Instantaneous Power Factor Determined by Instantaneous Phasors

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Abstract

The instantaneous power factor can be clearly understood from the instantaneous phasors. The unique property of instantaneous phasors is that at any instant the instantaneous three-phase currents and voltages can be represented by a set of balanced phasors. The instantaneous power factor resembles the familiar format of power factor derived from the conventional phasors. This new concept can be used for power quality monitoring, diagnostics, and compensations.

I. INTRODUCTION

The instantaneous phasor method originated by the author has a unique symmetrical property. Regardless of how unbalanced the three-phase situation is, the instantaneous phasors of one phase can be used to represent three phases[1-4]. Three-phase currents and voltages can be represented by a set of balanced instantaneous phasors, respectively.

Traditionally, the concept of power factor is the ratio of active power to the apparent power averaged over a period of time. For instance, Fig. 1 shows the waveforms of a voltage and a φ-degree lagging current. The classical phasors, V and I, of the voltage and current are also shown. The active power equals the integration of v and i products averaged over a cycle. Because of the phase angle difference between v and i, the sign of vi product changes, as illustrated in the figure. This integration can only be less or equal to the integration of v and i products of voltage and current that are in phase. The latter integration represents the apparent power. The traditional power factor can also be presented by the cosine of the angle between the voltage and current phasors, V and I. The instantaneous power factor concept is different from the traditional concept of power factor that is assessed over the period of a cycle.

Recent interesting developments on the instantaneous reactive power and instantaneous power concept [5-7] have been proven to be useful for power quality and utilization improvements.

The unique instantaneous phasors discussed in this paper not only provide a clear picture of the instantaneous power factor, they also give a clear overall power quality picture. The roundness of the trajectory of instantaneous phasors of a fundamental cycle indicates the quality of currents and voltages. The instantaneous phasors provide theoretical foundation for power quality monitoring, diagnostics, and improvements.

The instantaneous phasors of voltages and currents derived in [1] can either be presented in a vector format or

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in a complex number format. The arbitrarily chosen complex number format of the instantaneous phasors, such as the voltages, $V_a$, $V_b$, and $V_c$, are given in (1). They have the same magnitude but are 120-degrees apart.

$$V_a = (v_a - v_0) + jv_{aq},$$
$$V_b = (v_b - v_0) + jv_{bq},$$
$$V_c = (v_c - v_0) + jv_{cq}.$$  

where the zero-sequence component for the three-phase voltages is

$$v_0 = \frac{1}{3}(v_a + v_b + v_c).$$  

Alternatively, a more general expression of (1) to include the zero-sequence components in the equations by shifting the origins of phasors can be adapted.

The real values of the instantaneous phasors are simply the instantaneous phase values without the zero-sequence component as given in (3).

$$(v_a - v_0),$$
$$(v_b - v_0),$$ and
$$(v_c - v_0).$$

The instantaneous phasors' imaginary values denoted as $v_{aq}$, $v_{bq}$, and $v_{cq}$ can be obtained from (4)

$$v_{aq} = \frac{v_b - v_c}{\sqrt{3}},$$
$$v_{bq} = \frac{v_c - v_a}{\sqrt{3}},$$
$$v_{cq} = \frac{v_a - v_b}{\sqrt{3}}.$$  

The numerators of equation (4) are actually the instantaneous line-to-line voltages that are not affected by the zero-sequence component.

The instantaneous phasor magnitude, $V$, of $V_a$, $V_b$, and $V_c$ can be derived from (2), (3), and (4). The result is that

$$\|V_a\| = \|V_b\| = \|V_c\| = V.$$  

where

$$V = \sqrt{\frac{2}{9}[(v_a - v_b)^2 + (v_b - v_c)^2 + (v_c - v_a)^2]}.$$  

Alternatively, the instantaneous phasor magnitude $V$, of $V_a$, $V_b$, and $V_c$ can be derived from a phase, for instance, from phase-a

$$V = \|V_a\| = \sqrt{(v_a - v_0)^2 + v_{aq}^2},$$  

from phase-b

$$V = \|V_b\| = \sqrt{(v_b - v_0)^2 + v_{bq}^2},$$  

and from phase-c

$$V = \|V_c\| = \sqrt{(v_c - v_0)^2 + v_{cq}^2}. $$

II. ROUNDNESS OF TRAJECTORIES OF INSTANTANEOUS PHASORS

The voltages and currents obtained from a field test of a 50-hp, 4-pole induction motor are shown in Fig. 1. The voltages are slightly unbalanced, and the currents are significantly more unbalanced. The roundness of a trajectory indicates how good the power quality of three phases is.

Fig. 1 Phase voltages and currents at full load
The trajectories of instantaneous voltage phasors, \( V_a, V_b, \) and \( V_c \), of (1) are shown in Fig. 2, where the three phasors are always identical in magnitude but are 120-degrees apart. The same observations can be drawn from the trajectories of instantaneous current phasors, \( i_a, i_b, \) and \( i_c \), shown in Fig. 3. Mathematical proof of this 120-degree symmetry property is given in the Appendix.

Figs. 2 and 3 also show that for the polluted voltages and currents, the instantaneous phasor trajectories are not circles because of harmonics and negative-sequence content. The instantaneous phasors are not rotating at a constant speed [1, 2]. The non-circular shape of the current trajectories suggests that the three-phase currents are significantly unbalanced and with certain harmonics.

The unique symmetrical property of the instantaneous phasors of three phases permits taking the voltage and current phasors of one phase to calculate the various power components of the entire motor.

### III. INSTANTANEOUS ROOT-MEAN-SQUARE (rms) VALUES OF VOLTAGES OR CURRENTS

From Fig. 4 and (1) the instantaneous value of phase currents or voltages can be expressed by the projections of instantaneous phasors to the real axis.

\[
\frac{1}{3} \left( V \cos \theta \right)^2 + \left[ V \cos \left( \theta + \frac{4\pi}{3} \right) \right]^2 + \left[ V \cos \left( \theta + \frac{2\pi}{3} \right) \right]^2
\]

\[
= \frac{V}{\sqrt{3}} \quad \text{(10)}
\]
Detailed derivation proves that the instantaneous current or voltage phasor magnitude divided by \( \sqrt{2} \) equals the instantaneous rms value of three-phase currents or voltages excluding their zero-sequence component.

Combining (6), (7), (8), (9), and (10) the following relationship given in (11) holds true.

\[
\text{Magnitude of instantaneous phasor of voltage} \\
= \sqrt{\frac{2}{9}} \left[ (v_a - v_b)^2 + (v_b - v_c)^2 + (v_c - v_a)^2 \right] \\
= \sqrt{\frac{2}{9}} \left[ (v_a - v_b)^2 + \frac{(v_b - v_c)^2}{\sqrt{3}} \right] \\
= \sqrt{\frac{2}{9}} \left[ (v_b - v_0)^2 + \frac{(v_c - v_a)^2}{\sqrt{3}} \right] \\
= \sqrt{\frac{2}{3}} \sqrt{(v_a - v_0)^2 + (v_b - v_0)^2 + (v_c - v_0)^2} \\
= \sqrt{2} \cdot \text{Instantaneous rms value of three-phase voltages.} \quad (11)
\]

Similar expression can be derived for the instantaneous current magnitude, \( I \).

IV. INSTANTANEOUS ACTIVE AND REACTIVE POWERS

A. Instantaneous Active Power Excluding Zero-Sequence Components

The instantaneous active power excluding zero-sequence components of three phases is the summation of products of phase voltages and currents without zero-sequence components.

\[
P_{\text{phasor}} = (v_a - v_0) \cdot (i_a - i_0) + (v_b - v_0) \cdot (i_b - i_0) + (v_c - v_0) \cdot (i_c - i_0). \quad (12)
\]

The products on the right side of the equal sign of (12) are given in the left-hand side of the equal sign of the following equation, (13). The symmetrical property of instantaneous phasors having the same magnitude and being 120-degree apart among phasors is used.

\[
\begin{align*}
P_{\text{phasor}} &= \frac{3}{2} V I \cos \phi. \\
&= \|V_a\| \cos(\theta + \phi) I_a \cos \theta \\
&+ \|V_b\| \cos(\theta + \frac{4\pi}{3}) I_b \cos(\theta + \frac{4\pi}{3}) \\
&+ \|V_c\| \cos(\theta + \frac{2\pi}{3}) I_c \cos(\theta + \frac{2\pi}{3}) \\
&= \frac{3}{2} V I \cos \phi. \\
\end{align*} \quad (13)
\]

Simplifying the left-hand portion of the equal sign of (13) gives the right-hand term of (13).

From (12) and (13) we have

\[
P_{\text{phasor}} = \frac{3}{2} V I \cos \phi. \quad (14)
\]

This equation, (14), says that the instantaneous active power of three-phase phasors equals 3 times \( \frac{V}{\sqrt{2}} \) times \( \frac{I}{\sqrt{2}} \) times the cosine of the angle between the voltage and current phasors. The \( V \) and \( I \) are the magnitudes of voltage and current phasors excluding the zero-sequence components. This expression of instantaneous phasor power for either balanced or unbalanced situations is similar to the format of conventional average power of balanced three phases.

B. Three-Phase Instantaneous Active Power Including Zero-Sequence Components

The three-phase instantaneous active power, \( p \), calculated from the real instantaneous voltages and currents including zero-sequence components is

\[
p = v_d' i_d + v_b i_b + v_c i_c. \quad (15)
\]

We have the three-phase instantaneous power

\[
p = P_{\text{phasor}} + 3v_0i_0. \quad (16)
\]
C. Three-Phase Instantaneous Apparent Power of Instantaneous Phasors

The three-phase instantaneous apparent power, \( s_{\text{phasor}} \), is the product of 3 times the rms voltage and current.

\[
s_{\text{phasor}} = \frac{3}{2} V I
\] (17)

D. Three-Phase Instantaneous Reactive Power of Instantaneous Phasors

From Fig. 4 the instantaneous reactive power, \( q_{\text{phasor}} \), of the phasors is given by.

\[
q_{\text{phasor}} = \frac{3}{2} V I \sin \phi
\] (18)

E. Instantaneous Power Factor

The instantaneous power factor, \( \cos \phi \), is defined by the ratio of the active and the apparent instantaneous powers.

\[
\cos \phi = \frac{P_{\text{phasor}}}{s_{\text{phasor}}}
\] (19)

V. PHYSICAL MEANING OF INSTANTANEOUS POWER COMPONENTS

The properties of the conventional active and reactive power theory still hold true. From (16), (17), and (18) we have

\[
s_{\text{phasor}}^2 = P_{\text{phasor}}^2 + q_{\text{phasor}}^2
\] (20)

From (19) when the instantaneous power factor equals one, the instantaneous active power, \( P_{\text{phasor}} \), equals the instantaneous apparent power, \( s_{\text{phasor}} \). This is only possible if the instantaneous current phasor coincides with the instantaneous voltage phasor shown in Fig. 4. For a given instantaneous active power, the required magnitude of the current phasor is the smallest one as compared with those when the instantaneous power factor is not one. From (10), the magnitude of instantaneous phasor is proportional to the instantaneous rms value of three phases. The smallest rms value of three phases for a given active power means the losses associated with the rms value is the smallest one, and the power delivery is at its most efficient manner.

VI. CONCLUSIONS

The instantaneous power factor can be clearly understood from the instantaneous phasors. The unique property of instantaneous phasors is that at any instant the instantaneous three-phase currents and voltages can be represented by a set of balanced phasors. The instantaneous power factor resembles the familiar format of power factor derived from the conventional phasors. This new concept can be used for power quality monitoring, diagnostics, and compensations.

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VIII. REFERENCES


IX. APPENDIX

Symmetry Property of Instantaneous Phasors

The 120-degree symmetry property of instantaneous phasors is proven as follows.

The three-phase phasors are defined by (1). Substituting (4) into (1) gives

\[ V_a = (v_a - v_0) + j \frac{v_b - v_c}{\sqrt{3}}, \]
\[ V_b = (v_b - v_0) + j \frac{v_c - v_a}{\sqrt{3}}, \]
\[ V_c = (v_c - v_0) + j \frac{v_a - v_b}{\sqrt{3}}. \]  

(21)

To prove \( V_c \) equals \( V_a \) turned 120 degrees ahead, we can simply turn \( V_a \) 120 degrees ahead. This indeed results in \( V_c \). The turning is conducted by multiplying \( V_a \) with \( e^{j\frac{2\pi}{3}} \). Simplification gives

\[ \left[ (v_a - v_0) + j \frac{v_b - v_c}{\sqrt{3}} \right] e^{j\frac{2\pi}{3}} = (v_c - v_0) + j \frac{v_a - v_b}{\sqrt{3}}. \]  

(22)

In a like manner, we can prove that \( v_b \) is \( v_a \) turned 120 degrees behind.

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