medium dispersion in the observed frequency band is the necessary condition for the presence of dependence of blackening on the type of distribution function.

3. OTR AS COMBINED WITH LENS

If an optical lens is placed between the boundary and photofilm, then the mentioned effect will be observed in the absence of medium dispersion [$U(\theta, \omega) = U(\theta)$ in (3)]. If the lens is round and its axis coincides with the z axis, then the field on the film surface and its blackening will be expressed again through formulae (1) and (4) where now

$$A(\rho, \omega) = \frac{i\omega}{2\pi c} \int_0^a U(\tilde{\theta}) \exp\left(i\frac{\omega}{c}gr^2\right) J_0\left(\frac{\omega}{c} \frac{r\rho}{z_0 - s}\right) r dr. \quad (5)$$

$$\varphi(\rho) = \beta \left(z_0 + (n-1)\Delta + \frac{\rho^2}{2(z_0 - s)} \right) \quad (6)$$

$$\tilde{\theta} = \arctg \frac{r}{z_0 - s}$$

$$g = 1/s + 1/(z_0 - s) + 1/f.$$

$J_n(x)$ is a first-order Bessel function ($n=1,2,3,\dots$), s - the distance from the lens to the boundary, a - the lens radius, f - its focal length, Δ - its thickness, n - frequency-independent refractive index of the lens material.

Expression (5) is obtained in the paraxial approximation ($s \gg a$, $z_0 - s \gg \rho$) for a thin lens. If the photofilm surface is brought into coincidence with the radiation point image plane ($g=0$), then (5) turns into Fourier-Bessel transformation of frequency-angular characteristic of a single particle.

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4. OTR DIFFRACTION ON THE SCREEN HOLE

At $f \rightarrow \infty$ in (5) and (6) formulae (1) and (4) correspond to the diffraction of OTR field on the round hole in the flat screen in the paraxial approximation. In this case the effect of dependence of film blackening on the type of distribution is preserved. To calculate the field in the screen shadow we'll use the geometric diffraction theory approximation [5]. For the round hole in the screen placed at the distance s from the boundary the field on the surface of the film and its blackening will again be expressed through formulae (1) and (4) where we should put

$$A(\rho, \omega) \approx -U(\tilde{\theta}) \left\{ \frac{f_1(x)}{\sin \frac{\theta}{2}} + \frac{f_2(x)}{\cos \frac{\theta}{2}} \right\}, x = \frac{\omega}{c} a \sin \theta \dots (7)$$

$$\varphi(\rho) = \sqrt{(z-s)^2 + \rho^2}; \tilde{\theta} = \arctg \frac{a}{s}; \theta = \arctg \frac{\rho}{z_0 - s}$$

Here a is the hole radius, $\tilde{\theta}$ is the angle of OTR incidence on the hole edge ($\tilde{\theta} < \theta$);

$$f_1(x) = J_1(x), f_2 = J_0(x) \quad (8)$$

In the case of flat slot of $2a$ width, for the plane normal to the screen and containing the z axis we again can use formula (7) putting

$$f_1(x) = -[\sin x / (a\omega/c)]^{1/2}; f_2(x) = \cos x / (a\omega/c)^{1/2} \quad (9)$$

In calculating formula (7) we used along with the geometrical theory of diffraction the method of representation of axially symmetric fields using focal expansion in terms of Bessel functions.

5. RELATIONSHIP BETWEEN THE FILM BLACKENING AND DISTRIBUTION FUNCTION ENVELOPE

In the above considered cases (Sections 2 - 4) the field in the film plane (1) and the blackening caused by it (4) contain high-frequency Fourier spectrum of distribution function stipulated by the bunch microstructure. In order to express the film blackening function through the Fourier spectrum of distribution function envelope stipulated by macroscopic bunch charge distribution, we transform function $Q(\rho, t)$ (2) as

$$Q(\rho, t) = \int_{-\infty}^{\infty} \hat{A}(\rho, L - q) \hat{Z}(q) dq \dots \dots \dots (10)$$

where

$$\hat{A}(\rho) = \int_{-\infty}^{\infty} A(\rho, \omega) \exp(i \frac{\omega}{v} x) d \frac{\omega}{v};$$

$$\hat{Z}(x) = \int_{-\infty}^{\infty} Z(y) \frac{\sin \frac{\Omega}{v} (y - x)}{y - x} \exp(i \frac{\omega}{v} (y - x)) dy$$

Assuming that the Fourier spectrum of envelope $\tilde{Z}(x)$ of distribution function $Z(x)$ lies in the frequency interval 2Ω we can write

$$\tilde{Z}(x) = \int_{-\infty}^{\infty} Z(y) \frac{\sin \frac{\Omega}{v} (y - x)}{y - x} dy \quad (12)$$

Taking into account that because of fast-oscillating nature of the integrand exponent in (11)

$$|\hat{Z}(x)| \leq \tilde{Z}(x) \quad (13)$$

we can claim that

$$\operatorname{Re} \hat{Z}(x) = B \tilde{Z}(x) \cos(\frac{\omega}{v} X(x)); \quad (14)$$

$$\operatorname{Im} \hat{Z}(x) = B \tilde{Z}(x) \sin(\frac{\omega}{v} X(x))$$

where $X(x)$ and $\tilde{X}(x)$ are some continuous functions of x , B is const. Further, comparing the first-order derivatives of the real and imaginary parts of function $\tilde{Z}(x)$ defined by (11) and (14) in the approximation of large ω_0 (i.e. discarding in them zero-order terms relative to ω_0) we obtain (at $\omega_0 \rightarrow \infty$)

$$X(x) = \tilde{X}(x) = \tilde{X} + \text{const} \quad (15)$$

Substituting the asymptotic value of function $\hat{Z}(x)$ (with account of (14) and (15)) into (10) we arrive at the following expression for function $Q(\rho, t)$

$$Q(\rho, t) \approx B \int_{\omega_0 - \Omega}^{\omega_0 + \Omega} A(\rho, \omega) F(\frac{\omega - \omega_0}{v}) \exp(i \frac{\omega}{v} L + \text{const}) d \frac{\omega}{v} \quad (16)$$

The blackening function (4) takes the form

$$I(\rho) \approx B^2 \cos \theta \int_{\omega_0 - \Omega}^{\omega_0 + \Omega} |A(\rho, \omega)|^2 \left| F(\frac{\omega - \omega_0}{v}) \right|^2 d \frac{\omega}{v} \quad (17)$$

which contains a low-frequency part of Fourier spectrum of distribution function with the help of which we can determine its envelope.

6. NUMERICAL CALCULATION

The efficiency of the method confirms the essential difference in the character of the photofilm blackening at different initial bunch distribution functions. Fig.1 presents calculated normalized intensities

$$\bar{I}(\theta) = \frac{I(\theta) - I_{\min}(\theta)}{I(\theta) - I_{\max}(\theta)}$$

of the film blackening for three different shapes of distribution function: uniform (1), cosinusoidal (2) and Gaussian (3) for the case of flat slot in the screen. Calculations are done for the

wave length $\lambda=500 \text{ nm}$ and $2\Delta\lambda=10 \text{ nm}$. The slot half width s is taken equal to the bunch length: $a=d=100\mu\text{m}$.

[6] E.D. Gazazian, B.E. Kinber, "Asymptotics of Axially Symmetric Beams of Electromagnetic Waves", *Izvestia VUZ-ov SSSR, Radiofizika*, 14, No 8, p.p. 1219-1223, (1971).

Fig.1

As can be seen from Fig. 1, the difference in the character of blackening displays in different periodicity of their oscillations, in the extent of rapidity of their attenuation as well as in the availability of phase shifts of oscillations relative to each other.

7. CONCLUSION

In this work we considered three different approaches for the determination of modulus of Fourier spectrum envelope of bunch charge distribution function united by common technique: OTR exposed film, its photometry with subsequent mathematical processing of results, which reduces to solution of Fredholm's integral first-kind equation [4].

The problems related to the photofilm sensibility can be solved by detecting the radiation from a train of bunches in the beam: in that case the mathematical processing of photometry results will yield bunch-ensemble-averaged charge distribution.

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