LA-UR- 98-1695

Approved for public
distribution is unlimited.

Title:

THE SCALAR GLUEBALL MASS IN REGGE
PHENOMENOLOGY

CONF-980267--

Author(s):

BURAKOVSKY, LEONID/T8

RECEIVED

OCT 05 1998

OSTI

Submitted to:

First International Conference on
Parameterized Relativistic Quantum
Theory, PRQT '98, Houston, Texas, USA,
Feb 9-11, 1998

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

Los Alamos
NATIONAL LABORATORY

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the U.S. Department of Energy under contract W-7405-ENG-36. By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. The Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

©The Scalar Glueball Mass in Regge Phenomenology*

L. Burakovsky[†]

Theoretical Division, T-8
Los Alamos National Laboratory
Los Alamos NM 87545, USA

Abstract

We show that linear Regge trajectories for mesons and glueballs, and the cubic mass spectrum associated with them, determine a relation between the masses of the ρ meson and the scalar glueball, $M(0^{++}) = 3/\sqrt{2} M(\rho)$, which implies $M(0^{++}) = 1620 \pm 10$ MeV. We also discuss a relation between the masses of the scalar and tensor glueballs, $M(2^{++}) = \sqrt{2} M(0^{++})$, which implies $M(2^{++}) = 2290 \pm 15$ MeV.

Key words: Regge phenomenology, glueballs, mesons, mass spectrum

PACS: 12.39.Mk, 12.40.Nn, 12.40.Yx, 12.90.+b, 14.40.Cs

1 Introduction

The existence of a gluon self-coupling in QCD suggests that, in addition to the conventional $q\bar{q}$ states, there may be non- $q\bar{q}$ mesons: bound states including gluons (glueballs and $q\bar{q}g$ hybrids). However, the theoretical guidance on the properties of unusual states is often contradictory, and models that agree in the $q\bar{q}$ sector differ in their predictions about new states. Moreover, the abundance of $q\bar{q}$ meson states in the 1-2 GeV region and glueball-quarkonium mixing makes the identification of the would-be lightest non- $q\bar{q}$ mesons extremely difficult. To date, no glueball state has been firmly established yet.

*Presented at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT '98, Houston, Texas, USA, Feb 9-11, 1998

[†]E-mail: BURAKOV@QMC.LANL.GOV

Although the current situation with the identification of glueball states is rather complicated, some progress has been made recently in the 0^{++} scalar and 2^{++} tensor glueball sectors, where both experimental and QCD lattice simulation results seem to converge [1]. Recent lattice calculations predict the 0^{++} glueball mass to be 1600 ± 100 MeV [1, 2, 3, 4]. Accordingly, there are two experimental candidates [5], $f_0(1500)$ and $f_0(1710)$, in this mass range which cannot both fit into the scalar meson nonet, and this may be considered as strong evidence for one of these states being a scalar glueball (and the other being dominantly $s\bar{s}$ scalar quarkonium).

It is known that in both lattice QCD [3, 6, 7, 8] and pure Yang-Mills [9] calculations, the mass of the scalar glueball is determined by the dimensionless ratio

$$\gamma \equiv \frac{M(0^{++})}{\sqrt{\sigma}}, \quad (1)$$

where σ is the string tension, and the following values of γ have been claimed:

$$\gamma = \begin{cases} 3.95, & \text{ref. [3]} \\ 4.77 \pm 0.05, & \text{ref. [6],} \\ 3.88 \pm 0.11, & \text{ref. [8],} \\ \approx 3.3, & \text{ref. [9],} \end{cases} \quad (2)$$

so that the data cluster around $\gamma \simeq 4$, with uncertainty of $\sim 15\%$. The use of the value

$$\sqrt{\sigma} \approx 0.375 \text{ GeV}, \quad (3)$$

extracted from the light quark Regge phenomenology relation [10, 11]

$$\alpha' = \frac{1}{8\sigma} \simeq 0.9 \text{ GeV}^{-2}, \quad (4)$$

in Eq. (1) with $\gamma \approx 4$ leads to the following estimate for the scalar glueball mass:

$$M(0^{++}) \simeq 1500 \text{ MeV}. \quad (5)$$

Here we wish to suggest the following formula for the scalar glueball mass,

$$M(0^{++}) = 3\sqrt{2}\sqrt{\sigma} \approx 4.24\sqrt{\sigma}, \quad (6)$$

which, with $\sqrt{\sigma}$ given in (3), predicts

$$M(0^{++}) \simeq 1600 \text{ GeV}. \quad (7)$$

A naive way to obtain the formula (6) is to follow the procedure of the minimisation of the energy of a bound state of two massless gluons suggested by West [12]:

$$E = 2p + \frac{9}{4}\sigma r - \frac{\alpha}{r}, \quad (8)$$

where p is the gluon momentum, α is the strong coupling constant, and $9/4$ is a color factor in the long-range confining piece of the two-body Coulomb + linear potential. It follows from the uncertainty principle $pr \gtrsim 1$ that

$$E \geq \frac{2-\alpha}{4} + \frac{9}{4}\sigma r, \quad (9)$$

and minimising the lower bound of (9) gives

$$E = 3\sqrt{(2-\alpha)\sigma} \approx 3\sqrt{2}\sqrt{\sigma}. \quad (10)$$

We note that the procedure of West gives a reasonable result for, e.g., ordinary mesons: The analog of (8) in this case is

$$E = 2\sqrt{p^2 + m^2} - 2m + \sigma r - \frac{\alpha}{r}, \quad (11)$$

where m is the constituent quark mass, and minimising the lower bound of (11) leads to the solution (in the nonrelativistic approximation $\sqrt{p^2 + m^2} \approx m + p^2/(2m)$)

$$E \simeq 3 \left(\frac{\sigma^2}{4m} \right)^{1/3}, \quad r \simeq \left(\frac{2}{m\sigma} \right)^{1/3}, \quad (12)$$

which, with $m \simeq 300$ MeV, gives

$$E \simeq 765 \text{ MeV}, \quad (13)$$

in agreement with the ρ meson mass of ~ 770 MeV, and $r \simeq 0.7$ fm.

The way to derive Eq. (6) we suggest here is the use of the hadronic resonance spectrum. The idea of the spectral description of a strongly interacting gas which is a model for hot hadronic matter was suggested by Belenky and Landau [13] and consists in considering the unstable particles (resonances) on an equal footing with the stable ones in the thermodynamic quantities, by means of the resonance spectrum; e.g., the expression for pressure in such a resonance gas reads (in the Maxwell-Boltzmann approximation)

$$p = \sum_i g_i p(m_i) = \int_{M_l}^{M_h} dm \tau(m) p(m), \quad p(m) = \frac{T^2 m^2}{2\pi^2} K_2 \left(\frac{m}{T} \right), \quad (14)$$

where M_l and M_h are the masses of the lightest and heaviest species, respectively, and g_i are particle degeneracies.

Phenomenological studies [14] have suggested that the cubic density of states, $\tau(m) \sim m^3$, for each isospin and hypercharge provides a good fit to the observed hadron spectrum. Let us demonstrate here that this cubic spectrum is intrinsically related to collinear Regge trajectories (for each isospin and hypercharge).

It is very easy to show that the mass spectrum of an individual Regge trajectory is cubic. Indeed, consider, e.g., a model linear trajectory with negative intercept:

$$\alpha(t) = \alpha' t - 1. \quad (15)$$

The integer values of $\alpha(t)$ correspond to the states with integer spin, $J = \alpha(t_J)$, the masses squared of which are $m^2(J) = t_J$. Since a spin- J state has multiplicity $2J+1$, the number of states with spin $0 \leq J \leq \mathcal{J}$ is

$$N(\mathcal{J}) = \sum_{J=0}^{\mathcal{J}} (2J+1) = (\mathcal{J}+1)^2 = \alpha'^2 m^4(\mathcal{J}), \quad (16)$$

in view of (15), and therefore the density of states per unit mass interval (the mass spectrum) is

$$\tau(m) = \frac{dN(m)}{dm} = 4\alpha'^2 m^3. \quad (17)$$

It is also clear that for a finite number of collinear trajectories, the resulting mass spectrum is

$$\tau(m) = 4N\alpha'^2 m^3, \quad (18)$$

where N is the number of trajectories, and does not depend on the numerical values of trajectory intercepts, as far as its asymptotic form $m \rightarrow \infty$ is concerned.

It turns out further that the cubic spectrum of the family of collinear Regge trajectories enables one to determine the mass of the state this family starts with. Before we dwell upon this point, let us make the following remark.

It is widely believed that pseudoscalar mesons are the Goldstone bosons of broken $SU(3) \times SU(3)$ chiral symmetry of QCD, and that they should be massless in the chirally-symmetric phase. Therefore, it is not clear how well would the resonance spectrum be suitable for the description of the pseudoscalar mesons. Indeed, as we have tested in [15], this nonet is *not* described by the resonance spectrum. Moreover, pseudoscalar mesons are extremely narrow (zero width) states to fit into a resonance description.

Thus, the resonance description should start with vector mesons, and the cubic spectrum of a linear trajectory enables one to determine the mass of the ρ meson, as follows:

Since the ρ meson has the lowest mass which the resonance description starts with, let us locate this state by normalizing the ρ trajectory to one state in the characteristic mass interval $(\sqrt{M^2(\rho) - 1/(2\alpha')}, \sqrt{M^2(\rho) + 1/(2\alpha')})$. With the cubic spectrum (17) of a linear trajectory, one has¹

$$1 = 4\alpha'^2 \int_{\sqrt{M^2(\rho) - 1/(2\alpha')}}^{\sqrt{M^2(\rho) + 1/(2\alpha')}} m^3 dm = 2\alpha' M^2(\rho), \quad (19)$$

and therefore

$$M^2(\rho) = \frac{1}{2\alpha'}. \quad (20)$$

We note that Regge slope is known to have a weak flavor dependence in the light quark sector [11]: (in GeV^{-2}) $\alpha'_{n\bar{n}} = 0.88$, $\alpha'_{s\bar{n}} = 0.85$, $\alpha'_{s\bar{s}} = 0.81$. With $\alpha' = 0.85 \text{ GeV}^{-2}$, as the average of the above three values, Eq. (20) gives

$$M(\rho) = 767 \text{ MeV}, \quad (21)$$

¹Since the ρ trajectory starts with a spin-1 isospin-1 state (ρ), it corresponds to the spectrum $\tau(m) = 9 \times 4\alpha'^2 m^3$. There is therefore no difference in normalizing this trajectory to 9 states, or (17) to one state, in the vicinity of the ρ mass.

in excellent agreement with the measured ρ meson mass of 768.5 ± 0.6 MeV [5].

It is now tempting to apply similar arguments to glueballs. The slope of glueball trajectories can be related to that of meson ones by the product of two, color and mechanical, factors:

$$\alpha'_{glue} = \gamma_c \gamma_{mech} \alpha', \quad (22)$$

where

$$\gamma_c = \frac{4}{9}, \quad (23)$$

and

$$\gamma_{mech} = \frac{1}{2}. \quad (24)$$

Eq. (23) is easily understood by looking at the confining pieces of the two-body potentials in (8), (11), and Eq. (4). The mechanical factor is easily understood also, by noting that in the string model glueball represents a closed string, while ordinary meson an open string, and a spin-energy relation for a closed string has an extra factor of $1/2$ as compared to that for an open string (e.g., without color, $J = \alpha' E^2$ for an open string, and $J = 1/2 \alpha' E^2$ for a closed string, in the classical case) [16]. We note that the relation $\alpha'_{glue} = 4/9 \alpha'$, derived by Simonov via the vacuum correlators method [17], capitalizes only the color factor, (23), but not the mechanical one, (24). As we see here, both factors should be taken into account in the correct formula for the glueball slope, which is

$$\alpha'_{glue} = \frac{2}{9} \alpha' \simeq 0.2 \text{ GeV}^{-2}. \quad (25)$$

We now apply a procedure similar to that above for locating the ρ meson mass, in order to locate the lightest state which lie on glueball trajectories. It is firmly established that the lightest glueball state is the scalar glueball [12, 18]. We therefore locate the scalar glueball mass as that for which its trajectory is normalized to one state in the mass interval $(M^2(0^{++}) - 1/(2\alpha'_{glue}), M^2(0^{++}) + 1/(2\alpha'_{glue}))$. Similar to (20), this procedure leads to

$$M^2(0^{++}) = \frac{1}{2\alpha'_{glue}}, \quad (26)$$

which reduces, through (20), (25), to

$$M(0^{++}) = \frac{3}{\sqrt{2}} M(\rho). \quad (27)$$

We take $M(\rho) = 764 \pm 5$ MeV, to accommodate the value given in [5] and results on the ρ^0 meson mass, both theoretical [19] and experimental [20], which concentrate around 760 MeV. With this $M(\rho)$, the above relation yields

$$M(0^{++}) = 1620 \pm 10 \text{ MeV}, \quad (28)$$

in excellent agreement with QCD lattice results 1600 ± 100 MeV [1, 2, 3, 4].

Finally, we note that it follows from (4), (25), (26) that

$$M^2(0^{++}) = 18\sigma, \quad (29)$$

which is equivalent to Eq. (6).

2 Concluding remarks

Again, as in a previous paper [21] where we discussed the linear mass spectrum of an individual hadronic multiplet first established in [22] and then applied to the problem of the correct $q\bar{q}$ assignments for problematic meson nonets, and the cubic spectrum of a strongly interacting gas as a model for hot hadronic matter, we have again demonstrated that a mass spectrum is power tools for hadron spectroscopy. We have shown that linear trajectories for mesons and glueballs, and the cubic mass spectrum associated with them, determine a relation between the masses of the lightest states lying on these trajectories, the ρ meson and the scalar glueball, respectively, which implies the mass of the latter in the vicinity of 1600 MeV. We have also established a relation between the scalar glueball mass and string tension (Eq. (29)) which is the subject of lattice QCD and pure Yang-Mills calculations.

We note that the mass spectrum can also establish a relation between the scalar and tensor glueball mass. Indeed, as discussed in [21], mass splitting between S -wave spin-0 and spin-1 meson states (ρ and π , or K and K^*) is well reproduced by the linear and cubic spectra of the corresponding multiplets and Regge trajectories, respectively, leading to the relations

$$M^2(\rho) - M^2(\pi) = M^2(K^*) - M^2(K) = \frac{1}{2\alpha'},$$

in good agreement with data (first of these relations is consistent with (20), since $M(\pi) \ll M(\rho)$). Similar procedure in the glueball sector will lead to the relation between the masses of the spin-0 scalar and spin-2 tensor glueballs:

$$M^2(2^{++}) - M^2(0^{++}) = \frac{1}{2\alpha'_{glue}}, \quad (30)$$

which implies, through (20), (25),

$$M(2^{++}) = \sqrt{2} M(0^{++}) = 3 M(\rho) = 2290 \pm 15 \text{ MeV}, \quad (31)$$

in excellent agreement with QCD lattice simulations [4, 23] which give 2390 ± 120 MeV for the tensor glueball mass, and corresponding three experimental candidates in this mass region [5], $f_J(2220)$, $J = 2$ or 4 , $f_2(2300)$ and $f_2(2340)$, the first of which is seen in $J/\psi \rightarrow \gamma + X$ transitions but not in $\gamma\gamma$ production [5], while the remaining two are observed in the OZI rule-forbidden process $\pi p \rightarrow \phi\phi n$ [5], which favors the gluonium interpretation of all three states.

References

- [1] R. Landua, "New Results in Spectroscopy", plenary talk at the International Conference on High Energy Physics (ICHEP96), Warsaw, Poland, July 1996, and references therein

- [2] UKQCD Coll. (G.S. Bali *et al.*), Phys. Lett. B **309** (1993) 378
- [3] J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. **75** (1995) 4563
- [4] C.J. Morningstar and M. Peardon, Phys. Rev. D **56** (1997) 4043
- [5] Particle Data Group (R.M. Barnett *et al.*), , Phys. Rev. D **54** (1996) 1
- [6] M. Teper, Phys. Lett. B **289** (1992) 115; Nucl. Phys. B (Proc. Suppl.) **30** (1993) 529
- [7] C.J. Hamer, M. Sheppeard, Zheng Weihong and D. Schütte, Phys. Rev. D **54** (1996) 2395
- [8] X.-Q. Luo and Q.-Z. Chen, Mod. Phys. Lett. A **11** (1996) 2435
- [9] S. Samuel, Phys. Rev. D **55** (1997) 4189
- [10] J.S. Kang and H. Schnitzer, Phys. Rev. D **12** (1975) 841
- [11] J.-L. Basdevant and S. Boukraa, Z. Phys. C **28** (1985) 413; Ann. Phys. (Paris) **10** (1985) 475
- [12] G.B. West, Nucl. Phys. B (Proc. Suppl.) **54** (1997) 353
- [13] S.Z. Belenky and L.D. Landau, Sov. Phys. Uspekhi **56** (1955) 309; Suppl. Nuovo Cim. **3** (1956) 15
- [14] E.V. Shuryak, Sov. J. Nucl. Phys. **16** (1973) 220
L. Burakovsky and L.P. Horwitz, Nucl. Phys. A **614** (1997) 373
- [15] L. Burakovsky and L.P. Horwitz, Nucl. Phys. A **609** (1996) 585
- [16] X. Artru, Nucl. Phys. B **85** (1975) 442; Phys. Rep. **97** (1983) 147
- [17] Yu.A. Simonov, Phys. Lett. B **249** (1990) 514
- [18] G.B. West, Phys. Rev. Lett. **77** (1996) 2622
- [19] S. Dubnicka and L. Martinovic, J. Phys. G **15** (1989) 1349
A. Bernicha, G. Lopez Castro and J. Pestieau, Phys. Rev. D **50** (1994) 4454
- [20] L. Capraro *et al.*, Nucl. Phys. B **288** (1987) 659
LEBC-EHS Coll. (M. Aguilar-Benitez *et al.*), Z. Phys. C **50** (1991) 405
- [21] L. Burakovsky, Relativistic Statistical Mechanics and Particle Spectroscopy, Talk at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT '98, Houston, Texas, USA, Feb 9-11, 1998, [hep-ph/9804xxx], and Found. Phys., *in press*
- [22] L. Burakovsky and L.P. Horwitz, Found. Phys. Lett. **9** (1996) 561, **10** (1997) 61
- [23] M. Peardon, Coarse lattice results for glueballs and hybrids, hep-lat/9710029