OVERVIEW ON THE NRC-BNL BENCHMARK EVALUATION PROGRAM OF SEISMIC ANALYSIS METHODS FOR NON-CLASSICALLY DAMPED COUPLED SYSTEMS

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ABSTRACT

This paper provides an overview of a comprehensive benchmark program for evaluation of state-of-the-art analysis methods and computer programs for seismic analysis of coupled structures with non-classical damping. The benchmark program, which was sponsored by the United States Nuclear Regulatory Commission (USNRC), was developed by Brookhaven National Laboratory to evaluate the applicability and limitations of state-of-the-art analysis methods to analyze typical coupled nuclear power plant (NPP) structures with non-classical damping. The benchmarking process, benchmark structural models, and the analysis methodologies applied by the program participants are described. The discussions on the benchmark results and findings are then presented. Finally, the principal conclusions and recommendations are provided. It is expected that the findings and insights, as well as recommendations from this program will be useful in developing new acceptance criteria and providing guidance for future regulatory activities involving licensing applications of these alternate methods to coupled systems.

1.0 INTRODUCTION

In the nuclear industry, coupled seismic analysis of major subsystems with different damping (such as the NSSS system and Reactor Building) has been performed by applying approximate schemes (References 1, 2 and 3) to estimate equivalent modal damping ratios of the coupled system as weighted sums of the component damping ratios based on some weighting functions using the component mass or stiffness or combination of both. While these methods may provide reasonable approximations of the diagonal terms of the damping matrix, they ignore the effects of the off-diagonal terms. In more recent years, more rigorous approaches have been developed based on a method first proposed by Foss (Reference 4) in which the equations of motion of the non-classically damped systems are uncoupled by a transformation to the damped modal coordinate system. Unlike the traditional methods, the solution involves complex-valued eigenvalues and eigenvectors (Reference 5). In contrast to classical response spectrum methods, in addition to a displacement spectrum input, the response spectrum methods for non-classically damped systems require a velocity spectrum input, which is not explicitly provided in design practice. These newly evolved methods
appear more powerful, but are mathematically more complicated, and require greater computational effort than the traditional methods, and to date have not been widely applied or accepted for general use in the nuclear industry. While current regulatory requirements do not prohibit the use of coupled analysis, there is no guidance on the implementation of these new methods. From the regulatory standpoint, it is important to understand the applicability and limitations of these methods to assure that they produce reasonable results with acceptable safety margins.

Under the auspices of the US Nuclear Regulatory Commission (NRC), Office of Nuclear Regulatory Research, Brookhaven National Laboratory (BNL) developed a two-phase benchmark program for evaluation and verification of state-of-the-art analysis methods and computer programs for performing seismic analyses of coupled structures with non-classical damping. Under Phase I, a series of benchmark problems that cover various aspects of application and complexity of typical coupled nuclear power plant (NPP) structures with non-classical damping were first developed. These benchmark problems were subsequently analyzed by BNL using the direct integration time history analysis method with a rigorous formulation for the explicit damping matrices. A preliminary report, which included the detailed descriptions of the benchmark problems, the BNL analysis method, and the necessary inputs, was distributed to program participants for their independent analyses using alternate state-of-the-art analysis methods. In Phase II of the program, the analysis results were submitted to BNL by the participants. These results were then compared to and evaluated against the BNL “exact” solutions. This paper provides a description of the benchmark process, discussions of the benchmark results and findings with respect to the applicability and limitations of various alternate state-of-the-art analysis methods to coupled NPP structures with non-classical damping, as well as principal conclusions and recommendations.

This paper is organized in five sections. The benchmark process, objectives and the benchmark structural models are described in Section 2. In Section 3, a brief outline of the participants’ methodologies is presented. Section 4 provides discussions of the benchmark results and significant findings. Finally, principal conclusions and recommendations are discussed in Section 5.

2.0 BENCHMARK OBJECTIVES AND BENCHMARK PROCESS

The objective of this program was to evaluate state-of-the-art methods for performing seismic analysis of coupled NPP structures with non-classical damping. The program was focused on the analysis of a coupled primary-secondary system consisting of two subsystems with different modal damping ratios. A typical NPP application is the seismic analysis of a coupled model of a piping system with 2% damping supported by a reinforced concrete building with 7% damping. In order to evaluate the methods, BNL developed a series of benchmark problems designed to cover various aspects of application and complexity. BNL generated a series of “exact” solutions to these problems using the direct integration time history analysis method. Developers of alternate analysis methods applied their methods to analyze the benchmark problems and provided solutions for comparison to the BNL solutions. The latter was used to evaluate the applicability and limitations of various alternate state-of-the-art analysis methods to coupled NPP structures with non-classical damping. The findings and insights learned from this program could be used to develop new acceptance criteria to provide NRC staff guidance for evaluation of future licensing submittals involving the application of these alternate analysis methods to coupled systems.

The benchmark process includes developing the structural models of the benchmark problems, establishing the “exact” benchmark problem solutions, and then inviting program participants to analyze the benchmark problems with their own methodologies in either frequency or time domains, finally comparing the participants’ analysis results to the BNL “exact” solutions to draw insights into the applicability and limitations of these methods to coupled structures with non-classical damping.

Benchmark Models

Four benchmark problem configurations were developed for this program. They include three simple models and one complex model, each representing a coupled primary-secondary (P-S) system. The dynamic properties of the models are representative of NPP structures, systems and components. For each simple
model, a number of load cases covering variations in model properties were analyzed. In addition, for all
configurations, multiple load cases were analyzed for different earthquake loads corresponding to both real
and artificial earthquake records. Figures 1, 2 and 3 depict the structural configurations of the three simple
models, which have the same primary component (building) model that consists of weightless shear beam
elements and lumped masses and is fixed at the ground. The model has five degrees of freedom with each
node free to translate in one horizontal direction. The secondary components were designed differently for
each of the three benchmark problems for the purpose of evaluating the effect of multiple support excitations
of the secondary component (spatial coupling effect). The benchmark problem no.1 has an S-component
representative of base-supported equipment. The S-component consists of four weightless shear beam
elements and four lumped masses, and has four degrees of freedom with each node free to translate in one
horizontal direction. The base of the S component is coupled to a mid-elevation primary component node.
The S-component for the benchmark problem no.2 is representative of a multiply supported piping system,
which consists of eight shear beam elements and six lumped masses, and has six degrees of freedom with each
node free to translate in one horizontal direction. In contrast to benchmark problem no.1, the S-component
for benchmark problem no.2 is connected to the primary system building model at three different nodal
elevations. The S-component for the benchmark problem no.3 is similar to benchmark problem no.2, except
that in this case, the secondary component is attached to the building at two elevations and to the ground.
Both the building and piping system are subjected to the same ground motion at their ground support points.

The fourth benchmark model is shown in Figure 4. This model is representative of a realistic complex
model of a coupled NPP building and piping system which utilizes the same type of elements that would be
used in a design analysis. In this model, the primary system (building model) consists of seven weightless
3-D flexural beam elements and seven lumped masses. Each node has six DOF and the bottom node is fixed.
The secondary component (the piping model) consists of twenty-three straight and curved SAP piping
elements. Each node also has six DOF. The pipe is supported by anchors at its end points and by two
directional guides at intermediate points. Rigid weightless beam elements are used to support and couple the
piping to the building as shown in Figure 4. To model the guide constraints, the SAP beam element end
release option is used at the piping connection points to provide translational restraint in two directions
perpendicular to the axis of the pipe. At the anchor points, the rigid beams provide full six DOF constraint.
The model uses realistic piping and building material and cross-sectional properties. The properties and
support configuration were selected to provide equal fundamental frequencies for the uncoupled building and
the uncoupled piping system. Two configurations were selected, which provide uncoupled fundamental
natural frequencies of 8.24 Hz (No. 4a) and 4.60 Hz (No. 4b), respectively.

Modal damping ratios of 7% for the uncoupled building and 2% for the uncoupled multiply supported piping
system were assigned in both cases. The input ground motion is applied at the base of the primary component
(Building) in the global Y direction for the benchmark model #4a, and in the global X direction for the
benchmark model #4b.

“Exact” Solutions

To obtain the exact solutions to the benchmark problems, BNL selected the step-by-step direct integration
time history analysis method. In order to apply this method to non-classically damped coupled systems, a
synthesis formulation developed by BNL (Reference 6) for generating the fully populated damping matrix of
the coupled system from the damping ratios of its subsystems was applied. The formulation was programmed
into a series of preprocessor codes, which interface with the BNL in-house modified version of the SAP
program to perform direct integration time history analysis of the coupled models. The BNL programs for
developing the “exact” benchmark problem solutions were tested and verified by comparison to other
published solutions.

Benchmark Load Cases

Sixteen load cases were designed for the three simple benchmark models to investigate the applicability
of various analysis methods to problems with a range of parameter variations and multiple earthquake
excitations. A baseline case was selected for the three simple models such that both the uncoupled primary
component and the uncoupled fixed-base secondary component would have fundamental frequency equal to 5.0 Hz, the ratio of secondary/primary (S/P) component mass was selected as .005 (on an individual mass basis), and modal damping ratio of 7% was assigned for the uncoupled P-component and 2% for the uncoupled fixed-base S-component. The El Centro (1940) earthquake record was applied as input to the baseline case. The parametric variations, which were performed only for the three simple benchmark models with single earthquake ground motion (El Centro, 1940), included secondary to primary system frequency ratio, mass ratio, and different modal damping ratios. Six recorded earthquake ground motion acceleration time histories plus one artificial acceleration time history compatible to Regulatory Guide 1.60 response spectrum were used for the multiple earthquake excitation analyses, which were performed using the baseline structural model. Table 1 provides a typical matrix of load cases.

For benchmark problems no. 4a and no. 4b, only the baseline models with multiple earthquake excitations were analyzed. A total of seven load cases were designed. The baseline structural properties were provided in the previous section.

3.0 OUTLINE OF PARTICIPANTS' METHODOLOGIES

Developers of alternate analysis methods were invited to apply their methods to analyze the benchmark problems and provide solutions for comparison to the BNL solutions. Eleven organizations originally accepted the BNL invitation and four participants returned their analysis results in the required format to BNL. They include: C. Chen of Apollo Consulting, Inc. (Apollo), A. K. Gupta, A. Gupta and M. K. Bose of North Carolina State University (NCSU), T. Igusa of Johns Hopkins University and A. Der Kiureghian of University of California at Berkeley, A. Berkovski, O. Kireev, V. Kostarev and J. Stevenson of Stevenson and Associates, Russian Office (S&A). This section provides a brief description of the methods that participants applied to the benchmark problems.

**Chen's Method**

To produce solutions to the benchmark problems, Chen performed coupled analyses based on the classical normal mode response spectrum method (Reference 7). The calculation of damping for the coupled structure was performed using weighted damping values based on energy principle with strain energy based on ASCE 4-86 standard (Reference 8). For modal combinations, Chen applied the Complete Quadratic Combination (CQC) rule with cross coupling terms. Rigid modes were accounted for using the procedure described in NRC Standard Review Plan.

**Gupta's Method**

Gupta’s method is based on the approach that Gupta and colleagues (References 9, 10 an 11) at NCSU developed to obtain the coupled frequencies, damping ratios and mode shapes of the nonclassically damped systems using the modal properties of the uncoupled primary and secondary systems. They simplified the Foss formulation (Reference 4) by algebraically replacing the complex mode shape by two real modal vectors and further extended their formulation to the response spectrum method, which was implemented in the computer program CREST. The Gupta method does not require formulation of the system damping matrix in the time domain, since the equations of motion are first transformed in the modal coordinate system and only the modal damping of the coupled system needs be computed. The system modal damping is obtained based on the fixed-constraint modal damping of its components, which are assumed to be classically damped. The component damping and system modal damping are related through a transformation that is obtained through a set of static constraints which provide kinematic dependencies relating the motion of the component to other components that constrain it via the constraints. It was shown by Xu (Reference 6) that the system damping derived by Gupta’s method, if transformed to time domain, assumes the same form reduced from the BNL method. Gupta’s method also takes into account issues associated with complex valued response spectrum approaches such as procedures for estimating velocity spectrum, and for combining maximum modal responses, as well as incorporating residual response.
**Igusa and Der Kiureghian Method**

The method used by Igusa and Der Kiureghian for the benchmark analysis is based on random vibrations of non-classically damped systems and reflects improvements on formulations for the correlation coefficients developed in their past research efforts (Reference 12). The major improvement in the current method is that filtered white noise is used instead of white noise to determine the correlation coefficients for modal combinations. Therefore, seismic events can be modeled with non-stationary properties, and maximum response, which is computed separately for each seismic event, can be compared directly with time history analysis. In addition, Igusa and Der Kiureghian also provided treatment for the velocity spectrum input, and for the coupled system-damping matrix, the BNL formulation was applied without modification.

**Stevenson & Associates Method**

The method applied by the Stevenson & Associates (S&A) follows closely to that of Gupta (Reference 9), such as the system damping and complex-valued eigen problem solution. The analysis for the coupled system was performed using in-house computer codes. Two sets of analyses using response spectrum methods were performed, one with exact correlation coefficients and maximum modal responses directly computed from the modal time history analysis (RSM-I), and the second with an approximation for correlation coefficients and the velocity response spectrum input (RSM-II).

The RSM-I analysis, which used the exact correlation coefficients and maximum modal responses directly computed from the modal time history analysis, is not a practical approach since it is generally assumed that when performing a response spectrum analysis, the corresponding modal time history responses are not available.

The RSM-II analysis was performed based on approximations for correlation coefficients and the velocity response spectrum input. The formulation for correlation coefficients proposed by Igusa (Reference 13) was applied. The pseudo velocity response spectrum was used in place of the velocity response spectrum.

### 4.0 DISCUSSIONS OF BENCHMARK RESULTS AND FINDINGS

Sixteen load cases were analyzed for each of three simple benchmark problems Nos. 1, 2, 3, including parametric variations and the baseline model subjected to a suite of ground input motions. Seven load cases for each of benchmark problems Nos. 4a and 4b were analyzed using the baseline models and a suite of earthquake ground motions. Results from the first three benchmark problems include maximum nodal displacements and maximum element forces. Results from benchmark problems No. 4a and No. 4b, which were intended to represent realistic NPP building-piping systems, include nodal displacements in both translations and rotations, element forces and moments, and support forces in the coupling elements. The participants’ analysis solutions were compared to the BNL’s “exact” solutions by computing the ratio of the participant maximum response to the BNL maximum response for each load case, except for moments in the piping elements for problems No. 4a and No. 4b. In conventional piping analysis, Square-Root-of-Sum-of-Squares (SRSS) of all three components of the maximum element moment are used to calculate stresses which are compared against code requirements. Therefore, for the secondary component of the benchmark problems No. 4a and No. 4b, the ratios were computed for the participant SRSS maximum moments to the corresponding BNL SRSS maximum moments. In addition to comparisons for single ground motion, statistical estimates such as mean and COV were also computed and evaluated for a suite of ground motion inputs, as well as the corresponding maxima and minima of the response ratios.

Three participants: S&A, Igusa/Der Kiureghian and Gupta et al, performed both modal superposition time history and response spectrum analyses using complex modes. Chen analyzed the benchmark problems using the classical response spectrum method with composite modal damping. Due to limited space, only the significant findings are presented. The detailed comparisons of the benchmark results can be found in the benchmark final report (Reference 14). As a result of the comparisons and evaluations of the finalized benchmark analysis results, the following significant observations were made:
1. For the smaller benchmark problems nos. 1, 2, and 3, the complex-mode time history analysis results provided by S&A, Igusa, & Gupta were in excellent agreement with the BNL direct integration time history results. The comparisons of the displacements and forces for the baseline case for benchmark problem no. 1 are shown in Figures 5 and 6. Based on these results, it is concluded that for these problems, the participants' complex-mode time history analysis methods provide results comparable to those generated by the benchmark direct integration time history analysis methods.

2. For the larger benchmark problems no. 4a and no. 4b, which represent realistic coupled NPP building-piping systems, the complex-mode time history analysis results provided by S&A, Igusa & Gupta were in good agreement with the BNL direct integration time history results. The comparisons of the displacements and the piping SRSS moments for benchmark problem no.4a and 4b are shown in Figures 7 through 10. As also shown in Figure 11, which is plotted in terms actual displacements for benchmark problem no.4a, the large deviations in Figure 7 correspond to values too small to be of practical significance. Based on the overall results for both the small and large benchmark problems, it is concluded that the participants' complex-mode time history analysis methods provide results comparable to those generated by the benchmark direct integration time history analysis methods and are acceptable.

3. For the smaller benchmark problems nos. 1, 2, and 3, the complex-mode response spectrum analysis results provided by S&A, Igusa, & Gupta showed larger deviations when individual load case results were compared against the corresponding BNL direct integration time history analysis results. However, due to inherent differences between response spectrum and time history analysis methods, exact one-to-one correspondence of solutions is not expected for a specific load case. In comparing response spectrum to time history analysis results, one should expect that the average responses using a suite of input ground motions should be close to the responses from the corresponding response spectrum analysis. For this benchmark program, using a suite of seven earthquake input motions, it was shown that the mean values of participants' to BNL responses were in reasonably good agreement. All methods predicted conservative displacements and forces, as shown in Figures 12 and 13 for the mean response comparisons for benchmark problem no. 1. Based on these results, it is concluded that for these problems, the participants' complex-mode response spectrum analysis methods provide reasonably accurate and generally conservative results.

4. For the larger benchmark problems nos. 4a and 4b, the comparisons of complex-mode response spectrum analysis results provided by S&A, Igusa, & Gupta to BNL time history results were generally consistent with the smaller problem comparisons, as depicted in Figures 14 through 17 for benchmark problems no.4a and no.4b. Based on the overall results for both the small and large benchmark problems, it is concluded that the participants' complex-mode response spectrum analysis methods provide reasonably accurate and generally conservative results.

5. For the smaller benchmark problems, parametric studies were performed by varying model properties to investigate potential limitations of alternate analysis methods in predicting the response of coupled systems with: (a) tuned, near-tuned and untuned subsystems; (b) low to high subsystem mass ratios; and (c) low to high secondary system damping. The complex-mode time history analysis methods provided very good agreement with the benchmark results over the entire range of parameter variations. The response spectrum analysis methods showed larger deviations but since the comparisons were based on single earthquake input motions, it is not possible to determine whether the deviations are due to the parametric variations or to the normal differences in results due to the different analysis methods. It is reasonable, however, to assume that the findings and conclusions from the modal time history analysis parametric studies can be extended to response spectrum analysis. On that basis, no significant limitations were identified within the range of frequency and mass ratios investigated. Based on limited trends in the data and currently accepted practice, it would be prudent to limit the damping ratio to 20%.

6. The solutions provided by Chen using the classical response spectrum method with composite modal damping provided interesting comparisons with the non-classical complex mode method solutions. For the small benchmark problems nos. 1, 2, and 3, Chen's results were comparable and in several cases slightly
better than the responses generated using the complex-mode based response spectrum methods. However, for the larger, more complicated coupled models of benchmark problems 4a and 4b, the results showed much larger deviations with significant over-prediction of responses in many locations. The reason for these inconsistent findings is not clear and warrants further investigation.

5.0 PRINCIPAL CONCLUSIONS AND RECOMMENDATIONS

Based on the results of this benchmark program, the following principal conclusions and recommendations were made with respect to the acceptability and application of the complex-mode and normal-mode solution methods to non-classically damped coupled systems:

1. The complex-mode time history analysis methods applied by Stevenson and Associates, Igusa and Der Kiureghian, and Gupta, Gupta and Bose are acceptable methods for predicting the seismic response of non-classically damped coupled nuclear power plant (NPP) structures, systems and components when the ground input motion is defined as an acceleration time history. The complex-mode methodology is based on well-established theoretical principles and the close agreement between the participants' solutions and the BNL direct integration time history analysis solutions has validated each participant's implementation and application of the basic methodology.

2. The complex-mode response spectrum analysis methods applied by Stevenson and Associates, Igusa and Der Kiureghian, and Gupta, Gupta and Bose are acceptable methods for predicting the seismic response of non-classically damped coupled NPP structures, systems and components when the input motion is defined as an acceleration response spectrum. By comparing individual participant response spectrum solutions to corresponding BNL time history solutions for a suite of earthquakes and averaging the ratios of results, it was shown that the participant methods provided reasonably accurate and generally conservative results consistent with the accuracy of classical normal mode response spectrum analysis methods. The S&A RSM-I, Igusa, and Gupta methods provided generally closer agreement than the S&A RSM-II method.

3. The classical normal-mode response spectrum analysis method with composite modal damping as applied by Chen has been considered an acceptable method and has been previously applied in seismic design analysis of non-classically damped coupled NPP structures, systems and components for coupled subsystems with different modal damping ratios. In this program, the smaller problem solutions were in good agreement with the BNL benchmark solutions, and the larger problem solutions were generally very conservative.

4. For both the complex-mode and normal-mode methods, the modal damping values of either the primary or secondary system of the coupled model should be in accordance with regulatory requirements and should not exceed 20%.

5. For NPP structures, systems and components, both the complex-mode and normal mode analysis methods must be implemented in accordance with the requirements of the USNRC Standard Review Plan and Regulatory Guides for seismic analysis. For complex-mode methods, care must be applied in properly formulating the coupled system damping matrix, estimating the relative velocity-based response spectrum, and using appropriate modal combination procedures. In addition, since coupled analyses do not require the generation of floor response spectra, the effects of variations in seismic response due to modeling uncertainties normally addressed by the floor spectrum broadening requirements, need to be appropriately considered.

The primary goal of this benchmark program was to evaluate the acceptability of state-of-the-art complex-mode analytical methods for predicting the seismic response of coupled NPP structures with non-classical damping. The program goal was achieved by demonstrating that the methods provide results which are comparable to those obtained using the direct integration time history analysis method which has been long been considered an acceptable "exact" method of seismic analysis in the nuclear industry. In addition, during the course of this program, a significant amount of analytical data was generated using a variety of state-of-the-
art analysis methods and computer programs. Details of the analysis results can be found in the benchmark final report (Reference 14).

NOTICE
This work was performed under the auspices of the U.S. Nuclear Regulatory Commission, Washington, D.C. The findings and opinions expressed in this paper are those of the authors, and do not necessarily reflect the views of the U.S. Nuclear Regulatory Commission or organizations of authors.

REFERENCES


Table 1. Typical Matrix for BNL Benchmark Problem Load Cases

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<th>Load Cases</th>
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<td>Frequency Ratio (S/P)</td>
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<td>X</td>
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<td>7%, 5%</td>
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<td>7%, 20%</td>
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Figure 1. BNL benchmark model no. 1

Figure 2. BNL benchmark model no. 2
Figure 3. BNL benchmark model no. 3

Figure 4. BNL benchmark model no. 4a & 4b

Figure 5. Comparison of maximum displacements for benchmark problem No. 1, Baseline Case
Modal superposition time history analyses
Figure 6. Comparison of maximum element shear forces for benchmark problem No. 1, Baseline Case Modal superposition time history analyses.

Figure 7. Comparison of maximum displacements $U_y$ for benchmark problem No. 4a, Case a---Modal superposition time history analyses.
Figure 8. Comparison of maximum SRSS element moments of S-component for benchmark problem No. 4a, Case a—Modal superposition time history analyses.

Figure 9. Comparison of maximum displacements Ux for benchmark problem No. 4b, Case a—Modal superposition time history analyses.
Figure 10. Comparison of maximum SRSS element moments of S-component for benchmark problem No. 4b, Case a—Modal superposition time history analyses

Figure 11. Comparison of actual maximum displacements Uy for benchmark problem No. 4a, Case a—Modal superposition time history analyses
Figure 12. Comparison of mean responses of maximum displacements for benchmark problem No. 1 for seven earthquakes—Response spectrum methods.

Figure 13. Comparison of mean responses of maximum element shear forces for benchmark problem No. 1 for seven earthquakes—Response spectrum methods.
Figure 14. Comparison of mean responses of maximum displacements $U_y$ for benchmark problem No. 4a for seven earthquakes—Response spectrum methods.

Figure 15. Comparison of mean responses of maximum displacements $U_x$ for benchmark problem No. 4b for seven earthquakes—Response spectrum methods.
Figure 16. Comparison of mean responses of maximum SRSS element moments of S-component for benchmark problem No. 4a for seven earthquakes—Response spectrum methods.

Figure 17. Comparison of mean responses of maximum SRSS element moments of S-component for benchmark problem No. 4b for seven earthquakes—Response spectrum methods.