THE USE OF THE GOLDEN PROPORTION IN PAINTINGS

BY TITIAN AND RAPHAEL

APPROVED:

[Signatures]

Major Professor

Minor Professor

Director of the Department of Art

Chairman of the Graduate Council
THE USE OF THE GOLDEN PROPORTION IN PAINTINGS

BY TITIAN AND RAPHAEL

THESIS

Presented to the Graduate Council of the North Texas State Teachers College in Partial Fulfillment of the Requirements

For the Degree of

MASTER OF ARTS

By

Barbara Ruth Hamilton, B.A.

Dallas, Texas
August, 1942

100454
TABLE OF CONTENTS

LIST OF ILLUSTRATIONS.............................................. v

Chapter
I. INTRODUCTION................................................. 1

Research Done by Scholars Since 1830
Procedure of This Analysis
Raphael's Life and Works
Titian's Life and Works

II. GEOMETRY OF ENCLOSING RECTANGLES BY
TITIAN AND RAPHAEL............................................. 10

Illustrations and Explanations of
Geometrical Constructions Used by
Titian in Establishing the Rectangles of Fifteen of his Compositions
Illustrations and Explanations of
Geometrical Constructions Used by
Raphael in Establishing the Rectangles of Fourteen of his Compositions

III. GEOMETRICAL ANALYSES OF THREE PAINTINGS
BY TITIAN.......................................................... 20

"Madonna of the Cherries"
"The Entombment"
"Ecce Homo"

IV. GEOMETRICAL ANALYSES OF THREE PAINTINGS
BY RAPHAEL......................................................... 58

"Madonna del Granduca"
"Sistine Madonna"
"Madonna of the Chair"

V. CONCLUSION....................................................... 96

How and Why Geometry Was Used by the
Italian Renaissance Artists
How Geometrical Analyses of Classical
Paintings Increases Ability to
Appreciate the Work of the Masters
Suggestions for Further Study in the
Field of Geometry in Art
TABLE OF CONTENTS—Continued

APPENDIX.................................................. 100
BIBLIOGRAPHY.............................................. 120
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Illustrations of Geometrical Constructions Used by Titian in Establishing the Rectangles for Fifteen of his Compositions</td>
<td>11</td>
</tr>
<tr>
<td>2. Illustrations of Geometrical Constructions Used by Raphael in Establishing the Rectangles for Fourteen of his Compositions</td>
<td>15</td>
</tr>
<tr>
<td>3. Analysis Showing How Geometrically Determined Vertical and Horizontal Lines Were Used in the Composition of &quot;Madonna of the Cherries&quot;</td>
<td>26</td>
</tr>
<tr>
<td>4. Analysis Showing How Diagonal Lines Joining Points Geometrically Determined Were Used in the Composition of &quot;Madonna of the Cherries&quot;</td>
<td>29</td>
</tr>
<tr>
<td>5. Analysis Showing How Circles and Arcs Related in the Divine Proportion Were Used in the Composition of &quot;Madonna of the Cherries&quot;</td>
<td>32</td>
</tr>
<tr>
<td>6. Analysis Showing Figures 3, 4, and 5 Superimposed</td>
<td>33</td>
</tr>
<tr>
<td>7. Analysis Showing How Geometrically Determined Vertical and Horizontal Lines Were Used in the Composition of &quot;The Entombment&quot;</td>
<td>33</td>
</tr>
<tr>
<td>8. Analysis Showing How Diagonal Lines Joining Points Geometrically Determined Were Used in the Composition of &quot;The Entombment&quot;</td>
<td>42</td>
</tr>
<tr>
<td>9. Analysis Showing How Arcs Related in the Divine Proportion Were Used in the Composition of &quot;The Entombment&quot;</td>
<td>44</td>
</tr>
<tr>
<td>10. Analysis Showing Figures 7, 8, and 9 Superimposed</td>
<td>45</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>11. Analysis Showing How Geometrically Determined Vertical and Horizontal Lines Were Used in the Composition of &quot;Ecce Homo&quot;</td>
<td>50</td>
</tr>
<tr>
<td>12. Analysis Showing How Diagonal Lines Joining Points Geometrically Determined Were Used in the Composition of &quot;Ecce Homo&quot;</td>
<td>53</td>
</tr>
<tr>
<td>13. Analysis Showing How Circles Related in the Divine Proportion Were Used in the Composition of &quot;Madonna del Granduca&quot;</td>
<td>55</td>
</tr>
<tr>
<td>14. Analysis Showing Figures 11, 12, and 13 Superimposed</td>
<td>56</td>
</tr>
<tr>
<td>15. Analysis Showing How Geometrically Determined Vertical and Horizontal Lines Were Used in the Composition of &quot;Madonna del Granduca&quot;</td>
<td>62</td>
</tr>
<tr>
<td>16. Analysis Showing How Diagonal Lines Joining Points Geometrically Determined Were Used in the Composition of &quot;Madonna del Granduca&quot;</td>
<td>64</td>
</tr>
<tr>
<td>17. Analysis Showing How Circles Related in the Divine Proportion Were Used in the Composition of &quot;Madonna del Granduca&quot;</td>
<td>67</td>
</tr>
<tr>
<td>18. Analysis Showing Figures 15, 16, and 17 Superimposed</td>
<td>68</td>
</tr>
<tr>
<td>19. Analysis Showing How Geometrically Determined Vertical and Horizontal Lines Were Used in the Composition of &quot;Sistine Madonna&quot;</td>
<td>74</td>
</tr>
<tr>
<td>20. Analysis Showing How Diagonal Lines Joining Points Geometrically Determined Were Used in the Composition of &quot;Sistine Madonna&quot;</td>
<td>77</td>
</tr>
<tr>
<td>21. Analysis Showing How Circles Related in the Divine Proportion Were Used in the Composition of &quot;Sistine Madonna&quot;</td>
<td>80</td>
</tr>
</tbody>
</table>
Figure

<table>
<thead>
<tr>
<th>22. Analysis Showing Figures 19, 20, and 21 Superimposed</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. Analysis Showing How Geometrically Determined Vertical and Horizontal Lines Were Used in the Composition of &quot;Madonna of the Chair&quot;</td>
<td>87</td>
</tr>
<tr>
<td>24. Analysis Showing How Diagonal Lines Joining Points Geometrically Determined Were Used in the Composition of &quot;Madonna of the Chair&quot;</td>
<td>90</td>
</tr>
<tr>
<td>25. Analysis Showing How Circles Related in the Divine Proportion Were Used in the Composition of &quot;Madonna of the Chair&quot;</td>
<td>93</td>
</tr>
<tr>
<td>26. Analysis Showing Figures 23, 24, and 25 Superimposed</td>
<td>94</td>
</tr>
</tbody>
</table>

Plate

<p>| I. Titian, Madonna With Saint Anthony | 101 |
| II. Titian, Gipsy Madonna | 102 |
| III. Titian, Assumption of the Virgin | 103 |
| IV. Titian, Madonna of the Cherries | 104 |
| V. Titian, Bacchus and Ariadne | 105 |
| VI. Titian, The Entombment | 106 |
| VII. Titian, La Bella | 107 |
| VIII. Titian, The Tribute Money | 108 |
| IX. Raphael, Donna Velata | 110 |
| X. Raphael, Madonna Garvagh | 111 |
| XI. Raphael, Saint Cecilia | 112 |
| XII. Raphael, Portrait of Angelo Doni | 113 |
| XIII. Raphael, Solly Madonna | 114 |</p>
<table>
<thead>
<tr>
<th>Plate</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>XIV. Raphael, <em>Sistine Madonna</em></td>
<td>115</td>
</tr>
<tr>
<td>XV. Raphael, <em>Madonna del Granduca</em></td>
<td>116</td>
</tr>
<tr>
<td>XVI. Raphael, <em>Madonna dell' Impannata</em></td>
<td>117</td>
</tr>
<tr>
<td>XVII. Raphael, <em>Madonna of Francis I</em></td>
<td>118</td>
</tr>
<tr>
<td>XVIII. Raphael, <em>Galatea</em></td>
<td>119</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

Research Done by Scholars Since 1830

During the Renaissance in Europe there developed two Schools of Art—the Florentine and the Venetian. Outstanding in the first was Raphael Santi, and in the latter, Titian Vecelli. These two artists, together with many others of the same period, produced masterpieces which have never been surpassed in their popularity with the general public and in the excellence of their composition from the artist's viewpoint.

Beginning with the classical art of ancient Egypt and continuing in the art of Greece and Rome, it appeared that there were certain traditions resulting in excellence of proportion and composition, which were handed down to Renaissance artists but later lost. At different times since the Renaissance, scholars have attempted analyses of Italian paintings of the period without being able to find the underlying cause of the enduring appeal of these works. Jay Hambidge, the first to arrive at a conclusion, applicable to all works of art that he studied, advanced the theory that the rhythm of all the early arts was controlled by an inner law of growth based upon geometrical relations developing from the square and its diagonals. According to his theory, the enclosing rectangle...
and all arrangements within were dependent upon this geometrical principle. This law he called "Dynamic Symmetry," a term found to have been used by the Greeks long before. To them dynamic symmetry was expressive of the proportions of what they termed the golden rectangle. A golden rectangle is obtained by extending the base of a square ABNM (see Fig. 1, [A], p. 11) to point D, determined by the arc described from the mid-point O as a center, with the radius OX.

From the geometry of this figure a certain proportion was obtained which to the Greek mind exemplified a perfect relationship. In the golden rectangle a mean and extreme ratio exists, wherein the short section is to the long section as the long section is to the whole line. If the short section of the line is measured off on the long section, the remaining length is the next unit in an indefinite series.

The division between the long and the short section of a line is called the median point. When the long side of the golden rectangle is divided at the median point, the resulting rectangles are a square and a golden rectangle, and further divisions will continue this relationship indefinitely.

In explaining that lines, angles and curves are defining areas which compose the units of an arrangement, Hambidge says:

1Jay Hambidge, Dynamic Symmetry in Composition, p. 67.
Symmetry shows that these limits have a direct bearing upon all arrangements of form within the enclosed area. When the composition of a picture is developed in accordance with this idea the result is a unity comparable to that of an organism; every part is related to every other part and all parts are definite and, more or less logical elements of the entire pattern.  

In addition to many pictures, Hambidge made a thorough investigation of Greek vases, analyzing a large number of them by applying geometrical principles. As a result, he concluded that if the over-all proportions of a vase can be expressed in terms of rectangles derived from the square, then the ratio of all its parts can be expressed likewise.  

Inspired by the studies of Hambidge, L. D. Caskey, although he does not have as broad a scope, has analyzed the majority of Greek vases in the Museum of Fine Arts, Boston, Massachusetts. He found that each vase could be fitted exactly into either the golden rectangle or one of the root rectangles. The interrelation of details is shown by subdivisions and intersections of diagonals of the rectangles.

---

2Ibid., p. 6.

3Hambidge, Dynamic Symmetry: The Greek Vase, p. 44.

4L. D. Caskey, Geometry of Greek Vases.

5The root rectangles are derived from diagonals of a series of rectangles starting with a square, the length of which remains the short side of the succeeding rectangles. The long side of a root-two rectangle is equal to the diagonal of the square. The long side of a root-three rectangle is equal to the diagonal of a root-two rectangle. The long side of a root-four rectangle is equal to the diagonal of a root-three rectangle. The long side of a root-five rectangle is equal to the diagonal of a root-four rectangle (see Hambidge, Dynamic Symmetry: The Greek Vase, pp. 23-24).
Another group of geometrical analyses based on the golden proportion has been made by Irma Richter.⁶ Pictures of the Italian Renaissance were her chief interest and to many of these she applied the basic scheme consisting mostly of pentagons, stars and circles, having their radii in a geometrical series.⁷ The divisions of lines connecting points of a five-pointed star are also expressed in the mean and extreme ratio. Richter found that the pentagon was frequently used to enclose a star, a side of the pentagon being also one measure of the series represented in the star construction.

Other works of art which Richter used to prove her theory are Greek vases, paintings of various periods, the doorway of Chartres Cathedral and the floor plan of the Parthenon. While her discovery of the structural use of circles in classical compositions shows extensive study, her analyses are incomplete because she does not relate vertical, horizontal and diagonal structural lines to the circular scheme.

Of great assistance in geometrical analysis of paintings are the four laws discovered by J. de Vianna Kelsch. He did not study pictures for the purpose of determining their basic geometrical plans as did the above mentioned authors, but he proved that several important canonical points of figures within a composition were always located on a straight line.

⁶Irma Richter. Rhythmic Form in Art.
⁷Ibid., pp. 101-111.
His study was not carried far enough to discover how the positions of these lines were determined, or how the points at which they touched the enclosed rectangles were established. The four rules follow:

1. Law of Isocaphaly—determined by a straight line binding certain points of the body:
   A. Perfect—when the line binds only nasal points with each other;
   B. Broken—when the line binds indifferently nasal points, sub-nasal points, metopic points, or mental points with each other; and also when it passes by the middle cross section of the nose, of the forehead, by the point of the origin of the hair, or by the vertex.

2. Law of the Classical Portrait—A line of coordination binds the nasal point of the person represented to one of the canonical points of the middle finger of one, and, sometimes, of both hands, following exactly the longitudinal axis of the ridge of the nose.

3. Law of the Madonnas—The center of expression of the Madonna is almost always bound to one of the canonical points of each of her hands, the Child Jesus being enfolded by the maternal arms.

4. Law of the Annunciation—Facial points and canonical points of the hands, respectively, are bound by two parallel lines.

Kelsch applied his four laws to an extensive range of paintings from the early Egyptian period through the Renaissance. In many of these only one rule was applied, while in others two, or even three, were used.

Theodore Cook based his study on the belief that in many living organisms the relationship of parts expresses the golden

---

8G. de Vienna Kelsch, Canon Picturium of Composition Harmony and Rhythm, p. 43.
9Ibid., p. 219.
10Ibid., p. 373.
11Ibid., p. 415.
proportion and the resulting geometrical progression. It was his conclusion that man's creative art developed this structural scheme as the most satisfying because it is found so frequently in nature. Selecting natural formations—such as shells, flowers and horns—Cook successfully applied geometrical plans based on the golden proportion.\(^{12}\) Bringing one of these into practical use, he illustrated the relationship between the design of the spiral stairway at Blois and the series of diameters found in the spiral shell (*Voluta vespertilio*).\(^{13}\)

Claude Bragdon was another scholar who concluded, after an intensive study of the expressiveness of architecture, that there is a relation between the rhythmic organization of living organisms and the golden proportion found in the classical arts.\(^{14}\) He used simple shapes such as the circle, the square and the equilateral triangle in analyzing living organisms, cathedral floor plans and facades, and pictures. Although his studies are brief and indefinite, they show the latent geometry that forms the basis of unity.

Walter Darwin Teague,\(^{15}\) Charles Martin,\(^{16}\) and Walter Pach\(^{17}\) have made studies and have given enlightening reviews of geometry in art.

---


\(^{13}\)Ibid., p. 361.

\(^{14}\)Claude Bragdon, *The Beautiful Necessity*.

\(^{15}\)Walter Darwin Teague, *Design This Day*, pp. 141-161.


Procedure of This Analysis

Inspired by the studies of the above scholars, it seemed reasonable to the writer that all the artists of the Fifteenth Century may have used, either entirely or in part, geometrical plans to assure excellence of proportions in their works. For that reason, paintings of Raphael and Titian were chosen for study (1) to ascertain the extent to which they used geometry, and (2) to determine, if possible, the differences and likenesses in their underlying schemes, and (3) to determine how geometrical divisions, if used, affected the character of their paintings.

Twelve pictures by each artist were chosen in order to find out how the proportions of their enclosing rectangles were established. Chapter II gives an account of this experiment. Then, the structural lines governing the compositions of three pictures by each artist were completely analyzed. Chapter III deals with analyses of Titian's paintings and Chapter IV, with analyses of Raphael's paintings.

In the Conclusion, the author (1) states her opinion of the reason that, and the extent to which, geometry was used by the Italian Renaissance artists, (2) shows how geometrical analyses of classical paintings increase ability to appreciate the work of the Masters, and (3) offers suggestions for further study in the field of geometry in art.

Reproductions of eighteen of the thirty paintings chosen for study are comprised in the Appendix.
Raphael's Life and Works

That the reader may have a better understanding of the importance of the works of these two artists, a brief sketch concerning each is presented.

Raphael is and has always been acclaimed the greatest Madonna painter of all times. His pictures are simple and direct in their appeal and they portray a grace and charm that have given him an enduring popularity equalled by no other artist.

Raphael was born in Urbino, in 1483. From his artist father, Giovanni Santi, Raphael inherited a poetic nature, his strength of will and character, and a desire to create. His first apprenticeship was served with Timoteo Viti, from whom, although he later acquired traits from other teachers, basic qualities were absorbed which remained in all his best works. Perugino, Raphael's second teacher, was at the time, the best known artist in Urbino and he surpassed all with whom he worked, except his pupil. By Raphael he was left far behind.

In 1504, Raphael went to Florence, where he lived and worked the remainder of his life. During his thirty-seven years he reached a height shared by no other artist of Florence. His pictures have been before the eyes of the world's great critics and to the present day he still retains the title, "Master of Madonnas."
Titian's Life and Works

With Titian Vecelli, Venetian painting reached the zenith of its achievements. Here was an artist who lived to be almost one hundred years old, and during that time practice and experience improved his work to his last picture, which was painted after he passed eighty.

Titian was born in Pieve, about 1477, but he received his training and did his work in Venice. His first teacher was Sebastiano Zucato, from whom he passed on to the instruction of Giovanni Bellini. Studying under the same artist was Giorgione, whose work Titian admired and studied very carefully. From Bellini, Titian acquired such qualities as rich coloring, dignity, vigorous outlines, and harmony and strong contrasts in light and shade.

Far different from the simple life of Raphael, Titian enjoyed the visits of Kings and Emperors and painted the most famous people of his time. He was overwhelmed with orders and commissions. With equal success he painted many subjects. Altar-pieces, portraits, historical, mythological, and allegorical themes were all portrayed in the splendid style which made him the greatest Venetian painter of the Renaissance.
CHAPTER II

GEOMETRY OF ENCLOSING RECTANGLES

The study of fifteen pictures painted by Titian and fourteen by Raphael shows that in every instance the rectangles enclosing the compositions were derived geometrically. This procedure resulted in proportions that allowed a continuation of rhythmic arrangements throughout their areas.

Although each artist, in constructing his rectangles had his own method of handling the geometrical principles, they frequently used similar basic constructions. Both started with squares and added geometrical measures of these squares to determine the proportions of the final rectangles. The similarities and differences between the rectangles used by the two artists are shown in Figures 1 and 2.

To one side of a square, Titian frequently joined (1) a geometrical section of one of its sides, (2) a fraction of one of these measures, or (3) both a geometrical section and a fractional part of one side of the square (Fig. 1, [A]). By using the first procedure, Titian determined the rectangle for "Madonna With Saint Anthony" (Plate I, p.101), which is a square plus the short section of its length. By using the second procedure, the rectangle of "Ecce Homo" (Fig. 11, p.50) was constructed. To one side of a square was added a distance
Fig. 1—Illustrations of geometrical constructions used by Titian in establishing the rectangles for fifteen of his compositions. One side of a square was extended by adding a distance equal to (1) a geometrical section of one of its sides, (2) a fraction of one of these sections, (3) a fractional part of one of its sides, or (4) a geometrical section and a fractional part of one of its sides. A) To the square AFGM', was added rectangle GEDM'. M'D, the short side of the rectangle GEDM', is equal to MM', the long section of AM'. To the golden rectangle AFED, rectangle FBCE was added. FB, the short side of rectangle FBCE, is equal to one fourth of AF, a side of the square AFGM'.

B) Two equal squares, ABGM and EFCD, were horizontally superposed at M, the median point of ED. C) Two equal golden rectangles, ABGH and FECD, were horizontally superposed. FH, the overlapped distance, is equal to one third of MD, the short section of CD. D) To the square AFED was added the rectangle FBCE. FB, the short side of rectangle FBCE, is equal to one half of AF. E) To the golden rectangle FECD, rectangle ABEF was added. AF, the short side of rectangle ABEF, is equal to M'C, the fourth unit in the geometrical series of measures of line DC.
equal to one third of its short section. By using the third procedure, Titian determined the rectangle of "Gipsy Madonna" (Plate II, p. 102). To one side of a square was added a distance equal to its short section. To the short side of the golden rectangle, thus constructed, was added a distance equal to one-fourth of the length of the basic square.

In the group of fifteen paintings by Titian, four of the rectangles resulted from extending a square on one side by adding a fractional part of its length (Fig. 1, [9]). The rectangles of "The Virgin and Child," "The Assumption of the Virgin," and "The Madonna of the Cherries" illustrate this method.

The rectangle of "The Virgin and Child" resulted from extending a square on one side by adding a measure equal to one fifth of its length.

The rectangle of "The Assumption of the Virgin" (Plate II, p. 103) resulted from extending a square on one side by adding a measure equal to one half of its length.

The rectangle of "The Madonna of the Cherries" (Fig. 3, p. 26, and Plate IV, p. 104) resulted from extending a square on one side by adding a distance equal to one fourth of its length.

A variation of the method illustrated by Figure 1, (9) was used by Titian in establishing the rectangle of "Sacred and Profane Love," which is composed of two equal, horizontally...
superimposed squares plus one fifth of their total length.

In some instances, Titian first constructed a golden rectangle. A second, equal rectangle was overlapped so that the superimposed space is equal to (1) a measure in the geometrical progression of a side of one rectangle, (2) a fraction of one of these measures, or (3) the combined length of two of the measures (Fig. 1, [C]). Sometimes, equal squares were substituted for golden rectangles (Fig. 1, [B]).

By using the first procedure mentioned in the preceding paragraph, Titian established the rectangles of "Bacchus and Ariadne" and "The Holy Family." The rectangle of "Bacchus and Ariadne" (Plate V, p.105) was made by overlapping two equal golden rectangles. The overlapped distance is equal to the fifth unit in the geometrical series of measures of the long side of one of the rectangles.

By using the second procedure, the rectangle of "The Punishment of Actaeon" was established. The overlapped distance of two equal golden rectangles is equal to one third of the short section of the long side of one rectangle.

By using the third procedure, the rectangle of "The Entombment" (Fig. 7, p. 38, and Plate VI, p.106) was established. The overlapped distance of two equal squares is equal to the short section plus the fifth unit in the geometrical series of measures of a side of one square.

---

3 For reproduction see Titian Drawings and Paintings, Fig. 233.
The rectangle of "The Holy Family" was made by overlapping two equal squares at the median point.

Figure 1, E, resulting from the combination of a golden rectangle and a unit in the geometrical series of measures of one of its sides, or a fraction of one of these measures, is a scheme occasionally found in Titian's pictures, while a majority of Raphael's compositions were enclosed by its usage (Fig. 2, [A]). Titian used this plan to form the rectangles of "La Bella," "The Tribute Money," "The Annunciation," and "Venus and Adonis." Raphael used it for the rectangles of "Agony in the Garden," "Donna Velata," "Madonna Garvagh," "Saint Cecilia," "Madonna di Casa Tempi," "Portrait of Angelo Doni," "The Solly Madonna," "The Cowper Madonna," "Sistine Madonna," and "Madonna del Granduca." Some of Raphael's rectangles had a very short geometrical measure added to their areas. One painting, "Madonna del Impannata" (Plate XVI, p. 117) is contained in a rectangle which resulted from two equal, vertically superposed golden rectangles (Fig. 2, [C]).

The rectangle of "La Bella" (Plate VII, p. 107) was constructed by extending the short side of a golden rectangle a distance equal to the fifth unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "Tribute Money" (Plate VIII, p. 108) was constructed by extending the short side of a golden rectangle

---

\(^4\) For reproduction see Bensusan, Titian, p. 70.
Fig. 2—Illustrations of geometrical constructions used by Raphael in establishing the rectangles for fourteen of his compositions. One side of a square was extended by adding a distance equal to, (1) a geometrical section of one of its sides, (2) a fraction of one of these sections, (3) a fractional part of one of its sides, or (4) a geometrical section and a fractional part of one of its sides. A) AM, a side of square ABEM, was extended to point D, determined by an arc described from midpoint O as a center, with the radius OE. Golden rectangle ABCD was thus constructed. To rectangle ABCD was added rectangle FADB'. DB', the short side of rectangle FADB', is equal to one and one-half times MD, the short section of line AD. B) To the square FEEA was added rectangle DCEF. DF, the short side of rectangle DCEF, is equal to one half of FA. C) A side of the square ABEM was extended to make the golden rectangle ABCD. A second, equal rectangle AB'C'D was drawn vertically tangent to rectangle ABCD. D) Two equal squares, ABFE and M'GCD were overlapped. M'E, the superimposed distance, is equal to AM, the long section of AB.
a distance equal to one half of the third unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "The Annunciation" was constructed by extending the short side of a golden rectangle a distance equal to the third unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "Venus and Adonis" was constructed by extending the short side of a golden rectangle a distance equal to one half of the long side of the rectangle.

The rectangle of "Agony in the Garden" was constructed by extending the short side of a golden rectangle a distance equal to one and one-half times the short section of the long side of the rectangle.

The rectangle of "Donna Velata" (Plate IX, p. 110) was constructed by extending the short side of a golden rectangle a distance equal to one third of the short section of the long side of the rectangle.

The rectangle of "Madonna Garvagh" (Plate X, p. 111) was constructed by extending the short side of a golden rectangle a distance equal to one fourth of the long side of the rectangle.

A variation of the method illustrated by Figure 2, A) is the rectangle of "Saint Cecilia" (Plate XI, p. 112), which

---

5 For reproduction see Titian Paintings and Drawings, Fig. 62.
6 Ibid., Fig. 229.
7 For reproduction see Singleton, Old World Masters, p. 91.
resulted from two equal vertically superposed golden rectangles plus one half of the long section of a long side of one of the rectangles.

The rectangle of "Madonna di Casa Tassi" was constructed by extending the short side of a golden rectangle a distance equal to the seventh unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "Portrait of Angelo Doni" (Plate XII, p. 113) was constructed by extending the short side of a golden rectangle a distance equal to the sixth unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "The Holy Madonna" (Plate XIII, p. 114) was constructed by extending the short side of a golden rectangle a distance equal to the fifth unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "The Cowper Madonna" was constructed by extending the short side of a golden rectangle a distance equal to one half of the fourth unit in the geometrical series of measures of the long side of the rectangle.

The rectangle of "Sistine Madonna" (Fig. 19, p. 74 and Plate XIV, p. 115) was constructed by extending the short side of a golden rectangle a distance equal to the sixth unit in the geometrical series of measures of the long side of the rectangle.

For reproduction see University Prints, Series C, VII, 153.

For reproduction see Singleton, Old World Masters, p. 87.
The rectangle of "Madonna del Granduca" (Fig. 15, p. 62 and Plate XV, p. 116) was constructed by extending the short side of a golden rectangle a distance equal to the seventh unit in the geometrical series of measures of the long side of the rectangle.

Like Titian, Raphael established some of his rectangles by adding to one side of a square, a fractional part of its length (Fig. 2, B). The rectangle of "Madonna of Francis I" (Plate XVII, p. 116) was derived from a square plus one half of its length. Employing the same principle as that of Figure 2, B), the rectangle of "Galatea" (Plate XVIII, p. 119) was constructed. To the short side of a rectangle, resulting from two vertically tangent squares, one half of a side of one of the squares was added.

Figure 2, D) shows the method used by Raphael in determining the proportions for the rectangle of "The Niccolini Madonna."[10](Two squares were overlapped at the median point.) Although Titian used a greater variety, both artists were variable in the methods used to establish the proportions of their enclosing rectangles. Of the examples of the following methods based on analyses of twenty-nine paintings, Titian used Figure 1, A), once; B), twice; C), twice; D), six times; and E), twice. Raphael used Figure 2, A) ten times; B), twice; C), once; and D), once.

---

This analysis of geometrical constructions used by Titian and Raphael in establishing rectangles shows that each artist was versatile and individual, to a certain extent, in applying geometry to his compositions. Yet, similarity of their basic structural plans shows that the same geometrical principles were practiced by both artists. Predominant in all the works analyzed is the use of the golden proportion, while the use of root rectangles has not been discovered in these analyses of Titian's and Raphael's paintings. This probably points to a prevailing tendency during the Fifteenth Century to base the structural compositions of all works of art on proportions related in a geometrical series.
CHAPTER III

GEOMETRICAL ANALYSIS OF THREE PAINTINGS BY TITIAN

I. Madonna of the Cherries

By referring to the construction of the enclosing rectangle of "The Madonna of the Cherries" (Fig. 3, p. 26), it will be seen that a line drawn from point E, which joins the rectangle EDD'D to the square ABB'E, is an important line in the composition. By placing canonical points of the man and Saint John on this line, the locations of the two figures were determined.

The Madonna, the most important figure in the composition, was placed between vertical lines drawn from N and N', which are median points of BC, the long side of the rectangle ABCD. Touching these lines are canonical points of the Madonna, the Baby, and Saint John. Of equal importance are horizontal lines drawn from M and M', median points of CD, the short side of rectangle ABCD. A line drawn from point M was used to establish the levels of the heads of the Madonna, the two men, and the Baby. A line drawn from point M' was used to determine the levels of Saint John's head, and the hands of the Madonna and of one of the men.

Importance of the use of diagonal lines in determining directions in the composition is shown in Figure 4, page 29.
A line illustrating Kelsch's "Law of the Madonnas," when extended to the sides of the enclosing rectangle, touches AD at N"', one of its median points, and touches AB at M''', a division in the geometrical series of line AB. From N, a median point of AD, twelve diagonal lines were drawn which touch the enclosing rectangle at its geometrical divisions. Directions and canonical points of the figures placed on these lines resulted in strong diagonal movement within the composition.

Circles, having as their radii the measures of the rectangle, were used to locate canonical points not determined by straight lines (Fig. 5, p.32).

Following are complete analyses showing the use of vertical, horizontal, and diagonal lines, and circles related in the divine proportion.

Analysis of Vertical and Horizontal Structural Lines (Fig.3)

Rectangle ABCD was made by adding to the square ABD'E, the rectangle ECD'D. The short side of rectangle ECD'D is equal to one-fourth of a side of ABD'E. BC, the long side of rectangle ECD'D, and CD, the short side, were divided geometrically. Through these points of division the artist drew vertical lines from BC and horizontal lines from CD to help determine directions and areas within the composition.

1Kelsch, Canon Tirbutius, p. 373.
Divisions of BC, the long side of rectangle ABCD.—

N and N' are the median points of BC.
N'' is the median point of NC.
L' is the median point of BN'.
I is the median point of L'N.
K' is the median point of NB.
K'' is the median point of K'N.
J is the median point of BK''.
J' is the median point of BK'.
K is the median point of N'C.
H' is the median point NC.
H is the median point of N''N'.'
I' is the median point of NL.
N'L = N''K.

The Madonna was placed so that Her right eyebrow and an edge of Her cap touch a line drawn from point N. Crossing the same line are the Baby's elbows and cherry leaves lying on the table.

A line drawn from point N' touches a scroll in Saint John's left hand, goes through a twig in the Madonna's left hand, and crosses the center of Her shirred neckline.

A line drawn from point N'' touches Saint John's left elbow, the last finger of the Madonna's left hand, and the right side of the face of the man on the right.

A line drawn from point I balances the Baby's body by going from His right eye to the toes of His outstretched left foot.

A line drawn from point K'' crosses the Baby's body at His left heel and His right ear.

The man on the left is leaning so that his right eye and his second left finger touch a line drawn from point J.
A line drawn from point J' establishes the position of the Baby's right foot, and crosses the right ear of the man on the left.

The man on the right was placed with a corner of his left eye touching a line drawn from point K. His right eye and a corner of his mouth lie along the side of square ABD'E, drawn from point K. Saint John's hairline, the tip of his left ear, and a point of his collar were placed touching a line drawn from point K, while his left eye, his throat, and the edge of his sleeve contact a line drawn from point E.

A line drawn from point H' goes across Saint John's head, the fingertips of the man on the right, a point of his coat, and a side of his cap.

The Madonna's cape touches the shoulder of the man on the right at a place on a line drawn from point H. After crossing the Madonna's first left finger at the knuckles, and her second left finger at the end of the nail, the line crosses Saint John's wrist.

A line drawn from point I' touches the corner of the Madonna's left eyebrow, the end of her nose, and a corner of her mouth. It goes across the Baby's first right finger, follows a line of his left arm, and goes through the cherry lying on the table.
Divisions of CD, the short side of rectangle ABCD.---

M and M' are median points of CD.
M'' is the median of CM.
O is the median of DM.
Q and Q' are median points of OD.
S and S' are median points of OM.
R' is the median of OS.
P' and P'' are median points of MM'.
R is the median of M'M''.
O' and P are median points of CM''.
T is the median of P'O'.

A line drawn from point M crosses the chins of the two men, locates the neckline of the Madonna's dress, goes across the Baby's first right finger, and touches His lower lip and the tip of His right ear.

A line drawn from point M' locates Saint John's hairline, crosses the Madonna's knuckle and the twig in Her left hand, and establishes the place where the man's hand touches the Baby's hips.

A line drawn from point M'' goes from Saint John's neckline and throat across the top of the Baby's right knee.

A line drawn from point O crosses the Madonna's right eye, the foreheads of the two men, and locates the end of the pole in the man's hand.

Between lines drawn from points Q and Q' are the hairlines of the Madonna and the man on the right, and the top of the other man's head.

A line drawn from point S crosses a lower lid of one man's eye, touches the Madonna's lower lip, crosses the Baby's forehead,
crosses the end of the nose of the man on the left, and touches the top of his ear and the end of his pole.

A line drawn from point S' locates the top of the Baby's right eye, crosses the Madonna's chin, the nose of the man on the right, and the nose and ear of the man on the left.

Drawn along a line from point R' are the eyebrows of the two men, the top of the Baby's head, and the end of the Madonna's nose. This line, together with lines drawn from points O, S, and S' are applications of Kelsch's "Law of Isocephaly." 2

A line drawn from point P' touches the top of one man's collar and the end of his beard, crosses the Baby's left elbow and the hands of the Madonna and the man on the left.

On a line drawn from point R rest the fingers of the Madonna's left hand, Saint John's lower lip, and the Baby's stomach.

Touching a line drawn from point O' are the Baby's right foot and left knee, an edge of the Madonna's sleeve, and the left hands of Saint John and the man on the right.

The line of the table is determined by a line drawn from point P. On this same line are the Baby's feet, cherry leaves, an edge of Saint John's left hand, and one finger of the man on the right.

A line drawn from point T touches Saint John's left eyebrow and his nose, crosses the Madonna's left hand at the third

2 Kelsch, Canon Turbutius, p. 43.
joint of Her third finger and the first joint of Her first finger, passes through the twig in Her left hand, locates the Baby's navel and the place where His hips touch the man's hand.

Analysis of Diagonal Structural Lines (Fig. 4)

To establish directions and canonical points which were not decided by vertical and horizontal lines, the artist employed the use of diagonals. These lines were drawn from the geometrical divisions of CD, the short side, and BC, the long side of rectangle ABCD.

Kelsch's "Law of the Madonna," in which a line may be drawn from the center of expression of the Madonna to a canonical point of Her hand, was applied by diagonal N"N". Bound by this same line are the Baby's nose and right shoulder, and the hands of the man on the left.

From N, a median point of AD, diagonals NP", NO", NP, NC, NN", NH, NN', NI, N", NN", NP, and NS were drawn.

Line NP" touches the top of the Madonna's head, and the nose of the man on the right.

Line NO" follows a line of the Madonna's cape, and passes across Saint John's forehead.

Line NP touches points of the Madonna's cap, runs parallel with Her left arm, locates Saint John's left eyebrow and the

---

3 Kelsch, Canon Tiburtius, p. 373.
top of his left ear, and crosses the left hand of the man on the right.

Lying along line NC are points of the Madonna's cap, the tip of Saint John's nose and his left ear, and three fingers of the left hand of the man on the right.

Line NN' touches the corner of the Madonna's left eye, determines the direction of the twig in Her left hand, and locates the edge of the table.

The scroll in Saint John's left hand and the Madonna's left eye are bound by line NH.

Line NN'' touches a corner of the Madonna's left eye and a corner of Her mouth. Touching the same line are an edge of the scroll and Saint John's first left finger.

The Madonna's forehead and the Baby's right cheek touch line NI. It crosses the Baby's arms at a line of His clothing and crosses His first left toe.

Line NJ' establishes the direction of the Baby's body. It passes from His first right toe, up through the center of His right hip, and touches the top of His right ear. The Madonna's right thumb is crossed by the same line.

Line NM'' touches the edge of the Baby's hair and crosses the chin and hand of the man on the left.

Line NP' determines the collar line of the man on the left and forms the line of his upper right eyelid. His hairline and the place where his pole touches the enclosing rectangle are determined by line NS.
Fig. 4.—Analysis showing how diagonal lines joining points geometrically determined were used in the composition of "Mona Lisa and the Cherries."
Analysis of Circles Related in the Divine Proportion (Fig. 5)

X is the point of intersection of lines drawn from points M and L. With X as a center, seven concentric circles related in the divine proportion were described. With L as a center, three arcs, likewise related, were described.

The radius of circle I is equal to CP.
The radius of circle II is equal to CD.
The radius of circle III is equal to CM.
The radius of circle IV is equal to CR.
The radius of circle V is equal to CM.
The radius of circle VI is equal to CP.
The radius of circle VII is equal to CM.
The radius of arc VIII is equal to BN.
The radius of arc IX is equal to BN.
The radius of arc X is equal to BN.

Circle I locates the Madonna's neckline, crosses Her chin, and crosses the Baby's fingers.

Circle II touches the base of the Madonna's nose, a leaf in the Baby's hands, and curves with the shirred neckline of the Madonna's dress.

Circle III passes above and parallel with the Madonna's eyebrows, forms an edge of Her cap, and crosses the Baby's right eye and left elbow.

Bound by circle IV are the Baby's right ear, an edge of the Madonna's cape, and the twig in Her left hand.

The eye, nose, and mouth of the man on the right touch circle V. The Madonna's left hand was placed so that Her third finger lies along its circumference on one side. Her sleeve and the man's beard touch its circumference on the other side.
Circle VI crosses an eye of one man and an eyebrow of the other man. The Baby stands so that his hips are parallel with its curve as it passes along the top of them. Touching the same circle are the Madonna's second right finger, Saint John's left eye and eyebrow, and the scroll in his left hand.

Bound by circle VII are a side of the man's cap, lines of Saint John's collar, an edge of his scroll, and a toe and knee of the Baby.

Arc VIII forms the front of the Baby's right thigh, crosses the Madonna's left cuff, and touches Saint John's upper lip. The Madonna's left hand was turned so that her knuckles lie parallel with the circle.

Arc IX, beginning at the collar of the man on the right, crosses his nose, goes across the Madonna's right eye and the top of the baby's head, and follows the dividing line of the mouth of the man on the right.

Arc X crosses the hairlines of the two men and determines the top of the Madonna's cap.

Superimposed Analyses

For each of the paintings included in Chapters III and IV the three analyses presented are shown superimposed in a fourth illustration (for example, see Fig. 6). These composite analyses are of value for study of the relationship between the straight and curved structural lines of the composition, and they also reveal the occasional importance of points of intersection of oppositional lines.
II. The Entombment

In the construction of the enclosing rectangle of "The Entombment" (Fig. 7, p. 38) EE'FF', the overlapped area of squares ABFF' and EE'DC, was used to contain the four most important figures of the composition. On lines EE' and FF', the overlapped sides of the squares, canonical points of two of the men were placed.

A line from M, a median point of AB, and a line from N', a median point of BC, cross at a corner of Christ's right eye, thus establishing the location of His head in a place of major importance. Important canonical points of other figures in the composition also touch a line from point M. Lines from M', a median point of AB, and N, a median point of BC, were used to determine the position of Christ's legs and hips, and canonical points of other figures.

While the location of the figures in the composition was achieved through the use of vertical and horizontal lines, the positions of the figures were determined almost completely by the use of diagonals (Fig. 8, p. 42). Twelve lines were drawn from S', a division in the geometrical series of measures of AD, to geometrically determined points on the sides of rectangle ABCD. Lines of the bodies in the composition were drawn either tangent or parallel to these diagonal lines, and important canonical points in the painting were placed on them.

With S' as a center, ten concentric arcs, the radii of which are equal to the geometrical measures of rectangle ABCD,
were described to locate canonical points in the composition which were not determined by the vertical, horizontal, and diagonal lines (Fig. 9, p. 44).

Following are complete analyses of "The Entombment" showing the author's applications of vertical, horizontal, and diagonal lines, and circles related in the divine proportion.

Analysis of Structural Vertical and Horizontal Lines (Fig. 7)

Rectangle ABCD was made by overlapping two equal squares, ABFF' and EE'DD'. EF, the distance overlapped, is equal to a geometrical measure of a side of one of the squares.

**Divisions of AB, the short side of rectangle ABCD.**

- M and M' are median points of AB.
- L is the median point of AM'.
- L' is the median point of MB.
- K is the median point of ML'.
- K' is the median point of M'L.
- J is the median point of L'B.
- I is the median point of JB.
- J' is the median point of AL.

EF, the overlapped area, is equal to BK'.

A line drawn from point M crosses the woman's folded hands, the elbow and chin of the man on the left, Christ's right elbow, His right eyebrow and left eyelid, the belt of the man on the right, and the top edge of the tomb in the background.

A line drawn from point M' crosses Christ's right foot, the tips of the man's fingers placed under Christ's knees, and goes through the center of Christ's hips and across His left elbow.
A line drawn from point L, illustrating Kelsch's "Law of Isocephaly," goes across the nose of the woman on the left, touches the other woman's neckline, touches the nose of the man in the center, crosses the top of the head and forms a line of the collar and shoulder of the man on the right.

Line L' crosses one man's foot and follows the direction of his leg, goes across Christ's left wrist and the left leg of the man on the right at an edge of his trousers.

Line K goes across the right fingers of the man on the left, crosses the top of Christ's left knee and His breast, and goes through the left elbow of the man on the right, touching the point where his arm meets Christ's left arm.

Line K' goes across the top of Christ's right knee, across His beard, crosses the right wrist of the man on the left, and touches the top of a woman's sleeve.

Line J crosses a toe of the man on the left and the ankles of the other two men carrying Christ. It crosses Christ's left hand, touching the spot of blood.

Line I goes along the bottom of the rock, crosses the crown of thorns, forms the top line of the left foot of the man on the right. His right foot rests on the line.

Line J' forms the top line of one woman's head and touches the part of the other woman's hair.

4Kelsch, Canon Tirbutius, p. 43.
Divisions of BC, the long side of rectangle ABCD:

N and N' are median points of BC.
O is the median point of NO.
O' is the median point of O'B.
P is the median point of O'M'.
Q is median point of N'C.
R is median point of N'B.
T and S are median points of NN'.
V is median point of I'E.
N' = NF

A line drawn from point F, which forms a side of square ABFF', runs parallel with the shoulders of the man on the right and crosses his right knee and foot. Touching the same line are edges of his scarf, the front of his left sleeve, and limbs of the trees in the background.

A line drawn from point N, a side of square NN'MM', crosses the right shoulder of the man on the left and runs lengthwise with his hand, and determines a corner of the rock.

A line drawn from point N' goes through the center of Christ's chest, touches his mouth and right eye, and crosses the head of the man on the right.

A line drawn from point N runs lengthwise with and parallel with the face of the man on the left, touching his beard at his temples. It crosses the center of Christ's right knee and the base of his left knee, and goes through the center of the crown of thorns.

A line drawn from point O lies tangent to the heel of, and to some folds in the clothing of the man on the right.

The woman on the right stands so that her left eye and the edge of her collar touch a line drawn from O'. This same line
Fig. 7.—Analysis showing how geometrically determined vertical and horizontal lines were used in the composition of "The Entombment."
crosses the right elbow of the man on the left, the toes of Christ's right foot and the spot of blood on his left foot.

A line drawn from point Q runs parallel with lines of the trousers of the man on the right and touches the edge of his belt.

The woman on the left was drawn leaning forward, allowing her eyebrow to touch a line drawn from point R. Folds in the coat of the man on the left lie along the line.

The man in the center stands so that his hair and the back of his leg touch a line drawn from point T.

A line drawn from point S touches the corner of one eye and a foot of the man in the center, and goes across Christ's stomach and right elbow.

A line drawn from point V crosses the heel of Christ's right foot and an edge of his shroud, and forms a line of the sleeve of the man on the left.

Analysis of Diagonal Structural Lines (Fig. 8)

With S'' as a center, twelve diagonal lines were drawn to geometrical divisions of the sides of the enclosing rectangle ABCD.

Line S''K'' follows the direction of the back of the man on the right, touching edges of his cloak.

Line S''C lies tangent to the back of the head of the man on the right and crosses the toes of his left foot.
Line S''Q touches Christ's left eyebrow and forms the line of His left arm above His elbow. The man on the right is standing so that the line touches the back of his right leg, a line of his scarf, and the base of his chin.

Line S''O runs lengthwise through the left side of Christ's face, touching the corners of His eye and mouth, and along the side of His left hand. On the right, the man was drawn so that the line crosses the bottom of his left ear, touches the edge of his trousers on his knee and runs parallel with his left leg.

Line S''F passes through the left side of the hair of the man in the center and across the forehead of the man on the right, touches points of his scarf and crosses his toe. It locates the spot where Christ's head falls against His right shoulder, and crosses the center of His chest.

Line S''N determines the direction of the center figure. It touches the right side of his face and goes through the crown of thorns.

The figure of the man on the left is balanced between diagonals S''R and S''M'. Line S''R forms the line of his hair above his forehead, crosses his beard and right eye, and forms the line of his first right finger. Also touching this line are Christ's first right finger and the back of His right heel. Line S''M' passes along the man's back, crossing his right sleeve at points in his folded cuff.
Line S''B crosses the right foot of the man on the left, touches a line of his right sleeve and follows the direction of his head. The first toe of each of Christ's feet is tangent to this line.

Line S''J touches the back of the man's head and the edge of the robe of the woman on the left.

Line S''K crosses the folded hands of one woman and follows the direction of the other woman's left arm.

Line S''L follows the direction of the women's heads, crossing the back of one's head and touching the ear and hairline of the other.

From M to J'' a line was drawn which goes across the heads of two of the men, touching an ear, the hairline, and rolled sleeve of one, and crosses the chin of the man in the center. The hands of Christ and of the woman on the left are also touched by this line.

A line was drawn from J' to K'' which crosses Christ's right eye, an eyebrow and ear of one man and an eyebrow of one of the women. It goes across the other woman's chin and the hands of Christ and the man in the center. This line and line MJ'' are applications of Kelsch's "Law of Isocephaly."5

From W' to Q a line crosses one of the women's ears and the end of her nose, follows the shoulders of the man on the left and Christ's right leg, crosses the spot of blood on His left hand, and goes across the left leg of the man on the right. Touching this line is a point in the folds of Christ's shroud.

5Kelsch, Canon Tirbutius, p. 43.
Analysis of Circles Related in the Divine Proportion

With $S$ as a center, ten concentric arcs related in the divine proportion were described.

The radius of arc I is equal to $AB$.
The radius of arc II is equal to $BM$.
The radius of arc III is equal to $BL$.
The radius of arc IV is equal to $BM$.
The radius of arc V is equal to $BN$.
The radius of arc VI is equal to $BE$.
The radius of arc VII is equal to $BM$.
The radius of arc VIII is equal to $BN$.
The radius of arc IX is equal to $BM$.
The radius of arc X is equal to $BS$.

Arc I touches the crown of thorns and crosses the feet of the two men on either side of the picture.

Arc II touches the top of the crown, the joint of Christ's last left finger and the spot of blood on His left hand. It goes through points of the scarf, forms the bottom line of the center man's left foot, and touches the edge of the robe of the woman on the left.

Touching arc III are Christ's right heel and a cuff line of the woman on the left.

Arc IV crosses the chin of the woman on the left, runs parallel with one man's arm, crossing the end of his knuckles, follows the curve of Christ's body and touches the point where His left arm meets the line of the left arm of the man on the right.

Following arc V are the shoulders of the woman on the right, the right arm of the man on the left, Christ's body from His knees to His breast, and the belt of the man on the right.
Touching the same line are Christ's navel and the point between the eyes of the woman on the left.

Arc VI touches the left eyebrow of the woman on the right, a rolled sleeve of the man on the left, and a folded edge in the clothing of the man in the center. It crosses Christ's right armpit and His chin.

Arc VII touches an ear and the nose of the man on the left, crosses the hands of Christ and the man in the center, goes across Christ's forehead, and touches the neckline of the man on the right.

Arc VIII touches the hairline of the man on the left, crosses the hands of Christ and the man in the center, forms the line above the eyes of, and crosses the left ear of the man on the right.

Arc IX is tangent to the base of the center man's nose, and the top of the man's head on the right.

Arc X touches the hairline of the man in the center.

III. ECCE HOMO

In the construction of the enclosing rectangle of "Ecce Homo" (Fig. 11, p. 50) the figure of Christ occupies the basic square BB'C'C. Line B'C', a side of the square, was used, not only as a structural line of the enclosing rectangle ABCD, but also to locate important canonical points of Christ's head.

Vertical lines drawn from M and M', median points of BC, and horizontal lines drawn from N and N', median points of AB,
were of great importance in determining Christ's position and location in the picture. The outstanding canonical points of His body are located on these lines. Other lines drawn from a series of geometrical measures of the sides of the enclosing rectangle were likewise used to determine other canonical points of the painting.

Titian used diagonal lines connecting geometrical divisions of the rectangle to establish lines of Christ's body, and also to locate canonical points in the composition (Fig. 12, p. 53). From M", a median point of AD, seven diagonal lines were drawn to geometrical divisions of the enclosing rectangle. One of these lines, M"H", is an application of Kelsch's "Law of the Classical Portrait." The remaining six lines drawn from M", together with four other diagonal lines connecting geometrical divisions of the rectangle, determine other canonical points and lines of Christ's body.

Figure 13, page 56, shows the use of circles, having as their radii geometrical measures of rectangle ABCD, in locating canonical points in the composition.

The complete analyses showing the use of vertical, horizontal, and diagonal lines, and circles related in the divine proportion follow.

---

6 Kelsch, Canon Tirbutius, p. 219.
Analysis of Vertical and Horizontal Structural Lines (Fig. 11)

Rectangle ABCD was made by adding to square BB'C'C the rectangle AB'DC'. AB', the short side of rectangle AB'DC', is equal to one-third of the long section of a side of square BB'C'C.

Divisions of BC, the short side of rectangle ABCD.—

M and M' are medians of BC.
E is the median of MC.
F is the median of EM.
G is the median of EF.
E'' is the median of G'E'.
H and H' are medians of HJ.
I is the median of GF.
E' is the median of M'B.
F' is the median of M'E'.
G' and G'' are medians of B'E'.
M'' is the median of FM.

A line drawn from point M' goes through Christ's body, touching the corner of His right eye, crossing His lips, and runs lengthwise with His left hand. A line drawn from point M forms one side of a curl, crosses the center of His right breast, and crosses a muscle of His right arm.

A line drawn from point K crosses the top of Christ's head, touching a part of the thorny crown. It touches folds in the sleeve of His robe, an edge of the rope knot, and locates the spot where His hands cross.

A line drawn from point F is a vertical line through Christ's body, going from His temple to His first left finger.

A line drawn from point G touches the back of Christ's left hand, crosses His right breast, and follows a line of his chest. It crosses His nose, His right eyebrow and touches ends of thorns in His crown.
A line drawn from point J' determines the front of Christ's right arm, touching lines in its muscles.

A line drawn from point E'' follows the line of His body and touches the shoulder bone.

A line drawn from point F' crosses His crown, touches the back of His right ear, a drop of blood, a side of His beard, and His collarbone.

A line drawn from point G' lies tangent to His shoulder.

**Divisions of AB, the long side of rectangle ABCD.**

N and N' are medians of AB.
C is the median of AN'.
O' is the median of NB.
P and T' are medians of ON'.
P' is the median of O'N.
Q and Q' are the median of O'B.
R and R' are medians of A0.
S is the median of P'O'.
S' is the median of Q'O'.
T is the median of N'O'.
V is the median of OT'.

A line drawn from point T crosses the center of Christ's right breast and follows a line of His left sleeve.

A line drawn from point V crosses Christ's face, from the edge of his hair across the lower lid of His right eye, and crosses drops of blood on His neck and right cheek.

A line drawn from point N crosses the top of His shoulders and His chin.

A line drawn from point N' crosses the top of His right breast and the middle of His left breast.
Fig. 11.—Analysis showing how geometrically determined vertical and horizontal lines were used in the composition of "Ecce Homo."
A line drawn from point 0 touches the bottom of His right ear, goes through His right eyebrow and across the hair on His forehead.

A line drawn from point 0' crosses His right arm at a line in a muscle, goes through the top of some folds of cloth across His stomach, and crosses His left arm at the edge of the sleeve.

A line drawn from point P' crosses His chest and shoulders and goes through two drops of blood on His chest.

A line drawn from point T' touches the spot where His hair falls against His right shoulder, crosses a curl, goes across His nose, and through the point where His beard touches His left cheek.

A line drawn from point Q follows the direction of the right arm, touching its top line, and crosses the left wrist on both sides as the rope encircles it.

A line drawn from point Q' follows the direction of the lower right arm, going through its center, and crosses His left hand where an angle is made by its bending.

A line drawn from R' crosses His hand, touching the spot where two thorns cross.

A line drawn from point S' touches a line of the right arm, and crosses the hands and the ends of the rope.

Analysis of Diagonal Structural Lines (Fig. 12)

From M' seven diagonals were drawn to geometrical divisions of the sides of rectangle ABCD.
Line M''V'' lies tangent to the top of Christ's head.

Line M''R'' crosses Christ's right eyebrow, touches an eyelid, forms a line along the side of His beard, and goes along a line of the second finger of His right hand. This line follows Kelsch's "Law of the Classical Portrait." 7

Line M''Q'' crosses Christ's head, touching an eyebrow and the thorny crown, and crosses His left elbow, forming a line along the sleeve.

Line M''G'' crosses Christ's right eye, touches a side of His nose, crosses His lips, follows drops of blood down His chest, and crosses the side of His left hand.

Line M''IB'' follows the direction of His head, crosses the right ear and an end of a curl, and crosses the right shoulder and arm.

Line M''P'' touches the right shoulder and the edge of the crown.

Line M''T'' goes through the edge of His hair.

From N'' to M a line was drawn which follows the direction of Christ's left arm.

A line drawn from H to F'' is tangent to the back of Christ's right arm.

A line drawn from H'' to O'' touches the edges of His left sleeve and shoulder, crosses the end of His nose, forms the lower right eyelid, and crosses the spot on the crown where two thorns cross.

7 Kelsch, Canon Tirbutius, p. 219.
Fig. 12.—Analysis showing how diagonal lines joining points geometrically determined were used in the composition of "Ecce Homo."
A line drawn from $H''$ to $E'$ follows the direction of Christ's face, touching the front of His hair, a thorn, the tip of His nose, the edge of His mustache and crosses His right breast and arm.

Analysis of Circles Related in the Divine Proportion (Fig. 13)

At the intersection of lines drawn from points $N'$ and $M'$ a progressive series of seven concentric circles was described. These circles are related in the divine proportion.

The radius of circle I is equal to $MM'$.
The radius of circle II is equal to $N'Q'$.
The radius of circle III is equal to $BT$.
The radius of circle IV is equal to $BN'$.
The radius of circle V is equal to $CM''$.
The radius of circle VI is equal to $CM$.
The radius of circle VII is equal to $CJ'$.

Circle I touches the top of Christ's left arm, follows the direction of the folds in the sleeve and crosses His beard and right breast.

Circle II goes through the muscle of His right arm, touches the collarbone, the tip of His nose, follows the direction of the left arm and crosses the rope knot and His left hand.

Circle III crosses a curl of hair and His nose, touches a spot of blood on His face, forms the line of the upper right eyelid, and crosses the rope and His left hand.

Circle IV crosses the fingers of both hands and goes through the center of the upper right arm, and through His eyebrow and the edge of His hair.
Circle V goes across Christ's right elbow, and touches the right shoulder, the top of the ear, and the edge of the crown. Circle VI touches the top of His crown and a line in His hair.

Circle VII touches the top of his head.

The movement and direction in the three paintings by Titian included in this chapter result from the structural use of vertical, horizontal, and diagonal lines. By placing important points of the compositions on lines drawn from, or connecting points of division in the geometrical series of measures of the rectangles, the eye is compelled to move from one important part of the picture to another. Circles and arcs, having their radii equal to geometrical measures of the rectangles, were used, not to develop movement but to establish important points in the compositions.
Fig. 10.—Analysis showing how circles related in the divine proportion were used in the composition of "Ecce Homo."
Fig. 14.—Analysis showing Figures 11, 12, and 13 superimposed.
CHAPTER IV

GEOMETRICAL ANALYSES OF THREE PAINTINGS BY RAPHAEL

I. Madonna del Granduca

In the composition of "Madonna del Granduca" (Fig. 15, p. 62) the locations of the Madonna and the Baby were governed by the placing of important canonical points on vertical lines drawn from G and G', median points of BC. Lines drawn from M and M', median points of CD, determined the positions of the Baby's face and left hand and leg. Other vertical and horizontal lines drawn from geometrical divisions of the rectangle were used to locate additional significant points in the composition. A line drawn from point P, illustrating Kelsch's "Law of Isocephaly," touches the enclosing rectangle at a geometrical division.

To establish further points and lines in the picture, ten diagonals were drawn from G', the median point of AD, to geometrical divisions of the rectangle. One of these diagonal lines is an application of Kelsch's "Law of the Madonnas."

Of chief importance in developing movement in his picture was Raphael's use of concentric circles having as their radii geometrical measures of the rectangle. By placing the most

1Kelsch, Canon Tirbutius, p. 43.
2Ibid., p. 373.
important canonical points of the painting along the circumferences of these circles, the lines of direction were established.

Following are complete analyses showing the use of vertical, horizontal, and diagonal lines, and circles related in the divine proportion in the composition of "Madonna del Granduca."

Analysis of Vertical and Horizontal Structural Lines (Fig. 15)

Rectangle ABCD was made by adding to the golden rectangle RFDC, the rectangle ABKF. The short side of the rectangle ABKF, which is BE, is equal to O'N', a geometrical division of CD.

Divisions of BC, the short side of rectangle ABCD—

G and G' are median points of BC.
H is the median point of GC.
I is the median point of HJ.
J is the median point of G'H.
H' is the median point of HJ'.
J' is the median point of GH'.
I' is the median point of H'G'.
V is the median point of CV'.
JK = JH = IQ = IK' = EL' = HV'.

The Madonna was placed so that her right eye lies on a line drawn from point G. It passes down the center of her right cuff and follows some lines of her robe.

A line drawn from point G' touches the Baby's right eye and the bend of his left elbow. Crossing the Madonna's right hand, it goes down the center of the Baby's left leg, passing the point where the Madonna's hand touches his leg, and the point where her finger touches his arm.

The Madonna's left hand and the Baby's left ear are bound by a line drawn from point H.
A line drawn from point I touches a finger, a side of the right hand, and a toe of the Baby, and the Madonna's left cheek and an edge of Her right sleeve.

A line drawn from point J touches the Baby's left eye, crosses the fingers of both of the Madonna's hands, and goes through the center of the Baby's left foot.

A line drawn from point I' forms the end of the Madonna's right sleeve, touches the Baby's left hand, and crosses the Madonna's mouth and nose.

A line drawn from point K crosses the Madonna's right hand, the toes of the Baby's right foot, the heel of His other foot, and goes through the center of His face.

A line drawn from point K touches the Baby's left ear and crosses the fingers of the Madonna's left hand.

A line drawn from point L follows a roll of the Madonna's hair, touches lines of Her robe, and locates the cuff of Her right sleeve.

The Baby was placed so that His back touches a line drawn from point V'. It also crosses the Madonna's left thumbnail.

The left side of the Madonna's robe was determined by a line drawn from point V.

A line drawn from point L' touches points in the folds of the Madonna's robe.

**Divisions of CD, the long side of rectangle ABCD.**

M and M' are the median points of CD.
N is the median point of MC.
N' is the median point of MN.
O is the median point of MN'.
O' is the median point of ON'.
P is the median of M'D.
Q is the median point of DP.
Q' is the median point of CN.
RM = O'N' = RH' = PQ = S'T' = NT = MT' = Q'P'.

The lower edge of the Baby's lip and the base of His second left finger lie on a line drawn from point M.

A line drawn from point M' touches the Madonna's arm and the Baby's leg.

A line drawn from point P' crosses the center of the Baby's right foot and touches four toes of His left foot.

A line drawn from point O runs parallel with the Baby's left leg, determines folds in the Madonna's right sleeve, and touches the base of the thumb and the tips of two fingers of Her left hand.

A line drawn from point O' goes along the top of the Madonna's right arm, touches a point of Her hand, and touches the edge of the cloth on the Baby's body.

A line drawn from point P passes across the Baby's head and the Madonna's chin. This line illustrates Kelsch's "Law of Isocephaly."\(^3\)

A line drawn from point Q goes from the Madonna's left eyelid to Her right eyebrow.

A line drawn from point R determines the Madonna's neckline, and crosses the Baby's eyes and His left ear.

\(^3\)Kelsch, Canon Tirbutius, p. 43.
Fig. 15.—Analysis showing how geometrically determined vertical and horizontal lines were used in the composition of 'Madonna del Granduca.'
A line drawn from point R' locates the point where the Madonna's hair touches Her cape, goes through the center of the Baby's right thumb and crosses His hairline.

A line drawn from point S goes across the Madonna's nose and through the Baby's halo.

A line drawn from point S' touches the Madonna's hairline.

A line drawn from point T touches the back of the Madonna's left hand and edges of Her right sleeve.

The Baby's elbow is tangent to a line drawn from point T'.

Analysis of Diagonal Structural Lines (Fig. 16)

G'' is the median point of AD. From G'' ten diagonal lines were drawn.

Line G'M touches the top of the Baby's head and goes through the edge of the Madonna's cap.

Line G'M' touches the Baby's clothing, the corner of His left eye, and crosses the Madonna's left temple.

Line G'P' forms a line of the Madonna's left thumb, touches the tips of two fingers of Her right hand, crosses the Baby's mouth, nose, and right eye, and crosses the Madonna's left eye.

The Baby's left cheek and fingertips touch line G'C. Both hands and the left eye of the Madonna are crossed by the same line.
Fig. 10.—Analysis showing how diagonal lines joining geometrically determined points were used in the composition of "Madonna del Granduca."
The corner of the Madonna's left eye, the side of her nose, and a corner of her mouth touch line G''V'. It also goes across one hand and touches the end of one of her fingers, and goes through the back of the Baby's right leg. Lines G''G and G''V' conform to Kelsch's "Law of the Madonnas." 

Line G''G' touches a side of the Madonna's nose and a corner of her mouth, locates the edge of her right sleeve, and crosses the Baby's left fingers and toes.

Line G''L crosses the Madonna's right eye and goes along an edge of her right sleeve.

Line G''B touches a point at the Madonna's right elbow follows a line of her cape, and touches her hairline.

Line G''M'' determines the edge of the Madonna's right shoulder.

The Madonna's cap touches lines G''M'' and G''T''.

Analysis of Circles Related in the Divine Proportion(Fig.17) With the intersection of M and K' as a center nine concentric circles related in the divine proportion were described.

The radius of circle I is equal to BJ.
The radius of circle II is equal to BQ'.
The radius of circle III is equal to CM'.
The radius of circle IV is equal to CO.
The radius of circle V is equal to CN.
The radius of circle VI is equal to GJ.
The radius of circle VII is equal to CF'.
The radius of circle VIII is equal to GD'.
The radius of circle IX is equal to GI.

---

4Kelsch, Canon Tibutius, p. 373.
Circle I touches one of the Baby's left toenails and an edge of the Madonna's left sleeve.

Circle II touches the back of the Madonna's left hand and goes through the Baby's left ankle. Also, a point of the Madonna's robe touches the circle.

Circle III goes across an edge of the Madonna's right sleeve, crosses Her left hand at the first knuckle of Her thumb, and marks the place where Her halo touches the enclosing rectangle.

Circle IV crosses the top of the Madonna's head, touches the edge of Her left sleeve, crosses the nails of Her left hand, and locates an edge of Her right sleeve.

Curving with circle V are the Madonna's right wrist and the Baby's left leg. His head and the upper lids of the Madonna's eyes touch this line.

Circle VI curves with the Madonna's wrist and touches the tips of the last two fingers of Her right hand. The Baby's left ear and the Madonna's lower right eyelid lie along its curve.

Circle VII touches the top of the Madonna's right cuff, crosses Her right hand, crosses the Baby's hairline and the spot where His halo touches His head, and goes across the end of the Madonna's nose.

Circle VIII establishes the dividing line of the Madonna's mouth, crosses the Baby's left eye, and goes between the first two fingers of the Madonna's right hand.
Fig.17.--Analysis showing how circles related in the divine proportion were used in the composition of "Madonna del Granduca."
Fig. 18. -- Analysis showing Figures 15, 16, and 17 superimposed.
The Baby was placed so that circle IX passes through His right thumb, goes along the edge of His bangs and across His right eye, and touches His nose and a corner of His mouth.

II. Sistine Madonna

In the composition of "The Sistine Madonna" (Fig. 19, p.74) the placing of the Madonna and Baby and of the two angels was controlled by two vertical lines drawn from \( M \) and \( M' \), median points of BC. The attendant figures were placed within the two remaining vertical divisions of rectangle ABCD. Line \( A'B' \), which joins rectangle \( BAA'B' \) to golden rectangle \( A'B'CD \), was used to determine canonical points of the man in the picture. Horizontal lines drawn from \( N \) and \( N' \), median points of \( AB \), were employed in establishing the position of the man and the woman in the painting. Also touching these lines are folds in the Madonna's robe. Other vertical and horizontal lines drawn from a series of geometrical measures of the sides of the rectangle were used in determining other important canonical points in the composition.

Figure 20, page 77, shows that Raphael used diagonal lines to establish some of the canonical points in "The Sistine Madonna." With \( X' \) as a center, ten diagonal lines were drawn to geometrical divisions of rectangle ABCD. A line illustrating Kelsch's "Law of the Madonnas" touches the enclosing rectangle

---

Kelsch, Canon Tirbutius, p. 373.
at M', a median point of AD. A five-pointed star, its measures being equal to geometrical measures of the enclosing rectangle, shows the further use of diagonals in determining canonical points in the painting.

Movement in the composition is the result of seven concentric circles, their radii being equal to geometrical measures of the enclosing rectangle. These circles were described from the intersection of lines drawn from points X and Y. (Fig. 21, p. 60). The eye follows the most important points of the picture placed along the circumferences of these circles.

In the following complete analyses, the writer has applied vertical, horizontal, and diagonal lines, and circles related in the divine proportion to the composition of "The Sistine Madonna."

Analysis of Vertical and Horizontal Structural Lines(Fig.19)

Rectangle ABCD was made by adding rectangle BAA'B' to the golden rectangle A'B'C'D. B'B', the short side of rectangle BAA'B', is equal to a geometric measure of the long side of the golden rectangle A'B'C'D.

Divisions of BC, the short side of ABCD:--

X is the center of BC.
M and M' are median points of BC.
F is the median point of BM'.
F' is the median point of CM.
I and G are median points of BE.
I' and G' are median points of CE'.
F is the median point of MM'.
F'I = F'I = MJ'.
A line drawn from point X goes down the left side of the Madonna's face at the hairline, through the Baby's left thumb, and across the point where His right leg touches Mary's left arm. It crosses points in her robe, touches Her right leg at the edge of Her robe, and goes along Her first right toe. It touches the left shoulder of the angel on the left.

A line drawn from point M goes down one side of the Baby's face and body and crosses the head and a shoulder of the angel on the left.

A line drawn from point M' crosses a wrist, the mouth, the nose, and an eyebrow of the angel on the right, and crosses the Madonna's foot and elbow.

A line drawn from point E determines the position of the man on the left. It crosses his nose, mouth, beard, and one hand, and touches the front of his belt and points in the folds of his robe.

A line drawn from point E' touches the wing of the angel on the right, crosses the elbow of the woman on the right, and lies tangent to her right cheek.

A line drawn from point I' touches her hairline on the other side of her face and touches folds on her shoulder.

A line drawn from point G' forms a line of the building in the background.

A line drawn from point I makes a vertical line through the body of the man on the left, and touches a fold in his clothes and his upheld finger.
A line drawn from point F follows the direction of an angel's left arm, touching his mouth, his nose, and an eye. It hits the hem of the Madonna's robe, crosses Her left hand and a side of Her face, touching Her mouth, nose, and hair-line. The Baby's left arm and right leg are crossed by the line.

A line drawn from point J forms a line of Mary's robe, touches the head of the angel on the right, and crosses his left elbow.

A line drawn from point J' touches the right side of Mary's robe, crosses the edge of the Baby's right arm, and the right wing of the angel on the left.

Divisions of AB, the long side of rectangle ABCD.—

Y is the center of AB.
N and N' are median points of AB.
O is the median of AN'.
P is the median of ON'.
Q is the median of OP.
R and S are median points of OA.
T is the median of PN'.
U is the median of PQ.
V is the median of OQ.
O' is the median of N'B.
R' and S' are median points of O'B.
V' is the median of BS'.

BB', the short side of rectangle BAA'B', is equal to BS'.

A line drawn from point Y follows the line of the man's left hand, crosses the base of the woman's left thumb and ends at the spot where her robe touches the enclosing rectangle.

A line drawn from point N crosses from the man's right eyebrow to the woman's hair-line.

A line drawn from point N' follows the line of the man's belt, touches a fold of the Madonna's robe, and crosses the lower edge
of the woman’s left sleeve.

The Madonna’s right fingertips and the Baby’s left shoulder lie on a line drawn from point 0.

The man’s and woman’s shoulders are crossed by a line drawn from point P.

A line drawn from point Q crosses the Baby’s hips and follows the direction of the Madonna’s left hand, crossing Her arm at the end of Her sleeve.

A line drawn from point R is tangent to the top of the Baby’s head and the upper lids of the Madonna’s eyes.

The top of the Madonna’s head is crossed by a line drawn from point S.

A line drawn from point T goes through the man’s right hand and crosses the woman’s left elbow.

A line drawn from point V crosses the Baby’s right eye, touches the top of His left eye, and touches the Madonna’s chin and neckline.

A line drawn from point U crosses the man’s right jaw, touches the woman’s lower lip, and the roof of the building in the background. Lines U and V are illustrations of Kelsch’s "Law of Isocephaly."6

A line drawn from point O’ goes across the Madonna’s right foot and touches a toe of Her left foot.

A line drawn from point R’ begins at a fold in the man’s robe and touches the tops of some clouds.

6Kelsch, Canon Tiberius, p. 43.
Fig. 19.—Analysis showing how geometrically determined vertical and horizontal lines were used in the composition of "Sistine Madonna."
A line drawn from point S' crosses the forehead of the angel on the left, touching his bangs and one eyebrow, and touches the top of the other angel's head.

A line drawn from point V' touches the eyes and the top of a wing of the angel on the right, and touches the other angel's chin.

Analysis of Diagonal Structural Lines (Fig. 20)

With X' as a center, ten diagonal lines were drawn to geometrical divisions of the sides of the enclosing rectangle, ABCD.

Line X'P' touches the top of the woman's head and crosses the building in the background.

Line X'N' lies tangent to the side of the Madonna's robe and touches one of the lady's eyes, her headband, and her neck.

Line X'Z' follows the line of the Madonna's left arm, touches a side of Her dress, and hits the tip of the wing of the angel on the right.

Line X'J goes through the edge of the hair of the angel on the right, touches the last two toes of the Madonna's left foot, follows a fold in Her robe, and goes along the front of Her left sleeve.

Line X'M' crosses the mouth and right wrist and touches the corner of the right eye of the angel on the right. Crossing the Madonna's left ankle and touching the hem of Her robe, it also touches the base of Her neck and the left side of
Her head, and hits the fingers of the Baby's left hand.

Line X'E passes a corner of the man's robe and crosses the spot where his sleeve touches the Madonna's robe. It touches the left side of the Baby's face and goes from the corner of Mary's mouth along the side of her nose. A line in the clouds lies on the line also.

Line X'O' touches one finger of each of the man's hands, forms a line of his mouth, and passes the base of his chin. It crosses the Baby's right wrist, follows the line of the Madonna's right hand, and touches the hairline of the Baby and the right side of the Madonna's head.

Line X'Y crosses the top of the man's head and follows the line of his back.

Applying Kelsch's "Law of the Madonnas," a line was drawn from line M'' to T which crosses the Madonna's face between her eyes, touches the Baby's right eye, and goes down the second finger of the Madonna's right hand. The top of the man's right ear touches the same line.

Using twice the length of the geometrical division O'B, a star was drawn. Since the divisions of lines within a star are related in the divine proportion, the artists frequently used this construction to locate important canonical points of the composition.

The top point of the star lies on line XX'. Points III and IV would lie on lines drawn to the corners C and B of the

---

7Kelsch, Canon Tirbutius, p. 373.
Fig. 20.—Analysis showing how diagonal lines joining points geometrically determined were used in the composition of "Sistine Madonna."
rectangle ABCD. If line N were extended at each end, points II and V would lie on it.

The side I-III follows the direction of the Madonna's left arm and the woman's robe and left arm, touching the back of her hand.

Side I-IV touches the front of the man's blouse and cuff. It crosses the Baby's mouth and nose, touches His left eye, and crosses the Madonna's hairline and right eye.

Side II-IV crosses the lady's left shoulder, the tops of some clouds, and locates the spot where the man's robe touches a cloud.

Side II-V crosses the tops of the heads of the man and the woman. The back of the Madonna's left hand rests on the line.

A line from point II through the center of the star touches the woman's mouth, the cuffs of the man's sleeves and one of his fingers.

A line from point V through the center of the star goes from the man's collar, forms the line under his beard, touches folds of the Madonna's robe, and crosses the woman's left elbow.

Analysis of Circles Related in the Divine Proportion (Fig. 21)

With the intersection of lines X and Y as a center, a progressive series of seven concentric circles related in the divine proportion was described.
The radius of circle I is equal to BE.
The radius of circle II is equal to O'B.
The radius of circle III is equal to BM.
The radius of circle IV is equal to BF.
The radius of circle V is equal to AN.
The radius of circle VI is equal to MC.
The radius of circle VII is equal to the
diameter of circle E.

Circle I touches the backs of the left hands of the man
and woman, and follows the line of the Madonna's left thumb.

Circle II touches the man's belt, crosses the fingers of
his right hand, touches a finger of his left hand, and crosses
his right ear and eye. It goes through the Baby's right hand,
crosses the nose and right eyebrow of the woman, follows the
line of her left arm, crosses the Madonna's feet, and touches
a line in Her robe.

Circle III goes through the Madonna's toes, crosses the
fingers of Her right hand, and touches a fold in Her collar.
It crosses the man's head, and touches the woman's hairline
and sleeve.

Circle IV crosses the Baby's right eyebrow, touches the
Madonna's lower lip, and follows the line of the building in
the background.

Circle V touches the curtain on the left, crosses the Ma-
donna's forehead, and goes across the head of the angel on the
left.

Circle VI crosses the shoulders, chin, fingers, and wing
of the angel on the left, and touches the top of a wing and the
hairline of the angel on the right.
Fig. 31. — Analysis showing how circles related in the divine proportion were used in the composition of "Sistine Madonna."
Fig. 22.—Analysis showing Figures 10, 30, and 31 superimposed.
Along circle VII lie the lips and the top line of the wings of the angel on the right, and the bottom line of the other angel's right wing.

III. Madonna of the Chair

In the composition of "Madonna of the Chair" (Fig. 23, p. 87) bisecting lines BA and XZ were structurally significant, in that important canonical points of the three figures were placed on them.

The Madonna and Baby, the dominant parts of the painting, were located by placing canonical points of these figures on vertical lines drawn from B and B', median points of BA. Of equal structural importance in determining canonical points of the three figures are horizontal lines drawn from R and R', median points of XZ. Other canonical points were located by further divisions in the series of geometrical measures of lines BA and XZ. Lines illustrating Kelsch's "Law of isoccephaly"\(^8\) cross XZ at T and M, two of the geometrical divisions of line XZ.

Of equal importance in locating lines and canonical points within the composition was the use of diagonal lines joining geometrically determined points on the enclosing circle. These points resulted from vertical and horizontal lines drawn from the divisions in the series of geometrical measures of bisecting

\(^8\)Kelsch, Canon Tirbutius, p. 43.
lines BA and XZ. One of these diagonals, line F'G', illustrates Kelsch's "Law of the Madonnas." 9

Not only were circles used to establish canonical points of the painting, but also as the means of determining movement and direction within the composition. Lines of the three figures were drawn to follow the circumferences of a series of nine concentric circles, the radii of which are equal to the series of measures of lines BA and XZ.

Following are complete analyses showing how vertical, horizontal, and diagonal lines, and circles related in the divine proportion were used in the composition of "Madonna of the Chair."

Analysis of Vertical and Horizontal Structural Lines (Fig. 23)

The enclosing circle was bisected by lines BA and XZ. C is their point of intersection.

Divisions of BA, the horizontal bisecting line of the circle.--

E and E' are median points of BA.
D is the median point of BC.
CD = CD.
F is the median of DE.
DF = D'F' = DG = D'G'.
F'H = EF = E'H' = GI = DJ = JK = F'L = D'L'.

A line drawn through point E goes across the Baby's right foot, passes the point where His right leg touches the Madonna's

9 Ibid., p. 373.
right sleeve, and marks the neckline of Her robe. The faces of
the Madonna and the Baby were arranged so that they would fit
between lines drawn through points $E$ and $E'$.

A line drawn through point $D$ touches the back of the chair
and terminates at the point where the Madonna’s cap touches
the enclosing circle.

A line drawn through point $D'$ forms the edge of the Baby’s
clothing and passes through the corners of Saint John’s mouth
and left eye.

A line drawn through point $F$ touches the Madonna’s right
ear, lines of Her shawl, and an edge of the chair.

A line drawn through point $F'$ locates the end of the Ma-
donna’s right sleeve and goes down the center of Her left thumb.

A line drawn through point $H$ crosses the Baby’s left foot
at the base of His first toe, establishes the point where His
left arm meets the Madonna’s shawl, touches His right cheek,
crosses the Madonna’s nose, and touches lines of Her shawl.

A line drawn through point $H'$ touches the Madonna’s cap
and goes through the Baby’s halo, His left eye and His left
heel.

A line drawn through point $I$ passes through the Madonna’s
right eye, touches the side of Her nose, goes through Her mouth,
and locates the point where the Baby’s feet touch.

A line drawn through point $K$ touches the Baby’s right foot,
the Madonna’s neckline, and Her right ear.
The Madonna's hands were arranged so that a line drawn through point L crosses the knuckles of the right hand and the second finger of her left hand. Also touching this line are folds of the Baby's clothing, and Saint John's nose, hair-line, and right eye.

**Divisions of XZ, the vertical bisecting line of the circle.**

- R and R' are median points of XZ.
- M and P are median points of CX.
- CR = CM.
- CO = CF.
- CR' = S'Z = T'O = MY = W'V = W'V' = MV' = TR = SX.

The Baby's chin rests on a line drawn through point R. Touching the same line are Saint John's nose, his ear, and the top of his cross, and lines of the Madonna's shawl.

The Madonna's right arm was drawn so that a line drawn through point R' runs along its center to the elbow. Lines in the Baby's clothing and a border on the chair were made to touch the same line.

A line drawn through point P determines a point of the Madonna's hair-line and crosses her right eye.

A line drawn through point M crosses the Madonna's right shoulder at a spot where her cap falls against her shawl, passes through her chin, goes across the Baby's nose and left ear, and across Saint John's forehead.

A line drawn through point N crosses the top of a pattern on the chair, goes along an edge of the Madonna's right arm, and passes along lines of the Baby's clothing.
Along a line drawn through point O lies a line of the chair and folds of the Madonna's robe. The Baby's left foot was placed on this line.

The top of the Baby's left foot touches a line drawn through point S'. Of the same foot, the last toe and the top of the heel touch a line drawn through point T'. This line also makes a line on the chair.

A line drawn through point Y goes from an edge of a border on the Madonna's shawl across a line on the chair, runs parallel with Her right arm, and crosses the fingers of Her left hand.

A line drawn through point W goes across Saint John's fingers, crosses the Madonna's right hand at the edge of Her cuff, and runs parallel with the Baby's left arm.

A line drawn through point W' touches a stripe on the Madonna's shawl, forms the dividing line of the Baby's lips, crosses the top of His left shoulder and goes through Saint John's left eye.

A line drawn through point Y' crosses the Madonna's eyes and locates the Baby's hairline.

A line drawn through point V" touches the top of the Baby's left arm. The Madonna's right hand was drawn so that the line touches Her cuff and crosses the base of Her thumb. Saint John's chin, the tips of his fingers, and the middle of his cross lie in its path.

Touching a line drawn through point T are the corner of the Madonna's mouth, the tip of Her nose, the Baby's eyebrows, and the
Fig. 34. Analysis showing how geometrically set vertical and horizontal lines were used in the composition of "Madonna of the Chair."
top of Saint John's head. Lines drawn through points T and N are applications of Kelsch's "Law of Isocephaly."10

A line drawn through point S locates the Madonna's hairpart and the Baby's halo.

**Analysis of Diagonal Structural Lines (Fig. 94)**

The intersecting lines BA and XZ were divided geometrically in the first analysis. From points where lines drawn through these divisions touch the enclosing circle, diagonal lines were drawn which bind together canonical points of the composition.

Line J"R" runs along the back of the Madonna's head, Her dress, and a part of Her shoulder line.

Line H"S" leans with the Madonna's body, touches Her hairpart, goes down Her nose, across Her lips, through the corner of Her shawl, and locates lines on the chair.

Line H"T" marks the Baby's hair-line at His left temple, crosses His ear, and touches the cuff of the Madonna's right arm.

From F" three diagonals were drawn to determine canonical points.

Line F"R" crosses the Madonna's forehead and right hand, crosses the Baby's left ear, and follows the line of His shoulders. It is tangent to Saint John's right cheek.

---

10 Kelsch, Canon Tirbutius, p. 43.
Line F"N" crosses an eyebrow and eye of both the Madonna and Baby continues along the line of the little finger of the Madonna's right hand.

Kelsch's "Law of the Madonna"\textsuperscript{11} is demonstrated by the diagonal line from F" to G". The line goes from the center of expression in the Madonna's face across the Baby's face, and along the first finger of the Madonna's left hand, touching the nail of her second finger.

From K"F" a line crosses the Madonna's nose, touches the Baby's right cheek, crosses his left elbow, a fold in the Madonna's right sleeve, and a line of the Baby's left leg.

Line M"T" follows the direction of the Madonna's shawl and forms the band of the Baby's left knee. This line locates the top of the chair.

F"R" is a diagonal line illustrating Kelsch's "Law of Isocephaly."\textsuperscript{12} Touching the end of the Madonna's nose, it goes across the Baby's eyes, touches the upper lids, crosses his left ear, touches Saint John's hair-line and goes through his left eye and ear.

Line M"R" touches a line in the pattern on the Madonna's shawl, crosses the Baby's lower lip, and crosses Saint John's right eye and forms the end of his cross.

Line M"G" follows the direction of the Madonna's shawl.

\textsuperscript{11}Kelsch, Canon Tibutius, p. 373.
\textsuperscript{12}Ibid., p. 43.
Fig. 24.—Analysis showing how diagonal lines joining points geometrically determined were used in the composition of "Madonna of the Chair."
forms the top line of Her right arm, and crosses the Baby's left leg at the edge of His clothing.

The Baby's feet were drawn to follow the direction of a line drawn from H''B. His right foot lies directly along it and it crosses the toes of His left foot.

Line O'T' follows the top line of the Madonna's right arm, crosses the base of Her thumb, crosses Saint John's nose, and forms a line of his left eye.

Line R'S' touches an edge of the chair, and crosses the Baby's toes, the Madonna's hands and right elbow, and Saint John's fingers.

Diagonals from points H'', R'', H'', T'', and S'' form a five-pointed star. Connecting the points of the star, a pentagon is formed, the figures being arranged compactly within.

Analysis of Circles Related in the Divine Proportion (Fig. 25)

The geometrical diagram was based on a progressive series of nine concentric circles related in the divine proportion. The radius of circle I forms the enclosing frame of the composition. The other eight circles are equal to geometrical measures of the intersecting lines BA and ZX, explained in a former analysis.

The radius of Circle II is equal to BE.
The radius of Circle III is equal to BF.
The radius of Circle IV is equal to CF.
The radius of Circle V is equal to CE.
The radius of Circle VI is equal to CJ.
The radius of Circle VII is equal to CV'.
The radius of Circle VIII is equal to CG.
The radius of Circle IX is equal to the diameter of Circle VI.
The Madonna's head was drawn so that her hairline and the direction of her neck and shoulders follow circle II. Its circumference also passes through the place where the Baby's feet touch, and it crosses Saint John's fingers.

Touching circle III are the Madonna's right eye, a toe of the Baby's right foot, and a fingernail of the Madonna's left hand. Saint John's head was drawn so that the circle goes through the center of his lips, touches the corner of his right eye, and determines his hairline.

The Madonna's right arm curves with circle IV for a distance. Her left thumb follows its course exactly and it forms a part of the edge of her right cuff.

The Baby's chin rests on Circle V.

From his left knee, the Baby's body curves to fit circle VI. It touches an eyebrow and ear, the Madonna's lips, and a line at her neckline.

Saint John's right cheek touches circle VII. Other canonical points touching its circumference are the knuckles of the Madonna's right hand, her left eye, a toe of the Baby's left foot, and a stripe on the chair.

Saint John's head and neck were drawn to fit the curve of circle VIII. It touches lines of the Madonna's cap and marks an edge of a decorative border on her shawl. It runs the same direction with the Baby's left foot, touching the top of his heel and the end of his last toe.
Fig. 25.—Analysis showing how circles related in the Divine proportion were used in the composition of "Madonna of the Chair."
Fig. 26. - Analysis showing Figures 23, 24, and 25 superimposed
Circle IX determines an edge of Saint John's cross and the back of his head, and touches some lines of the Madonna's cap.

Raphael used circles to determine most of the movement in the three paintings included in this chapter. By placing the most important canonical points of the pictures on or along the circumferences of series of circles, their radii being equal to geometrical divisions of the compositions, the eye moves along these circular lines.

Although they were not used to the same extent as circles, diagonals drawn from geometrical divisions of the compositions affected some of the lines of direction in the three paintings. Canonical points were determined by the use of these diagonal lines and by the use of vertical and horizontal lines drawn from divisions in the geometrical series of measures of the enclosing rectangle, and of the bisecting lines of the circular picture.

The outstanding geometrical differences between the compositions of the three paintings by Titian and by Raphael which are comprised in this study are: (1) the movement in Titian's pictures is along straight lines, while the movement in Raphael's paintings is circular; (2) the structural lines of the enclosing rectangles of Titian's pictures form important lines in the compositions, but in Raphael's pictures the important structural lines are circles related in the divine proportion and straight lines drawn from divisions in the geometrical series of measures of the compositions.
CHAPTER V

CONCLUSION

The analyses comprising this study confirm the results of research done by Hambidge, Bragdon, Richter, and others in attempting to prove the theory that geometry was extensively used by the artists of the Fifteenth Century as a structural basis for composition.

The author does not contend that an intricate diagram, such as those presented in this analysis, was prepared before each picture was drawn. That would have been confusing to the artist. Neither is it likely that as many guide lines were actually used in determining points and relating forms as were employed by the writer in analyzing the pictures of these artists. Some of the lines were probably drawn, while others were only partially constructed or else extended merely in the imagination; moreover, it is possible that after the protracted use of geometrical measures, the artist could follow them subconsciously. The preceding analyses attempt to show lines, either actual or imaginary, which were used by Titian and Raphael in the composition of thirty of their paintings.

Proof that enclosing rectangles were determined by definite geometrical measures lies in the fact that every one of the twenty-nine used in this study can be so analyzed. The sides of
the rectangles were also divided geometrically; and from these
division points, vertical, horizontal, and diagonal lines were
drawn, when needed, to locate important points in the composi-
tion. Circles having as their radii measures which correspond
to those of the rectangles were likewise used. By basing their
compositions on these structural divisions, the artists achieved
works wherein every line of direction and movement was placed
according to geometrical rules and all parts were thus related
to one another.

Raphael's pictures were based chiefly upon circles. Al-
though he used vertical, horizontal, and diagonal lines in lo-
cating forms, the main movement in all of his compositions in-
cluded in this study is circular. In the author's analyses of
many of Raphael's pictures, the most important points lie on or
along the circumference of circles related in the divine propor-
tion. The center of several concentric circles is always an
intersection of vertical and horizontal lines drawn from geo-
metrical divisions of the sides of the enclosing rectangle.

Titian's compositions show that he used vertical, horizontal,
and diagonal lines to locate important points. Arcs described
from geometrical divisions of the sides of rectangles, and circles
described from an intersection of vertical and horizontal lines
were occasionally used, as in his "Assumption of the Virgin"
(Plate III); but as a rule, the eye moves through his paintings
in vertical, horizontal, and diagonal directions.
From geometrical analyses the author has acquired a deeper appreciation and better understanding of Italian art of the Fifteenth Century. It has answered a question which has frequently been expressed—why art of the Classic period has remained so popular while that of other periods has not. Upon the straight or circular lines derived geometrically, the artist placed several important points. These points developed movement in the composition and the eye was directed along disciplined directions rather than left to wander aimlessly from one part of the picture to another part. By using a geometrical series to decide their proportion, rhythm, and unity, these Masters produced results which are above reproach, and their popularity will continue, regardless of changing methods and styles.

Through geometrical analysis it is possible to re-create, in a degree, the thought-processes of the Masters. To find the particular plan that one of them used in a composition, one follows his procedure, his trend of reasoning, and feels the same sense of perfection which he must have felt in his completed work centuries ago.

During the course of this problem the possibilities of geometry in art of today is a factor upon which much thought has been placed. We live in an age when machinery and mechanism surround us on every side, and when the structure of art is receiving much emphasis. If man's work is to stand the test of time and the critical analyses to which it will be subjected, the
artist needs a system by which to adjust rhythmic relationships. The designer—whether he draws plans for airplanes, automobiles, or locomotives, or for furniture, or buildings, or whether he paints a composition, or makes jewelry, or designs book jackets—needs an accurate sense of proportion. Nothing can bring about a more satisfying proportion than the sensitive use of the mean and extreme ratio; that is, the use of the golden proportion.

The knowledge of such ratios, and experiences in creating them will refine the artist's sense of correctness in handling areas, and like Titian and Raphael, he will become accustomed to perfect rhythm of relationships to such an extent that it will not always be necessary to use guide lines. Rightness will be felt emotionally, and rhythmic relations created may be perceived without help of the ruler and compass. The extended use of this basic system for creating forms would lend unity and enduring quality to our surroundings.
APPENDIX

REPRODUCTIONS OF PAINTINGS

BY TITIAN
PLATE II

MADONNA AND CHILD (Gipsy Madonna)
Imperial Gallery, Vienna
PLATE VII

LA BELLA (DUCHESS OF URBINO)?
Pitti, Florence
PLATE VIII

THE TRIBUTE MONEY
Gallery, Dresden
REPRODUCTIONS OF PAINTINGS

BY RAPHAEL
PLATE XIII

SOLLY MADONNA
Kaiser Friedrich Museum, Berlin
PLATE XVI

MADONNA DELL' IMPANNATA
Pitti, Florence
BIBLIOGRAPHY

Books Cited


Caskey, Lacey Davis, Geometry of Greek Vases, Boston, Museum of Fine Arts, 1922.


Magazines and Yearbooks


Books From Which Tracings of Masterpieces Were Made

Ady, Julia Cartwright, Raphael, New York, Rand, McNally and Company.

Bensusan, S. L., Titian, New York, Frederick A. Stokes Company.


Titian Paintings and Drawings, George Allen and Unwin Ltd., 1937.