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Buckling and Dynamic Analysis of Drill Strings for Core Sampling

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Abstract: This supporting document presents buckling and dynamic stability analyses of the drill strings used for core sampling. The results of the drill string analyses provide limiting operating axial loads and rotational speeds to prevent drill string failure, instability and drill bit overheating during core sampling. The recommended loads and speeds provide controls necessary for Tank Waste Remediation System (TWRS) programmatic field operations.

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BUCKLING AND DYNAMIC ANALYSIS OF DRILL STRINGS FOR CORE SAMPLING

1.0 INTRODUCTION

This supporting document presents buckling and dynamic stability analyses of the drill strings used for core sampling. The results of the drill string analyses provide limiting operating axial loads and rotational speeds to prevent drill string failure, instability, and drill bit overheating during core sampling. The recommended loads and speeds provide controls necessary for Tank Waste Remediation System (TWRS) programmatic field operations.

The core sampling truck drill string is used to retrieve samples from single- and double-shell waste storage tanks at the Hanford Site. The drill string is operated in a push mode to retrieve liquid and highly viscous slurry type wastes. To retrieve harder saltcake type waste, the drill string is operated in a rotary mode using a rotating bit to penetrate the waste. Buckling of the drill string under either the push mode or the rotary mode of operation is addressed herein. Also, the critical speeds at which the rotary mode drill string can become dynamically unstable are addressed. Reasonable margins on both the buckling loads and critical speeds were determined to provide safe operation of the drill string.

The analysis was prompted because, during sampling (push-mode) of a Hanford Site waste tank, a drill string failed as the result of a structural instability (buckling) of the drill-string tube under an imposed load of 2,600 lbf. The drill string was over 41-ft long fixed at the top and pushing into the waste at the tank bottom. This failure warranted further investigation to establish limits for down forces and rotational speeds, to provide guidance and resolve concerns that may be raised during field operation. In addition, trend studies were conducted to investigate increasing the buckling loads with a 3-in. Schedule 80 pipe (sleeve) of 304 stainless steel. The drill string tube will be inserted through the sleeve with a fixed condition at the top of the tank riser. A detailed analyses was performed for 15-ft and 25-ft sleeves.

To ensure the core sampling does not create an ignition source, the operational downward force and the rotational speed should be within a certain operating envelope to prevent drill bit overheating. The operating envelope that was provided by WHC 1993 for safe operating conditions is a rotational speed not to exceed 55 r/min, a maximum downward force of 1,170 lbf, and a minimum-purge gas flow rate of 30 std ft³/min (SCFM).

The detailed buckling and dynamic analysis for drill strings, without and with a sleeve, appear in Appendix A. Closed form solutions for drill strings without a sleeve appear in Appendix B.

2.0 RECOMMENDATIONS AND DISCUSSION

The results of the analyses provide critical buckling loads and rotational speeds for drill string lengths ranging from 30-ft to 60-ft. In addition, some cases were studied for 20-ft and 70-ft lengths.

The recommended loads and speeds are provided in Tables 1 through 3 and Figures 1 through 5. All of the numerical values in the tables and figures are for fixed-pinned conditions.

For the push-mode core sampling, the critical buckling load of fixed-pinned condition is selected to be the limiting downward load for the drill strings. This limit was chosen because, when the drill string is in the waste, the bottom end will be somewhere between fixed-pinned and fixed-fixed condition. The fixed-fixed condition gives critical buckling loads twice as large as the fixed-pinned condition. So, the critical buckling load of the fixed-pinned condition should provide a reasonable margin of safety during operation, when the drill bit is already in the waste.

For rotary-mode core sampling, also, the fixed-pinned condition is selected for the limiting conditions of downward loads and rotational speeds. The recommended rotational speeds were selected to be within reasonable bounds with respect to the lower natural frequencies of the drill strings, to avoid whirling (dynamic instability). A factor of 1.5 to 2.0 away from the lowest natural frequencies are selected to provide reasonable bounds for the rotational speeds to avoid dynamic instability of the drill-string assembly.

Tables 1 through 3 include the numerical values of buckling loads at drill string intervals of 10-ft. These tables also provide recommended rotational speeds and associated range of compressive axial loads for stable rotary mode core sampling operations. These tables are provided for use during field operations. It is important to follow the notes of the tables to avoid any possibility of dynamic instability.

Figure 1 shows the compressive axial load versus drill string length for push-mode drill strings without a sleeve, with a 15-ft sleeve, and with a 25-ft sleeve. Figure 2 shows generic plots for critical buckling loads versus sleeve pipe length. Figures 3 through 5 provide plots for critical rotational speed (first lateral natural frequency) versus axial compressive load for drill strings without a sleeve, with a 15-ft sleeve, and with a 25-ft sleeve, respectively.

Figure 7 presents the buckling loads for different lengths of the drill string without a sleeve for both fixed-fixed and fixed-pinned conditions and also shows the location of the 2,600 lbf failure load in comparison to the buckling load curves. The failure experienced in truck #2 at a load of 2,600 lbf for a 41.6-ft (500-in.) drill string is close to the fixed-fixed estimate (the X point in Figure 7). This indicates that the lower end was

either deep in the solid waste or was trapped in some solid debris in the tank. These results also support the selection of the fixed-pinned condition to be the basis for the limiting buckling loads.

The following recommendations are provided to ensure the operational stability and integrity of the drill strings.

- Drill string operations should use load and speed value recommendations given in Tables 1 through 3. The loads of the longer drill strings should be used for any drill string length that falls between two given lengths in the tables (e.g., use the load of 50-ft length for 45-ft drill string). If field operations needs to gain the advantage of interpolated higher values, they should consult with the design engineer assigned from Characterization Equipment Design Group. Only the design engineer is authorized to allow the limits of the longer lengths to be exceeded as justified by field observations and by interpolating the analysis results provided in Figures 1, 2, and 7 through 10.
- Use sleeves for drill strings longer than 40-ft to attain higher loads than those given in Table 1.

PUSH MODE SAMPLING

- The operation personnel shall not apply loads above the limits predicted by the fixed-pinned boundary conditions with gravity loads as shown in Tables 1 through 3. If there is a need to go beyond these loads, only the design engineer is authorized to use loads in between fixed-pinned and fixed-fixed boundary conditions as justified by field observations, and by using the analysis results provided in Figures 7 through 10.

ROTARY MODE SAMPLING

- The drilling shall start only after the drill bit encounters and bites in the solid waste (i.e., it will not be rotating in a liquid or sludge waste). This prevents the lower end from behaving as a free or guided condition. This condition should be strictly followed during operation.
- The downward load should not exceed 500 lbf (except the 60-ft drill string) initially as the drill bit enters the solid waste, Ziada (1994). After engagement with solid waste, the load should not exceed the limiting buckling load that corresponds to the drill string length and the associated speed (see Tables 1 through 3). The load for 60-ft drill string should not exceed 200 lbf initially. Approval of the design engineer is required for higher loads.
- The operation personnel shall stay below the lowest critical rotational speed, as provided in Tables 1 through 3 and in Figures 3 through 5. For the long drill strings, such as the 60-ft case, it would be acceptable to quickly go to a higher speed (55 r/min) that will be above the lower natural frequency of the fixed-fixed condition and away from any lateral natural frequency by a factor not less than 1.5. These cases should be approved by the design engineer before applying them. These values are provided in Tables 1 through 3, with notes that prohibit using the values without the approval of the design engineer.

- The drill string shall not operate for long periods near the critical natural frequencies (i.e., shall not be close to the curves in Figures 3 through 5). If shifting from lower speed to higher speed is required (see the preceding recommendation), the shifting should be performed very fast, and should be approved by the design engineer.
- The operation of the drill string should be monitored very carefully for dynamic instability. The instability loads are provided in the last column of Tables 1 through 3. It is recommended that the design engineer be present to monitor these cases.
- In any case, the rotational speed should not exceed 55 r/min, and the downward force should not exceed 1,170 lbf (safety requirement, WHC 1993).
- The drill string should not be operated at speeds lower than 15 r/min. Approval of the design engineer is required before operating at lower speeds.

The above recommendations should be strictly followed.

3.0 CONFIGURATION AND LOADING

The drill string is made of steel pipe segments (5-ft segments) with an outer diameter of 2.25-in. and an inner diameter of 1.906-in. The total length may vary from 15-ft to 70-ft. The expected length of the drill string when the drill bit encounters the solid waste is between 30-ft to 50-ft. The drill string is supported by a riser adapter in a long bearing sleeve, making the constraint of the upper end very close to a fixed condition. At the lower end, the drill string has a spike (at the center of the drill bit) that will be pushed in the solid waste before drilling. This arrangement will not allow the lower end of the drill bit to wander around, and makes the lower end very close to a pinned condition during the initial penetration in the solid waste.

At the very beginning (when the spike encounters the solid waste), assuming the lower end of the drill string (spike) penetrates the waste at initial loads less than the elastic stability for a fixed-free column (about 1/8 of the fixed-pinned load), the lower end of the drill string will act somewhere between guided (i.e., free to move laterally) and pinned conditions. After the drill bit initially penetrates the solid waste, the constraint at the lower end is very close to a pinned condition. When the drill bit has penetrated deep into the solid waste, the constraint is very close to a fixed condition.

Two boundary condition cases were considered for the analysis of the push-mode drill string: fixed-pinned and fixed-fixed. These two cases should bound the elastic stability of the drill string after the drill bit is being engaged and pushed into the solid waste. In other words, the buckling loads of the fixed-pinned and fixed-fixed conditions are the bounding downward loads of the core sampling operations for both push and rotary modes. For the rotary mode, the drill strings should operate at rotational speed corresponding to the associated compressive axial load.

The drilling (rotary mode) shall start only after the drill bit encounters the solid waste and bites into the waste (the spike is already fully inserted into the solid waste). This precaution prevents the lower end from behaving as a free or guided condition. This condition should be strictly followed during operation.

To increase the buckling load, the 2.25-in drill string will be inserted through a sleeve. The sleeve is made of a 3-in. Schedule 80 pipe of 304 stainless steel material. The top of the sleeve is fixed at the riser drilling adapter. The radial clearance between the sleeve and the drill string is 0.325-in. The sleeve may have a length up to 25-ft. The lower end of the sleeve is free (i.e., not supported).

4.0 MATERIAL AND ACCEPTANCE CRITERIA

4.1 MATERIAL PROPERTIES

The material of the drill string is a steel with the following properties.

Yield Strength	= 76,000 lbf/in ²
Ultimate Tensile Strength	= 90,000 lbf/in ²
Modulus of Elasticity	= 29×10^6 lbf/in ²
Total Elongation	= 0.10 in./in.

The sleeve is made of 304 stainless steel.

Yield Strength	= 30,000 lbf/in ²
Ultimate Tensile Strength	= 75,000 lbf/in ²
Modulus of Elasticity	= 28×10^6 lbf/in ²
Total Elongation	= 0.34 in./in.

4.2 ACCEPTANCE CRITERIA

The following acceptance criteria are adopted. The details and discussions for the justification of these criteria appear in Section 6.0.

- The downward loads should not exceed the buckling load limits predicted by the fixed-pinned boundary conditions with gravity loads (without safety margin).
- The rotational speeds should have a factor of 1.5 to 2.0 below the lowest critical fixed-pinned natural frequency of the drill string. The drill string shall rotate only after the drill bit encounters the solid waste.

For the cases of very long drill strings, where the speeds will be very low as the downward load increases, the drill string is allowed to quickly go to higher speeds that will be above the lower natural frequency of the fixed-fixed condition and away from any natural frequency by a factor not less

than 1.5. The natural frequencies for the fixed-fixed condition are at least 1.5 times the fixed-pinned conditions. These cases should be approved by the design engineer before applying them.

5.0 ANALYSIS AND ANALYSIS RESULTS

Under the push mode operation, the drill string can buckle elastically as the push load is increased. Once buckling occurs the drill string will quickly exceed its yield strength under the resulting combined bending and axial load. The incremental load to attain full cross-section yielding and subsequent cross-section failure is relatively small.

Under rotary mode operation, the drill string can become dynamically unstable, resulting in a whirling vibratory motion. The critical speed at which this occurs is a function of the lateral natural frequency of the drill string, which is reduced as the push load is increased.

5.1 ANALYSIS METHODS

The numerical investigation included exact derived solutions for buckling. The exact solutions were checked with results of an ABAQUS finite element solution (ABAQUS 1994). The results matched and trend studies were generated using ABAQUS finite-element analysis. Textbook derived solutions, finite element numerical buckling, and full nonlinear large deflection theory loading to failure all give the same results for this problem (within 3 percent). As a check of tube bending section failure, the steel tube material was modeled with yield, strain hardening, and material failure at ultimate.

The results of this analysis was based on ABAQUS (1994) finite-element analysis. The details of the analysis appear in Appendix A.

Appendix B includes mathematical closed end derived solutions using formulas from Timoshenko and Gere (1961), Johnston (1976), and Blevins (1979).

5.2 FINITE ELEMENT MODELS

ABAQUS/Standard version 5.4, ABAQUS (1994), was used for the nonlinear structural analysis of the tank sampling drill string and the later addition of stiffening sleeves. Both the drill string and sleeve were modeled with beam elements with a pipe cross section. Modeling of the sleeve interaction with the drill string was addressed with cylindrical gap elements. The cylindrical gap element modeled the drill string cross section as a movable point inside a ring that represents the inside of the sleeve (the radius of the ring is set to correspond to the 0.325-in. radial free play between the drill string OD and the sleeve ID).

The drill string and sleeve materials were modeled as nonlinear materials with strain hardening and failure included for the large deflection analyses. But, the critical failure mode for the drill string is classic buckling. Classic buckling can be predicted simply for linear elastic

materials. The full nonlinear analysis capability allows elimination of other failure concerns and accounting for the free play between the sleeve and the drill string.

5.3 DRILL STRINGS WITHOUT SLEEVE

5.3.1 Axial Failure Load Estimate for the 41.67-ft Drill String

This section describes the failure analysis results of the axial compression failure load for the push mode drill string used for core sampling.

During sampling of a Hanford Site waste tank, a drill string failed as a result of a structural instability or buckling of the drill string tube under the imposed load of 2,600 lbf. This tube was 41.67-ft (500-in.) long fixed at the top and pushing into the waste at the tank bottom. A sketch of the tube configuration in the tank is shown in Figure 6.

The dotted curve in Figure 7 represents the load results when the drill string is in contact with the lower end of the 92-in. riser (point C in Figure 6). This contact is simulated by a roller-pin (restricted laterally and free vertically) at this location, and the resulting load is about 35 percent higher than the corresponding load for the fixed-pinned condition at 41.67-ft length.

The failure experienced in truck #2 at a load of 2,600 lbf for a 41.67-ft drill string is close to the fixed-fixed condition estimate (the X point in Figure 7). This indicates that the lower end of the failed drill string was either deep in the solid waste or trapped in some debris in the tank. In addition, the contact with the lower end of the 92-in. riser may have increased the load capability of the drill string to some extent. Actually, if 5-ft is in the solid waste, then the effective length is 36.67-ft with a partially fixed end condition.

This result also suggests that the drill string response is close to fixed-fixed end condition when it is in the waste. Accordingly, the selection of the buckling loads of fixed-pinned condition as the limiting loads for the drill string, is a reasonable criterion.

5.3.2 Axial Buckling Loads for Push Mode With Gravity Effects

Figure 7 provides the load results for drill strings up to 70-ft length with different boundary conditions. These loads correspond to the classic buckling failure loads without a factor of safety. The large deflection load increment (post buckling load) results shown in Figure 7 indicate that the large deflection results do not differ from the classic buckling results (not more than 3 percent increase). The drill string is so long that classic buckling accurately predicts the failure, and the nonlinear properties do not influence the axial load at which failure occurs.

Although the critical axial force required for long column buckling is the most important consideration in the drill string buckling, it is necessary to also include the effect of gravity on the drill string to predict the buckling load of the drill rig force measurements. Figure 7 shows that accounting for the gravity load slightly lowers the buckling load by about 100 lbf or less, as read on the down force instrument. It can be seen that for the fixed-fixed boundary condition of the 41.67-ft drill string the gravity effect results in a lowering of the analytical results to more closely match the observed failure.

For safe operations, the bottom heavy black line of Figure 7 should be used for guidelines on critical compressive drill string load versus length. This curve is reproduced in Figure 1, and is used for the recommended limit loads of the push mode drill string operations with no sleeve.

For a drill string inserted into liquid waste, the effective drill string gravity load would be reduced some, due to the waste floatation effects. The answers would depend on the length of drill string inserted into the liquid waste, but the answers would be bounded by the limits between the cases of axial loads only and the axial loads with gravity of Figure 7.

5.3.3 Rotational Speeds and Downward Loads for Rotary-Mode

Dynamic analyses were performed to determine rotational speeds and the associated downward loads for different lengths of drill string. In order to conduct successful safe operations of the drill string, the frequencies and mode shapes of the drill string were analyzed for various lengths, axial loads, and end conditions.

The dynamic analysis of the drill string employed ABAQUS (1994) with large deflection theory. These analysis trend cases include gravity loads. The solution approach is a combination of static load steps and dynamic modal analysis. The first static step applies the gravity loads. The next step is a dynamic modal analysis step that solve for the natural frequencies at zero axial force with gravity loads. The trend is then completed by alternating static load steps that apply axial loads with dynamic modal analysis steps. As the axial compressive loads are increased, the natural bending frequencies decrease. The frequency of each length of drill string goes to zero at the axial load corresponding to the buckling instability (see Appendix A, Section 3.0).

The torsional frequency of the 60-ft drill string is 42.7 Hz, that frequency would correspond to a critical r/min of 2,520. The operational rotational speed limits of 55 r/min or below already eliminate torsional excitation for all drill string lengths. The lowest lateral (bending) natural frequencies are used to determine the rotational speeds because, they may cause whirling (a form of dynamic instability) when they are associated with lateral deflection.

Figure 3 shows trends of the critical rotational speeds versus axial compression loads of drill strings of 30, 40, 50, and 60-ft lengths. This plot of critical rotational speeds was obtained by simply multiplying the lowest lateral modal frequency in cycles per second by 60 seconds per minute

to obtain r/min. The critical rotation speeds shown in Figure 3 are with boundary conditions of fixed-pinned.

Appendix A, Section 3.0, includes additional analysis results for fixed-fixed boundary conditions and second lateral frequencies. The lowest natural frequencies of the fixed-fixed condition are at least 1.5 times the fixed-pinned condition (lowest at no load). The second lowest natural frequencies are at least 3.0 times the corresponding first lower natural frequencies (lowest at no load).

Additional dynamic frequency studies were completed with the 41.67-ft long drill string inserted into 20-ft of liquid waste, with an assumed specific gravity of 1.7. The addition of fluid mass based on Belvins (1992) does reduce the drill stem frequency, but as the axial compression load increases, this reduction approaches zero. At 50 percent of the buckling load, the frequency reduction was found to be 26 percent.

A factor of 1.5 to 2.0 away from the natural frequencies will be sufficient to ensure stability of the drill string, see Section 6.0 for details.

5.4 DRILL STRINGS WITH SLEEVES

The field operations recognized a need to increase the buckling load capacity of the drill strings for core sampling. A design change, to increase the buckling load capacity, is to insert the 2.25-in. drill string through a 3-in. Schedule 80 pipe of 304 stainless steel material.

Trend studies were conducted to investigate the increase of the buckling load with the modified design. The top of the sleeve is fixed at the riser drilling adapter and, the lower end is free. The radial clearance between the sleeve and the drill string is 0.325-in.

Figure 2 shows trends of the critical buckling load versus sleeve length for drill strings of 30, 41.67, 50, and 60-ft long for fixed-pinned condition. The trend of these figures include large deflection theory, gravity loads, and radial gaps between the sleeve and drill string. The load increase for sleeve lengths between 15 to 25 ft increases slightly for drill strings longer than 40-ft.

The load trends for fixed-fixed condition appear in Figure 8. The loads in Figure 8 are only for the use of the design engineer.

5.5 DRILL STRINGS WITH 15-FT SLEEVE

5.5.1 Axial Buckling Loads for Push-Mode

Figure 9 shows the buckling load versus drill string length with a 3-in. Schedule 80 sleeve of 15-ft length for fixed-fixed and fixed-pinned boundary conditions. The results of this plot include radial pipe gaps and gravity loads. Large deflection theory with incremental loads was used for the solution procedure. Again, the fixed-fixed boundary conditions result in

critical buckling loads approximately a factor of 2 greater than the fixed-pinned boundary conditions. The fixed-pinned curve is reproduced in Figure 1 for field operation use.

5.5.2 Rotational Speeds and Downward Loads for Rotary-Mode

In order to conduct successful safe rotational operations of the drill string with a 15-ft sleeve, the frequencies and mode shapes of the drill string and sleeve were analyzed for various lengths and axial loads. The analysis approach is the same as that used for drill strings without a sleeve, see Section 5.3.3.

Figure 4 shows trends of the critical rotational speeds versus axial compression loads of drill strings of 20, 30, 40, 50, and 60-ft length with a 15-ft sleeve. This plot of critical rotational speeds was obtained by simply multiplying the modal frequency in cycles per second by 60 seconds per minute to obtain r/min. The results in Figure 4 are with fixed-pinned boundary conditions.

The frequency trend of the drill string with the sleeve can be seen to start out identical to the trend for the drill string without the sleeve for the 30, 40, 50, and 60-ft drill string, but as the radial gaps close between the drill string and the sleeve, the frequency trend changes. The trend with the 15-ft sleeve for the 20-ft drill string is initially modified by the lower frequency of the fixed-free 3-in. Schedule 80 15-ft sleeve, but after the sleeve and drill string make contact a similar trend is observed.

Appendix A, Section 7.0, includes additional analysis results for fixed-fixed boundary conditions.

5.6 DRILL STRING WITH 25-FT SLEEVE

5.6.1 Axial Buckling Loads for Push-Mode

Figure 10 shows the buckling load versus drill string length with a 3-in. Schedule 80 sleeve of 25-ft length for fixed-fixed and fixed-pinned boundary conditions. The results of this plot include radial pipe gaps and gravity loads. Large deflection theory with incremental loads was used for the solution procedures. Again, the fixed-fixed boundary conditions result in critical buckling loads approximately a factor of 2 greater than the fixed-pinned boundary conditions. The fixed-pinned curve is reproduced in Figure 1 for the field operation use.

5.6.2 Rotational Speeds and Downward Loads for Rotary-Mode

In order to conduct successful safety rotational operations of the drill string with a 25-ft sleeve, the frequencies and mode shapes of the drill string and sleeve were analyzed for various lengths and axial loads. The analysis approach is the same as that used for both drill strings without a sleeve (Section 5.3.3) and drill strings with a 15-ft sleeve (Section 5.5.2).

Figure 5 shows trends of the critical rotational speeds versus axial compression loads of drill strings of 30, 40, 50, and 60-ft length with a 25-ft sleeve. This plot of critical rotational speeds was obtained by simply multiplying the modal frequency in cycles per second by 60 seconds per minute to obtain r/min. The results in Figure 5 are with fixed-pinned boundary conditions.

The frequency trend of the drill string with the sleeve can be seen to start out identical to the trend for the drill string without the sleeve for the 40, 50, and 60-ft drill string, but as the radial gaps close between the drill string and the sleeve, the frequency trend changes. The trend with the 25-ft sleeve for the 30-ft drill string is initially modified by the lower frequency of the fixed-free 3-in. Schedule 80 25-ft sleeve, but after the sleeve and drill string make contact a similar trend is observed. The 60-ft long drill string with the sleeve has changes (not smooth curve) in the frequency trend as gaps close and open in the three dimensions.

Appendix A, Section 5.0, includes additional analysis results for fixed-fixed boundary conditions.

6.0 EVALUATION OF RESULTS AND RECOMMENDATIONS JUSTIFICATION

The analysis results provide critical buckling loads and critical speeds for different drill string and sleeve lengths, with different end conditions. These critical loads and speeds were used to determine safe operating conditions for the drill strings. The results of the analyses do not include factors of safety. For safe operating conditions, some factors have to be applied to the results in order to provide a reasonable factor of safety.

The factors of safety provided in the structural and design codes will be conservative if applied for components such as the drill string. For example, the American Institute of Steel Construction (AISC 1989) imposed factor of safety against buckling was developed to govern the design of buildings occupied by persons and subjected to loads with varying degrees of uncertainty. This level of protection does not appear to be justified for the drill string unless a failure of the drill string would lead to a credible safety concern within the tank or to operator personnel. If this is not the case then the safety factor could be relaxed to meet nonsafety related issues concerning time and economic losses in the event of a drill string failure. In either respect, there is an uncertainty which needs to be addressed as to the actual end support condition as the drill string contacts the waste or an object within the waste.

The factor of safety in AISC (1989) varies from about 1.67 to 1.92. According to Johnston (1976), the factor of safety tends to compensate for:

- increasing sensitivity of long columns to variations in the effective-length factor, K , which depends on the end conditions,
- the practical difficulty in determining the value of K in a real structure, and
- the initial crookedness of the column.

If the end conditions and initial imperfections can be identified and considered in the analysis, the factor of safety can be reduced considerably.

The finite-element large deformation and nonlinear analysis methods are powerful tools, in which imperfections and different complicated support conditions can be analyzed with high accuracy. The finite-element analysis was performed using full nonlinear large deflection theory, with incremental loading to failure under the resulting combined bending and axial loads. The analysis starts with lateral deflection (imperfection in rod straightness) of the drill strings. Some cases were studied for bottom-end misalignment up to 14-in. offset. These imperfections have slight effect on the critical buckling loads for strings equal to or longer than 40-ft without an offset. In addition, the design of the drill string provides definitive end conditions. For example, the upper end is supported by a riser adapter in a long bearing sleeve, making the constraint at this end very close to a fixed condition. The lower end has a spike that will be pushed in the solid waste, making the constraint at the lower end very close to a pinned condition during the initial contact of the drill bit with the solid waste. When the drill bit has penetrated deep into the solid waste, the constraint is very close to a fixed condition.

There is no safety factor included in the recommended values for fixed-pinned condition. However, the assumption of a fixed-pinned condition provides sufficient protection because as axial loads are applied to penetrate the waste, the drill string conditions approach a fixed-fixed condition where critical buckling loads are twice as large as the fixed-pinned condition buckling loads. Actually, if the lower end of a 40-ft drill string is 5-ft deep in the waste, the effective length is 35-ft with a partially fixed or fully fixed end condition (depends on waste condition). The analysis provides load limit for a 40-ft drill string with fixed-pinned condition, while the drill string is actually 35-ft with end conditions close to fixed-fixed. Accordingly, there is an indirect safety factor of at least 2 for this example (refer to Figure 7). Therefore, the fixed-pinned condition for the total length of the drill string is expected to provide a reasonable margin of safety on the axial loads for the drill string core sampling operation. In addition, this criteria is substantiated by the failure results for a 41.67-ft drill string at a load of 2,600 lbf. This load is close to the fixed-fixed condition buckling load estimate (the X point on Figure 7). This field experience is a good example that a fixed-pinned condition is conservative and provides a sufficient margin of safety for operation.

The load measurement reading on the down force instrument includes gravity load effects. The gravity load lowers the critical load measurements by about 100 lbf. Therefore, the loads provided on the curves correspond to the load instrument measurements.

For push mode operations, we recommend that operation personnel shall not apply load above the buckling load limits predicted by the fixed-pinned boundary conditions with gravity loads.

Under rotary mode operation, the drill string can become dynamically unstable, resulting in a whirling vibratory motion. The critical speed at which this occurs is a function of the lateral natural frequency of the drill string.

It is important to determine proper speeds and corresponding compressive loads that will prevent resonance or whirling of the drill string. The natural frequency of the drill string decreases by increasing the compressive load, and becomes zero at the critical buckling load. The torsional natural frequencies of the drill strings are high and may allow the drill string to operate at speeds faster than 2,000 r/min. On the other hand, the first lateral natural frequencies are low and may cause whirling when they are associated with lateral deflections. Thus, it is important to keep the operational speeds within reasonable bounds from the critical natural frequencies to avoid whirling.

Figure 3 through 5 provide plots for critical rotational speed (first lateral natural frequency) versus axial compressive load (fixed-pinned boundary conditions) for drill strings without a sleeve, with a 15-ft sleeve, and with a 25-ft sleeve, respectively. Appendix A includes critical speeds for fixed-fixed conditions. The lowest natural frequency of the fixed-fixed condition is about 1.5 times the fixed-pinned condition. The second lowest natural frequencies are about 3.0 times the corresponding first lower natural frequencies. The addition of fluid mass (equivalent to 20-ft waste depth), based on Belvins (1992), does reduce the drill string frequency, but as the axial compression load increases this reduction approaches zero. At 50 percent of the buckling load, the frequency reduction was found to be 26 percent.

Belvins (1979) shows that a rotational speed twice as much (or half) of the corresponding natural frequency almost eliminates the possibility of whirling. WHC (1993) specifies maximum speed limit of 55 r/min for safety purposes. This speed boundary makes it difficult to satisfy the factor of two on the critical speeds to avoid whirling. A factor of 1.5 (or 0.67) on the natural frequencies is sufficient to keep the drill string in controlled stable dynamic conditions, and satisfies the speed limits of the operating envelope. If the ratio is less than 1.5, the operation should be closely monitored for the possibility of dynamic instability.

Accordingly, the speed (r/min) should be a factor of 1.5 to 2.0 below the lowest critical fixed-pinned r/min to guarantee the stability of the drill string.

In addition, because the drill string is somehow between fixed-fixed and fixed-pinned boundary conditions, the speeds between the two boundary conditions cannot guarantee the stability of the drill string and should be avoided. For the longer drill strings such as the 60-ft case, as the load increases, the critical rotational speed will be very low. In this case, it would be acceptable to quickly go to a higher speed that will be above the lower natural frequency of the fixed-fixed condition and away from any lateral natural frequency by a factor of 1.5 to 2.0.

Dynamic instability may occur when either the rotational speed is below the lowest natural frequency by a factor less than 1.5 or when shifting from lower speed to higher speed. The operation of the drill string should be monitored very carefully for these cases (see last column of Tables 1 through 3).

6.1 RECOMMENDATIONS FOR AXIAL LOAD LIMITS

For push mode, the operation personnel shall not apply loads above the limits predicted by the fixed-pinned boundary conditions with gravity loads. The plots in Figure 1 illustrate these load limits. Tables 1 through 3 provide load limits for operations at drill string intervals of 10-ft.

The loads of the longer drill string should be used for any drill string length that falls between two given lengths in the tables (e.g., use the load of 50-ft length for 45-ft drill string). Only the design engineer is allowed to exceed the limits of the longer length as justified by field observations, and by interpolating the results provided in Figures 1, 2 and 7 through 10.

6.2 RECOMMENDATIONS FOR ROTATIONAL SPEED LIMITS

For rotary mode, the operation personnel shall stay below the lowest critical rotational speeds, as provided in Tables 1 through 3 and in Figures 3 through 5. For long drill strings such as the 60-ft case, it would be acceptable to quickly go to a higher speed (55 r/min) that will be above the lower natural frequencies of the fixed-fixed condition, and away from any natural frequency by a factor not less than 1.5. The values of these cases are provided in Tables 1 through 3, with notes that prohibit using the values without the approval of the design engineer. These cases should be monitored for instability with the presence of the design engineer.

Section 2.0 includes the required recommendations that shall be strictly followed to ensure safe and stable operations of the drill string during core sampling.

7.0 REFERENCES

- ABAQUS, 1994, *ABAQUS User's Manual*, Version 5.4, Hibbit, Karlsson, and Sorensen, Inc., Pawtucket, Rod Island.
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- Blevins, R. D., 1992, *Applied Fluid Dynamics Handbook*, Krieger Publishing Company, Malabar, Florida.
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- Johnston, B. G., 1976, *Guide to Stability Design Criteria for Metal Structures*, Structural Stability Research Council, John Wiley and Sons, New York, New York.
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- WHC, 1993, *Core Drill Operating Envelope Test Report*, WHC-SD-WM-TRP-123, Rev. 1, Westinghouse Hanford Company, Richland, Washington.
- Ziada, H. H., 1994, *Dynamic and Buckling Analysis for Rotary Mode Drill String for Core Sampling*, Internal memo CSA:HH2:99b:9413 to T. R. Farris, January 6, Westinghouse Hanford Company, Richland, Washington.

Table 1. Recommended Axial Compressive Loads and Rotational Speeds for Drill Strings Without a Sleeve.

Drill String Length	Push-Mode	Rotary-Mode		
	Buckling Load lbf	Speed r/min	Stable Load Range lbf	Unstable Load Range ⁽¹⁾ lbf
≤30-ft	2,600	55	0-1,170	---
≤40-ft	1,400	45	---	0-600
		40	0-300	600-800
		35	300-600	800-950
		30	600-800	950-1,170
		25	800-1,000	---
		20	1,000-1,170	---
≤50-ft	800	30	---	0-300
		25	0-200	300-500
		20	200-400	500-650
		15	400-600	650-750 ⁽²⁾
		10 ⁽³⁾	600-750	750-800 ⁽⁴⁾
≤60-ft	500	20	---	0-200
		15	0-200	200-350 ⁽²⁾
		10 ⁽³⁾	200-400 ⁽¹⁾	400-450 ⁽⁴⁾

- (1) Watch carefully for whirling. Approval of design engineer is required before operating at these loads. If whirling occurs, use lower speeds.
- (2) Limit load if cannot go below 15 r/min speed. To increase load, use appropriate sleeve.
- (3) The drill string should not operate at speeds lower than 15 r/min. Approval of design engineer is required before operating at lower speed.
- (4) Rotary mode load limit for fixed-hinged boundary conditions. To increase load, use appropriate sleeve.

General Note

The loads of the longer drill string should be used for any drill string length that falls between two given lengths (e.g., use the load of 50-ft length for 45-ft drill string). Only the design engineer, assigned from Characterization Equipment Design group, is authorized to allow the limits of the longer length to be exceeded as justified by field observations, and by interpolating the results provided in Figures 1 through 5 and 7 through 10.

Table 2. Recommended Axial Compressive Loads and Rotational Speeds for Drill Strings With a 15-ft Sleeve.

Drill String Length	Push-Mode	Rotary-Mode		
	Buckling Load lbf	Speed r/min	Stable Load Range lbf	Unstable Load Range ⁽¹⁾ lbf
≤30-ft	5,000	55	0-1,170	---
≤40-ft	2,400	45	---	0-600
		40	0-300	600-1,000
		35	300-600	1,000-1,170
		30	600-1,170	---
≤50-ft	1,400	30	---	0-300
		25	0-200	300-500
		20	200-400	500-750
		15	400-750	750-1,000 ⁽²⁾
		10 ⁽³⁾	750-1,000	1,000-1,170
≤60-ft	700	20	---	0-200
		15	0-200	200-400 ⁽²⁾
		10 ⁽³⁾	200-400	400-550
		55 ⁽⁴⁾	---	550-700 ⁽⁵⁾

- (1) Watch carefully for whirling. Approval of design engineer is required before operating at these loads. If whirling occurs, use lower speeds.
- (2) Limit load if cannot go below 15 r/min speed (or high to 55 r/min for 60-ft length). To increase load, use 25-ft sleeve.
- (3) The drill string should not operate at speeds lower than 15 r/min. Approval of design engineer is required before operating at lower speed.
- (4) Higher speeds should be shifted very fast (drill strings should not operate for long periods near the critical speed). Approval of design engineer is required before shifting to higher speed.
- (5) Rotary mode load limit for fixed-pinned boundary conditions. To increase load, use 25-ft sleeve.

General Note

The loads of the longer drill string should be used for any drill string length that falls between two given lengths (e.g., use the load of 50-ft length for 45-ft drill string). Only the design engineer, assigned from Characterization Equipment Design group, is authorized to allow the limits of the longer length to be exceeded as justified by field observations, and by interpolating the results provided in Figures 1 through 5 and 7 through 10.

Table 3. Recommended Axial Compressive Loads and Rotational Speeds for Drill Strings With 25-ft Sleeve.

Drill String Length	Push-Mode	Rotary-Mode		
	Buckling Load lbf	Speed r/min	Stable Load Range lbf	Unstable Load Range ⁽¹⁾ lbf
≤30-ft	8,000	55	0-1,170	---
≤40-ft	2,750	45	---	0-600
		40	0-300	600-1,000
		35	300-600	1,000-1,170
		30	600-1,170	---
≤50-ft	1,700	30	---	0-300
		25	0-200	300-500
		20	200-400	500-900
		15	400-900	900-1,170
≤60-ft	1,050	20	---	0-200
		15	0-200	200-400 ⁽²⁾
		10 ⁽³⁾	300-400	400-600
		55 ⁽⁴⁾	---	600-1,050 ⁽⁵⁾

- (1) Watch carefully for whirling. Approval of design engineer is required before operating at these loads. If whirling occurs, use lower speeds.
- (2) Limit load if cannot go below 15 r/min or high to 55 r/min.
- (3) The drill string should not operate at speeds lower than 15 r/min. Approval of design engineer is required before operating at lower speed.
- (4) Higher speeds should be shifted very fast (drill strings shall not operate for long periods near the critical speeds). Approval of design engineer is required before shifting to higher speed.
- (5) Rotary mode load limit for fixed-pinned boundary conditions.

General Note

The loads of the longer drill string should be used for any drill string length that falls between two given lengths (e.g., use the load of 50-ft length for 45-ft drill string). Only the design engineer, assigned from Characterization Equipment Design group, is authorized to allow the limits of the longer length to be exceeded as justified by field observations, and by interpolating the results provided in Figures 1 through 5 and 7 through 10.

Figure 1. Buckling Load Versus Drill String Length With
No Sleeve, 15-ft Sleeve, and 25-ft Sleeve,
Fixed-Pinned Condition.

Figure 2. Buckling Load Versus Sleeve Length
For Different Drill String Lengths,
Fixed-Pinned Condition.

Figure 3. Critical Rotational Speed Versus Axial Compressive Load for Drill String With No Sleeve, Fixed-Pinned Condition.

Figure 4. Critical Rotational Speed Versus Axial Compressive Load for Drill String With 15-ft Sleeve, Fixed-Pinned Condition.

Figure 5. Critical Rotational Speed Versus Axial Compressive Load for Drill String With 25-ft Sleeve, Fixed-Pinned Condition.

Figure 6. Drill String Configuration at Failure.

Figure 7. Buckling Load Versus Drill String Length With No Sleeve,
Including Failed Drill String Data,
Fixed-Fixed and Fixed-Pinned Conditions.

Figure 8. Buckling Load Versus Sleeve Length
for Different Drill String Lengths,
Fixed-Fixed Condition.

Figure 9. Buckling Load Versus Drill String Length With 15-ft Sleeve,
Fixed-Fixed and Fixed-Pinned Conditions.

Figure 10. Buckling Load Versus Drill String Length With 25-ft Sleeve,
Fixed-Fixed and Fixed-Pinned Conditions.

APPENDIX A
BUCKLING AND DYNAMIC ANALYSIS
OF DRILL STRINGS

Prepared By
C. J. Moore and B. L. Coverdell
ICF Kaiser Hanford Company

CHECKLIST FOR INDEPENDENT REVIEW

Appendix A

Document Reviewed Buckling and Dynamic Analysis of Drill Strings

Author C. J. Moore and B. L. Coverdell Doc. No. WHC-SD-WM-ANAL-047, Rev. 0

Reviewer H. H. Ziada

Yes No/N/A

- ☒ [ ] [ ] Problem completely defined.
- ☒ [ ] [ ] Necessary assumptions explicitly stated and supported.
- ☒ [ ] [ ] Computer codes and data files documented.
- ☒ [ ] [ ] Data used in calculations explicitly stated in document.
- ☒ [ ] [ ] Data checked for consistency with original source information as applicable.
- ☒ [ ] [ ] Mathematical derivations checked including dimensional consistency of results.
- ☒ [ ] [ ] Models appropriate and used within range of validity or use outside range of established validity justified.
- ☒ [ ] [ ] Hand calculations checked for errors.
- ☒ [ ] [ ] Code run streams correct and consistent with analysis documentation.
- ☒ [ ] [ ] Code output consistent with input and with results reported in analysis documentation.
- ☒ [ ] [ ] Acceptability limits on analytical results applicable and supported. Limits checked against sources.
- ☒ [ ] [ ] Safety margins consistent with good engineering practices.
- ☒ [ ] [ ] Conclusions consistent with analytical results and applicable limits.
- ☒ [ ] [ ] Results and conclusions address all points required in the problem statement.

H. H. Ziada H. H. Ziada 4-29-96
Reviewer Date

BUCKLING AND DYNAMIC ANALYSIS OF DRILL STRINGS

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2.0 Gravity Load Influence on Axial Failure Loads	A-9
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4.0 Axial Failure Load of a Drill String With Different Lengths of a Strengthening Sleeve, and with a 25-ft Sleeve	A-25
5.0 Dynamics of the Drill String with a 25-ft Sleeve	A-33
6.0 Axial Failure of Various Length Drill Strings with 15-ft Strengthening Sleeve	A-53
7.0 Dynamics of the Drill String with a 15-ft Sleeve	A-57

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Client	WO/Job No.	
Subject AXIAL FAILURE LOAD ESTIMATES	Date 12/18/95	By Carleton J. Moore
FOR PUSH MODE DRILL STRING	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

1.0 Axial Failure Load Estimates for Push Mode Drill String

This section describes the analysis results of the axial compression failure loads for the push mode drill string used for core sampling.

The drill string is made of steel pipe segments with an outer diameter of 2.25 inch and an inner diameter of 1.906 inch. The total length may vary from 15 ft to 70 ft. The expected length of the drill string when the drill bit encounters the solid waste is between 30 ft to 50 ft. The drill string is supported by a riser adapter in a long bearing sleeve, that makes the constraint of the upper end very close to a fixed condition. Assuming the lower end of the drill string penetrates the waste at initial loads less than the elastic stability for a fixed-free column (about 1/8 of the fixed-pinned loads), the lower end of the drill string will act somewhere between guided (i.e.; free to move laterally) and pinned conditions. After the drill bit penetrates the solid waste, the constraint at the lower end is very close to a pinned condition. When the drill bit has penetrated deep into the solid waste, the constraint is very close to a fixed condition.

Two boundary condition cases were considered for the analysis of the push mode drill string: fixed-pinned and fixed-fixed. These two cases should bound the elastic stability of the drill string after the drill bit has engaged and pushed into the solid waste.

During sampling of a Hanford Site waste tank, a drill string failed as a result of a structural instability or buckling of the pipe under the imposed load of 2,600 lbf. This pipe was over 41-ft long fixed at the top and pushing into the waste at the tank bottom. A sketch of the pipe configuration in the tank is shown in Figure A-1.

The numerical investigation began with using exact derived solutions for buckling. The exact solutions were checked with results of an ABAQUS finite element solutions. The results matched and trend studies were generated using ABAQUS. Textbook derived solutions, finite element numerical buckling, and full nonlinear large deflection theory loading to failure all give the same results for this problem (within 3 percent). As a check of pipe bending section failure, the pipe steel was modeled with yield, strain hardening, and material failure at ultimate.

1.1 Finite Element Model

ABAQUS/Standard version 5.4 was used for the nonlinear structural analysis of the tank sampling drill string and the later addition of stiffening sleeves. Both the drill string and sleeve were modeled with beam elements with a pipe cross section. Modeling of the sleeve interaction with the drill string was addressed with cylindrical gap elements. The cylindrical gap element modeled the drill string cross

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Subject AXIAL FAILURE LOAD ESTIMATES	Date 12/18/95	By Carleton J. Moore
FOR PUSH MODE DRILL STRING	Checked	By Hassan H. Ziada
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section as a movable point inside a ring that represents the inside of the sleeve (the radius of the ring is set to correspond to the freeplay between the drill string OD and the sleeve ID).

The drill string and sleeve materials were modeled as nonlinear materials with strain hardening and failure included for the large deflection analyses. But the critical failure mode for the drill string is classic buckling. Classic buckling can be predicted simply with linear elastic materials. The full nonlinear analysis capability allows elimination of other failure concerns and accounting for the freeplay between the sleeve and drill string.

1.2 Analysis Results

Figure A-2 and Table A-1 provide the load results for drill strings up to 70 ft length with different boundary conditions. These loads correspond to the buckling failure loads without a factor of safety. The large deflection load increment (post buckling load) results shown in Figure A-2 indicate that the large deflection results do not differ from the classic buckling results (not more than 3 percent increase). The pipe is so long that the classic buckling completely controls the failure, and the nonlinear properties do not influence the axial load at which failure occurs.

A case was completed with a 14 inch offset of the bottom of the drill string from vertical. This offset does not change the critical buckling load for pipe string equal to or greater than 40 feet. The study results gave the same results as the 40 foot pipe case without an offset.

The dotted curve in Figure A-2 represents the load results when the drill string is in contact with the lower end of the 92 inch riser (point C in Figure A-1). This contact is simulated by a roller-pin (restricted laterally and free vertically) at this location, and the resulting load is about 35 percent higher than the corresponding load for the fixed-pinned condition at 41.67 ft length.

The failure experienced in truck #2 at a load of 2,600 lbf for a 500 inch drill string is close to the fixed-fixed condition estimate (the X point in Figure A-2). This indicates that the lower end was either deep in the solid waste or trapped in some debris in the tank. In addition, the contact with the lower end of the 12 inch riser increased the load capability of the drill string to some extent.

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Client	WO/Job No.	
Subject AXIAL FAILURE LOAD ESTIMATES	Date 12/18/95	By Carleton J. Moore
FOR PUSH MODE DRILL STRING	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

Drill String Configuration at Failure

Figure A-1

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FOR PUSH MODE DRILL STRING	Checked	By Hassan H. Ziada
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CRITICAL BUCKLING LOAD VERSUS DRILL PIPE LENGTH
(10/18/95 CJM)

Figure A-2

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Client

WO/Job No.

Subject AXIAL FAILURE LOAD ESTIMATES

Date 12/18/95

By Carleton J. Moore

FOR PUSH MODE DRILL STRING

Checked

By Hassan H. Ziada

Location Tank Farms

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DRILL STRING LOADS FOR DIFFERENT
LENGTHS AND BOUNDARY CONDITIONS

LENGTH (ft)	LOADS (lbf)		
	FIXED(A) - PINNED(B)	FIXED(A) & (B)	FIXED(A) - PINNED(B) & (C)
15	10930	21348	28178
20	6155	12025	12253
25	3941	7701	6710
30	2737	5350	4219
35	2011	3932	2896
37.5		3425	2456
40	1540	3010	
41.67	1419	2775	1918
45	1217		
45.83		2293	1539
50	986	1927	1262
55	815	1593	1018
60	685	1338	839
65	583	1140	703
70	503	983	598

Table A-1

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Client	WO/Job No.	
Subject ABAQUS INPUT FOR CLASSIC BUCKLING	Date 10/24/95	By Carleton J. Moore
PREDICTION OF CRITICAL AXIAL LOAD	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

*HEADING

DRILL PIPE BUCKLING (FILE DPBFPF1.INP 10/24/95 CJM)

*NODE, NSET=ALL

1,0,0,0,0,0,0

201,0,0,500.,0.0

*NGEN,NSET=ALL

1,201,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,200,1,1,1

*MATERIAL,NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING

1.125,0.172

*BOUNDARY

**fixed-pinned

201,1,6

1,1

1,3

*STEP

*BUCKLE

6,

*MODAL FILE

*CLOAD

1,2,1.

*END STEP

** end of analysis

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Client	WO/Job No.	
Subject ABAQUS INPUT INCREMENTAL BUCKLING	Date 10/19/95	By Carleton J. Moore
LARGE DEF. CRITICAL AXIAL LOAD	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

```

*HEADING
  DRILL PIPE BUCKLING (LOAD INCR) (FILE DPLDFF.INP 10/19/95 CJM)
*NODE, NSET=ALL
1,0.0,0.0,0.0
101,0.25,250.,0.0
201,0.0,500.,0.0
*NGEN,NSET=ALL
1,101,1
101,201,1
*ELEMENT, TYPE=B31, ELSET=STRING
1,1,2
*ELGEN, ELSET=STRING
1,200,1,1,1
*MATERIAL,NAME=STEEL
*ELASTIC
29.E+06,0.3
*DENSITY
0.000734990
*PLASTIC
76000.,0.0
90000.,0.1
4000.,0.15
4000.,0.20
*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING
1.125,0.172
*BOUNDARY
**fixed-fixed
201,1,6
1,1
1,3,6
*RESTART,WRITE,FREQUENCY=1
*STEP,NLGEOM,INC=500
*STATIC
100.,4000., , 100.
*CLOAD
1,2,4000.
*NODE PRINT,FREQ=50
U,CF1
*END STEP
*** end of analysis

```

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Subject GRAVITY LOAD INFLUENCE	Date 12/18/95	By Carleton J. Moore	
ON AXIAL FAILURE LOADS	Checked	By Hassan H. Ziada	
Location Tank Farms	Revised	By	

2.0 Gravity Load Influence on Axial Failure Loads

Although the critical axial force required for long column buckling is the most important consideration in the drill pipe buckling, it is necessary to also include the effect of gravity on the drill stem to predict the buckling load of the drill rig force measurements. Figure A-3 shows that accounting for the gravity load slightly lowers by 100 lbf or less the buckling load as read on the down force instrument. It can be seen that for the fixed-fixed boundary condition of the 41.67 ft drill stem the gravity effect results in a lowering of the analytical results to more closely match the observed failure.

If the operations are being conducted almost to the limit of fixed-fixed buckling, the gravity load must be considered, but if operations are conducted with a factor of safety of 2 on buckling the gravity loads are not that important. We would like to recommend that operations not apply load above the limits predicted by the fixed-pinned boundary conditions with gravity loads. For safe operations the bottom heavy black line of Figure A-3 should be used for guidelines on critical compressive drill string load versus length.

Sections 4 and 5 of this appendix explore using a 3 inch schedule 80 stainless steel sleeve to increase the effective buckling loads to allow increased capability.

For a drill string inserted into liquid waste the effective drill string gravity load would be reduced some due to the waste floatation effects. The answers would depend on the length of drill string inserted into the liquid waste, but the answers would be bounded by the limits between the cases of axial loads only and the axial loads with gravity of Figure A-3.

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ON AXIAL FAILURE LOADS

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By Hassan H. Zaida

Location Trak Farms

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By

Figure A-3

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Client	WO/Job No.
Subject ABAQUS INPUT FOR CLASSIC BUCKLING	Date 10/24/95
CRITICAL AXIAL LOAD WITH GRAVITY	By Carleton J. Moore
Location Tank Farms	Checked By Hassan H. Ziada
	Revised By

*HEADING

DRILL PIPE BUCKLING (FILE DPBFPF2.INP 10/24/95 CJM)

*NODE, NSET=ALL

1,0,0,0,0,0,0

201,0,0,500.,0.0

*NGEN,NSET=ALL

1,201,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,200,1,1,1

*MATERIAL,NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING

1.125,0.172

*BOUNDARY

**fixed-pinned

201,1

201,3,6

1,1,3

*STEP

*STATIC

1.,1.

*DLOAD

STRING, GRAV, 386.088, 0.0, -1., 0.0

*END STEP

*STEP

*BUCKLE

6,

*MODAL FILE

*CLOAD

201,2,-1.

*END STEP

*** end of analysis

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Client	WO/Job No.	
Subject ABAQUS INPUT INCREMENTAL BUCKLING	Date 12/14/95	By Carleton J. Moore
LARGE DEFLECTION AXIAL LOAD WITH GRAVITY	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

*HEADING

DRILL PIPE BUCKLING (LOAD INCR) (FILE LD42FFG.INP 12/14/95 CJM)

*NODE, NSET=ALL

1,0.0,0.0,0.0

101,0.25,250.,0.0

201,0.0,500.,0.0

*NGEN,NSET=ALL

1,101,1

101,201,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,200,1,1,1

*MATERIAL,NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*PLASTIC

76000.,0.0

90000.,0.1

4000.,0.15

4000.,0.20

*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING

1.125,0.172

*BOUNDARY

**fixed-fixed

201,1

201,3,6

1,1,6

**RESTART,WRITE,FREQUENCY=1

*STEP,NLGEOM

Step #1 apply gravity load

*STATIC

1.,1.

** WEIGHT *****

*DLOAD

STRING, GRAV, 386.088, 0.0, -1.0, 0.0

*ELPRINT, FREQ=0

*NODE PRINT, FREQ=0

*END STEP

*STEP, NLGEOM, INC=500

*STATIC

10., 3000., , 10.

*CLOAD

201, 2, -3000.

*NODE PRINT, FREQ=0

*END STEP

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Client	WO/Job No.	
Subject DYNAMICS OF THE DRILL STRING	Date 12/18/95	By Carleton J. Moore
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Location Tank Farms	Revised	By

3.0 Dynamics of the Drill String

In order to conduct successful safe rotational operations of the drill string the frequencies and mode shapes of the drill string were analyzed for various lengths and axial loads. As the axial compressive loads are increased the natural bending frequencies decreases. It is interesting that the frequency, of the mode whose shape most closely matches classic first buckling mode, goes to zero at the large deflection theory predicted buckling load.

The dynamic analysis of the drill stem employed ABAQUS with large deflection theory. These analysis trend cases include gravity loads. The solution approach is a combination of static load steps and dynamic modal analysis. The first static step applies the gravity loads. The next step is a dynamic modal analysis step that solve for the natural frequencies at zero axial force with gravity loads. The trend is then completed by alternating static load steps that apply axial loads with dynamic modal analysis steps.

Figure A-4 shows trends of the lowest modal frequency versus axial compressive load for drill strings of 30, 40, 50, and 60 ft length. The frequency of each length of drill stem goes to zero at the axial load corresponding to the buckling instability. Figure A-4 gives the drill string frequencies with the conservative boundary conditions of fixed-pinned.

Figure A-5 shows trends of the critical rotation speed versus axial compression loads of drill strings of 30, 40, 50, and 60 ft length. This plot of critical rotational speeds was obtained by simply multiplying the modal frequency in cycles per second by 60 seconds per minute to obtain RPM. The critical rotation speeds shown in Figure A-5 are with assumed conservative boundary conditions of fixed-pinned.

Figure A-6 shows a trend of the first bending frequency (lowest modal frequency) and the second bending frequency versus axial compression load for a 41.67 ft drill stem with assumed fixed-pinned boundary conditions. For comparison, Figure A-7 shows the same trend with assumed boundary conditions of fixed-fixed. The frequencies in Figure A-7 can be seen to be higher with the fixed-fixed boundary conditions than the frequencies shown in Figure A-6 with the conservative fixed-pinned boundary conditions.

Figures A-6 and A-7 also show that the higher second bending modes must be considered if the drilling operations try to employ rotational speeds greater than the lowest critical RPM's. It is simpler to stay below the lowest critical rotational speed if practical for operations.

We recommend that the drill string not be rotated until it is into the solid waste with a compressive load applied. Then the RPM should be a

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factor of 1.5 to 2.0 below the lowest critical fixed-pinned RPM as documented in the attached curves. As the compressive load increases the critical RPM will decrease. For the longer drill strings such as the 60 ft case as the load increases the critical RPM will be very low, in this case it would be acceptable to quickly go to 55 RPM if it is above the first two bending modes or above the fixed-fixed condition, and it is away from the natural frequency by a factor of 1.5 to 2.0.

Additional dynamic frequency studies were completed with the 41.67 ft long drill stem inserted into 20 ft of liquid waste with an assumed specific gravity of 1.7. The addition of fluid mass based on Blevins book "Applied Fluid Dynamics Handbook" does reduce the drill stem frequency, but as the axial compression load increases this reduction approaches zero. At 50 percent of the buckling load the frequency reduction was found to be 26 percent.

The torsional frequency of the 60 ft drill string is 42.7 Hz, that frequency would correspond to a critical RPM of 2,520. The RPM operational limits of 55 RPM or below already eliminate torsional excitation for all drill string lengths.

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Figure A-4

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Location Tank Farms	Revised By Hassan H. Ziada

Figure A-5

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Subject DYNAMICS OF THE DRILL STRING	Date 12/18/95	By Carleton J. Moore
	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

DRILL STEM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
41.67 FT DRILL STEM, FIXED-PINNED
(11/22/95 CJM)

Figure A-6

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Subject DYNAMICS OF THE DRILL STRING	Date 12/18/95	By	Hassan H. Ziada
Location Tank Farms	Checked	Revised	

Figure A-7

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Client	WO/Job No.	
Subject ABAQUS INPUT FOR DYNAMIC MODAL	Date 11/21/95	By Carleton J. Moore
ANALYSIS	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

*HEADING

DRILL PIPE NATURAL FREQUENCIES (FILE FT42FP.INP 11/21/95 CJM)

*NODE, NSET=ALL

1,0.0,0.0,0.0

101,0.25,250.,0.0

201,0.0,500.,0.0

*NGEN, NSET=ALL

1,101,1

101,201,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,200,1,1,1

*MATERIAL, NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*BEAM SECTION, MATERIAL=STEEL, SECTION=PIPE, ELSET=STRING

1.125,0.172

*BOUNDARY

**fixed-pinned

201,1

201,3,6

1,1,3

*STEP, NLGEOM

Step #1 apply gravity loads

*STATIC

1.,1.

** WEIGHT *****

*DLOAD

STRING, GRAV, 386.088,0.0,-1.0,0.0

*ELPRINT, FREQ=0

*NODE PRINT, FREQ=0

*END STEP

*STEP, NLGEOM

Step #2 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT, FREQ=0

*NODE PRINT, FREQ=0

*END STEP

*STEP, NLGEOM

Step # 3 APPLY AXIAL LOAD of 100 lbs

*STATIC

1.,1.

*CLOAD

201,2,-100.

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Client	WO/Job No.	
Subject ABAQUS INPUT FOR DYNAMIC MODAL ANALYSIS	Date 11/21/95	By Carleton J. Moore
	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

```

*END STEP
*STEP,INC=100,NLGEOM
  Step #4 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #5 APPLY AXIAL LOAD of 200 lbs
*STATIC
1.,1.
*CLOAD
201,2,-200.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #6 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #7 APPLY AXIAL LOAD of 300 lbs
*STATIC
1.,1.
*CLOAD
201,2,-300.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #8 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #9 APPLY AXIAL LOAD of 400 lbs
*STATIC
1.,1.
*CLOAD
201,2,-400.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0

```

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```
*END STEP
*STEP,INC=100,NLGEOM
  Step #10 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #11 APPLY AXIAL LOAD of 500 lbs
*STATIC
1.,1.
*CLOAD
201,2,-500.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #12 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #13 APPLY AXIAL LOAD of 600 lbs
*STATIC
1.,1.
*CLOAD
201,2,-600.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #14 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #15 APPLY AXIAL LOAD of 700 lbs
*STATIC
1.,1.
*CLOAD
201,2,-700.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
```

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Subject ABAQUS INPUT FOR DYNAMIC MODAL ANALYSIS	Date 11/21/95
	By Carleton J. Moore
	Checked By Hassan H. Ziada
Location Tank Farms	Revised By

```

*END STEP
*STEP,INC=100,NLGEOM
  Step #16 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #17 APPLY AXIAL LOAD of 800 lbs
*STATIC
1.,1.
*CLOAD
201,2,-800.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #18 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #19 APPLY AXIAL LOAD of 900 lbs
*STATIC
1.,1.
*CLOAD
201,2,-900.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #20 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #21 APPLY AXIAL LOAD of 1000 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1000.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0

```

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Subject ABAQUS INPUT FOR DYNAMIC MODAL ANALYSIS	Date 11/21/95	By Carleton J. Moore
	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

```

*END STEP
*STEP,INC=100,NLGEOM
  Step #22 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #23 APPLY AXIAL LOAD of 1100 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1100.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #24 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #25 APPLY AXIAL LOAD of 1200 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1200.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #26 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
  Step #27 APPLY AXIAL LOAD of 1300 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1300.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0

```

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WO/Job No.

Subject ABAQUS INPUT FOR DYNAMIC MODAL

Date 11/21/95

By Carleton J. Moore

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Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

*END STEP
*STEP, INC=100, NLGEOM
Step #28 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*STEP, NLGEOM
Step #29 APPLY AXIAL LOAD of 1400 lbs
*STATIC
1., 1.
*CLOAD
201, 2, -1400.
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*STEP, INC=100, NLGEOM
Step #30 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*** end of analysis

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Client	WO/Job No.	
Subject	AXIAL FAILURE LOAD OF A DRILL STRING	Date 12/18/95
WITH LENGTHS OF STRENGTHENING SLEEVE		By Carleton J. Moore
Location	Tank Farms	Checked By Hassan H. Ziada
		Revised By

4.0 Axial Failure Load of a Drill String with Different Lengths of a Strengthening Sleeve and with a 25-ft Sleeve.

Trend studies were conducted to investigate increasing the buckling load with a 3 inch schedule 80 pipe of 304 stainless steel. The top of the sleeve is fixed at the riser drilling adapter. The 2.25 inch drill pipe is inserted through the sleeve with a fixed condition at the top of the riser.

Figure A-8 shows the critical buckling load versus sleeve length for a drill string of 41.67-ft long. The heavy dark line of straight segments is a classic buckling solution of an axial load for the fixed-pinned boundary condition with sleeve stiffness added to the drill stem. The classic buckling solution is used to check against the ABAQUS analysis with large deflection theory, gravity loads, axial loads, and radial gaps between the sleeve and drill stem. The addition of gaps and gravity loads can decrease the fixed-pinned trend as shown by the solid black circles. The thin dark line is the classic buckling solution for the fixed-fixed boundary condition. The addition of gaps and gravity loads decrease the fixed-fixed trend as shown by the solid black squares.

Figures A-9, A-10, and A-11 show trends of the critical buckling load versus sleeve length for drill strings of 30, 50, and 60 ft long respectively. The trend of these figures all include large deflection theory, gravity loads, and radial gaps between the sleeve and drill stem. The load increase for sleeve lengths between 15 to 25 ft increases slightly.

Figure A-12 shows a trend of critical buckling load versus drill pipe length with a 3 inch schedule 80 sleeve of 25 ft length. The results of this plot include radial pipe gaps and gravity loads. Large deflection theory with incremental loads was used for the solution procedure. The heavy dark line with the squares has fixed-pinned boundary conditions. The thin line with the circles has fixed-fixed boundary conditions. Again the fixed-fixed boundary conditions result in critical buckling loads approximately a factor of 2 greater than the fixed-pinned boundary conditions. This relation between the two boundary conditions matches classical Euler buckling relations for a long column.

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Client	WO/Job No.		
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WITH LENGTHS OF STRENGTHENING SLEEVE		Checked	By Carleton J. Moore
Location	Tank Farms	Revised	By Hassan H. Ziada

CRITICAL BUCKLING LOAD VERSUS SLEEVE PIPE LENGTH
41.67 FT OF DRILL PIPE WITH 3 INCH SCHEDULE 80 SLEEVE
LARGE DEFLECTION INCLUDES GRAVITY AND 0.325 SLEEVE GAPS
(10/31/95 CJM)

Figure A-8

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Subject AXIAL FAILURE LOAD OF A DRILL STRING

Date 12/18/95

By Carlton J. Moore

WITH LENGTHS OF STRENGTHENING SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

CRITICAL BUCKLING LOAD VERSUS SLEEVE PIPE LENGTH
30 FT OF DRILL PIPE WITH 3 INCH SCHEDULE 80 SLEEVE
LARGE DEFLECTION INCLUDES GRAVITY AND 0.325 IN SLEEVE GAPS
(10/31/95 CJM)

Figure A-9

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WO/Job No.

Subject AXIAL FAILURE LOAD OF A DRILL STRING

Date 12/18/95

By Carlton J. Moore

WITH LENGTHS OF STRENGTHENING SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

CRITICAL BUCKLING LOAD VERSUS SLEEVE PIPE LENGTH
50 FT OF DRILL PIPE WITH 3 INCH SCHEDULE 80 SLEEVE
LARGE DEFLECTION INCLUDES GRAVITY AND 0.325 IN SLEEVE GAPS
(10/31/95 CJM)

Figure A-10

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Subject AXIAL FAILURE LOAD OF A DRILL STRING

Date 12/18/95

By Carlton J. Moore

WITH LENGTHS OF STRENGTHENING SLEEVE

Checked

By Hassan H. Zaida

Location Tank Farms

Revised

By

Figure A-11

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Client

WO/Job No.

Subject AXIAL FAILURE LOAD OF A DRILL STRING

Date 12/18/95

By Cateion J. Moore

WITH LENGTHS OF STRENGTHENING SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

Figure A-12

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Client	WO/Job No.	
Subject ABAQUS INPUT INCREMENTAL BUCKLING	Date 10/30/95	By Carleton J. Moore
LARGE DEF. CRITICAL LOAD WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

```

*HEADING
  DRILL PIPE BUCKLING (FILE DPDFPS7.INP 10/30/95 CJM)
*NODE, NSET=ALL
1,0.0,0.0,0.0
101,0.25,250.,0.0
201,0.0,500.,0.0
202,0.20,200.,0.0
322,0.0,500.,0.0
*NGEN,NSET=ALL
1,101,1
101,201,1
202,322,1
*ELEMENT, TYPE=B31, ELSET=STRING
1,1,2
*ELGEN, ELSET=STRING
1,200,1,1,1
*ELEMENT, TYPE=B31, ELSET=SLEEVE
201,202,203
*ELGEN, ELSET=SLEEVE
201,120,1,1,1
*ELEMENT, TYPE=GAPCYL, ELSET=GAPC
321,81,202
*ELGEN, ELSET=GAPC
321,120
*MATERIAL,NAME=STEEL
*ELASTIC
29.E+06,0.3
*DENSITY
0.000734990
*PLASTIC
76000.,0.0
90000.,0.1
4000.,0.15
4000.,0.20
*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING
1.125,0.172
-1,0,0
*MATERIAL,NAME=SS304
*ELASTIC
28.E+06,.27
*DENSITY
0.000734990
*PLASTIC
29500.,0.0
75000.,0.34
4000.,0.38
4000.,0.42
*BEAM SECTION, MATERIAL=SS304,SECTION=PIPE,ELSET=SLEEVE

```

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Client

WO/Job No.

Subject ABAQUS INPUT INCREMENTAL BUCKLING

Date 10/30/95

By Carleton J. Moore

LARGE DEF. CRITICAL LOAD WITH 25 FT SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

1.75,0.3

-1,0,0

*GAP, ELSET=GAPC

.325,0.0,1.,0.0

*BOUNDARY

**fixed-pinned

1,1,3

201,1

201,3,6

322,1,6

*RESTART,WRITE,FREQUENCY=30

*STEP,NLGEOM

*STATIC

1.,1.

*DLOAD

STRING, GRAV, 386.088, 0.0, -1.0, 0.0

SLEEVE, GRAV, 386.088, 0.0, -1.0, 0.0

*END STEP

*STEP, NLGEOM, INC=500

*STATIC

100., 10000., , 100.

*CLOAD

201, 2, -10000.

*NODE PRINT, FREQ=50

U, CF1

*END STEP

*** end of analysis

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Client	WO/Job No.	
Subject DYNAMICS OF DRILL STRING	Date 12/18/95	By Carleton J. Moore
WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

5.0 Dynamics of the Drill String with a 25 ft Sleeve

In order to conduct successful safe rotational operations of the drill string with a 25 ft sleeve the frequencies and mode shapes of the drill string and sleeve were analyzed for various lengths and axial loads. As the axial compressive loads are increased the natural bending frequencies decreases. It is interesting that the frequency, of the mode whose shape most closely matches classic first buckling mode, goes to zero at the large deflection theory predicted buckling.

The dynamic analysis of the drill stem and sleeve employed ABAQUS with large deflection theory. These analysis trend cases include gravity loads, but the trends do not include waste liquid. The solution approach is a combination of static load steps and dynamic modal analysis. The first static step applies the gravity loads. The next step is a dynamic modal analysis step that solve for the natural frequencies at zero axial force with gravity loads. The trend is then completed by alternating static load steps that apply axial loads with dynamic modal analysis steps.

Figure A-13 shows trends of the lowest modal frequency versus axial compressive load for drill strings of 30, 40, 50, and 60 ft length with a 25 ft sleeve. The frequency of each length of drill stem goes to zero at the axial load corresponding to the buckling instability.

Figure A-14 shows trends of the critical rotation speed versus axial compression loads of drill strings of 30, 40, 50, and 60 ft length with a 25 ft sleeve. This plot of critical rotational speeds was obtained by simply multiplying the modal frequency in cycles per second by 60 seconds per minute to obtain RPM.

Figure A-15 shows a comparison of the lowest modal frequency versus axial compression loads of a 41.67 ft length drill string with fixed-pinned boundary conditions and fixed-fixed boundary conditions. The fixed-fixed boundary conditions result in higher frequencies at all axial compressive load values. The critical load or buckling occurs at the axial loads that result in zero frequency. The critical buckling crossings shift by a factor of 2 from the fixed-pinned case to the fixed-fixed boundary condition.

Figures A-16, A-17, A-18, and A-19 compare trends of the lowest frequency versus axial loads for drill strings of 30, 41.67, 50, and 60 ft length with and without a 25 ft sleeve, respectively. The frequency trend of the drill string with the sleeve can be seen to start out identical to the trend for the drill string without the sleeve for the 41.67, 50 and 60 ft drill string, but as the radial gaps close between the drill string and the sleeve the frequency trend changes. The trend with the 25 ft sleeve for the 30 ft drill string is initially modified by the lower frequency of the fixed-free 3 inch schedule 80 25 ft sleeve, but after the sleeve and

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Client	WO/Job No.	
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WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

drill string make contact a similar trend is observed. Figure A-19 shows that the 60 ft long drill string with the sleeve has changes in the frequency trend as gaps close and open in the three dimensions.

Figures A-20, A-21, A-22, and A-23 compare trends of the critical rotation speed versus axial loads for drill strings of 30, 41.67, 50, and 60 ft length with and without a 25 ft sleeve, respectively.

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WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

Figure A-13

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Client

Subject DYNAMICS OF DRILL STRING

Date 12/18/95

By Carleton J. Moore

WITH 25 FT SLEEVE

Checked

By Hassan H. Ziaa

Location Tank Farms

Revised

By

Figure A-14

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Client

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Subject DYNAMICS OF DRILL STRING

Date 12/18/95

By Cateion J. Moore

WITH 25 FT SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

Figure A-15

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Subject DYNAMICS OF DRILL STRING

Date 12/18/95

By Carlton J. Moore

WITH 25 FT SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

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By

Figure A-16

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Subject DYNAMICS OF DRILL STRING	Date 12/18/95	By Hassan H. Zia
WITH 25 FT SLEEVE	Checked	
Location Tank Farms	Revised	By

Figure A-17

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Client

WO/Job No.

Subject DYNAMICS OF DRILL STRING

Date 12/18/95

By Carleton J. Moore

WITH 25 FT SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

DRILL STEM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
50 FT DRILL STEM, WITH AND WITHOUT 25 FT SLEEVE
FIXED-PINNED (12/8/95 C.J.M.)

Figure A-18

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Client	WO/Job No.		
Subject DYNAMICS OF DRILL STRING	Date 12/18/95	By	Carleton J. Moore
WITH 25 FT SLEEVE	Checked	By	Hassan H. Ziada
Location Tank Farms	Revised	By	

DRILL STEM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
60 FT DRILL STEM, WITH AND WITHOUT 25 FT SLEEVE
FIXED-PINNED (12/8/95 CJM)

Figure A-19

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WHC-SD-WM-ANAL-047

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Client	WO/Job No.		
Subject	DYNAMICS OF DRILL STRING		
WITH 25 FT SLEEVE	Checked	By	Carlton J. Moore
Location	Tank Farms	Revised	By Hassan H. Zia

Figure A-20

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WO/Job No.

Subject DYNAMICS OF DRILL STRING

Date 12/18/95

By Carleton J. Moore

WITH 25 FT SLEEVE

Checked

By Hassan H. Zaida

Location Tank Farms

Revised

By

Figure A-21

DESIGN ANALYSIS

Client

WO/Job No.

Subject DYNAMICS OF DRILL STRING

Date 12/18/95

By Carleton J. Moore

WITH 25 FT SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

DRILL CRITICAL ROTATION SPEED VERSUS AXIAL COMPRESSIVE LOAD
50 FT DRILL STEM, WITH AND WITHOUT 25 FT SLEEVE
FIXED-PINNED (12/8/95 CJM)

Figure A-22

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Client	WO/Job No.	By	Catleon J. Moore
Subject DYNAMICS OF DRILL STRING	Date 12/18/95	Checked	By Hassan H. Ziaida
WITH 25 FT SLEEVE		Revised	By
Location Tank Farms			

Figure A-23

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Client	WO/Job No.	
Subject ABAQUS INPUT FOR DYNAMIC MODAL	Date 12/5/95	By Carleton J. Moore
ANALYSIS WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

*HEADING

DRILL PIPE BUCKLING (FILE F42S25FP.INP 12/5/95 CJM)

*NODE, NSET=ALL

1,0,0,0,0,0,0

101,0.25,250.,0.0

201,0.0,500.,0.0

202,0.20,200.,0.0

322,0.0,500.,0.0

*NGEN,NSET=ALL

1,101,1

101,201,1

202,322,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,200,1,1,1

*ELEMENT, TYPE=B31, ELSET=SLEEVE

201,202,203

*ELGEN, ELSET=SLEEVE

201,120,1,1,1

*ELEMENT, TYPE=GAPCYL, ELSET=GAPC

321,81,202

*ELGEN, ELSET=GAPC

321,120

*MATERIAL,NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*PLASTIC

76000.,0.0

90000.,0.1

4000.,0.15

4000.,0.20

*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING

1.125,0.172

-1,0,0

*MATERIAL,NAME=SS304

*ELASTIC

28.E+06,.27

*DENSITY

0.000734990

*PLASTIC

29500.,0.0

75000.,0.34

4000.,0.38

4000.,0.42

*BEAM SECTION, MATERIAL=SS304,SECTION=PIPE,ELSET=SLEEVE

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Client	WO/Job No.		
Subject ABAQUS INPUT FOR DYNAMIC MODAL	Date 12/5/95	By	Carleton J. Moore
ANALYSIS WITH 25 FT SLEEVE	Checked	By	Hassan H. Ziada
Location Tank Farms	Revised	By	

```

1.75,0.3
-1,0,0
*GAP, ELSET=GAPC
.325,0.0,1.,0.0
*BOUNDARY
**fixed-pinned
1,1,3
201,1
201,3,6
322,1,6
**RESTART,WRITE
*STEP,NLGEOM
  Step #1 apply gravity loads
*STATIC
1.,1.
*DLOAD
STRING, GRAV, 386.088, 0.0, -1.0, 0.0
SLEEVE, GRAV, 386.088, 0.0, -1.0, 0.0
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*STEP, NLGEOM
  Step #2 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*STEP, NLGEOM, INC=500
  Step # 3 APPLY AXIAL LOAD of 1400 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1400.
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*STEP, INC=100, NLGEOM
  Step #4 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT, FREQ=0
*NODE PRINT, FREQ=0
*END STEP
*STEP, NLGEOM
  Step #5 APPLY AXIAL LOAD of 1500 lbs
*STATIC
1.,1.

```

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ANALYSIS WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

*CLOAD
201,2,-1500.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #6 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
Step #7 APPLY AXIAL LOAD of 1600 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1600.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #8 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
Step #9 APPLY AXIAL LOAD of 1700 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1700.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #10 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM
Step #11 APPLY AXIAL LOAD of 1800 lbs
*STATIC
1.,1.

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Location Tank Farms	Revised	By

*CLOAD
201,2,-1800.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #12 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
Step #13 APPLY AXIAL LOAD of 1900 lbs
*STATIC
1.,1.
*CLOAD
201,2,-1900.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #14 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
Step #15 APPLY AXIAL LOAD of 2000 lbs
*STATIC
1.,1.
*CLOAD
201,2,-2000.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #16 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
Step #17 APPLY AXIAL LOAD of 2100 lbs
*STATIC
1.,1.

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Client	WO/Job No.
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ANALYSIS WITH 25 FT SLEEVE	By Carleton J. Moore
Location Tank Farms	Checked By Hassan H. Ziada
	Revised By

```

*CLOAD
201,2,-2100.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #18 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
  Step #19 APPLY AXIAL LOAD of 2200 lbs
*STATIC
1.,1.
*CLOAD
201,2,-2200.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
  Step #20 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
  Step #21 APPLY AXIAL LOAD of 2300 lbs
*STATIC
1.,1.
*CLOAD
201,2,-2300.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
  Step #22 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
  Step #23 APPLY AXIAL LOAD of 2400 lbs
*STATIC
1.,1.

```


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Subject ABAQUS INPUT FOR DYNAMIC MODAL	Date 12/5/95	By Carleton J. Moore
ANALYSIS WITH 25 FT SLEEVE	Checked	By Hassan H. Ziada
Location Tank Farms	Revised	By

*CLOAD
201,2,-2400.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #24 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
Step #25 APPLY AXIAL LOAD of 2500 lbs
*STATIC
1.,1.
*CLOAD
201,2,-2500.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #26 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
Step #27 APPLY AXIAL LOAD of 2600 lbs
*STATIC
1.,1.
*CLOAD
201,2,-2600.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #28 Extract first 21 natural frequencies
*FREQUENCY
21
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,NLGEOM,INC=100
Step #29 APPLY AXIAL LOAD of 2800 lbs
*STATIC
1.,1.

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WO/Job No.

Subject ABAQUS INPUT FOR DYNAMIC MODAL

Date 12/5/95

By Carleton J. Moore

ANALYSIS WITH 25 FT SLEEVE

Checked

By Hassan H. Ziada

Location Tank Farms

Revised

By

*CLOAD

201,2,-2800.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #30 Extract first 21 natural frequencies

*FREQUENCY

21

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*** end of analysis

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

6.0 Axial Failure of Various Length Drill Strings with 15-ft. Strengthening Sleeve

A Trend study was conducted to determine the buckling load for 20-ft, 30-ft, 40-ft, 50-ft and 60-ft lengths of drill string when used in conjunction with a 15-ft strengthening sleeve made of 3 in. schedule 80, 304 stainless steel pipe. The boundary conditions were also varied fixed-pinned to fixed-fixed to reflect drilling in either an aqueous or solid environment. Figure A-24 shows the buckling load for the various lengths and boundary conditions of drill string.

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

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By

CRITICAL BUCKLING LOAD VERSUS DRILL STRING LENGTH FOR A
15-FT SLEEVE AND INCLUDING GRAVITY EFFECTS

Figure A-24

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AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

**SAMPLE ABAQUS INPUT FILE DETERMINING THE CRITICAL
BUCKLING LOAD OF THE DRILL STEM COMBIND
WITH A 15 FT SLEEVE**

*HEADING

DRILL PIPE BUCKLING (FILE: DRIL_20.INP February 13, 1996 BLC)

*NODE, NSET=ALL

1,0,0,0,0,0,0

67,0.0825,66,0.0

121,0.15,120,0.0

241,0.0,240,0.0

242,0.0,66,0.0

416,0.0,240,0.0

*NGEN,NSET=ALL

1,67,1

67,121,1

121,241,1

242,416,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,240,1,1,1

*ELEMENT, TYPE=B31, ELSET=SLEEVE

241,242,243

*ELGEN,ELSET=SLEEVE

241,174,1,1,1

*ELEMENT,TYPE=GAPCYL,ELSET=GAPC

416,67,242

*ELGEN, ELSET=GAPC

416,174

*MATERIAL,NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*PLASTIC

76000.,0.0

90000.,0.1

4000.,0.15

4000.,0.20

*BEAM SECTION,MATERIAL=STEEL,SECTION=PIPE,ELSET=STRING

1.125,0.172

-1,0,0

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AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By	Carleton Moore
Location Tank Farms	Revised	By	

*MATERIAL,NAME=SS304

*ELASTIC

28.E+06.,.27

*DENSITY

0.000734990

*PLASTIC

29500.,0.0

75000.,0.34

4000.,0.38

4000.,0.42

*BEAM SECTION, MATERIAL=SS304,SECTION=PIPE,ELSET=SLEEVE

1.75,0.29

-1,0,0

*GAP, ELSET=GAPC

.325,0,0,1.,0.0

*BOUNDARY

**fixed-pinned

1,1,3

241,1

241,3,6

416,1,6

*RESTART,WRITE,FREQUENCY=30

*STEP,NLGEOM

*STATIC

1.,1.

*DLOAD

STRING,GRAV,386.088,0.0,-1.0,0.0

SLEEVE,GRAV,386.088,0.0,-1.0,0.0

*END STEP

*STEP,NLGEOM,INC=500

*STATIC

100.,15000.,,100.

*CLOAD

241,2,-15000.

*NODE PRINT,FREQ=100

U,CF1

*END STEP

*** end of analysis

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

7.0 Dynamics of the Drill String With a 15-ft Sleeve

In order to conduct successful safe rotational operations of the drill string with a 15-ft sleeve the frequencies and mode shapes of the drill string and sleeve were analyzed for various lengths and axial loads. As the axial compressive loads are increased the natural bending frequencies decreases. It is interesting that the frequency of the mode whose shape most closely matches classic first buckling mode goes to zero at the large deflection theory predicted buckling.

The dynamic analysis of the drill stem and sleeve employed ABAQUS with large deflection theory. The analysis trend cases include gravity loads. The solution approach is a combination of static load steps and dynamic modal analysis. The first static step applies the gravity loads. The next step is a dynamic modal analysis step that solve for the natural frequencies at zero axial force with gravity loads. The trend is then completed by alternating static load steps that apply axial loads with dynamic modal analysis steps.

Figures A-25, A-26, A-27, A-28, and A-29 compare trends of the lowest frequency versus axial loads for drill strings of 20-ft, 30-ft, 40-ft, 50-ft, and 60-ft length with fixed-pinned and fixed-fixed boundary conditions when used with a 15-ft sleeve.

The frequencies start out identical to the frequencies for drill strings without sleeves, except for the 20-ft drill string. The trend with the 15-ft sleeve for the 20-ft drill string is initially modified by the lower frequency of the fixed-free 15-ft sleeve, but after the sleeve and drill string make contact, a trend similar to the longer drill strings is observed.

Figures A-30 and A-31 show trends of the lowest modal frequency versus axial compressive load for drill strings of 20-ft, 30-ft, 40-ft, 50-ft, and 60-ft length with a 15-ft sleeve. Figure A-30 is of the drill strings modelled with a fixed-pinned boundary condition at the base and Figure A-31 is of the drill strings modelled with a fixed-fixed boundary condition at the base. The plot of critical rotational speeds, Figures A-32 and A-33, were obtained by simply multiplying the modal frequency in cycles per second by 60 seconds per minute to obtain RPM.

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

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Figure A-25

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

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By

MINIMUM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
30-FT DRILL STRING WITH 15-FT SLEEVE

Figure A-26

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

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MINIMUM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
40-FT DRILL STRING WITH 15-FT SLEEVE

Figure A-27

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

MINIMUM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
50-FT DRILL STRING WITH 15-FT SLEEVE

Figure A-28

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

MINIMUM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
60-FT DRILL STRING WITH 15-FT SLEEVE

Figure A-29

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

MINIMUM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
FOR 20-FT THROUGH 60-FT DRILL STRING WITH A 15-FT SLEEVE
AND FIXED-PINNED BOUNDARY CONDITIONS

Figure A-30

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

MINIMUM FREQUENCY VERSUS AXIAL COMPRESSIVE LOAD
FOR 20-FT THROUGH 60-FT DRILL STRINGS WITH A 15-FT SLEEVE
AND FIXED-FIXED BOUNDARY CONDITIONS

Figure A-31

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

Figure A-32

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Client

WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

MINIMUM CRITICAL RPM VERSUS AXIAL COMPRESSIVE LOAD
FOR 30-FT THROUGH 60-FT DRILL STRINGS WITH A 15-FT SLEEVE
AND FIXED-FIXED BOUNDARY CONDITIONS

Figure A-33

DESIGN ANALYSIS

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Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By	Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By	Carleton Moore
Location Tank Farms	Revised	By	

**SAMPLE ABAQUS INPUT FILE DETERMINING THE CRITICAL
FREQUENCY OF THE DRILL STEM WHEN COMBINED
WITH A 15 FT SLEEVE**

*HEADING

DRILL PIPE BUCKLING (FILE F20S15FF.INP February 20, 1996 BLC)

*NODE, NSET=ALL

1,0,0,0,0,0,0

67,0.0825,66,0.0

121,0.15,120,0.0

241,0.0,240,0.0

242,0.0,66,0.0

416,0.0,240,0.0

*NGEN,NSET=ALL

1,67,1

67,121,1

121,241,1

242,416,1

*ELEMENT, TYPE=B31, ELSET=STRING

1,1,2

*ELGEN, ELSET=STRING

1,240,1,1,1

*ELEMENT, TYPE=B31, ELSET=SLEEVE

241,242,243

*ELGEN, ELSET=SLEEVE

241,174,1,1,1

*ELEMENT, TYPE=GAPCYL, ELSET=GAPC

416,67,242

*ELGEN, ELSET=GAPC

416,174

*MATERIAL, NAME=STEEL

*ELASTIC

29.E+06,0.3

*DENSITY

0.000734990

*PLASTIC

76000.,0.0

90000.,0.1

4000.,0.15

4000.,0.20

*BEAM SECTION, MATERIAL=STEEL, SECTION=PIPE, ELSET=STRING

1.125,0.172

-1,0,0

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Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
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Location Tank Farms	Revised	By

*MATERIAL,NAME=SS304

*ELASTIC

28.E+06,.27

*DENSITY

0.000734990

*PLASTIC

29500.,0.0

75000.,0.34

4000.,0.38

4000.,0.42

*BEAM SECTION, MATERIAL=SS304,SECTION=PIPE,ELSET=SLEEVE

1.75,0.3

-1,0,0

*GAP, ELSET=GAPC

.325,0.0,1.,0.0

*BOUNDARY

**fixed-fixed

1,1,6

241,1

241,3,6

416,1,6

*PREPRINT,HISTORY=NO,MODEL=NO

*STEP,NLGEOM

Step #1 apply gravity loads

*STATIC

1.,1.

** WEIGHT *****

*DLOAD

STRING, GRAV,386.088,0.0,-1.0,0.0

SLEEVE, GRAV,386.088,0.0,-1.0,0.0

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #2 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step # 3 APPLY AXIAL LOAD of 800 lbs

*STATIC

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Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By	Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By	Carleton Moore
Location Tank Farms	Revised	By	

1.,1.

*CLOAD

241,2,-800.

*END STEP

*STEP,INC=100,NLGEOM

Step #4 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #5 APPLY AXIAL LOAD of 1600 lbs

*STATIC

1.,1.

*CLOAD

241,2,-1600.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #6 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #7 APPLY AXIAL LOAD of 2400 lbs

*STATIC

1.,1.

*CLOAD

241,2,-2400.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #8 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

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Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

*STEP,INC=100,NLGEOM
Step #9 APPLY AXIAL LOAD of 3200 lbs
*STATIC
1.,1.
*CLOAD
241,2,-3200.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #10 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #11 APPLY AXIAL LOAD of 4000 lbs
*STATIC
1.,1.
*CLOAD
241,2,-4000.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #12 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #13 APPLY AXIAL LOAD of 4800 lbs
*STATIC
1.,1.
*CLOAD
241,2,-4800.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #14 Extract first 20 natural frequencies

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AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

*FREQUENCY

20

*ELPRINT,FREQ=0***NODE PRINT,FREQ=0*****END STEP*****STEP,INC=100,NLGEOM**

Step #15 APPLY AXIAL LOAD of 5600 lbs

*STATIC

1.,1.

*CLOAD

241,2,-5600.

*ELPRINT,FREQ=0***NODE PRINT,FREQ=0*****END STEP*****STEP,INC=100,NLGEOM**

Step #16 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0***NODE PRINT,FREQ=0*****END STEP*****STEP,INC=100,NLGEOM**

Step #17 APPLY AXIAL LOAD of 6400 lbs

*STATIC

1.,1.

*CLOAD

241,2,-6400.

*ELPRINT,FREQ=0***NODE PRINT,FREQ=0*****END STEP*****STEP,INC=100,NLGEOM**

Step #18 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0***NODE PRINT,FREQ=0*****END STEP*****STEP,INC=100,NLGEOM**

Step #19 APPLY AXIAL LOAD of 7200 lbs

*STATIC

1.,1.

*CLOAD

241,2,-7200.

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Client	WO/Job No.		
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AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By	Carleton Moore
Location Tank Farms	Revised	By	

*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #20 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #21 APPLY AXIAL LOAD of 8000 lbs
*STATIC
1.,1.
*CLOAD
241,2,-8000.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #22 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #23 APPLY AXIAL LOAD of 8800 lbs
*STATIC
1.,1.
*CLOAD
241,2,-8800.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #24 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

Step #25 APPLY AXIAL LOAD of 9600 lbs

*STATIC

1.,1.

*CLOAD

241,2,-9600.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #26 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #27 APPLY AXIAL LOAD of 10400 lbs

*STATIC

1.,1.

*CLOAD

241,2,-10400.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #28 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #29 APPLY AXIAL LOAD of 11200 lbs

*STATIC

1.,1.

*CLOAD

241,2,-11200.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #30 Extract first 20 natural frequencies

*FREQUENCY

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #31 APPLY AXIAL LOAD of 12000 lbs

*STATIC

1.,1.

*CLOAD

241,2,-12000.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #32 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #33 APPLY AXIAL LOAD of 12800 lbs

*STATIC

1.,1.

*CLOAD

241,2,-12800.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #34 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #35 APPLY AXIAL LOAD of 13600 lbs

*STATIC

1.,1.

*CLOAD

241,2,-13600.

*ELPRINT,FREQ=0

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

```

*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #36 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #37 APPLY AXIAL LOAD of 14400 lbs
*STATIC
1.,1.
*CLOAD
241,2,-14400.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #38 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #39 APPLY AXIAL LOAD of 15200 lbs
*STATIC
1.,1.
*CLOAD
241,2,-15200.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #40 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #41 APPLY AXIAL LOAD of 16000 lbs

```

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

*STATIC

1.,1.

*CLOAD

241,2,-16000.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #42 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #43 APPLY AXIAL LOAD of 16800 lbs

*STATIC

1.,1.

*CLOAD

241,2,-16800.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #44 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #45 APPLY AXIAL LOAD of 17600 lbs

*STATIC

1.,1.

*CLOAD

241,2,-17600.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #46 Extract first 20 natural frequencies

*FREQUENCY

20

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #47 APPLY AXIAL LOAD of 18400 lbs
*STATIC
1.,1.
*CLOAD
241,2,-18400.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #48 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #49 APPLY AXIAL LOAD of 19200 lbs
*STATIC
1.,1.
*CLOAD
241,2,-19200.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #50 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
Step #51 APPLY AXIAL LOAD of 20000 lbs
*STATIC
1.,1.
*CLOAD
241,2,-20000.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

*END STEP

*STEP,INC=100,NLGEOM

Step #52 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #53 APPLY AXIAL LOAD of 20800 lbs

*STATIC

1.,1.

*CLOAD

241,2,-20800.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #54 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #55 APPLY AXIAL LOAD of 21600 lbs

*STATIC

1.,1.

*CLOAD

241,2,-21600.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #56 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #57 APPLY AXIAL LOAD of 22400 lbs

*STATIC

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

1.,1.

*CLOAD

241,2,-22400.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #58 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #59 APPLY AXIAL LOAD of 23200 lbs

*STATIC

1.,1.

*CLOAD

241,2,-23200.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #60 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #61 APPLY AXIAL LOAD of 24000 lbs

*STATIC

1.,1.

*CLOAD

241,2,-24000.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #62 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

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Client	WO/Job No.	
Subject ABAQUS DYNAMIC, BUCKLING	Date 2/22/96	By Brad L. Coverdell
AND MODAL ANALYSIS WITH 15 FT SLEEVE	Checked	By Carleton Moore
Location Tank Farms	Revised	By

```

*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #63 APPLY AXIAL LOAD of 24800 lbs
*STATIC
1.,1.
*CLOAD
241,2,-24800.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #64 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #65 APPLY AXIAL LOAD of 25600 lbs
*STATIC
1.,1.
*CLOAD
241,2,-25600.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #66 Extract first 20 natural frequencies
*FREQUENCY
20
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
*STEP,INC=100,NLGEOM
  Step #67 APPLY AXIAL LOAD of 26400 lbs
*STATIC
1.,1.
*CLOAD
241,2,-26400.
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP

```

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WO/Job No.

Subject ABAQUS DYNAMIC, BUCKLING

Date 2/22/96

By Brad L. Coverdell

AND MODAL ANALYSIS WITH 15 FT SLEEVE

Checked

By Carleton Moore

Location Tank Farms

Revised

By

*STEP,INC=100,NLGEOM

Step #68 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #69 APPLY AXIAL LOAD of 27200 lbs

*STATIC

1.,1.

*CLOAD

241,2,-27200.

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*STEP,INC=100,NLGEOM

Step #70 Extract first 20 natural frequencies

*FREQUENCY

20

*ELPRINT,FREQ=0

*NODE PRINT,FREQ=0

*END STEP

*** end of analysis

APPENDIX B
SUPPLEMENTAL CLOSED FORM CALCULATIONS FOR
DRILL STRINGS WITHOUT A SLEEVE

Prepared By
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CHECKLIST FOR INDEPENDENT REVIEW

Appendix B

Document Reviewed Supplemental Closed Form Calculations for Drill Strings Without a Sleeve

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Yes No N/A

- ☒ [ ] [ ] Problem completely defined.
- ☒ [ ] [ ] Necessary assumptions explicitly stated and supported.
- [ ] [ ] ☒ Computer codes and data files documented.
- ☒ [ ] [ ] Data used in calculations explicitly stated in document.
- ☒ [ ] [ ] Data checked for consistency with original source information as applicable.
- ☒ [ ] [ ] Mathematical derivations checked including dimensional consistency of results.
- ☒ [ ] [ ] Models appropriate and used within range of validity or use outside range of established validity justified.
- ☒ [ ] [ ] Hand calculations checked for errors.
- [ ] [ ] ☒ Code run streams correct and consistent with analysis documentation.
- [ ] [ ] ☒ Code output consistent with input and with results reported in analysis documentation.
- ☒ [ ] [ ] Acceptability limits on analytical results applicable and supported. Limits checked against sources.
- ☒ [ ] [ ] Safety margins consistent with good engineering practices.
- ☒ [ ] [ ] Conclusions consistent with analytical results and applicable limits.
- ☒ [ ] [ ] Results and conclusions address all points required in the problem statement.

H. H. Ziada
Reviewer

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1.0 INTRODUCTION

Supplemental "hand calculations" are provided to determine the axial elastic buckling load of the core sampling truck drill string which is used to retrieve waste samples from single- and double-shell waste storage tanks at the Hanford Site. The drill string is operated in a push mode to retrieve liquid and highly viscous slurry type wastes. To retrieve harder saltcake type waste the drill string is operated in a rotary mode using a rotating bit to penetrate the waste. Buckling of the drill string under either the push mode or the rotary mode of operation is addressed herein on the basis of buckling equations provided in the open literature. This provides a simple means to assess the sensitivity of the drill string buckling load to the drill string end support conditions, the length of the drill string inserted into the waste tank, the initial out-of-straightness of the drill string, and the speed of rotation of the drill string. The effect of the surrounding liquid is also considered in determining the critical speed of the drill string in rotary mode. In order to simplify the analysis for the critical speed the drill string is assumed to be submerged for the full length deployed within the tank.

2.0 SUMMARY OF RESULTS

The axial load to cause buckling of the drill string or to cause full-section yielding of the drill string under push mode operation are summarized in Section 2.1. Critical speeds under axial compression loading during rotary mode operation of the drill string are summarized in Section 2.2.

2.1 PUSH MODE

Figure B-1 provides a summary of the elastic buckling load and full-section yield load of the drill string under axial compression load as a function of the deployed free length for various assumed end support conditions. The buckling load is based on an initially straight drill string; the prebuckled stress state is uniform axial compression. Once the drill string buckles bending stresses develop along the length of the drill string but the drill string is capable of resisting additional axial load until full-section yielding (plastic hinge) occurs. The axial load to achieve full-section yielding is also tabulated in Table B-1. The stresses in the postbuckled regime are calculated with the aid of the exact solution for large deflections of a buckled bar (the elastica problem) from Timoshenko and Gere 1961. However, as shown in Figure B-1, the increase in axial load capacity before full-section yielding occurs is small relative to the buckling load. Results are given for an assumed yield strength for the drill string of 76,000 and 90,000 lbf/in². The effect of yield strength for this range of yield strength is also shown to be small (see Figure B-1). The results from these simplified analyses are consistent with the finite-element results given in Appendix A.

The effect of initial out-of-straightness of the drill string on the axial load to reach full-section yielding for a yield strength of 76,000 lbf/in² for the fixed-pinned end condition is shown in Figure B-2 and is tabulated in Table B-2. These results were obtained from the approximate Perry-Robertson formula as given in Johnston 1976. The reduction in axial

load capacity for an initial out-of-straightness relative to the buckling load of an initially straight drill string is apparent in Figure B-2. The effect of the out-of-straightness is reduced as the deployed length of the drill string is increased. At an initial 12-in. of out-of-straightness the reduction in axial load capacity is 24 and 16 percent for a deployed drill string length of 40 and 50 ft, respectively. Whereas, at an initial 2-in. of out-of-straightness, the reduction in axial load capacity is 5 and 3 percent for a deployed drill string length of 40 and 50 ft, respectively. These simplified analysis results show a greater sensitivity to initial out-of-straightness than exhibited in the finite-element results given in Appendix A. However, large out-of-straightness is more likely for a long drill string where the effect on the axial load capacity is reduced.

2.2 ROTARY MODE

Under a rotary mode of operation the drill string can develop an unstable whirling motion. The critical speed at which this whirling motion initiates is a function of the end support conditions of the drill string, the magnitude of the axial compressive load applied, and whether the drill string is submerged in the waste or not. Figure B-3 and Table B-3 present results for the first mode critical speed of the drill string in a dry condition as a function of axial compression load at various end support conditions and selected deployed lengths (35, 40, 45, and 50 ft). Similar results are presented in Figure B-4 and Table B-4 for the drill string fully submerged (wet condition). In the fully submerged case the waste specific gravity is taken as 1.7. The axial compression load greatly reduces the critical speed as the buckling load is approached. The effect of the waste on the critical speed increases as the axial compression load decreases (compare Figures B-3 and B-4). In these simplified calculation the drill string is assumed to be fully submerged in the "wet" condition case. This is not likely even with a tank at its full waste storage capacity because the section of the drill string in the riser and in the dome plenum region of the tank would still be "dry." These results are in general agreement with the corresponding finite-element results in Appendix A.

3.0 ANALYSIS

Simplified analyses were conducted in Section 3.1 to determine the elastic (Euler) buckling load of the drill string under axial compression loading (push mode) as a function of deployed length and assumed end support conditions. In Section 3.2 the post buckled axial load capacity is determined as the drill string approaches full-section yielding (plastic hinge). The effect of initial out-of-straightness is presented in Section 3.3. The critical speed under rotary mode operation is determined in Section 3.4.

3.1 ELASTIC BUCKLING ANALYSIS

Under push mode operation the drill string can buckle elastically as the push load is increased. Once buckling occurs the drill string will quickly exceed its yield strength under the resulting combined bending and axial load. The incremental load to subsequent full cross-section yielding and subsequent cross-section failure is relatively small. The elastic (Euler) buckling load

for a long slender column under an axial compressive end load and its own weight is given by Timoshenko and Gere 1961 as

$$P_{cr} = \pi^2 \frac{EI}{K^2 L^2} - \alpha w L \quad (B-1)$$

where

- E = Modulus of elasticity (= 29×10^6 lbf/in²)
- I = Cross-section area moment of inertia (in⁴) = $\frac{\pi}{64}(D_o^4 - D_i^4)$
- D_o = Outer diameter of drill string (= 2.25 in.) (Ziada 1994)
- D_i = Inner diameter of drill string (= 1.906 in.)
- K = End condition effective-length factor (see Table B-5)
- L = Length (in.) of drill string deployed
- α = Effective weight factor accounting for distributed weight of drill string (see Table B-5)
- w = Weight of drill string per unit length (lbf/in.) = γA
- A = Cross-sectional area (in²) = $\frac{\pi}{4}(D_o^2 - D_i^2)$
- γ = Weight per unit volume for steel drill string (= 0.2836 lbf/in³).

The effect of the various end support conditions is accounted for by the introduction of the effective length (end condition) factor, K. The effect of the distributed weight of the drill string is accounted for by the introduction of the effective weight factor, α . The end condition factor, K, and effective weight factor, α , are given in Table B-5. Only the theoretical K value is applied for the various end support conditions of interest. An allowable axial buckling load in accordance to AISC 1989 is given by

$$P_{allow} = \frac{P_{cr}}{FS} \quad (B-2)$$

where

- FS = Variable factor of safety as a function of column slenderness ratio (KL/r) (Johnston 1976)
- = $\frac{5}{3} + \frac{3}{8} \frac{KL/r}{C_c} - \frac{1}{8} \left(\frac{KL/r}{C_c} \right)^3$ for $KL/r < C_c$
- = $\frac{23}{12}$ for $KL/r > C_c$

$$r = \text{Radius of gyration of drill string cross section (in.)} = \sqrt{\frac{I}{A}}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

F_y = Yield strength of drill string material (= 76,000 to 90,000 lbf/in²).

Hence, the factor of safety varies from about 1.67 (as KL/r approaches zero) to 1.92 (for $KL/r > C_c$). According to Johnston (1976) the factor of safety tends to compensate for

- increasing sensitivity of long columns to variations in the effective-length factor, K ,
- the practical difficulty in determining the value of K in a real structure, and
- the initial crookedness of the column.

The AISC imposed factor of safety against buckling was developed to govern the design of buildings occupied by persons and subjected to loads with varying degrees of uncertainty. This level of protection does not appear to be justified for the drill string unless a failure of the drill string would lead to a credible safety concern within the tank or to operator personnel. If this is not the case then the safety factor could be relaxed to meet non-safety related issues concerning time and economic losses in the event of a drill string failure. In either respect, there is an uncertainty which needs to be addressed as to the actual end support condition as the drill string contacts the waste or an object within the waste. No safety factor has been applied to the results summarized in Section 2.0.

3.2 POSTBUCKLED ANALYSIS

In the preceding section, the assumption of small displacement was applied in determining the curvature to obtain the elastic Euler buckling load given by Equation B-1. If the exact expression for the beam curvature is introduced, the following nonlinear equilibrium differential equation for a beam under axial load (P) is obtained:

$$EI \frac{d\theta}{ds} + Py = 0 \quad (B-3)$$

where

E = Modulus of elasticity (= 29×10^6 lbf/in²)

I = Cross-section area moment of inertia (in⁴) = $\frac{\pi}{64} (D_o^4 - D_i^4)$

y = Lateral displacement of beam column

s = Length coordinate measured from one end along deflection curve

θ = Slope angle of beam along deflection curve.

Taking derivative of Equation B-3 with respect to s and realizing that

$$\frac{dy}{ds} = \sin\theta \quad (B-4)$$

we obtain

$$EI \frac{d^2\theta}{ds^2} + P \sin\theta = 0. \quad (B-5)$$

The shape of the elastic curve obtained from the solution of this exact differential equation is called the elastica (Timoshenko and Gere 1961). The solution can be obtained by using elliptical integrals, such that

$$\frac{P(p)}{P_{cr}} = \frac{4}{\pi^2} E_1^2(p) \quad (B-6)$$

$$\delta_{\max}(p) = \frac{pKL}{E_1(p)} \quad (B-7)$$

$$M_{\max}(p) = \delta_{\max}(p)P(p) \quad (B-8)$$

where

$P(p)$ = Axial compression load greater than P_{cr} as a function of p

P_{cr} = Elastic (Euler) buckling load (Equation B-1)

$\delta_{\max}(p)$ = Maximum lateral displacement as a function of p

$M_{\max}(p)$ = Maximum moment as a function of p

p = $\sin(\theta_o/2)$

θ_o = End slope angle of beam column at effective length KL

K = End condition effective-length factor (see Table B-5)

L = Length (in.) of drill string deployed

$$E_1(p) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - p^2 \sin^2 \phi}} d\phi \quad \text{complete elliptic integral of the first kind}$$

$$E_2(p) = \int_0^{\frac{\pi}{2}} \sqrt{1 - p^2 \sin^2 \phi} d\phi \quad \text{complete elliptic integral of the second kind.}$$

The resulting postbuckled behavior is stable up to the point at which the full section yields (plastic hinge forms).

By iterating on θ_0 such that the total maximum stress is equal to the yield strength, i.e.,

$$\sigma_T(p) = S_y \quad (B-9)$$

where

$\sigma_T(p) = \sigma_m(p) + \sigma_b(p)$ total stress at critical section of beam column with effective length KL.

$$\sigma_m(p) = \frac{P(p)}{A} \quad \text{membrane stress}$$

$$\sigma_b(p) = \frac{M_{\max}(p)}{S_p} \quad \text{bending stress}$$

$$A = \text{Cross-sectional area (in}^2\text{)} = \frac{\pi}{4}(D_o^2 - D_i^2)$$

$$I = \text{Cross-section area moment of inertia (in}^4\text{)} = \frac{\pi}{64}(D_o^4 - D_i^4)$$

$$D_o = \text{Outer diameter of drill string (= 2.25 in.) (Ziada 1994)}$$

$$D_i = \text{Inner diameter of drill string (= 1.906 in.)}$$

$$S_p = K_p S \quad \text{plastic section modulus}$$

$$S = \frac{2I}{D_o} \quad \text{elastic section modulus}$$

$$K_p = \frac{16}{3\pi} D_o \frac{D_o^3 - D_i^3}{D_o^4 - D_i^4} \quad \text{plastic bending factor for tubular section}$$

the axial load at full-section yielding can be obtained from Equation B-6. These calculations are given in Attachment B1 to Appendix B. The results are summarized in Section 2.1.

3.3 EFFECT OF INITIAL OUT-OF-STRAIGHTNESS

Initial out-of-straightness can significantly reduce the axial compression load capacity of the drill string. The effect of initial out-of-straightness can be approximated by extending the Perry-Robertson formula (Johnston 1976) for the maximum stress in an initially curved column with hinged end supports to an initially curved column with general end supports. This is done by applying the effective length concept introduced in Section 3.1 for buckling. The initial out-of-straightness is assumed to be sinusoidal over the effective length of the column (drill string). The maximum stress is given by

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{\delta_o D_o}{2r^2 K_p} \frac{P_{cr}}{P_{cr} - P} \right) \quad (B-10)$$

where

P = Axial compression load (lbf)

P_{cr} = Elastic (Euler) buckling load (Equation B-1)

δ_o = Initial out-of-straightness (in.)

A = Cross-sectional area (in²) = $\frac{\pi}{4}(D_o^2 - D_i^2)$

r = Radius of gyration of drill string cross section (in.) = $\sqrt{\frac{I}{A}}$

I = Cross-section area moment of inertia (in⁴) = $\frac{\pi}{64}(D_o^4 - D_i^4)$

D_o = Outer diameter of drill string (= 2.25 in.) (Ziada 1994)

D_i = Inner diameter of drill string (= 1.906 in.)

$K_p = \frac{16}{3\pi} D_o \frac{D_o^3 - D_i^3}{D_o^4 - D_i^4}$ plastic bending factor for tubular section.

By setting $\sigma_{max} = S_y$ and solving for P in Equation B-10 we obtain the axial compression load at full-section yielding for an initial out-of-straightness, δ_o , that is

$$P = \frac{1}{2}(b - \sqrt{b^2 - 4c}) \quad (B-11)$$

where

$$b = S_y A + \left(1 + \frac{\delta_o D_o}{2r^2 K_p}\right) P_{cr}$$

$$c = S_y A P_{cr}$$

Results are given in Attachment B2 to Appendix B. The results are summarized in Section 2.1 for the fixed-pinned end support condition.

3.4 CRITICAL SPEED

Under rotary mode operation the drill string can become dynamically unstable resulting in a whirling vibratory motion. The critical speed at which this occurs is a function of the lateral natural frequency of the drill string which is reduced as the push load is increased. The i^{th} mode critical speed, f_i , is given by Blevins 1979 as

$$f_i = \frac{60}{2\pi} \left(\frac{\lambda_i}{L}\right)^2 \sqrt{\frac{EIg}{w + w_f} \left[1 - \frac{P}{P_{cr}} \left(\frac{\lambda_i}{\lambda_1}\right)^2\right]} \quad (B-12)$$

where

- f_i = i^{th} mode critical speed (rpm)
- λ_i = i^{th} mode natural frequency parameter (dependent on end support conditions) (see Table B-6)
- E = Modulus of elasticity ($= 29 \times 10^6$ lbf/in²)
- I = Cross-section area moment of inertia (in⁴)
- g = Acceleration due to gravity ($= 386.4$ in/s²)
- L = Length of drill string (in.)
- w = Weight of drill string per unit length (lbf/in.) $= \gamma A$
- A = Cross-sectional area (in²) $= \frac{\pi}{4}(D_o^2 - D_i^2)$
- γ = Weight per unit volume for steel drill string ($= 0.2836$ lbf/in³).
- w_f = Displaced fluid weight per unit length (lbf/in.) $= \lambda_f A_o$
- A_o = Displaced fluid cross-sectional area wetted by liquid waste (in²) $= \frac{\pi}{4} D_o^2$
- λ_f = Weight per unit volume of fluid waste $= SG \lambda_{\text{water}}$
- SG = Specific gravity of the waste ($= 1$ to 1.7)
- P = Applied axial push load (lbf)
- P_{cr} = Axial buckling load (lbf) given by Equation B-1 (dependent on end support conditions).

For the dry condition where the drill string not submerged in waste, $w_f = 0$. In the wet condition the full deployed length of the drill string is assumed submerged in the waste for simplicity. Results are summarized in Section 2.2.

Figure B-1. Drill String Push-Mode Axial Compression Elastic Buckling Load and Full-Section Yield Load as a Function of Drill String Deployed Length and End Support Conditions.

Figure B-2. Drill String Push Mode Axial Compression Load at Full-Section Yield for Fixed-Pinned End Support Conditions and Initial Out-of-Straightness.

Figure B-3. Drill String Rotary Mode First Mode Critical Speed (Dry) as a Function of Axial Compression Load, End Support Conditions, and Deployed Free Length.

Figure B-4. Drill String Rotary Mode First Mode Critical Speed (Wet) as a Function of Axial Compression Load, End Support Conditions, and Deployed Free Length.

Table B-1. Drill String Push Mode Axial Compression Full-Section Yield Load
as a Function of Drill String Deployed Free Length
and End Support Conditions.

Based on Post-Buckled Elastica Solution (Timoshenko and Gere 1961)

Sy (lb/in ²)	Fixed- Free		Fixed- Guided		Fixed- Pinned		Fixed- Fixed		Pinned- Pinned	
	76,000	90,000	76,000	90,000	76,000	90,000	76,000	90,000	76,000	90,000
L (ft)	Axial Load at Full Section Yield (lbf)									
17	1,053	1,062	4,186	4,195	8,383	8,392	16,770	16,780	4,186	4,195
19	841	851	3,345	3,354	6,706	6,714	13,420	13,430	3,345	3,354
21	687	697	2,732	2,741	5,483	5,492	10,980	10,990	2,732	2,741
23	571	581	2,271	2,280	4,564	4,573	9,147	9,155	2,271	2,280
25	481	491	1,915	1,925	3,856	3,865	7,735	7,743	1,915	1,925
27	410	420	1,635	1,644	3,299	3,308	6,624	6,633	1,635	1,644
29	353	364	1,410	1,419	2,852	2,861	5,734	5,743	1,410	1,419
31	306	317	1,226	1,236	2,488	2,497	5,010	5,019	1,226	1,236
33	268	279	1,074	1,084	2,188	2,197	4,413	4,422	1,074	1,084
35	236	248	947	957	1,937	1,946	3,915	3,924	947	957
37	209	221	840	850	1,725	1,734	3,495	3,504	840	850
39	186	200	748	758	1,544	1,554	3,137	3,146	748	758
41	167	181	669	679	1,389	1,398	2,830	2,839	669	679
43	150	167	600	611	1,254	1,264	2,564	2,573	600	611
45	137	156	540	551	1,136	1,146	2,332	2,342	540	551
47	127	148	487	498	1,033	1,043	2,129	2,139	487	498
49	120	145	440	452	942	952	1,950	1,960	440	452
51	117	145	398	410	861	871	1,791	1,801	398	410
53	117	152	361	374	788	799	1,649	1,659	361	374
55	126	173	328	341	723	734	1,522	1,532	328	341
57	153	215	298	312	665	676	1,408	1,418	298	312
59	205	277	271	286	612	623	1,305	1,315	271	286

Table B-2. Drill String Push Mode Axial Compression Load at Full-Section Yield for Fixed-Pinned End Support Conditions and Initial Out-of-Straightness.

Length L (ft)	Buckling	Full-Section Yielding, $S_y = 76,000 \text{ lb/in}^2$					
		Initial Out-of-Straightness, δ_o (in.)					
		0	2	4	8	12	16
		Axial Compression Load, P (lbf)					
16	9,448	6,929	5,512	3,936	3,069	2,517	2,135
18	7,455	5,811	4,784	3,550	2,829	2,353	2,015
20	6,028	4,916	4,163	3,197	2,600	2,192	1,896
22	4,971	4,196	3,636	2,878	2,385	2,037	1,779
24	4,167	3,611	3,190	2,592	2,185	1,890	1,665
26	3,540	3,133	2,812	2,337	2,001	1,751	1,556
28	3,042	2,737	2,490	2,111	1,833	1,621	1,453
30	2,639	2,408	2,215	1,910	1,680	1,500	1,355
32	2,309	2,130	1,978	1,732	1,540	1,388	1,263
34	2,034	1,895	1,774	1,573	1,414	1,284	1,176
36	1,804	1,693	1,596	1,432	1,299	1,189	1,096
38	1,608	1,520	1,441	1,306	1,195	1,101	1,021
40	1,440	1,369	1,305	1,194	1,100	1,020	951
42	1,295	1,238	1,185	1,093	1,014	945	886
44	1,169	1,122	1,079	1,002	935	876	825
46	1,059	1,020	984	920	863	813	769
48	961	930	900	845	797	754	716
50	875	849	824	778	737	700	667
52	798	776	755	717	682	650	621
54	729	711	693	660	631	604	579
56	667	651	637	609	584	560	539
58	611	598	585	562	540	520	502
60	559	548	538	518	500	483	467

Table B-3. Drill String Rotary Mode First Mode Critical Speed (Dry) as a Function of Axial Compression Load, End Support Conditions, and Deployed Free Length.

K = α = L (ft) = P _a (lb/ft) = λ _c = f (rpm) =	Fixed-Free				Fixed-Fixed	Fixed-Pinned				Fixed-Fixed	Fixed-Fixed			
	2	2	2	2		0.707	0.707	0.707	0.707		0.5	0.5	0.5	0.5
	50	45	40	35		50	45	40	35		50	45	40	35
600	600	540	480	420	600	540	480	420	600	540	480	420	600	540
1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875
13.67	16.87	21.36	27.89	37.14	21.74	26.84	33.97	44.37	59.93	73.99	93.65	122.32	160.77	217.49
Axial P (lb)					Axial P (lb)					Axial P (lb)				
f (rpm)					f (rpm)					f (rpm)				
0	13.7	16.9	21.4	27.9	0	21.7	26.8	34.0	44.4	0	59.9	74.0	93.6	122.3
10	12.6	16.0	20.6	27.2	25	21.0	26.2	33.3	43.8	40	58.2	72.3	92.0	120.7
20	11.3	15.1	19.4	26.5	50	20.3	25.5	32.7	43.2	100	56.4	70.6	90.3	119.1
30	10.0	14.1	18.0	25.8	75	19.5	24.8	32.0	42.5	150	54.6	68.8	88.6	117.4
40	8.4	13.0	18.1	25.1	100	18.7	24.1	31.4	41.9	200	52.8	67.0	86.9	115.7
50	6.4	11.8	17.2	24.3	125	17.9	23.3	30.7	41.3	250	50.7	65.1	85.1	114.0
60	3.4	10.5	16.3	23.5	150	17.1	22.6	30.0	40.6	300	48.6	63.2	83.3	112.3
62	2.4	10.2	16.1	23.4	175	16.1	21.8	29.3	39.9	350	46.4	61.3	81.5	110.6
64	0.0	10.0	15.9	23.2	200	15.2	21.0	28.6	39.3	400	44.2	59.2	79.6	108.8
90	4.9	13.1	21.0	22.5	225	14.1	20.1	27.8	38.6	450	41.8	57.1	77.7	107.0
94	3.3	12.6	20.6	22.0	250	13.0	19.2	27.0	37.9	500	39.2	54.9	75.7	105.1
98	0.0	12.0	20.2	21.8	275	11.8	18.3	26.2	37.2	550	36.5	52.6	73.6	103.3
120	8.7	18.1	30.0	21.4	300	10.4	17.3	25.4	36.5	600	33.6	50.2	71.5	101.3
130	6.6	17.0	32.5	20.9	325	8.9	16.2	24.6	35.7	650	30.4	47.7	69.4	99.4
140	3.4	15.9	35.0	20.4	350	6.9	15.1	23.7	35.0	700	26.8	45.0	67.1	97.4
142	2.3	15.7	33.5	20.2	375	4.2	13.9	22.8	34.2	750	22.7	42.2	64.8	95.4
144	0.0	15.5	38.5	20.0	400	2.4	12.4	23.4	33.9	800	17.6	39.2	62.4	93.3
170	11.8	39.0	0.0	0.0	425	13.2	22.2	33.7	85.0	850	10.1	35.9	59.9	91.2
180	8.1	47.5	0.0	0.0	450	9.4	19.8	31.8	87.0	900	4.6	34.5	58.0	90.3
190	5.3	50.8	0.0	0.0	475	7.3	18.7	30.9	87.5	950	0.0	34.2	56.7	90.1
200	3.0	51.0	0.0	0.0	500	4.7	17.5	30.0	1,000	23.5	51.8	84.5	1,000	58.9
205	0.0	51.1	0.0	0.0	525	2.1	17.0	29.9	1,050	17.5	48.7	82.2	1,050	57.1
207	0.0	51.1	0.0	0.0	550	16.9	29.8	1,100	1,100	1.8	45.5	79.8	1,100	55.3
					575	13.4	27.3	1,110	1,110	3.4	44.8	79.3	1,150	53.4
					600	11.8	26.3	1,112	1,112	0.0	44.7	79.2	1,200	51.4
					625	9.8	25.2	1,250	1,250	34.0	72.0	1,250	1,250	49.4
					650	7.3	24.1	1,300	1,300	29.2	69.3	1,300	1,300	47.3
					675	5.3	23.0	1,350	1,350	23.4	66.4	1,350	1,350	45.1
					699	2.3	22.8	1,400	1,400	15.6	63.4	1,400	1,400	42.7
					682	0.0	22.7	1,430	1,430	7.9	61.5	1,450	1,450	40.2
					750	10.2	1,440	1,440	1,440	60.0	1,500	1,500	1,500	37.6
					775	17.8	1,450	1,450	1,450	53.3	1,550	1,550	1,550	35.7
					800	16.2	1,460	1,460	1,460	49.5	1,600	1,600	1,600	33.7
					825	14.5	1,480	1,480	1,480	45.4	1,650	1,650	1,650	31.7
					850	12.5	1,500	1,500	1,500	40.9	1,700	1,700	1,700	29.4
					875	10.1	1,520	1,520	1,520	35.8	1,750	1,750	1,750	26.9
					900	7.0	1,500	1,500	1,500	29.8	1,800	1,800	1,800	24.4
					918	3.3	1,850	1,850	1,850	22.4	1,840	1,840	1,840	21.5
					921	0.0	1,900	1,900	1,900	18.5	1,845	1,845	1,845	19.0
										5.6	1,950	1,950	1,950	16.5
										0.0	2,000	2,000	2,000	14.0
											2,050	2,050	2,050	11.5
											2,100	2,100	2,100	9.0
											2,150	2,150	2,150	6.5
											2,200	2,200	2,200	4.0
											2,250	2,250	2,250	1.5
											2,300	2,300	2,300	0.0
											2,350	2,350	2,350	0.0
											2,400	2,400	2,400	0.0
											2,450	2,450	2,450	0.0
											2,500	2,500	2,500	0.0
											2,550	2,550	2,550	0.0
											2,600	2,600	2,600	0.0
											2,650	2,650	2,650	0.0
											2,700	2,700	2,700	0.0
											2,750	2,750	2,750	0.0
											2,800	2,800	2,800	0.0
											2,850	2,850	2,850	0.0
											2,900	2,900	2,900	0.0
											2,950	2,950	2,950	0.0
											3,000	3,000	3,000	0.0
											3,050	3,050	3,050	0.0
											3,100	3,100	3,100	0.0
											3,150	3,150	3,150	0.0
											3,200	3,200	3,200	0.0
											3,250	3,250	3,250	0.0
											3,300	3,300	3,300	0.0
											3,350	3,350	3,350	0.0
											3,400	3,400	3,400	0.0
											3,450	3,450	3,450	0.0
											3,500	3,500	3,500	0.0
											3,550	3,550	3,550	0.0
											3,600	3,600	3,600	0.0
											3,650	3,650	3,650	0.0
											3,700	3,700	3,700	0.0
											3,750	3,750	3,750	0.0
											3,800	3,800	3,800	0.0
											3,850	3,850	3,850	0.0
											3,900	3,900	3,900	0.0
											3,950	3,950	3,950	0.0

Drill String First Critical Speed (Dry) as a Function of Axial Compressive Force and
Drill String Free Length on Contact with Solid Waste - Best Estimate, No Safety Factor Applied

D = 2.25 in.	E = 29000000 lb/in ²
P = 1.906 in.	γ = 0.2856 lb/in ³
A = 1.1229 in ²	w = 0.3184 lb/in
r = 0.6102 in ³	g = 386.4 in/s ²
c = 0.7372 in	n(E = 174657803)
A _i = 0 in ²	SG = 0
A _d = 3.9761 in ²	γ _d = 0 lb/in ³
A _e = 3.9761 in ²	w _d = 0 lb/in
(Elgth=w)/w/2 = 1.97415	

Elastic (Euler) Buckling Axial Compression Load

$$P_c = \pi^2 E I (K^2 L)^{-1} = \alpha L$$

Critical First Speed

$$f_1 (\text{rpm}) = (60 \lambda_c / 2 \pi L) \sqrt{E I g / (w \alpha)} \sqrt{(1 - P/P_c)^{1/2}}$$

A_i = internal cross-sectional area of drill string wetted by liquid waste.

A_d = displaced fluid cross-sectional area wetted by liquid waste.

$$A_e = A_i + A_d$$

Table B-4. Drill String Rotary Mode First Mode Critical Speed (Wet) as a Function of Axial Compression Load, End Support Conditions, and Deployed Free Length.

K = α = L (ft) = L _{eff} (ft) = P _{ax} (lb) = λ ₁ = f ₁ (rpm) =	Fixed-Free				Fixed-Fixed	Fixed-Pinned				Fixed-Fixed	Fixed-Pinned								
	2	2	2	2		0.707	0.707	0.707	0.707		0.5	0.5	0.5	0.5					
	0.5	0.5	0.5	0.5		0.5	0.5	0.5	0.5		0.5	0.5	0.5	0.5					
50	45	40	35	50		45	40	35	50		45	40	35						
600	540	480	420	600		540	480	420	600		540	480	420						
64	98	144	207	875		1,112	1,440	1,914	1,845		2,310	2,596	3,494						
1.875	1.875	1.875	1.875	2.17		2.365	2.37	2.37	3.927		3.927	3.927	3.927						
10.24	12.72	16.07	20.99	16.36		20.20	24.56	33.34	45.09		55.67	70.46	92.01						
Axial P (lb)						f ₁ (rpm)					Axial P (lb)					f ₁ (rpm)			
10	10.5	12.7	16.1	21.0	0	16.4	20.5	25.6	33.4	0	45.1	55.7	70.5	92.0	0	64.4	80.8	102.2	133.7
20	9.4	12.0	15.5	20.5	25	15.8	19.7	25.1	32.9	50	43.8	54.4	69.2	90.8	50	64.5	79.9	101.4	132.7
30	8.5	11.3	14.9	19.9	50	15.3	19.2	24.6	32.5	100	42.4	53.1	68.0	89.6	100	63.6	79.0	100.5	131.8
40	7.5	10.6	14.3	19.4	75	14.7	18.7	24.1	32.0	150	41.0	51.8	66.7	88.3	150	62.7	78.1	99.6	130.7
50	6.8	9.8	13.4	18.9	100	14.1	18.1	23.6	31.5	200	39.4	50.4	65.3	87.1	200	61.8	77.2	98.7	129.6
60	6.4	8.9	13.0	18.3	125	13.5	17.6	23.1	31.0	250	38.1	49.0	64.1	85.8	250	60.8	76.3	97.8	128.5
70	2.6	7.9	12.3	17.7	150	12.8	17.0	22.6	30.6	300	36.6	47.6	62.7	84.5	300	59.9	75.4	96.9	128.1
80	1.8	7.1	12.1	17.6	175	12.1	16.4	22.0	30.3	350	34.9	46.1	61.3	83.2	350	58.9	74.4	96.0	127.9
90	0.0	7.5	12.0	17.5	200	11.4	15.8	21.5	29.5	400	33.2	44.9	59.9	81.8	400	57.5	73.5	95.1	126.8
100	0.0	3.7	9.8	15.8	225	10.6	15.1	20.9	29.0	450	31.4	43.0	58.4	80.5	450	56.0	72.5	94.1	126.6
110	0.0	1.5	9.3	15.4	250	9.8	14.5	20.3	28.5	500	29.5	41.3	56.9	79.1	500	55.0	71.5	93.2	124.7
120	0.0	6.5	15.6	30.0	275	8.9	13.8	19.7	28.0	550	27.5	39.6	55.4	77.5	550	54.8	70.5	92.2	123.7
130	0.0	5.0	12.8	32.5	300	7.8	13.0	19.1	27.4	600	25.3	37.8	53.8	76.2	600	53.8	69.5	91.3	122.8
140	0.0	2.6	12.0	35.0	325	6.7	12.2	18.5	26.9	650	22.9	35.9	52.2	74.8	650	52.7	68.5	90.3	121.9
150	0.0	1.8	11.8	37.5	350	5.2	11.4	17.8	26.3	700	20.2	33.9	50.5	73.3	700	51.5	67.4	89.3	120.9
160	0.0	1.1	11.6	38.5	375	3.8	10.1	16.0	25.5	750	17.0	31.8	48.8	71.8	750	50.4	66.4	88.3	120.0
170	0.0	0.9	9.9	39.0	400	0.0	9.9	16.7	25.4	800	17.2	29.5	47.0	70.2	800	49.2	65.3	87.3	119.0
180	0.0	7.6	45.0	50	425	7.1	14.9	23.9	87.0	850	7.6	27.0	45.1	68.6	850	48.1	64.2	86.3	118.1
190	0.0	6.1	47.8	55	450	5.5	14.1	23.3	87.5	0	25.4	41.7	67.8	69.0	0	46.8	63.1	85.3	117.1
200	0.0	5.0	40.0	60	475	3.2	13.2	22.6	88.0	10.0	17.7	39.0	63.6	1,000	44.3	60.8	83.2	115.1	
210	0.0	2.3	51.0	65	500	1.5	12.8	22.3	1,054	13.2	36.7	61.8	1,050	41.0	39.7	82.1	114.1		
220	0.0	51.3	0.0	70	525	10.7	12.7	22.3	1,100	5.9	34.2	60.0	1,100	41.4	38.5	81.0	113.1		
230	0.0	57.5	0.0	75	550	10.1	10.5	1,110	1,110	2.5	33.7	59.6	1,100	40.2	37.2	79.9	112.1		
240	0.0	60.0	0.0	80	575	1.6	17.2	1,400	1,112	0.0	33.6	59.6	1,200	36.7	36.0	78.8	111.1		
250	0.0	62.5	0.0	85	600	7.4	19.0	1,250	1,250	25.6	54.2	1,250	37.2	54.7	77.7	110.0			
260	0.0	65.0	0.0	90	625	5.5	18.2	1,300	1,300	22.0	52.1	1,300	35.6	53.4	76.5	109.0			
270	0.0	67.5	0.0	95	650	2.5	17.3	1,350	1,350	17.6	50.0	1,250	33.9	52.1	75.4	107.9			
280	0.0	70.0	0.0	100	675	1.6	17.2	1,400	1,400	11.8	47.7	1,400	32.1	50.7	74.2	106.9			
290	0.0	72.5	0.0	105	700	0.0	17.1	1,433	1,433	4.2	46.0	1,400	30.3	49.3	73.0	105.8			
300	0.0	75.0	0.0	110	725	14.5	14.0	1,450	1,450	0.0	45.8	1,500	28.3	47.8	71.8	104.8			
310	0.0	77.5	0.0	115	750	12.2	14.0	1,500	1,500	40.1	43.5	1,500	26.2	46.3	70.5	103.6			
320	0.0	80.0	0.0	120	775	10.9	14.0	1,550	1,550	37.3	42.0	1,600	23.8	44.8	69.2	102.5			
330	0.0	82.5	0.0	125	800	10.9	14.5	1,600	1,600	34.2	41.5	1,650	21.3	43.2	68.0	101.4			
340	0.0	85.0	0.0	130	825	9.4	14.0	1,700	1,700	30.9	40.0	1,700	19.4	41.5	66.6	100.2			
350	0.0	87.5	0.0	135	850	7.6	13.8	1,750	1,750	26.9	38.5	1,750	14.0	39.8	65.3	99.1			
360	0.0	90.0	0.0	140	875	5.5	13.8	1,800	1,800	22.5	37.0	1,800	10.2	38.0	63.9	97.9			
370	0.0	91.8	0.0	145	900	2.5	13.8	1,850	1,850	16.8	35.1	1,850	4.1	36.5	62.9	97.0			
380	0.0	92.5	0.0	150	925	0.0	13.8	1,900	1,900	7.9	33.8	1,850	0.0	36.3	62.7	96.0			
390	0.0	1,910	0.0	155	1,910	31.9	33.9	1,950	1,950	4.2	32.0	1,950	31.9	33.9	61.3	94.3			
400	0.0	1,914	0.0	160	1,914	0.0	33.9	2,000	2,000	29.6	31.8	2,000	29.6	31.8	60.1	93.3			
410	0.0	2,050	0.0	165	2,050	27.1	36.9	91	91	21.1	36.9	91	21.1	36.9	91	91	91	91	
420	0.0	2,100	0.0	170	2,100	24.4	35.0	96	96	24.4	35.0	96	24.4	35.0	96	24.4	35.0	96	
430	0.0	2,150	0.0	175	2,150	21.3	33.4	89.4	89.4	17.6	34.1	89.4	17.6	34.1	89.4	17.6	34.1	89.4	
440	0.0	2,200	0.0	180	2,200	13.0	30.0	86.8	86.8	13.0	30.0	86.8	13.0	30.0	86.8	13.0	30.0	86.8	
450	0.0	2,300	0.0	185	2,300	5.3	48.2	85.4	85.4	5.3	48.2	85.4	5.3	48.2	85.4	5.3	48.2	85.4	
460	0.0	2,308	0.0	190	2,308	2.3	47.9	85.2	85.2	2.3	47.9	85.2	2.3	47.9	85.2	2.3	47.9	85.2	
470	0.0	2,310	0.0	195	2,310	0.0	47.8	85.2	85.2	0.0	47.8	85.2	0.0	47.8	85.2	0.0	47.8	85.2	
480	0.0	2,450	0.0	200	2,450	42.3	41.3	81.3	81.3	42.3	41.3	81.3	42.3	41.3	81.3	42.3	41.3	81.3	
490	0.0	2,450	0.0	205	2,450	40.1	39.9	79.9	79.9	40.1	39.9	79.9	40.1	39.9	79.9	40.1	39.9	79.9	
500	0.0	2,460	0.0	210	2,460	35.5	37.7	78.4	78.4	35.5	37.7	78.4	35.5	37.7	78.4	35.5	37.7	78.4	
510	0.0	2,650	0.0	215	2,650	32.9	35.5	75.5	75.5	32.9	35.5	75.5	32.9	35.5	75.5	32.9	35.5	75.5	
520	0.0	2,650	0.0	220	2,650	30.1	33.9	72.9	72.9	30.1	33.9	72.9	30.1	33.9	72.9	30.1	33.9	72.9	
530	0.0	2,750	0.0	225	2,750	27.5	32.0	70.4	70.4	27.5	32.0	70.4	27.5	32.0	70.4	27.5	32.0	70.4	
540	0.0	2,800	0.0	230	2,800	21.5	30.0	67.0	67.0	21.5	30.0	67.0	21.5	30.0	67.0	21.5	30.0	67.0	
550	0.0	2,850	0.0	235	2,850	19.1	28.1	63.9	63.9	19.1	28.1	63.9	19.1	28.1	63.9	19.1	28.1	63.9	
560	0.0	2,850	0.0	240	2,850	16.1	26.1	60.9	60.9	16.1	26.1	60.9	16.1	26.1	60.9	16.1	26.1	60.9	
570	0.0	2,900	0.0	245	2,900	13.1	24.1	57.9	57.9	13.1	24.1	57.9	13.1	24.1	57.9	13.1	24.1	57.9	
580	0.0	2,950	0.0	250	2,950	10.1	22.1	54.9	54.9	10.1	22.1	54.9	10.1	22.1	54.9	10.1	22.1	54.9	
590	0.0	2,950	0.0	255	2,950	7.1	20.1	51.9	51.9	7.1	20.1	51.9	7.1	20.1	51.9	7.1	20.1	51.9	
600	0.0	3,000	0.0	260	3,000	4.1	18.1	48.9	48.9	4.1	18.1	48.9	4.1	18.1	48.9	4.1	18.1	48.9	
610	0.0	3,050	0.0	265	3,050	1.1	16.1	45.9	45.9	1.1	16.1	45.9	1.1	16.1	45.9	1.1	16.1	45.9	
620	0.0	3,100	0.0	270	3,100	0.0	14.1	42.9	42.9	0.0	14.1	42.9	0.0	14.1	42.9	0.0	14.1	42.9	
630	0.0	3,150	0.0	275	3,150	0.0	12.1	39.9	39.9	0.0	12.1	39.9	0.0	12.1	39.9	0.0	12.1	39.9	
640	0.0	3,200	0.0	280	3,200	0.0	10.1	36.9	36.9	0.0	10.1	36.9	0.0	10.1	36.9	0.0	10.1	36.9	
650	0.0	3,250	0.0	285	3,250	0.0	8.1	33.9	33.9	0.0	8.1	33.9	0.0	8.1	33.9	0.0	8.1	33.9	
660	0.0	3,300	0.0	290	3,300	0.0	6.1	30.9	30.9	0.0	6.1	30.9	0.0	6.1	30.9	0.0	6.1	30.9	
670	0.0	3,350	0.0	295	3,350	0.0	4.1	27.9	27.9	0.0	4.1	27.9	0.0	4.1	27.9	0.0	4.1	27.9	
680	0.0	3,400	0.0	300	3,400	0.0	2.1	24.9	24.9	0.0	2.1	24.9	0.0	2.1	24.9	0.0	2.1	24.9	
690	0.0																		

Table B-5. End Condition Factor (K) and Effective Weight Factor (α)
for Buckling of a Slender Vertical Column Under an
Axial End Load and Its Own Weight.

Top restraint	Fixed	Fixed	Fixed	Pinned	Fixed	Guided ¹
Bottom restraint	Fixed	Pinned	Guided ¹	Pinned	Free	Pinned
Theoretical K ²	0.5	0.7	1.0	1.0	2.0	2.0
Recommended K value when ideal conditions are approximated ³	0.65	0.8	1.2	1.0	2.1	2.0
Effective distributed weight factor α ⁴	0.5	0.5	0.5	0.5	0.3	0.5

¹ Fixed against rotation but free to translate.

² Timoshenko and Gere 1961, AISC 1989, and Johnston 1976.

³ AISC 1989 and Johnston 1976.

⁴ Timoshenko and Gere 1961.

Table B-6. Natural Frequency Parameter (λ_i)
for Lateral Vibration of Slender Vertical Column.
(Blevins 1979)

Top restraint	Fixed	Fixed	Fixed	Pinned	Fixed	Guided ¹
Bottom restraint	Fixed	Pinned	Guided ¹	Pinned	Free	Pinned
λ_1	4.73004	3.92660	2.36502	3.14159	1.87510	1.57080
λ_2	7.85320	7.06858	5.49780	6.28319	4.69409	4.71239
λ_3	10.99560	10.21017	8.63938	9.42478	7.85475	7.85398
λ_4	14.13716	13.35177	11.78097	12.56637	10.99554	10.99557

¹ Fixed against rotation but free to translate.

4.0 REFERENCES

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ATTACHMENT B1
POSTBUCKLED SOLUTION FOR AXIALLY COMPRESSED DRILL STRING
(Mathcad¹ Work Sheets)

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¹ Mathcad is a registered trademark of MathSoft Inc.,
Mathcad, 1994, *Mathcad Plus 5.0 User's Guide*, Mathsoft Inc., Cambridge,
Massachusetts.

Postbuckled Solution for Axial Compressed Drill String

$D_o = 2.25 \text{ in}$ $S_y = 90 \cdot 10^3 \frac{\text{lb}}{\text{in}^2}$ $E = 29 \cdot 10^6 \frac{\text{lb}}{\text{in}^2}$ **Specific weight of steel**
 $D_i = 1.906 \text{ in}$ $\gamma = 490 \frac{\text{lb}}{\text{ft}^3}$ $\gamma = 0.2836 \frac{\text{lb}}{\text{in}^3}$
 $A = \frac{\pi}{4} (D_o^2 - D_i^2)$ $I = \frac{\pi}{64} (D_o^4 - D_i^4)$ $w = \gamma A$ $w = 0.3184 \frac{\text{lb}}{\text{in}}$
 $A = 1.1229 \cdot \text{in}^2$ $I = 0.6102 \cdot \text{in}^4$ $S = \frac{I}{\left(\frac{D_o}{2}\right)}$ $S = 0.5424 \cdot \text{in}^3$
 $r = \sqrt{\frac{I}{A}}$ $r = 0.7372 \cdot \text{in}$

$i = 1..5$ End condition effective-length factor, K

i	K_i	α_i	Effective weight factor, α
1	2	0.3	Fixed-Free Fixed-Guided Fixed-Pinned Fixed-Fixed Pinned-Pinned
2	1	0.5	
3	0.707	0.5	
4	0.5	0.5	
5	1	0.5	

Plastic bending factor for pipe section

$K_p = \frac{16}{3\pi} D_o \frac{D_o^3 - D_i^3}{D_o^4 - D_i^4}$ $K_p = 1.372$
 $S_p = K_p \cdot S$ $S_p = 0.744 \cdot \text{in}^3$
Elastic (Euler) buckling load
 (Timoshenko and Gere 1961)
 $P_E(L, K, \alpha) = \frac{\pi^2 E I}{K^2 L^2} - \alpha w L$

Complete elliptic integral of the first kind

$$E_1(p) := \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - p^2 \sin^2(\phi)}} d\phi$$

Complete elliptic integral of the second kind

$$E_2(p) := \int_0^{\frac{\pi}{2}} \sqrt{1 - p^2 \sin^2(\phi)} d\phi$$

Approximate formula (Timoshenko and Gere 1961)

$$\delta_{\max, \text{approx}}(P, L, K, \alpha) = K L \frac{\sqrt{8}}{\pi} \sqrt{\frac{P}{P_E(L, K, \alpha)}} - 1 \left[1 - \frac{1}{2} \left(\frac{P}{P_E(L, K, \alpha)} - 1 \right) \right]$$

$$P(p, L, K, \alpha) = \frac{4}{\pi^2} P_E(L, K, \alpha) \cdot E_1(p)^2$$

Maximum lateral deflection

$$\delta_{\max}(p, L, K, \alpha) = \frac{P K L}{E_1(p)}$$

End axial deflection

$$\Delta_{\text{end}}(p, L, K, \alpha) = 2 \cdot L \left(1 - \frac{E_2(p)}{E_1(p)} \right)$$

$$M_{\max}(p, L, K, \alpha) = \delta_{\max}(p, L, K, \alpha) \cdot P(p, L, K, \alpha)$$

$$\sigma_m(p, L, K, \alpha) = \frac{P(p, L, K, \alpha)}{A}$$

$$\sigma_b(p, L, K, \alpha) = \frac{M_{\max}(p, L, K, \alpha)}{S_p}$$

$$\sigma_T(p, L, K, \alpha) = \sigma_m(p, L, K, \alpha) + \sigma_b(p, L, K, \alpha)$$

$N = 10$ $j = 1$ Fixed-free end support condition
 $n = 0..N$ $k = K_j$ $a = \alpha_j$
 $\theta_{o_n} = \frac{n}{N} \cdot 40\text{-deg}$ $k = 2$ $a = 0.3$

$L = 42\text{-ft}$ $P_E(L, k, a) = 124\text{-lbf}$

j = 1 Fixed-free end support condition

$$k := K_j \quad a := \alpha_j \quad S_y = 76000 \frac{\text{lb} \cdot \text{f}}{\text{in}^2}$$

$$k = 2 \quad a = 0.3$$

$$L_{\text{start}} = 15 \cdot \text{ft} \quad L_{\text{end}} = 60 \cdot \text{ft} \quad L_{\text{inc}} = 2 \cdot \text{ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n := 1..N \quad L_n := (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 := 10 \cdot \text{deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) := \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n := \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F1\_76) := \frac{P(p(\theta_n), L_n, k, a)}{\text{lb} \cdot \text{f}}$$

θ_n deg	L_n ft	$P(p(\theta_n), L_n, k, a)$ lb f	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
23.909	17	1053	1.022	0.261
26.905	19	841	1.028	0.292
30.037	21	687	1.035	0.324
33.277	23	571	1.043	0.357
36.655	25	481	1.053	0.39
40.205	27	410	1.064	0.424
43.965	29	353	1.078	0.459
47.985	31	306	1.093	0.495
52.322	33	268	1.113	0.532
57.045	35	236	1.136	0.57
62.243	37	209	1.165	0.61
68.022	39	186	1.201	0.65
74.517	41	167	1.248	0.69
81.897	43	150	1.311	0.729
90.372	45	137	1.397	0.764
100.197	47	127	1.521	0.792
111.658	49	120	1.708	0.806
125.012	51	117	2.013	0.796
138.727	53	117	2.486	0.756
153.591	55	126	3.38	0.674
168.149	57	152	5.442	0.543
182.824	59	205	10.501	0.393

j := 1 Fixed-free end support condition

$$k := K_j \quad a := \alpha_j \quad S_y = 90000 \frac{\text{lbf}}{\text{in}^2}$$

$$k = 2 \quad a = 0.3$$

$$L_{\text{start}} = 15 \text{ ft} \quad L_{\text{end}} = 60 \text{ ft} \quad L_{\text{inc}} = 2 \text{ ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10 \text{ deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) = \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F1\_90) = \frac{P(p(\theta_n), L_n, k, a)}{\text{lbf}}$$

θ_n	L_n	$P(p(\theta_n), L_n, k, a)$	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
deg	ft	lbf		
28.364	17	1062	1.031	0.307
31.848	19	851	1.04	0.343
35.529	21	697	1.05	0.379
39.332	23	580	1.061	0.416
43.29	25	491	1.075	0.453
47.44	27	420	1.091	0.49
51.826	29	363	1.11	0.528
56.501	31	317	1.133	0.566
61.526	33	279	1.16	0.605
66.975	35	248	1.194	0.643
72.937	37	221	1.236	0.681
79.52	39	199	1.29	0.717
86.851	41	181	1.359	0.751
95.079	43	167	1.453	0.779
104.373	45	155	1.583	0.799
114.897	47	148	1.772	0.806
126.762	49	145	2.062	0.793
138.721	51	144	2.485	0.756
151.672	53	152	3.227	0.687
164.258	55	173	4.646	0.585
174.082	57	215	7.677	0.459
181.222	59	277	14.241	0.337

j = 2 Fixed-guided end support condition

$$k = K_j \quad a = \alpha_j \quad S_y = 76000 \frac{\text{lbf}}{\text{in}^2}$$

$$k = 1 \quad a = 0.5$$

$$L_{\text{start}} = 15\text{-ft} \quad L_{\text{end}} = 60\text{-ft} \quad L_{\text{inc}} = 2\text{-ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10\text{-deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) = \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F2\_76) = \frac{P(p(\theta_n), L_n, k, a)}{\text{lbf}}$$

θ_n	L_n	$P(p(\theta_n), L_n, k, a)$	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
deg	ft	lbf		
11.39	17	4185	1.005	0.063
12.899	19	3345	1.006	0.071
14.415	21	2732	1.008	0.08
15.945	23	2271	1.01	0.088
17.497	25	1915	1.012	0.096
19.078	27	1635	1.014	0.105
20.696	29	1410	1.017	0.113
22.36	31	1226	1.019	0.122
24.078	33	1074	1.022	0.131
25.86	35	947	1.026	0.141
27.719	37	840	1.03	0.15
29.668	39	748	1.034	0.16
31.723	41	669	1.039	0.171
33.9	43	600	1.045	0.182
36.222	45	540	1.052	0.193
38.712	47	487	1.059	0.205
41.402	49	440	1.068	0.218
44.326	51	398	1.079	0.231
47.529	53	361	1.092	0.246
51.063	55	327	1.107	0.261
54.994	57	298	1.125	0.277
59.407	59	271	1.148	0.294

j = 2 Fixed-guided end support condition

$$k = K_j \quad a = \alpha_j \quad S_y = 90000 \cdot \frac{\text{lb} \cdot \text{ft}}{\text{in}^2}$$

$$k = 1 \quad a = 0.5$$

$$L_{\text{start}} = 15 \text{ ft} \quad L_{\text{end}} = 60 \text{ ft} \quad L_{\text{inc}} = 2 \text{ ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10 \text{ deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) = \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F2\_90) = \frac{P(p(\theta_n), L_n, k, a)}{\text{lb} \cdot \text{ft}}$$

θ_n deg	L_n ft	$P(p(\theta_n), L_n, k, a)$ lbft	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
13.593	17	4194	1.007	0.075
15.364	19	3354	1.009	0.085
17.147	21	2741	1.011	0.094
18.948	23	2280	1.014	0.104
20.776	25	1924	1.017	0.114
22.638	27	1644	1.02	0.124
24.543	29	1419	1.023	0.134
26.502	31	1236	1.027	0.144
28.524	33	1084	1.032	0.154
30.621	35	957	1.037	0.165
32.807	37	850	1.042	0.176
35.098	39	758	1.049	0.187
37.509	41	679	1.056	0.199
40.064	43	610	1.064	0.211
42.784	45	551	1.073	0.224
45.698	47	498	1.084	0.237
48.839	49	451	1.097	0.251
52.248	51	410	1.112	0.266
55.973	53	374	1.13	0.281
60.071	55	341	1.152	0.297
64.614	57	312	1.179	0.313
69.69	59	286	1.213	0.33

j = 3 Fixed-pinned end support condition

$$k = K_j \quad a = \alpha_j \quad S_y = 76000 \frac{\text{lb}}{\text{in}^2}$$

$$k = 0.707 \quad a = 0.5$$

$$L_{\text{start}} = 15\text{-ft} \quad L_{\text{end}} = 60\text{-ft} \quad L_{\text{inc}} = 2\text{-ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10\text{-deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) = \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F3\_76) = \frac{P(p(\theta_n), L_n, k, a)}{\text{lb}} \quad \text{lb}$$

θ_n deg	L_n ft	$P(p(\theta_n), L_n, k, a)$ lb	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
7.611	17	8382	1.002	0.03
8.703	19	6705	1.003	0.034
9.786	21	5482	1.004	0.038
10.865	23	4564	1.005	0.043
11.944	25	3856	1.005	0.047
13.028	27	3298	1.006	0.051
14.12	29	2851	1.008	0.055
15.223	31	2488	1.009	0.059
16.341	33	2187	1.01	0.064
17.478	35	1936	1.012	0.068
18.637	37	1725	1.013	0.072
19.821	39	1544	1.015	0.077
21.037	41	1388	1.017	0.081
22.287	43	1254	1.019	0.086
23.577	45	1136	1.021	0.091
24.913	47	1033	1.024	0.096
26.302	49	942	1.027	0.101
27.749	51	861	1.03	0.106
29.263	53	788	1.033	0.112
30.853	55	723	1.037	0.118
32.529	57	664	1.041	0.124
34.304	59	611	1.046	0.13

j := 3 Fixed-pinned end support condition

$$k := K_j \quad a := \alpha_j \quad S_y = 90000 \frac{\text{lb}}{\text{in}^2}$$

$$k = 0.707 \quad a = 0.5$$

$$L_{\text{start}} := 15\text{-ft} \quad L_{\text{end}} := 60\text{-ft} \quad L_{\text{inc}} := 2\text{-ft}$$

$$N := \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n := 1..N \quad L_n := (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 := 10\text{-deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) := \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F3\_90) := \frac{P(p(\theta_n), L_n, k, a)}{\text{lb}} \text{ lb}$$

θ_n deg	L_n ft	$P(p(\theta_n), L_n, k, a)$ lb	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\text{max}}(p(\theta_n), L_n, k, a)}{L_n}$
9.162	17	8391	1.003	0.036
10.44	19	6713	1.004	0.041
11.709	21	5491	1.005	0.046
12.974	23	4572	1.006	0.051
14.242	25	3864	1.008	0.056
15.516	27	3307	1.009	0.06
16.8	29	2861	1.011	0.065
18.099	31	2497	1.013	0.07
19.415	33	2197	1.015	0.075
20.754	35	1946	1.017	0.08
22.118	37	1734	1.019	0.086
23.514	39	1553	1.021	0.091
24.945	41	1398	1.024	0.096
26.417	43	1264	1.027	0.101
27.936	45	1146	1.03	0.107
29.508	47	1043	1.034	0.113
31.141	49	952	1.038	0.119
32.843	51	871	1.042	0.125
34.622	53	799	1.047	0.131
36.49	55	734	1.053	0.137
38.457	57	675	1.059	0.144
40.538	59	623	1.065	0.151

j = 4 Fixed-fixed end support condition

$$k = K_j \quad a = \alpha_j \quad S_y = 76000 \frac{\text{lb}}{\text{in}^2}$$

$$k = 0.5 \quad a = 0.5$$

$$L_{\text{start}} = 15\text{-ft} \quad L_{\text{end}} = 60\text{-ft} \quad L_{\text{inc}} = 2\text{-ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10\text{-deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) := \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F4\_76) := \frac{P(p(\theta_n), L_n, k, a)}{\text{lb}} \frac{\text{lb}}{\text{ft}}$$

θ_n	L_n	$P(p(\theta_n), L_n, k, a)$	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
deg	ft	lb		
4.793	17	16770	1.001	0.013
5.616	19	13419	1.001	0.016
6.423	21	10978	1.002	0.018
7.216	23	9145	1.002	0.02
7.999	25	7734	1.002	0.022
8.776	27	6623	1.003	0.024
9.55	29	5733	1.003	0.026
10.322	31	5010	1.004	0.029
11.096	33	4413	1.005	0.031
11.873	35	3915	1.005	0.033
12.654	37	3494	1.006	0.035
13.441	39	3137	1.007	0.037
14.236	41	2830	1.008	0.039
15.041	43	2564	1.009	0.041
15.857	45	2332	1.01	0.044
16.686	47	2129	1.011	0.046
17.53	49	1950	1.012	0.048
18.39	51	1791	1.013	0.051
19.27	53	1649	1.014	0.053
20.17	55	1522	1.016	0.055
21.094	57	1408	1.017	0.058
22.044	59	1305	1.019	0.06

j := 4 Fixed-fixed end support condition

$$k := K_j \quad a := \alpha_j \quad S_y = 90000 \frac{\text{lbf}}{\text{in}^2}$$

$$k = 0.5 \quad a = 0.5$$

$$L_{\text{start}} := 15\text{-ft} \quad L_{\text{end}} := 60\text{-ft} \quad L_{\text{inc}} := 2\text{-ft}$$

$$N := \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n := 1..N \quad L_n := (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 := 10\text{-deg}$$

$$\beta(L, \theta, k, a, S_y) := \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n := \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F4\_90) := \frac{P(p(\theta_n), L_n, k, a)}{\text{lbf}}$$

θ_n	L_n	$P(p(\theta_n), L_n, k, a)$	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
deg	ft	lbf		
5.887	17	16777	1.001	0.016
6.842	19	13427	1.002	0.019
7.779	21	10987	1.002	0.022
8.703	23	9154	1.003	0.024
9.617	25	7742	1.004	0.027
10.526	27	6632	1.004	0.029
11.432	29	5742	1.005	0.032
12.338	31	5018	1.006	0.034
13.246	33	4422	1.007	0.037
14.158	35	3924	1.008	0.039
15.076	37	3503	1.009	0.042
16.002	39	3146	1.01	0.044
16.938	41	2839	1.011	0.047
17.885	43	2573	1.012	0.049
18.846	45	2341	1.014	0.052
19.822	47	2138	1.015	0.054
20.815	49	1959	1.017	0.057
21.829	51	1800	1.018	0.06
22.865	53	1659	1.02	0.062
23.925	55	1532	1.022	0.065
25.013	57	1418	1.024	0.068
26.132	59	1315	1.026	0.071

j = 5 Pinned-pinned end support condition

$$k = K_j \quad a = \alpha_j \quad S_y = 76000 \frac{\text{lb}}{\text{in}^2}$$

$$k = 1 \quad a = 0.5$$

$$L_{\text{start}} = 15\text{-ft} \quad L_{\text{end}} = 60\text{-ft} \quad L_{\text{inc}} = 2\text{-ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10\text{-deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) = \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F5\_76) := \frac{P(p(\theta_n), L_n, k, a)}{\text{lb}} \quad \text{lb}$$

θ_n deg	L_n ft	$P(p(\theta_n), L_n, k, a)$ lb	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
11.39	17	4185	1.005	0.063
12.899	19	3345	1.006	0.071
14.415	21	2732	1.008	0.08
15.945	23	2271	1.01	0.088
17.497	25	1915	1.012	0.096
19.078	27	1635	1.014	0.105
20.696	29	1410	1.017	0.113
22.36	31	1226	1.019	0.122
24.078	33	1074	1.022	0.131
25.86	35	947	1.026	0.141
27.719	37	840	1.03	0.15
29.668	39	748	1.034	0.16
31.723	41	669	1.039	0.171
33.9	43	600	1.045	0.182
36.222	45	540	1.052	0.193
38.712	47	487	1.059	0.205
41.402	49	440	1.068	0.218
44.326	51	398	1.079	0.231
47.529	53	361	1.092	0.246
51.063	55	327	1.107	0.261
54.994	57	298	1.125	0.277
59.407	59	271	1.148	0.294

j = 5 Pinned-pinned end support condition

$$k = K_j \quad a = \alpha_j \quad S_y = 90000 \frac{\text{lb}}{\text{in}^2}$$

$$k = 1 \quad a = 0.5$$

$$L_{\text{start}} = 15\text{-ft} \quad L_{\text{end}} = 60\text{-ft} \quad L_{\text{inc}} = 2\text{-ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 22.5$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

$$\theta_0 = 10\text{-deg} \quad \text{initial guess}$$

$$\beta(L, \theta, k, a, S_y) = \text{root} \left(\frac{\sigma_T(p(\theta), L, k, a)}{S_y} - 1, \theta \right)$$

$$\theta_n = \beta(L_n, \theta_{n-1}, k, a, S_y)$$

$$\text{WRITEPRN}(F5\_90) = \frac{P(p(\theta_n), L_n, k, a)}{\text{lb}} \quad \text{lb}$$

θ_n	L_n	$P(p(\theta_n), L_n, k, a)$	$\frac{P(p(\theta_n), L_n, k, a)}{P_E(L_n, k, a)}$	$\frac{\delta_{\max}(p(\theta_n), L_n, k, a)}{L_n}$
deg	ft	lb		
13.593	17	4194	1.007	0.075
15.364	19	3354	1.009	0.085
17.147	21	2741	1.011	0.094
18.948	23	2280	1.014	0.104
20.776	25	1924	1.017	0.114
22.638	27	1644	1.02	0.124
24.543	29	1419	1.023	0.134
26.502	31	1236	1.027	0.144
28.524	33	1084	1.032	0.154
30.621	35	957	1.037	0.165
32.807	37	850	1.042	0.176
35.098	39	758	1.049	0.187
37.509	41	679	1.056	0.199
40.064	43	610	1.064	0.211
42.784	45	551	1.073	0.224
45.698	47	498	1.084	0.237
48.839	49	451	1.097	0.251
52.248	51	410	1.112	0.266
55.973	53	374	1.13	0.281
60.071	55	341	1.152	0.297
64.614	57	312	1.179	0.313
69.69	59	286	1.213	0.33

ATTACHMENT B2

DRILL STRING FULL-SECTION YIELD LOAD
UNDER AXIAL COMPRESSION FOR
INITIAL OUT-OF-STRAIGHTNESS

(Mathcad² Work Sheets)

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² Mathcad is a registered trademark of MathSoft Inc.,
Mathcad, 1994, *Mathcad Plus 5.0 User's Guide*, Mathsoft Inc., Cambridge,
Massachusetts.

Maximum Axial Compression Load for Drill String with Initial Out-of-Straightness

$$\begin{aligned} D_o &:= 2.25 \cdot \text{in} & S_y &:= 76 \cdot 10^3 \frac{\text{lb}}{\text{in}^2} & E &:= 29 \cdot 10^6 \frac{\text{lb}}{\text{in}^2} & \text{Specific weight of steel} \\ D_i &:= 1.906 \cdot \text{in} & & & & & \gamma = 490 \frac{\text{lb}}{\text{ft}^3} \quad \gamma = 0.2836 \frac{\text{lb}}{\text{in}^3} \\ A &:= \frac{\pi}{4} (D_o^2 - D_i^2) & I &:= \frac{\pi}{64} (D_o^4 - D_i^4) & w &:= \gamma A & w = 0.3184 \frac{\text{lb}}{\text{in}} \\ A &:= 1.1229 \cdot \text{in}^2 & I &:= 0.6102 \cdot \text{in}^4 & S &:= \frac{I}{\left(\frac{D_o}{2}\right)} & S = 0.5424 \cdot \text{in}^3 \\ r &:= \sqrt{\frac{I}{A}} & r &:= 0.7372 \cdot \text{in} & & & \end{aligned}$$

i = 1..5 End condition effective-length factor, K

i	$K_i :=$	$\alpha_i :=$	Effective weight factor, α
1	2	0.3	Fixed-Free
2	1	0.5	Fixed-Guided
3	0.707	0.5	Fixed-Pinned
4	0.5	0.5	Fixed-Fixed
5	1	0.5	Pinned-Pinned

Plastic bending factor for pipe section

$$K_p = \frac{16}{3 \cdot \pi} D_o \frac{D_o^3 - D_i^3}{D_o^4 - D_i^4} \quad K_p = 1.372$$

Elastic (Euler) buckling load
(Timoshenko and Gere 1961)

$$S_p = K_p S$$

$$S_p = 0.744 \cdot \text{in}^3$$

$$P_E(L, K, \alpha) = \frac{\pi^2 E I}{K^2 L^2} - \alpha w L$$

Initially Curved Column with Hinged Ends (Pinned-Pinned) extended to general end support conditions

δ_o = initial out-of-straightness

P = axial load

$$\sigma_{\max}(P, L, k, a, \delta_o) = \frac{P}{A} \left(1 + \frac{\delta_o \cdot D_o}{2 r^2 \cdot K_p} \frac{P_E(L, k, a)}{P_E(L, k, a) - P} \right) \quad (\text{Johnston 1976})$$

Solving for P such that the maximum stress is equal to S_y gives

$$b(L, k, a, \delta_o) = S_y \cdot A \cdot \left(1 + \frac{\delta_o \cdot D_o}{2 r^2 \cdot K_p} \right) \cdot P_E(L, k, a)$$

$$c(L, k, a, \delta_o) = S_y \cdot A \cdot P_E(L, k, a)$$

$$P(L, k, a, \delta_o) = \frac{1}{2} \left(b(L, k, a, \delta_o) - \sqrt{b(L, k, a, \delta_o)^2 - 4 c(L, k, a, \delta_o)} \right)$$

Johnston, B. G. (ed.), 1976, Structural Stability Research Council, Guide to Stability Design Criteria for Metal Structures, John Wiley & Sons, New York, New York.

$j = 1$ Fixed-free end support condition

$k = K_j$ $a = \alpha_j$

$k = 2$ $a = 0.3$

$K_p = 1.372$

$S_y = 76000 \frac{\text{lbf}}{\text{in}^2}$

$L_{\text{start}} = 14 \cdot \text{ft}$ $L_{\text{end}} = 60 \cdot \text{ft}$ $L_{\text{inc}} = 2 \cdot \text{ft}$

$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 23$

$n = 1 \dots N$ $L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$

L_n	$P(L_n, k, a, 2 \cdot \text{in})$	$P(L_n, k, a, 4 \cdot \text{in})$	$P(L_n, k, a, 8 \cdot \text{in})$	$P(L_n, k, a, 12 \cdot \text{in})$	$P(L_n, k, a, 16 \cdot \text{in})$	$P(L_n, k, a, 20 \cdot \text{in})$	$P E(L_n, k, a)$
ft	lbf	lbf	lbf	lbf	lbf	lbf	lbf
16	1119	1076	999	933	875	823	1166
18	886	859	809	765	726	690	915
20	716	699	665	635	608	583	735
22	589	577	554	533	513	495	601
24	490	482	466	451	437	424	499
26	413	407	395	384	374	365	419
28	350	346	338	330	322	315	355
30	299	296	290	284	279	273	303
32	257	255	250	246	242	238	259
34	222	220	216	213	210	207	223
36	191	190	188	185	183	180	193
38	165	164	163	161	159	157	166
40	143	142	141	139	138	137	144
42	123	123	122	121	120	119	124
44	106	105	105	104	103	102	106
46	90	90	89	89	88	88	91
48	76	76	76	75	75	75	77
50	64	64	63	63	63	63	64
52	52	52	52	52	52	52	53
54	42	42	42	42	42	41	42
56	32	32	32	32	32	32	33
58	24	24	24	24	23	23	24
60	15	15	15	15	15	15	15

**Axial Compression Load to Reach Full Plastic Section
with Initial Out-of-Straightness, δ_o , for
Fixed-Free End Condition**

$k = 2$

$a = 0.3$

j = 2 Fixed-guided end support condition

$$K_p = 1.372$$

$$k = K_j \quad a = \alpha_j$$

$$S_y = 76000 \cdot \frac{\text{lbf}}{\text{in}^2}$$

$$k = 1 \quad a = 0.5$$

$$L_{\text{start}} := 14 \text{ ft} \quad L_{\text{end}} := 60 \text{ ft} \quad L_{\text{inc}} := 2 \text{ ft}$$

$$N := \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 23$$

$$n := 1..N \quad L_n := (L_{\text{start}} + n \cdot L_{\text{inc}})$$

L_n	$P(L_n, k, a, 2\text{-in})$	$P(L_n, k, a, 4\text{-in})$	$P(L_n, k, a, 8\text{-in})$	$P(L_n, k, a, 12\text{-in})$	$P(L_n, k, a, 16\text{-in})$	$P(L_n, k, a, 20\text{-in})$	$P_E(L_n, k, a)$
ft	lbf	lbf	lbf	lbf	lbf	lbf	lbf
16	4008	3495	2788	2323	1992	1744	4707
18	3264	2917	2409	2054	1791	1588	3709
20	2699	2458	2088	1816	1607	1442	2994
22	2262	2091	1817	1608	1442	1307	2464
24	1917	1793	1588	1426	1294	1185	2060
26	1641	1550	1395	1268	1163	1074	1745
28	1417	1349	1230	1131	1046	974	1494
30	1233	1181	1089	1010	943	883	1290
32	1080	1040	968	905	850	802	1123
34	951	920	863	813	768	729	984
36	841	817	772	731	695	662	867
38	747	728	692	659	630	603	767
40	665	650	621	595	571	549	682
42	594	582	559	538	518	499	607
44	532	522	504	486	470	455	542
46	477	469	454	440	426	414	485
48	428	422	409	398	387	377	435
50	384	379	369	360	351	342	390
52	345	341	333	325	318	311	349
54	309	306	299	293	287	282	313
56	277	274	269	264	259	255	280
58	248	245	241	237	233	229	250
60	221	219	216	212	209	206	222

**Axial Compression Load to Reach Full Plastic Section
with Initial Out-of-Straightness, δ_o , for
Fixed-Guided End Condition** $k=1$ $a=0.5$

j = 3 Fixed-pinned end support condition

$$K_p = 1.372$$

$$k = K_j \quad a = \alpha_j$$

$$S_y = 76000 \cdot \frac{\text{lbf}}{\text{in}^2}$$

$$k = 0.707 \quad a = 0.5$$

$$L_{\text{start}} = 14\text{-ft} \quad L_{\text{end}} = 60\text{-ft} \quad L_{\text{inc}} = 2\text{-ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 23$$

$$n = 1..N \quad L_n = (L_{\text{start}} + n \cdot L_{\text{inc}})$$

L_n	$P(L_n, k, a, 2\text{-in})$	$P(L_n, k, a, 4\text{-in})$	$P(L_n, k, a, 8\text{-in})$	$P(L_n, k, a, 12\text{-in})$	$P(L_n, k, a, 16\text{-in})$	$P(L_n, k, a, 20\text{-in})$	$P_E(L_n, k, a)$
ft	lbf	lbf	lbf	lbf	lbf	lbf	lbf
16	6929	5512	3936	3069	2517	2135	9448
18	5811	4784	3550	2829	2353	2015	7455
20	4916	4163	3197	2600	2192	1896	6028
22	4196	3636	2878	2385	2037	1779	4971
24	3611	3190	2592	2185	1890	1665	4167
26	3133	2812	2337	2001	1751	1556	3540
28	2737	2490	2111	1833	1621	1453	3042
30	2408	2215	1910	1680	1500	1355	2639
32	2130	1978	1732	1540	1388	1263	2309
34	1895	1774	1573	1414	1284	1176	2034
36	1693	1596	1432	1299	1189	1096	1804
38	1520	1441	1306	1195	1101	1021	1608
40	1369	1305	1194	1100	1020	951	1440
42	1238	1185	1093	1014	945	886	1295
44	1122	1079	1002	935	876	825	1169
46	1020	984	920	863	813	769	1059
48	930	900	845	797	754	716	961
50	849	824	778	737	700	667	875
52	776	755	717	682	650	621	798
54	711	693	660	631	604	579	729
56	651	637	609	584	560	539	667
58	598	585	562	540	520	502	611
60	548	538	518	500	483	467	559

**Axial Compression Load to Reach Full Plastic Section
with Initial Out-of Straightness, δ_o , for
Fixed-Pinned End Condition**

$k = 0.707$ $a = 0.5$

j := 4 Fixed-fixed end support condition

$$K_p = 1.372$$

$$k := K_j \quad a := a_j$$

$$S_y = 76000 \cdot \frac{\text{lbf}}{\text{in}^2}$$

$$k = 0.5 \quad a = 0.5$$

$$L_{\text{start}} = 14 \text{ ft} \quad L_{\text{end}} = 60 \text{ ft} \quad L_{\text{inc}} = 2 \text{ ft}$$

$$N = \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 23$$

$$n := 1..N \quad L_n := (L_{\text{start}} + n \cdot L_{\text{inc}})$$

L_n	$P(L_n, k, a, 2\text{-in})$	$P(L_n, k, a, 4\text{-in})$	$P(L_n, k, a, 8\text{-in})$	$P(L_n, k, a, 12\text{-in})$	$P(L_n, k, a, 16\text{-in})$	$P(L_n, k, a, 20\text{-in})$	$P_E(L_n, k, a)$
ft	lbf	lbf	lbf	lbf	lbf	lbf	lbf
16	10720	7661	4928	3644	2894	2400	18921
18	9376	6949	4620	3472	2783	2324	14940
20	8209	6288	4317	3297	2669	2243	12091
22	7205	5684	4023	3122	2553	2160	9982
24	6347	5139	3742	2949	2436	2076	8377
26	5613	4649	3476	2781	2320	1991	7127
28	4986	4213	3226	2619	2206	1906	6135
30	4449	3824	2994	2464	2095	1822	5333
32	3986	3478	2778	2316	1987	1740	4677
34	3585	3170	2578	2175	1883	1660	4132
36	3238	2896	2395	2043	1783	1581	3675
38	2934	2651	2225	1919	1687	1506	3287
40	2668	2433	2069	1802	1596	1433	2956
42	2434	2237	1926	1692	1510	1363	2670
44	2226	2060	1794	1590	1428	1296	2422
46	2042	1902	1673	1494	1350	1231	2205
48	1877	1758	1561	1404	1276	1170	2014
50	1730	1629	1458	1320	1207	1111	1845
52	1597	1511	1363	1242	1141	1055	1695
54	1478	1403	1275	1168	1079	1002	1561
56	1369	1305	1194	1100	1020	951	1440
58	1271	1215	1118	1035	964	902	1331
60	1181	1133	1048	975	912	856	1233

**Axial Compression Load to Reach Full Plastic Section
with Initial Out-of Straightness, δ_o , for
Fixed-Fixed End Condition**

$k = 0.5$ $a = 0.5$

j = 5 Pinned-pinned end support condition

$K_p = 1.372$

$k := K_j$ $a := \alpha_j$

$S_y = 76000 \cdot \frac{\text{lb} \cdot \text{ft}}{\text{in}^2}$

$k = 1$ $a = 0.5$

$L_{\text{start}} := 14 \cdot \text{ft}$ $L_{\text{end}} := 60 \cdot \text{ft}$ $L_{\text{inc}} = 2 \cdot \text{ft}$

$N := \frac{L_{\text{end}} - L_{\text{start}}}{L_{\text{inc}}} \quad N = 23$

$n := 1..N$ $L_n := (L_{\text{start}} + n \cdot L_{\text{inc}})$

L_n	$P(L_n, k, a, 2 \cdot \text{in})$	$P(L_n, k, a, 4 \cdot \text{in})$	$P(L_n, k, a, 8 \cdot \text{in})$	$P(L_n, k, a, 12 \cdot \text{in})$	$P(L_n, k, a, 16 \cdot \text{in})$	$P(L_n, k, a, 20 \cdot \text{in})$	$P_E(L_n, k, a)$
ft	lbf	lbf	lbf	lbf	lbf	lbf	lbf
16	4008	3495	2788	2323	1992	1744	4707
18	3264	2917	2409	2054	1791	1588	3709
20	2699	2458	2088	1816	1607	1442	2994
22	2262	2091	1817	1608	1442	1307	2464
24	1917	1793	1588	1426	1294	1185	2060
26	1641	1550	1395	1268	1163	1074	1745
28	1417	1349	1230	1131	1046	974	1494
30	1233	1181	1089	1010	943	883	1290
32	1080	1040	968	905	850	802	1123
34	951	920	863	813	768	729	984
36	841	817	772	731	695	662	867
38	747	728	692	659	630	603	767
40	665	650	621	595	571	549	682
42	594	582	559	538	518	499	607
44	532	522	504	486	470	455	542
46	477	469	454	440	426	414	485
48	428	422	409	398	387	377	435
50	384	379	369	360	351	342	390
52	345	341	333	325	318	311	349
54	309	306	299	293	287	282	313
56	277	274	269	264	259	255	280
58	248	245	241	237	233	229	250
60	221	219	216	212	209	206	222

**Axial Compression Load to Reach Full Plastic Section
with Initial Out-of Straightness, δ_o , for
Pinned-Pinned End Condition**

$k = 1$ $a = 0.5$

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