Instantaneous Phasor Method under Severely Unbalanced Situations

John S. Hsu
Senior Member
Oak Ridge National Laboratory*
Post Office Box 2009, MS 8038
Oak Ridge, Tennessee 37831-8038

Key words: Instantaneous phasors, Method, Initial phasor, Balanced fundamental component, Power quality, Control, Continuous diagnostics, Severely unbalanced.

Abstract

The new instantaneous phasor method originated by the author for obtaining instantaneous balanced fundamental components is further studied under severely unbalanced situations. Selection of the initial and synchronization phasors becomes important. An example is presented. This technique may substitute for the fast Fourier transformation (FFT) plus symmetrical component method for active power quality control and for continuous diagnostics. The influence of high magnitudes of harmonics will be discussed in a separate paper.

I. INTRODUCTION

The benefit of knowing instantaneous balanced fundamental frequency components has been discussed [1]. The references of potential application areas such as the air-gap torque method, power quality, and instantaneous power components are listed [2-5]. This paper presents a further study of the instantaneous phasor method [1] originated by the author. For mildly polluted voltages and currents with harmonics and unbalance, such as the example given in [1], the use of instantaneous phasor method incurs negligible phasal error. However, the phasal error increases under severely polluted situations. This paper addresses the severely unbalanced situations. A separate paper will discuss the strong harmonic content situations.

The immediate history of fundamental balanced components of three-phase voltages or currents that have three equivalent equal root-mean-square (rms) values can be obtained through the classical fast Fourier transformation (FFT) plus symmetrical component method. Four steps are required. They are: (1) conducting FFT of the past-cycle waveforms for each phase individually; (2) shifting phase b and c by 120 and 240 degrees, respectively, in forward and backward directions to find their symmetrical components; (3) calculating values for each of the three phases; and (4) computing the equivalent rms value of three phases. This time-consuming classical approach is not the method introduced in this paper.

II. EXAMPLE OF A SEVERELY UNBALANCED SITUATION

Fig. 1 An example of severely unbalanced three-phase voltages

The instantaneous phasor method considers three phases simultaneously. This method produces the instantaneous fundamental balanced components of the polluted voltages or currents. Fig. 1 shows a set of severely unbalanced three-phase fundamental-frequency voltages.

This example shows that under severely unbalanced situations the voltage magnitudes differ a great deal, and the phase displacements are very different from the normal 120 degrees.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
III. EQUATIONS OF INSTANTANEOUS PHASOR METHOD

The instantaneous phasors derived in [1] can either be presented in a vector format or in a complex number format. The arbitrarily chosen complex number format of the instantaneous phasors, \( V_a, V_b, \) and \( V_c \), are given in equation (1). They are 120 degrees apart from each other.

\[
\begin{align*}
V_a &= (v_a - v_0) + jv_{aq}, \\
V_b &= (v_b - v_0) + jv_{bq}, \text{ and} \\
V_c &= (v_c - v_0) + jv_{cq},
\end{align*}
\]

where, the zero-sequence component for the three-phase voltages is

\[
v_0 = \frac{1}{3}(v_a + v_b + v_c)
\]

(2)

The real values of the instantaneous phasors are simply the instantaneous phase values without the zero-sequence component as given in equation (3).

\[
\begin{align*}
(v_a - v_0), \\
(v_b - v_0), \text{ and} \\
(v_c - v_0).
\end{align*}
\]

(3)

The instantaneous phasors’ imaginary values denoted as \( v_{aq}, v_{bq}, \) and \( v_{cq} \) can be obtained from equation (4).

\[
\begin{align*}
v_{aq} &= \frac{v_b - v_c}{\sqrt{3}}, \\
v_{bq} &= \frac{v_c - v_a}{\sqrt{3}}, \text{ and} \\
v_{cq} &= \frac{v_a - v_b}{\sqrt{3}}.
\end{align*}
\]

(4)

The numerators of equation (4) are actually the instantaneous line to line voltages that are not affected by the zero-sequence component.

The instantaneous phasor magnitude, \( V \), of \( V_a, V_b, \) and \( V_c \) can be derived from equations (4), (3), and (2). The result is that

\[
\|V_a\| = \|V_b\| = \|V_c\| = V,
\]

(5)

where

\[
V = \sqrt{\frac{2}{9}} \left[ (v_a - v_b)^2 + (v_b - v_c)^2 + (v_c - v_a)^2 \right].
\]

(6)

The meaning of \( V \) can be seen by rearranging equation (6).

\[
V = \sqrt{\frac{2}{3}} \sqrt{\frac{(v_a - v_b)^2 + (v_b - v_c)^2 + (v_c - v_a)^2}{3}}.
\]

(7)

It is the equivalent phase magnitude of the three-phase line-to-line voltages averaged in an rms manner.

IV. INSTANTANEOUS PHASORS OF UNBALANCED VOLTAGES OR CURRENTS

The trajectories of instantaneous phasors, \( V_a, V_b, \) and \( V_c \), of severely unbalanced three phase voltages

The trajectories of instantaneous phasors, \( V_a, V_b, \) and \( V_c \), of equation (1) for the sample of severely unbalanced voltages, given in Fig. 1, are shown in Fig. 2, where the horizontal axis is for the real portion and the vertical axis is for the imaginary portion (or x and y axes if one prefers not to use imaginary numbers).

For polluted voltages and currents, the instantaneous phasor trajectories, \( V_a, V_b, \) and \( V_c \), are not circles, and they are not rotating at a constant speed because of harmonics and negative-sequence contents. However, the trajectories of phase b and c are the same as phase a except that they are shifted 120 and 240 degrees, respectively.
V. FIRST AND PAST CYCLE INFORMATION

1. Phase Sequence From First Cycle
   The phase sequence of supply voltages normally would not be changed during a continuous operation. It is obtained from the first cycle and can be used throughout the operation. The sequence of positive or negative peak magnitudes of each phase voltage is sensed [1].

2. Instantaneous Average Magnitude of Phasor
   The term “fundamental” inherently relates to periodic quantities. Therefore, the instantaneous fundamental components correspond to the immediate history of the fundamental components of the past cycle with the adjustment of the present data points. The magnitude and the additional synchronization for severely polluted cases are constantly adjusted.

   After passing the first cycle, whenever a new instantaneous phasor is acquired the corresponding phasor data of a cycle earlier are dropped, so that the average phasor magnitude reflects the immediate change of the fundamental-frequency component. Detailed equations for implementation are given [1].

VI. PHASAL RELATIONSHIP BETWEEN INSTANTANEOUS PHASOR AND FUNDAMENTAL-FREQUENCY POSITIVE-SEQUENCE PHASOR

   In order to synchronize the output of this method with the fundamental-frequency positive-sequence component, the phasal relationship between the instantaneous phasor and the fundamental-frequency positive-sequence phasor must be understood.

   The example given in Fig. 1 contains severely unbalanced fundamental-frequency components. The instantaneous phasor of phase-a calculated from equation (1) is plotted in Fig. 3. The phasor's magnitude can be viewed as the summation of the positive-sequence and the negative-sequence components that are equivalently rotating in opposite directions.

   Fig. 4 shows that when the positive-sequence component and the negative-sequence component are coincided in the same direction, the instantaneous phasor has the maximum magnitude and is in phase with the positive-sequence component. This will be used as an indicator for lining up the proper phasal relationship.

   Fig. 5 shows that when the positive-sequence component and the negative-sequence component are coincided in opposite directions, the instantaneous phasor has the minimum magnitude and is again in phase with the positive-sequence component. This will be used as an alternative indicator for lining up the proper phasal relationship.
The maximum and minimum magnitudes of instantaneous phasors are detected and used to synchronize the phasal relationship between the output and the fundamental-frequency positive-sequence component.

The trajectory of adjusted instantaneous phasor \( V_{a\,adj} \) corresponding to phase-a voltage, \( v_a \), shown in Fig. 1 is illustrated in Fig. 7.

In order to modify the instantaneous phasor, certain first-cycle information described in [1] is needed. Subsequent instantaneous data readings will instantaneously either modify or retain the first-cycle information.

VII. OBTAINING INSTANTANEOUS FUNDAMENTAL COMPONENTS THROUGH ADJUSTED PHASORS

1. Adjusted Instantaneous Phasors of Unbalanced Voltages or Currents

The instantaneous phasors of unbalanced voltages given in equation (1) are adjusted to give balanced phasors.

\[
V_{a\,adj} = V_a \frac{V_{ave}}{V}, \quad V_{b\,adj} = V_b \frac{V_{ave}}{V}, \quad V_{c\,adj} = V_c \frac{V_{ave}}{V}.
\]  

The maximum and minimum points of the magnitude of instantaneous phasors of the example given in Fig. 1. After the first cycle, the maximum phasor magnitudes occur at 8.639, 11.781, and 14.923 radians of time, respectively. A separate paper will introduce additional steps for obtaining initial and synchronization phasors when voltages have a strong harmonic content.

Fig. 5 When the positive-sequence component and the negative-sequence component are coincided in opposite directions, the instantaneous phasor has the minimum magnitude and is again in phase with the positive-sequence component.

Fig. 6 The maximum and minimum points of the magnitude of instantaneous phasors of the example given in Fig. 1.
The nearly circular-shaped trajectory reveals that the adjusted magnitude of the instantaneous phasor is quite uniform. However, because of the negative-sequence component the adjusted instantaneous phasor shown in Fig. 7 is not rotating at the fundamental-frequency synchronous speed at all times. It is important to synchronize the adjusted instantaneous phasor with the phasor of the positive-sequence fundamental frequency component.

2. Instantaneous Balanced Three-Phase Fundamental Components

Once the phase sequence, which was detected in the first cycle, the instantaneous average magnitude of the phasor, and the initial phasors corresponding to the instantaneous phasors with maximum or minimum magnitudes are known, the instantaneous balanced three-phase fundamental components can be obtained through the equations given (1). The maximum and minimum points are used to synchronize the phasal relationship between the adjusted instantaneous phasors and the phasor of the positive-sequence fundamental frequency component as discussed earlier and illustrated in Figs. 3, 4, and 5.

VIII. COMPARISON OF RESULTS OBTAINED THROUGH INSTANTANEOUS PHASOR METHOD AND FOURIER PLUS SYMMETRICAL COMPONENT METHOD

Fig. 8 shows the comparison of balanced fundamental-frequency components of three phases obtained from the new instantaneous phasor method versus the traditional Fourier analysis plus symmetrical component method. For the traditional analysis, the positive sequence component was further adjusted to the equivalent phase magnitude of the three-phase, line-to-line voltages and averaged in an rms manner. For severely unbalanced phase voltages, the initial phasors must be used to synchronize the phasal relationship. If there is a question about unbalance, use the proper initial phasor. The discrepancies of phase and magnitude shown in Fig. 8 are practically negligible.

Fig. 9 shows the comparison between the positive-sequence component obtained from the traditional FFT plus symmetrical component method and the balanced fundamental-frequency equivalent phase magnitude of the three-phase, line-to-line voltages averaged in an rms manner obtained from the instantaneous phasor method. The magnitude ratio of the phase magnitude of the three-phase, line-to-line voltages averaged in an rms manner and the positive-sequence values is 1.062. It should be noted that the instantaneous phasor method can also provide the positive- and negative-sequence values based on the maximum and minimum magnitudes of the instantaneous phasors. They are practically the same as those obtained from the traditional Fourier analysis plus symmetrical component method.
X. CONCLUSIONS

The new and simple instantaneous phasor method to obtain instantaneous balanced fundamental-frequency voltages or currents is further studied for situations with severe unbalances. The potential applications of this technique are for power quality control and for continuous diagnostics.

Unlike the mildly polluted situations [1], when the voltages or currents are severely unbalanced, it is necessary to use the phasors with the maximum and the minimum magnitudes to initiate and to synchronize the output phase with the positive-sequence fundamental-frequency component.

The method is simple and provides sufficiently accurate results. It may replace the FFT plus symmetrical component method in many practical situations. Further study on situations of strong harmonics content will be discussed in a separate paper.

X. ACKNOWLEDGMENT

Encouragement from the Power Electronics Group headed by Mr. Donald Adams and the helpful discussions from Dr. John McKeever are gratefully acknowledged.

XI. REFERENCES


Dr. John S. Hsu (M’61, SM’89) worked with the Emerson Electric Company, Westinghouse Electric Corporation, and later with the University of Texas at Austin. He is currently a Senior Staff Engineer at the Oak Ridge National Laboratory. He has published nearly forty refereed papers and over one hundred technical publications. He holds eight patents in rotating machines and power electronics.

*Prepared by the Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, managed by Lockheed Martin Research Corp. for the U. S. Department of Energy under contract DE-AC05-96OR22464.

The submitted manuscript has been authored by a contractor of the U. S. Government under contract No. DE-AC05-96OR22464. Accordingly, the U. S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U. S. Government purposes.