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Mechanical Properties of Cellular Materials

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Abstract

We calculated the stress-strain relation for elastomeric foam from an *ab initio* theory, which shows that the "plateau" and "densification" regions should be described by a hyperbola. The theory seems to agree reasonably well with experiment.

Background and Research Objectives

Dienes[1] has suggested that the principle of *superposition of strain rates* has much wider application than usually assumed, and that it can serve as a basis for formulating constitutive relations for highly nonlinear material behavior. In general, the relations involve tensors, but for this project we considered isotropic behavior, which requires only scalars. For hydrostatically stressed isotropic foam, the strain rate can be expressed as the sum of a term involving a changing stress and a term that represents the changing compliance due to compaction, so

$$\dot{\epsilon} = C\dot{\sigma} + \dot{C}\sigma, \quad (1)$$

where ϵ is the strain, C is the compliance, σ is the stress, and a dot denotes the rate of the underlying quantity.

In this project, we applied the principle of superposition of strain rates to obtain the nonlinear behavior of certain elastomeric foams.

Importance to LANL's Science and Technology Base and National R&D Needs

Understanding of cellular solids is important to Science-Based Stockpile Stewardship. Any elaboration cannot be discussed in an unclassified context.

Scientific Approach and Accomplishments

We extended the theory of a spherical void in calculations combining the behavior of an ensemble of voids. We demonstrated the validity of the results in limit of infinite bulk modulus and we applied the method to the most frequently encountered case for elastomeric foams: finite bulk modulus, large compared to shear modulus. In this case we found the stress-strain curve to be a hyperbola. We then computed the effective shear modulus, and we showed the results to be consistent with experimental data at large compressions for three types of foam. We offered a tentative explanation for why the results of an apparently naive theory agree so well with data in a highly nonlinear regime.

The essential results are embodied in a simple hyperbolic relation for the compressive stress

$$\sigma = \frac{4}{3} \frac{\mu_s \epsilon (1 - \bar{f})^\alpha}{\bar{f} - \epsilon} + c. \quad (2)$$

where $\bar{f}$ is the void fraction, ϵ is the nominal strain, μ_s is the shear modulus of the solid (matrix) material, and

$$\alpha = \frac{15(1 - \nu)}{7 - 5\nu}, \quad (3)$$

where ν is Poisson's ratio.

Figure 1a is a family of experimental stress-strain curves for open-cell polyethylene foam[2], taken at low strain rate ($\dot{\epsilon} \sim 10^{-2} \text{ s}^{-1}$) so the material is essentially in equilibrium. The densities of the uncompressed foams are 138, 120, 69.2, and $29.4 \text{ kg} \cdot \text{m}^{-3}$. Figure 1b is a family of theoretical stress-strain curves for the same densities as calculated from Eq. (2), assuming a solid polymer density of $1 \text{ Mg} \cdot \text{m}^{-3}$ and solid shear modulus[3] of $\mu_s = 10 \text{ MN} \cdot \text{m}^{-2}$ and a Poisson's ratio of $\nu = \frac{1}{2}$. The curves are cut off below a strain of 0.2. In the low strain region, there is a linear response of stress to strain until buckling occurs, at which point the curve rapidly turns down to the plateau region. We use the term *buckling constant* for the stress at the onset of the plateau region. This is the same constant c as first mentioned near the end of §4. In Fig. 1, the buckling constants for the curves are all left at zero to emphasize the semiquantitative agreement between the theoretical and experimental results. The buckling constant will increase with density, causing the curves in Fig. 1b to more closely agree with the curves in Fig. 1a. The low-stress region is not applicable to the theory presented here, and analysis of buckling will be addressed in a future paper.

Figure 2a is a family of experimental stress-strain curves for open-cell polyurethane foam⁴, taken at low strain rate ($\dot{\epsilon} \sim 10^{-3} \text{ s}^{-1}$) so the material is essentially in equilibrium. The densities of the uncompressed foams are 51.7, 32.4, and $14.1 \text{ kg} \cdot \text{m}^{-3}$. Figure 2b is a family of theoretical stress-strain curves for the same densities as calculated from Eq. (2), assuming a solid polymer density of $1.2 \text{ Mg} \cdot \text{m}^{-3}$ and $\mu_s = 0.8 \text{ MN} \cdot \text{m}^{-2}$. The curves are cut off below a strain of 0.2 as that is the region where buckling dominates. The buckling constants for the curves are 0.002, 0.002, and $0.004 \text{ MN} \cdot \text{m}^{-2}$ respectively.

Figure 3a is a pair of experimental stress-strain curves for open-cell silicone foam[4], also taken at low strain rate. The thickness of both samples was about 1 mm. The porosities $\bar{f}$ are 0.6 and 0.7. Figure 3b is a pair of theoretical stress-strain curves as calculated from Eq. (2), assuming $\mu_s = 0.27 \text{ MN} \cdot \text{m}^{-2}$. The curves are cut off below a strain of 0.2 as that is the region where buckling dominates. The buckling constants for the curves are 0.02 and $0.005 \text{ MN} \cdot \text{m}^{-2}$ respectively.

The success of our simple calculation can be partially explained by referring to some comments by Kachanov, although strictly speaking, they apply only to linear media. First, if his method of superposition[5] were applied to find the interactions within an ensemble of spherical holes, it would hinge on finding the mean stress at a cavity location (without the void) due to each of its cavity neighbors. This mean stress is uniform, however, and unaffected by the presence of a hole. Thus his superposition method, which is very similar to those described by a number of authors cited by Kachanov, would lead to the conclusion that the interaction has no effect, at least in that approximation. In a similar vein, Kachanov notes that competing effects of stress shielding and stress amplification balance each other — at least in the cases investigated.[6]

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Figure 1

Figure 2

Figure 3