BNL-65444

CONF-980806--

A COMPUTATIONAL ANALYSIS OF NATURAL CONVECTION IN A VERTICAL CHANNEL WITH A MODIFIED POWER LAW NON-NEWTONIAN FLUID *

Shah-Rong Lee and Thomas F. Irvine, Jr.
Department of Mechanical Engineering
State University of New York
Stony Brook, New York 11794-2300
USA

George Alanson Greene
Department of Advanced Technology
Brookhaven National Laboratory
Upton, New York 11973-5000
USA

BNL-65444

A COMPUTATIONAL ANALYSIS OF NATURAL CONVECTION IN A VERTICAL CHANNEL WITH A MODIFIED POWER LAW NON-NEWTONIAN FLUID *

Shah-Rong Lee and Thomas F. Irvine, Jr.
Department of Mechanical Engineering
State University of New York
Stony Brook, New York 11794-2300
USA

George Alanson Greene
Department of Advanced Technology
Brookhaven National Laboratory
Upton, New York 11973-5000
USA

ABSTRACT

An implicit finite difference method was applied to analyze laminar natural convection in a vertical channel with a modified power law fluid. This fluid model was chosen because it describes the viscous properties of a pseudoplastic fluid over the entire shear rate range likely to be found in natural convection flows since it covers the shear rate range from Newtonian through transition to simple power law behavior. In addition, a dimensionless similarity parameter is identified which specifies in which of the three regions a particular system is operating.

The results for the average channel velocity and average Nusselt number in the asymptotic Newtonian and power law regions are compared with numerical data in the literature. Also, graphical results are presented for the velocity and temperature fields and entrance lengths. The results of average channel velocity and Nusselt number are given in the three regions including developing and fully developed flows.

As an example, a pseudoplastic fluid (carboxymethyl cellulose) was chosen to compare the different results of average channel velocity and Nusselt number between a modified power law fluid and the conventional power law model. The results show, depending upon the operating conditions, that if the correct model is not used, gross errors can result.

1. INTRODUCTION

The problem under consideration is laminar natural convection between vertical parallel plates. The plates are of height H of infinite width with a spacing of 2b and both have constant and equal temperatures, T_w. Figure 1 is a schematic of the physical and coordinate systems.

The fluid between the plates has a modified power constitutive equation given by Eq. (1) and illustrated in Fig. 2 for a pseudoplastic fluid. As reported by Park et al. (1993), this equation shows good agreement with experimental viscosity measurements.

$$\eta_a = \frac{\eta_0}{1 + \frac{\eta_0}{K}(\dot{\gamma})^{1-n}} \quad (1)$$

Examination of Eq. (1) and Fig. 2 reveals that for "low values" of the shear rate ($\dot{\gamma}$), Region I, Eq. (1) becomes Newtonian in that the apparent viscosity becomes independent of shear rate. At large values of shear rate, Region III, Eq. (1) becomes,

$$\eta_a = K(\dot{\gamma})^{n-1} \quad (2)$$

which is the constitutive equation for a conventional power law fluid. Between these two extremes is a transition region (Region II). Because non-Newtonian natural convection flows have characteristically low velocities (and thus low shear rates), they often operate in Regions I or II in Fig. 2 even though they might be power law fluids at higher shear rates. Equation (1) used in the appropriate field equations will yield solutions for all three regions in Fig. 2. In such flows as described above, it would be most advantageous to know in which of the three regions a particular system is operating. As a result of the analysis to follow, a similarity parameter, α, will be identified which specifies that particular region.

A number of investigations have been published on the flow of Newtonian fluids between parallel vertical plates (Aung, 1972; Aung et al., 1972; Bodia and Osterle, 1962; Miyatake et al., 1972). However, only a few studies have been reported where a power law fluid is considered (Irvine et al., 1982; Irvine and Schneider, 1984). Both Newtonian and power law solutions are asymptotic solutions in this analysis and can therefore be used to validate the solutions to be presented. The quantities of most interest in this problem are the average flow velocity between the plates and the average Nusselt number. These will be presented along with the hydrodynamic and thermal entrance lengths for a variety of operating conditions.

*This work was performed under the auspices of the U.S. Department of Energy.

2. ANALYSIS

The appropriate field equations will be given here only in dimensionless form (see Nomenclature). A more detailed derivation can be found in the dissertation by Lee (1992).

Continuity:

$$\frac{\partial u^+}{\partial x^+} + \frac{\partial v^+}{\partial y^+} = 0 \tag{3}$$

Momentum:

$$u^+ \frac{\partial u^+}{\partial x^+} + v^+ \frac{\partial u^+}{\partial y^+} = -\frac{\partial P'^+}{\partial x^+} + T^+ + \frac{1}{Gr_M^{1/2}} \frac{\partial}{\partial y^+}\left[\eta_a^+ \frac{\partial u^+}{\partial y^+}\right] \tag{4}$$

Energy:

$$u^+ \frac{\partial T^+}{\partial x^+} + v^+ \frac{\partial T^+}{\partial y^+} = \frac{1}{Bo^{1/2}} \frac{\partial^2 T^+}{\partial y^{+2}} \tag{5}$$

Global Continuity:

$$\int_0^1 u^+ dy^+ = u_0^+ \tag{6}$$

where $x^+ = x/H$, $y^+ = y/b$, $u^+ = u/\dot{u}$ and $v^+ = (v/\dot{u}) \cdot (H/b)$. The boundary conditions for Eqs. (3-5) are,

$$\begin{aligned} &x^+ = 0: \quad v^+ = T^+ = 0 \\ &y^+ = 0: \quad \frac{\partial u^+}{\partial y^+} = v^+ = \frac{\partial T^+}{\partial y^+} = 0 \\ &y^+ = 1: \quad u^+ = v^+ = 0,\ T^+ = 1 \\ &x^+ = 0 \text{ and } x^+ = 1: \quad P'^+ = 0 \end{aligned} \tag{7}$$

Because of the temperature and velocity coupling between Eqs. (4) and (5), both the dimensionless average velocity, u_0^+, and the average Nusselt number, $\overline{Nu}$, will be functions of a large number of parameters, i.e., Gr_N, Bo, α, n. This makes it difficult to present comprehensive results of the analysis. Because of space limitations, only characteristic solutions in tabular and graphical form will be given and discussed. More comprehensive results can be found in Lee (1992).

Equations (3-6) and the boundary conditions (7) were cast in an implicit numerical form and solved by iteration. The parameters Bo, Gr_N, n and α were specified, and the number of dimensionless mesh points was 100 in the y^+ direction and 140 in the x^+direction. It should be noted that the entrance velocity u_0^+ is not a boundary condition but is obtained from the numerical solution as follows. A value of u_0^+ was assumed at $x^+ = 0$ and the equations solved to $x^+ = 1$ where the value of P'^+ is again required to be zero. Different values of u_0^+ were assumed until $P'^+ = 0$ (in this case, $P'^+ < 10^{-7}$). Using the solution for the correct u_0^+, the average Nusselt numbers were then calculated from the exit temperature and velocity fields from the equations,

$$\overline{Nu} = (Bo)^{1/2}\, Q^+_{Total} \tag{8}$$

where

$$Q^+_{Total} = \int_0^1 u^+(1,y^+)\, T^+(1,y^+)\, dy^+ \tag{9}$$

3. RESULTS

3.1 Asymptotic Solutions

A variety of asymptotic solutions are available to confirm the validity of the modified power law analysis. These include fully developed and developing flows for Newtonian and power law fluids. Such asymptotic solutions can be obtained from Eqs. (3-7) by specifying the values of Gr_N, n, Bo and α. For large values of α, Eqs. (3-7) describe the flow of a power law fluid while small values of α describe a Newtonian fluid. The shear rate parameter α and its following ranges are applicable to any purely viscous non-Newtonian fluid whose constituitive equation is described by Eq. (1). More specifically, if

$\alpha \le 10^{-2}$	Newtonian fluid, Region I, Fig. 2
$\alpha \ge 10^{2}$	Power law fluid, Region III, Fig. 2
$10^{-2} \le \alpha \le 10^{2}$	Transition region, Region II, Fig. 2

3.2 Fully Developed Flow

Several asymptotic solutions for fully developed Newtonian and power law flows can be obtained by direct integration of Eqs. (3-7). If Gr_N is low enough or if the channel height is large compared to the plate spacing, fully developed flow will occur. Under these conditions, the fluid temperature approaches the wall temperature and the pressure gradient becomes zero (Aung, 1972). Thus, an energy balance on the fluid yields,

$$Q = \bar{h}\, H\, (T_w - T_\infty) = \rho\, c_p\, u_0\, b (T_w - T_\infty) \tag{10}$$

The solutions for the dimensionless average velocity and Nusselt numbers for Newtonian (Eqs. 11, 12) and power law fluids (Eqs. 13, 14) are:

$$u_0^+/Gr_N^{1/2} = 1/3 \tag{11}$$

$$\overline{Nu}/(Gr_N \cdot Pr_N) = u_0^+/Gr_N^{1/2} \tag{12}$$

$$u_0^+/Gr_N^{1/2n} = \frac{n}{2n+1}(1+\alpha)^{1/n} \tag{13}$$

$$\overline{Nu}/(Gr_N^{\frac{n+1}{2n}} \cdot Pr_N) = u_0^+/Gr_N^{1/2n} \tag{14}$$

DISCLAIMER

Note that Eqs. (13, 14) revert to Eqs. (11, 12) if n = 1 and α approaches zero. The above exact solutions were checked against the present numerical solutions from Eqs. (3-7) by specifying that $T^+ = 1$ and $\partial P'^+/\partial x^+ = v^+ = \partial u^+/\partial x^+ = 0$ and, $\eta_a^+ = 1$ (Newtonian) and $\eta_a^+ = (\partial u^+/\partial y^+)^{n-1}$ (power law). The comparisons are shown in Tables 1 and 2 where it is seen that satisfactory agreement occurs.

TABLE 1
Comparison between numerical and exact solutions for $\alpha = 10^{-4}$ in fully-developed-flow case.

Gr_M	u_0^+	
	Numerical	Exact
0.1	0.10540	0.10541
1.0	0.33333	0.33333
10.	1.05412	1.05402
10^2	3.33360	3.33333

TABLE 2
Comparison between the numerical and exact solutions for $\alpha = 10^4$ and n = 0.7 in fully-developed-flow case.

Gr_M	u_0^+	
	Numerical	Exact
0.1	0.056315	0.056312
1.0	0.291655	0.291667
10.	1.510511	1.510680
10^2	7.823352	7.824529

In addition, Fig. 3 shows the results from the numerical analysis if all three regions shown in Fig. 2 are considered for fully developed flow. It can be seen in the figure that both u_0^+ and $\overline{Nu}$ are quite sensitive to the value of the shear rate parameter, α.

3.3 Developing Flow Literature Comparisons

Figure 4 and Table 3 compare the Newtonian results with those of Bodia and Osterle (1962) where Gr_M is used as the variable parameter.

TABLE 3
Comparison of present numerical results to Bodia and Osterle (1962) for a Newtonian fluid: $\alpha = 10^{-4}$ and $Pr_N = 0.7$.

Gr_M	u_0^+	u_0^+ (B-O)	$\overline{Nu}$	$\overline{Nu}$ (B-O)
10^2	0.6094	0.6064	1.9687	1.9537
10.	0.5099	0.5114	0.9522	0.9493
1.0	0.2938	0.2916	0.2056	0.2041
0.1	0.1034	0.1039	0.02288	0.02299
10^{-2}	0.03333	0.03328	0.002333	0.00233

Figure 4 includes calculations of the thermal and hydrodynamic entrance lengths x^+_{ET} and x^+_{EM} where $\partial T^+/\partial x^+ = 10^{-2}$ at x^+_{ET} and $\partial u^+/\partial x^+ = 10^{-2}$ at x^+_{EM}. It should be noted that these entrance lengths have not previously been reported in the literature. Figure 5 compares the power law calculations from the present analyses with those of Irvine et al. (1982) where Gr_M is the variable parameter on the abscissa. The agreement between the two analyses is again reasonable.

In Figure 5, the agreement between the numerical analyses is not as good as in the Newtonian case. The maximum deviation is 1.9% but the data in Irvine et al. (1982) are only given in the form of a log-log graph and it is possible that a portion of the differences comes from reading-errors. As was the case for Newtonian channel flow, the entrance lengths for power law fluids presented in Fig. 5 have not previously been reported in the literature.

3.4 Modified Power Law Solutions

Because of the large number of parameters involved in the solutions of Eqs. (3-7), only a few representative results can be presented here. For those interested in a particular case, the equations must be solved independently. Details of the solution method are given in Lee (1992). In general, the solution method consists of specifying the properties η_0, K, n, k, c_p, β, ρ, and $(T_w - T_\infty)$ plus the geometric quantities b and H. Then it is possible to specify the parameters Gr_N, Gr_M, α, n, Bo for a particular solution. This will be illustrated in a numerical example to be presented later.

Figure 6 illustrates the developing velocity and temperature profiles for a modified power law fluid in the transition region (Region II) where the shear rate parameter $\alpha = 10$. It is seen in the figure that most of the flow is in the thermally developing region and only becomes fully developed at the exit ($T^+ = 1$). The hydrodynamic entrance length is small because of the large Prandtl number ($Pr \approx 5$).

Figure 7 shows representative values of u_0^+, $\overline{Nu}$, x^+_{ET} and x^+_{EM} for flow in the transition region, $\alpha = 10^{-1}$. This is the type of graph that can be obtained by solving Eqs. (3-7) for particular parametric values. One check on the validity of the solution is that at $Gr_M = 10^{-4}$ where the flow is approximately fully-developed hydrodynamically, the value of u_0^+ agrees with Eq. (11) for fully developed flow.

3.5 Numerical Example

Equations (3-7) were solved for the following practical example using the properties, geometric dimensions and temperature differences for a fluid consisting of 2500 ppm of CMC in water. The viscous properties were obtained from Park, et al. (1993) which agreed well with the modified power law equation.

$\eta_0 = 0.06454$ Ns/m^2	$\rho = 999.1$ kg/m^3
$K = 1.0261$ Ns^n/m^2	$\beta = 2.06 \times 10^{-4}$ K^{-1}
n = 0.5	$b = 10^{-2}$ m
k = 0.597 W/mK	H = 0.20 m
$c_p = 4.18 \times 10^3$ J/kgK	ΔT = 1, 10 and 100 K

Using the above quantities resulted in the following values of the shear rate parameter, α:

ΔT(K)	α
1	0.0905
10	0.1649
100	0.3003

All of the above are in the transition region and numerical calculations using Eqs. (3-7) were made to compare the modified power law results for u_0^+ and $\overline{Nu}$ with those if the power law equations were incorrectly used. The results are shown in Fig. 8. From Fig. 8, as expected, the differences between the two models increase as the temperature differences decrease ($\alpha_{MPL} < \alpha_{PL}$). It is also clear that large errors can occur if the incorrect model is used. For example, from Fig. 8 at a temperature difference of 10 K, the average velocities differ by over an order of magnitude. These discrepancies can be eliminated if the correct model which specifies the appropriate shear rate range as determined by α is used.

4. CONCLUSIONS

Numerical results have been presented for the free convection flow and heat transfer characteristics of a modified power law non-Newtonian fluid between vertical parallel plates. The results indicate that if a simple power law constitutive equation is used, under some operating conditions gross errors can occur. A shear rate similarity parameter is presented which determines when it is necessary to use the modified power law model.

NOMENCLATURE

b half-channel spacing (m)
Bo Boussinesq number = $Gr_N \cdot Pr_N^2$
Gr_M Modified Grashof number = $Gr_N \cdot (1+\alpha)^2$
Gr_N Newtonian Grashof number = $(g\beta(T_w - T_\infty)b^4)/(\nu_0 H)$
H duct height (m)
K power law consistency (Ns^n/m^2)
n flow index
$\overline{Nu}$ Nusselt number = $\bar{h} \cdot b/k$
P channel pressure (N/m^2)
P' modified channel pressure (N/m^2) = $P(x)-P_\infty+\rho_\infty gx$
P'^+ dimensionless channel pressure = $P'/\rho u^{*2}$
Pr_N Prandtl number = ν_0/α_T
Q heat transfer per unit width (W/m)
T^+ dimensionless fluid temperature = $(T(x,y) - T_\infty)/(T_w - T_\infty)$
u^* reference velocity (m/s) = $[g\beta(T_w-T_\infty)H]^{1/2}$

Greek Symbols

α shear rate parameter = $(\eta_0/K) \cdot (u^*/b)^{1-n}$
α_T thermal diffusivity (m^2/s)
β thermal expansion coefficient (K^{-1})
$\dot{\gamma}$ shear rate (s^{-1})
η_a apparent viscosity (Ns/m^2)
η_a^+ dimensionless apparent viscosity = $(1+\alpha)/[1+\alpha(\partial u^+/\partial y^+)^{1-n}]$

Superscripts

$+$ dimensionless quantity
$-$ average quantity

Subscripts

N refers to Newtonian region
M modified, as in Gr_M
∞, 0 entrance conditions
w wall

REFERENCES

Aung, W., 1972, Fully Developed Laminar Free Convection Between Heated Vertical Plates, *Int. J. Heat Mass Transfer*, vol. 15, pp. 1577-1580.

Aung, W., Fletcher, L. S. & Sernas, V., 1972, Developing Laminar Free Convection Between Vertical Flat Plates with Asymmetric Heating, *Int. J. Heat Mass Transfer*, vol. 15, pp. 2293-2308.

Bodia, J. R. & Osterle, J. F., 1962, Development of Free Convection Between Vertical Heated Plates, *J. Heat Transfer*, vol. 84, pp. 40-44.

Irvine, T. F., Wu, K. C. & Schneider, W. J., 1982, Vertical Channel Free Convection for a Power Law Fluid, ASME Paper 82-WA/HT-69.

Irvine, T. F. & Schneider, W. J., 1984, Vertical Channel Free Convection for a Power Law Fluid with a Constant Heat Flux, ASME Paper 84-HT-69.

Lee, S. R., 1992, A Computational Analysis of Natural Convection in a Vertical Channel with a Modified Power Law Fluid, Ph. D. Dissertation, Mech. Engr. Dept., State University of New York, Stony Brook, NY.

Miyatake, O., Fujii, T., Fujii, M. & Tanaka, H., 1972, Natural Convection Heat Transfer Between Vertical Parallel Plates-One Plate with a Uniform Heat Flux and the Other Plate Insulated, *Trans. JSME*, pp. 25-32.

Park, S. S., Irvine, T. F. & Capobianchi, M., 1993, Experimental and Numerical Study of Friction Factor for a Modified Power Law Fluid in a Rectangular Duct, Proceedings of the 3rd World Conference on Experimental Heat Transfer, Fluid Mechanics and Thermodyamics, vol. 1, pp. 900-908, Elsevier.

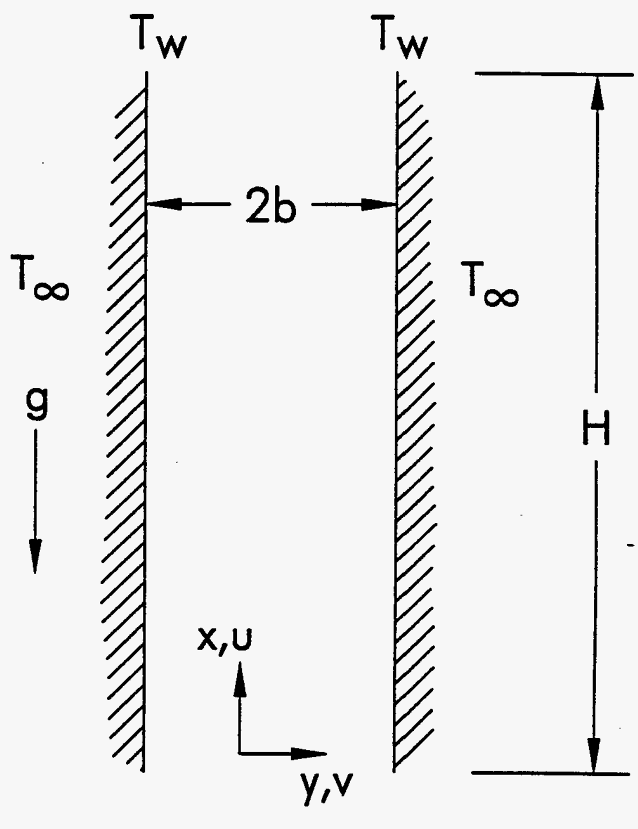

FIGURE 1
Schematic of vertical channel.

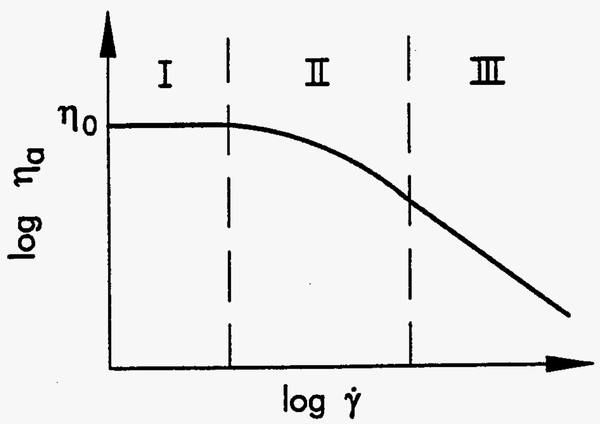

FIGURE 2
Typical flow curve for a pseudoplastic fluid: I-Newtonian region, II-Transition region, III-Power law region.

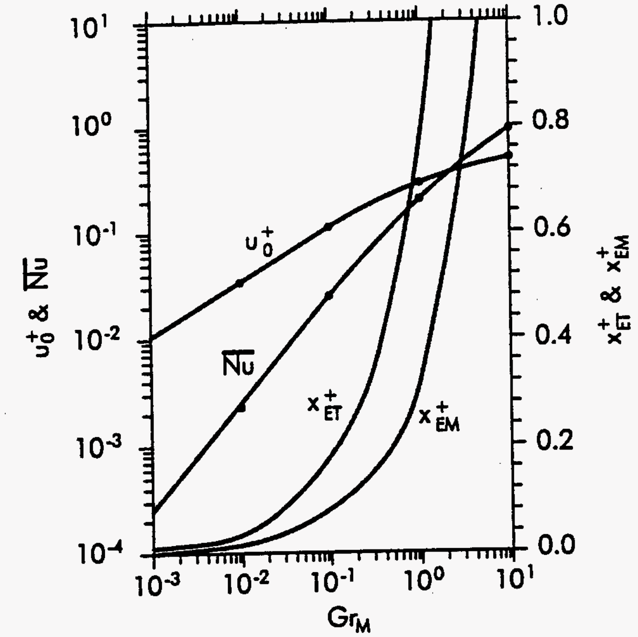

FIGURE 4
Average velocities, Nusselt numbers and entrance lengths. Pr=0.7, Newtonian region. Data (dots) from Bodia & Osterle (1962).

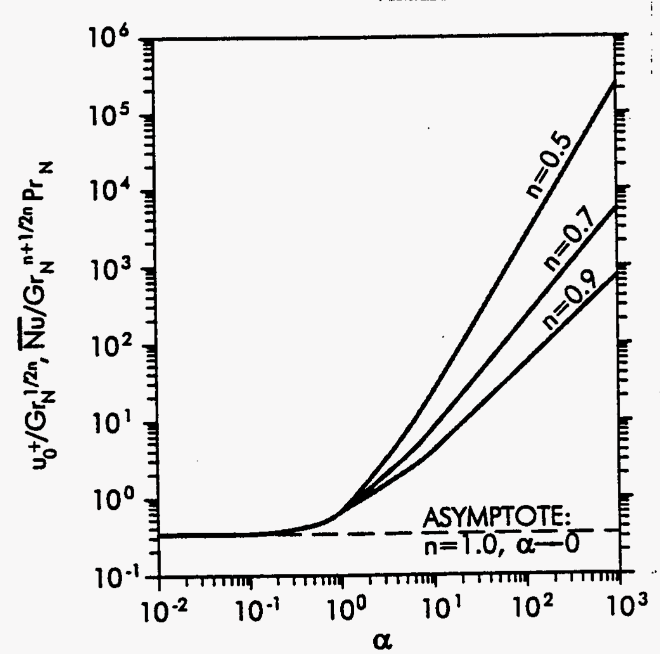

FIGURE 3
Variation of u_0^+ and $\overline{Nu}$ with α and Gr_N for fully-developed-flow.

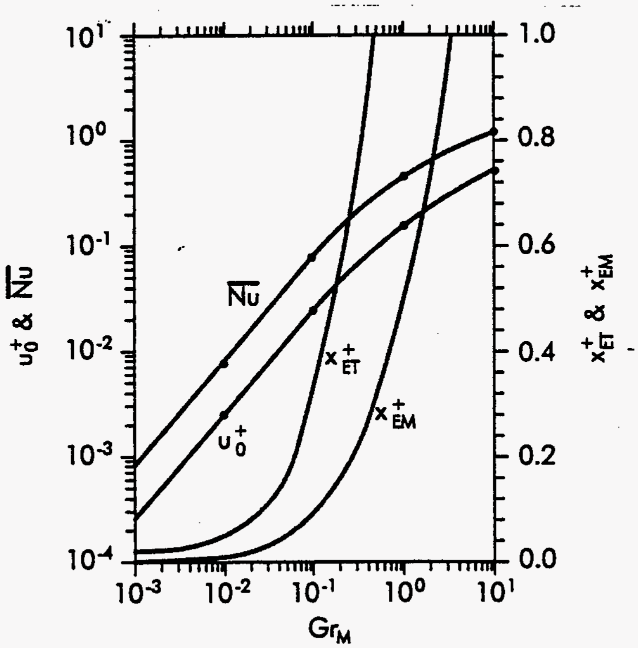

FIGURE 5
Average velocities, Nusselt numbers and entrance lengths, power law region. Data (dots) from Irvine et al. (1982).

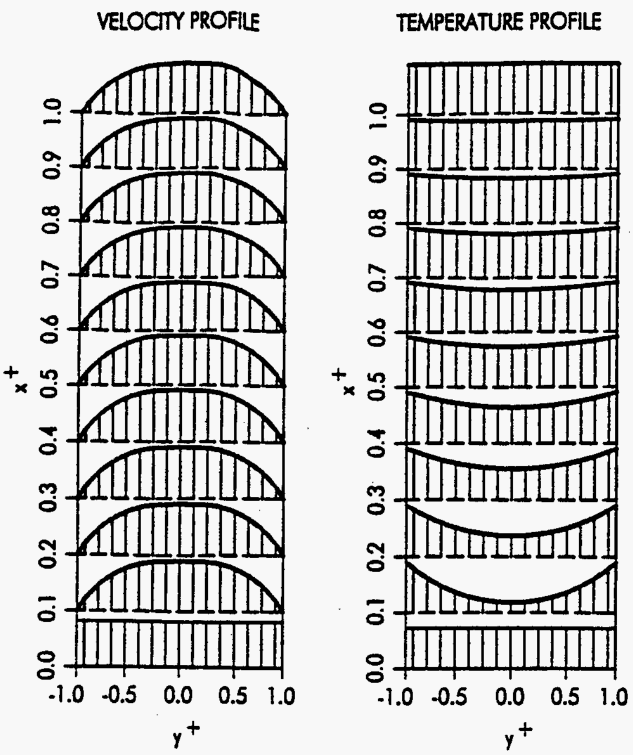

FIGURE 6
The variation of temperature and velocity profiles through a vertical channel: $Gr_M=1$. Bo=1, n=0.5, $\alpha=10$.

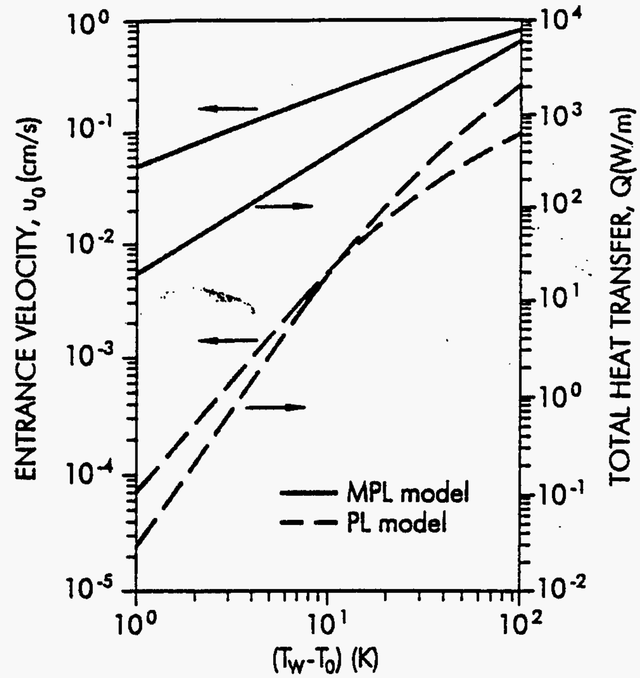

FIGURE 8
Comparison of the entrance velocities and total heat transfer per unit width by using modified power law (MPL) and power law (PL) models in CMC 2500 wppm solution.

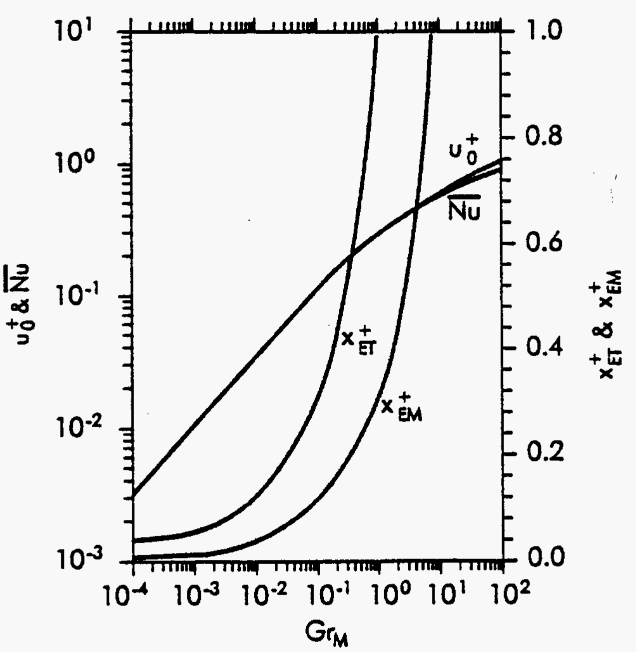

FIGURE 7
Entrance velocities, Nusselt numbers and entrance lengths: Bo=1, n=0.8, $\alpha=0.1$.

M98004981

Report Number (14) BNL--65444

CONF-980806--

Publ. Date (11) 199804

Sponsor Code (18) DOE/DP, XF

UC Category (19) UC-700, DOE/ER

19980707 009

DTIC QUALITY INSPECTED 1

DOE