ANALYSES OF INTERFACIAL SHEAR DEBONDING IN FIBER-REINFORCED CERAMIC COMPOSITES

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An important toughening mechanism in fiber-reinforced ceramic composites is pullout of fibers from the matrix during matrix cracking. This relies on mode II (i.e., shear) debonding at the fiber/matrix interface which can be analyzed using either the strength-based or the energy-based criterion. In the strength-based approach, debonding occurs when the maximum interfacial shear stress induced by the applied load reaches the interfacial shear strength, $\tau_s$. In the energy-based approach, a mode II crack propagating along the interface is considered, and debonding occurs when the energy release rate due to crack propagation reaches the interface debond energy, $\Gamma_I$. Based on the above two criteria, the applied stress on the fiber to initiate debonding (i.e., the initial debond stress), $\sigma_d$, can be derived. The first issue considered in the present study is the relation between $\tau_s$ and $\Gamma_I$. Also, for a monolithic ceramic, the tensile strength can be related to its defect size based on the Griffith theory [1]. A question is hence raised as to whether the initial debond stress for fiber pullout in a fiber-reinforced ceramic composite can be related to any defect at the interface.

Considering two semi-infinite elastic materials bonded at the interface, the crack propagation problem has been analyzed by He and Hutchinson [2]. When a crack reaches the interface, the crack either deflects along the interface or penetrates into the next layer depending upon the ratio of the energy release rate due to debonding to that due to crack penetration. This criterion [2] has been used extensively to predict interfacial debonding versus fiber fracture for a crack propagating in a fiber-reinforced ceramic composite. However, the crack propagation problem in fiber-reinforced composites is three-dimensional. For an embedded fiber of a finite radius, there are three options when a matrix crack reaches the interface: interface debonding, fiber fracture, or crack circumventing the fiber. The analysis by He and Hutchinson focuses on the case that the crack does not circumvent the fiber. However, when the crack circumvents the fiber, the crack is bridged by intact fibers, and the fiber-pullout geometry can be used to analyze this problem. Hence, the second issue considered in the present study is how the condition of interfacial debonding versus fiber fracture is modified for a bridging-fiber case.

An idealized fiber-pullout geometry is shown in Fig. 1a, in which a fiber with a radius, $a$, is embedded in a coaxial cylindrical shell of matrix with a radius, $b$, and is subjected to a tensile stress at one end in its axial direction. The concept of Griffith theory is adopted to derive the relation between $\tau_s$ and $\Gamma_I$ for the fiber-pullout geometry. In the Griffith theory, a monolithic ceramic subjected to a uniform tension is considered. Crack propagation occurs at the existing crack tip, and the tensile strength of the material can be related to the fracture energy and the crack size. In the fiber-pullout case, the fiber has different material properties from the matrix and is subjected to a tensile load. Debonding initiates at the circumference where the fiber enters the matrix. The

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Fig. 1. Schematic drawings showing (a) the fiber-pullout geometry, and (b) the effective circumferential defect introduced at the interface to account for the presence of the fiber in the matrix and the fiber-pullout geometry.
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stress intensity at this circumference is related to the fiber-pullout geometry. To account for this stress intensity, it is assumed that the presence of the fiber in the pullout case is equivalent to the introduction of an effective circumferential defect at the interface which extends from the surface to a depth $h$ (Fig. 1b). Also, similar to the Griffith theory, it is assumed that the effective defect is subjected to a uniform shear stress, $\tau$ (Fig. 1b). When $\tau$ reaches $\tau_s$, crack propagation occurs. Hence, $\tau_s$ can be related to $\Gamma_l$ and $h$.

The effective defect length, $h$, can be determined by equating $\sigma_d$ derived from the two debonding criteria, and the result is shown in Fig. 2. When the fiber and the matrix have similar Young's modulus (e.g., for ceramic composites), $h$ is in the order of the fiber radius. The normalized effective defect length, $h/a$, increases with an increase in either the Young's modulus ratio of fiber to matrix, $E_f/E_m$, or the radius ratio of matrix to fiber, $b/a$. For the material design, knowing the effective defect length in a fiber-reinforced ceramic composite is as useful as knowing the defect size in a monolithic ceramic.

![Fig. 2. The normalized effective defect length, $h/a$, as a function of $E_f/E_m$ for $\nu_f=\nu_m=0.25$.](image)

Using the energy-based criterion, the relation between $\sigma_d$ and $\Gamma_l$ has been defined. When $\sigma_d$ is greater than the fiber strength, $\sigma_s$, fiber fracture occurs prior to interfacial debonding. However, construction of the diagram of interfacial debonding versus fiber fracture needs not only $\Gamma_l$ but also the fiber fracture energy, $\Gamma_f$. To achieve this, the relation between $\sigma_s$ and $\Gamma_f$ is required. Combining the $\sigma_d-\Gamma_l$ and the $\sigma_s-\Gamma_f$ relations, a critical ratio for $\Gamma_l/\Gamma_f$ can be defined, such that interfacial debonding or fiber fracture occurs when $\Gamma_l/\Gamma_f$ is smaller or greater than the critical ratio (Fig. 3). Here, $c/a$ is the relative defect size in the fiber, $\lambda$ is the defect-geometry factor, and $\alpha$ is the Dundurs' parameter [3] defined by

$$\alpha = \frac{E_f(1-\nu_m^2) - E_m(1-\nu_f^2)}{E_f(1-\nu_m^2) + E_m(1-\nu_f^2)}$$

where $\nu$ is Poisson's ratio, and the subscripts, f and m, denote the fiber and the matrix, respectively. The critical ratio decreases with the increase in the Dundurs' parameter, $\alpha$. Also, the curve in Fig. 3 becomes flatter when the radius ratio of matrix to fiber, $b/a$, increases.

![Fig. 3. The diagram of interfacial debonding versus fiber fracture for the bridging-fiber geometry.](image)

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