4
DOE/OR/00033--T 786

UCLA-FNT-46 DOE/OR/00033--T78

Development of an Analytic Core Flow Approximation for a Square Duct in an Oblique Magnetic Field

March 1991

Neil B. Morley, Mark S. Tillack,
and Mohamed A. Abdou

Fusion Technology Program
University of California, Los Angeles
Los Angeles, CA 90024

UCLA-FNT-46 **DOE/OR/00033-T786**

Development of an Analytic Core Flow Approximation for a Square Duct in an Oblique Magnetic Field

March 1991

Neil B. Morley, Mark S. Tillack,
and Mohamed A. Abdou

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

I. INTRODUCTION

The "core flow" approximation for liquid metal (LM), magnetohydrodynamic (MHD) duct flow is a method that ignores the contributions of viscous forces in the fluid. For a fully developed, steady state flow situation, this approximation leaves the magnetic forces to be balanced only by the pressure gradient and results in a greatly simplified momentum equation. The velocity field predicted by the core flow equations is obtained much more easily than that described by the "full" solution, which usually requires a numerical approach. For this reason it is desirable to use the core flow method for flow situations in which viscosity has little effect. Developed here is an analytic core flow solution for a square duct in an obliquely incident magnetic field which omits any special treatment of boundary layers. This solution is compared to the full solution method developed in the code MH2D¹ and a parametric comparison is performed.

This general analytic approach can be expanded to consider rectangular ducts or walls of different thickness and electrical conductivity. The latter, however, will greatly complicate the equations presented here.

19980507 003

THIS DOCUMENT IS UNCLASSIFIED

II. CORE FLOW SOLUTION

The core flow fluid equations are the starting point in this effort to determine the current, potential, and velocity field of a liquid metal flowing through a square duct in the presence of an obliquely incident magnetic field. These equations are:

$$\nabla p = \underline{J} \times \underline{B} \quad \text{Momentum Eq.} \quad (1)$$

$$\nabla \cdot \underline{V} = 0 \quad \text{Mass Conservation Eq.} \quad (2)$$

$$\nabla \cdot \underline{J} = 0 \quad \text{Current Conservation Eq.} \quad (3)$$

$$\underline{J} = \sigma_f(-\nabla \phi + \underline{V} \times \underline{B}) \quad \text{Ohm's Law} \quad (4)$$

where p is the pressure, $\underline{J}$ is the current density, $\underline{B}$ is the magnetic field, $\underline{V} = (u,v,w)$ is the velocity, σ is the electrical conductivity, and ϕ is the electric potential. Notice that these equations are for the steady state case and the field induced by the currents is neglected.

The duct geometry for the problem solved here is given in figure 1 below.

Figure 1. Duct Geometry

For the fully developed flow situation both u and v , as well as J_z , are zero. It can be shown that by expanding the curl of the momentum equation that $(\underline{B} \cdot \nabla)\underline{J} = 0$. From this is deduced the fact that J_x and J_y are not functions of y , since $\underline{B} = (0, B, 0)$ only has a component in the y direction. Using the fact of no y dependence in the current conservation equation then tells us that J_x is not a function of x either, and is thus constant throughout the duct. Ohm's Law in the y direction is

$$J_y = -\sigma_f \frac{\partial \phi}{\partial y} \quad (5)$$

If this is integrated over y from the left wall to the point y in the fluid, it is seen that

$$\frac{J_y \delta(x, y)}{\sigma_f} = -(\phi(x, y) - \phi_L),$$

or

$$\phi(x, y) = g(x) - \frac{\delta(x, y) \cdot f(x)}{\sigma_f} \quad (\phi(x, y) = g(x) - \delta(x, y) \cdot f(x)) \quad (6)$$

where $f(x)$ is defined as J_y , $g(x)$ is defined as the potential along the left wall ϕ_L , and the quantity in brackets is the nondimensional form of the equation (this will be discussed more later). This quantity g may seem to be a function of both x and y but by specifying the left wall only, x and y are no longer independent. It is chosen as a function of x (instead of y) for convenience. Lastly, we can rearrange the x component of Ohm's Law to get the following expression for the velocity distribution:

TABLE I. - Summary of Core Approximations

$$\begin{aligned} J_x &\equiv J_{x0} \text{ is a constant} \\ J_y &\equiv f(x) \text{ is a function of } x \text{ only} \\ J_z = u = v &= 0, \text{ fully developed flow} \\ \phi(x, y) &= g(x) - \frac{\delta(x, y) \cdot f(x)}{\sigma_f} \text{ where } \phi_L \equiv g(x), \text{ a function of } x \text{ only} \\ w(x, y) &= -\frac{1}{B_y} \left(\frac{\partial \phi}{\partial x} + \frac{J_x}{\sigma_f} \right) \end{aligned}$$

$$w(x,y) = -\frac{1}{B_y} \left(\frac{\partial \phi}{\partial x} + \frac{J_x}{\sigma_f} \right) \quad \left(w(x,y) = - \left(\frac{\partial \phi}{\partial x} + J_x \right) \right) \quad (7)$$

The results obtained from the core flow equations are summarized in Table I.

From above we see that it is possible to find the velocity distribution provided the functions $f(x)$ and $g(x)$ are known. To do this we need to impose electric boundary conditions at the left and right walls and use the information given about f and g in Table I to determine their behavior inside the duct. The thin wall boundary condition²

$$\sigma_w \frac{\partial^2 \phi_w}{\partial s^2} = \frac{J \cdot \underline{n}}{t}$$

where $\underline{n}$ is the unit vector normal to the wall and directed into the fluid, s is the distance along the wall, and t is the wall thickness; will accomplish this purpose. The normals and derivatives are given in Table II.

TABLE II. - Normals and Derivatives at the Walls.

	Left wall	Right wall	Separation
Region I	$\underline{n}_L = \underline{x} \sin \theta + \underline{y} \cos \theta$ $\frac{\partial}{\partial s} = \cos \theta \frac{\partial}{\partial x}$	$\underline{n}_R = \underline{x} \cos \theta - \underline{y} \sin \theta$ $\frac{\partial}{\partial s} = \sin \theta \frac{\partial}{\partial x}$	$\delta_{L,R} = \frac{x}{\sin \theta \cos \theta}$
Region II	$\underline{n}_L = \underline{x} \sin \theta + \underline{y} \cos \theta$ $\frac{\partial}{\partial s} = \cos \theta \frac{\partial}{\partial x}$	$\underline{n}_R = -\underline{x} \sin \theta - \underline{y} \cos \theta$ $\frac{\partial}{\partial s} = \cos \theta \frac{\partial}{\partial x}$	$\delta_{L,R} = \frac{a}{\cos \theta}$

Using these definitions and the fact that the potential on the right wall can be found in terms of f and g by using the δ 's from Table II and equation 6, it is possible to construct from the thin wall boundary condition a coupled set of differential equations for f and g in each region. Nondimensionalizing the variables with

$$J = J^* \sigma_f V_0 B, \quad \phi = \phi^* a V_0 B, \quad \text{and} \quad x = x^* a,$$

the above mentioned equations take on the form

Region I

$$\frac{d^2g}{dx^2} = \frac{1}{\Phi \cos\theta} (J_{x0} \tan\theta + f),$$

$$\frac{d^2g}{dx^2} - \frac{1}{\sin\theta \cos\theta} \left(2 \frac{df}{dx} + x \frac{d^2f}{dx^2} \right) = \frac{1}{\Phi \sin\theta} \left(\frac{J_{x0}}{\tan\theta} - f \right) \quad (8,9)$$

Region II

$$\frac{d^2g}{dx^2} = \frac{1}{\Phi \cos\theta} (J_{x0} \tan\theta + f),$$

$$\frac{d^2g}{dx^2} - \frac{1}{\cos\theta} \frac{d^2f}{dx^2} = \frac{-1}{\Phi \cos\theta} (J_{x0} \tan\theta + f) \quad (10,11)$$

where $\Phi \equiv \frac{\sigma_{wt}}{\sigma_{fa}}$ is defined as the wall conductance ratio, and the *'s have been dropped for

convenience (see appendix I for a more detailed discussion of this development).

The construction of a solution for these DE's requires that boundary conditions for f and g be given at $x=0$ and $x=x_m$. Also, some sort of matching conditions are needed at the region boundary. Since the potential can be specified at any one point in the duct, it is convenient to set it to zero at x (and y) = 0. From this we can say that $\lim_{x \rightarrow 0} \phi_{L,R} = 0$. Expanding this out gives two conditions at $x = 0$.

$$\lim_{x \rightarrow 0} \phi_L = g(x=0) = 0 \text{ (bc1)}$$

$$\lim_{x \rightarrow 0} \phi_R = \lim_{x \rightarrow 0} (g - \delta_R f) = \lim_{x \rightarrow 0} (xf) = 0 \quad (bc2)$$

At the mid-plane, $x = x_m$, a symmetry condition exists for the current such that $\frac{\partial f}{\partial x} = 0$

(bc3). Another condition here is conservation of current. All the current going up through the mid-plane in the fluid, must return through the walls. This condition is expressed by

$$\frac{J_{x0}}{\Phi \cos\theta} - \cos\theta \left(\frac{d\phi_L}{dx} + \frac{d\phi_R}{dx} \right)_{x=x_m} = 0$$

or substituting for $\phi_{L,R}$;

$$2\cos^2\theta \frac{dg}{dx} - \frac{J_{x0}}{\Phi} = 0 \quad \text{at } x = x_m. \quad (\text{bc4})$$

These conditions are augmented by continuity conditions at the boundary interface, $x = x_1$. At this interface the potentials and the currents in the left and right walls are assumed to be continuous. This fact gives three simple conditions;

$$g_I = g_{II}, \quad \frac{dg_I}{dx} = \frac{dg_{II}}{dx}, \quad \text{and } f_I = f_{II}; \quad (\text{bc5,6,7})$$

and one more complicated condition;

$$(\sin\theta - \cos\theta) \frac{dg_{I \text{ or } II}}{dx} = \frac{1}{\cos\theta} \frac{d(xf)}{dx} - \frac{df_{II}}{dx}. \quad (\text{bc8})$$

These eight conditions and equations 8-11 completely describe the functions f and g and can be solved analytically.

The solution of these equations is somewhat long but generally straightforward. In each region it is possible to substitute the first equation into the second in order to eliminate g . The resulting equation contains only f and can be solved. This result is then substituted back into the first equation, which is now solved for g . The form of the solution and the four constants of integration associated with it will be different in each region. The integration constants are then determined through the application of the eight boundary conditions given above. These solutions for f and g are given on the following page where I_n denotes the modified Bessel function of the first kind of order n . The C constants are groupings that appear after the substitutions described above are executed, while the A 's are constants of integration that were too lengthy to be put into the solutions directly. Notice that the A 's are solved for in terms of each other, but if they are evaluated in the order given, then all information necessary has already been computed.

Region I

$$f = A_5 \frac{I_1(2\sqrt{C_2 x})}{\sqrt{x}} - \frac{C_3}{C_2}, \quad (12)$$

$$g = \frac{A_5}{\Phi C_2 \cos \theta} \sqrt{x} I_1(2\sqrt{C_2 x}) + \frac{C_4 x^2}{2} + A_7 x \quad (13)$$

Region II

$$f = A_2 \cosh[C_1(x_m - x)] - J_{x0} \tan \theta, \quad (14)$$

$$g = \frac{1}{2 \cos \theta} \left(A_2 \cosh[C_1(x_m - x)] + \frac{J_{x0} x}{\Phi \cos \theta} \right) + A_4, \quad (15)$$

Constants

$$C_1 = \sqrt{\frac{2}{\Phi}}$$

$$C_2 = \frac{1}{\Phi} (\sin \theta + \cos \theta)$$

$$C_3 = \frac{J_{x0}}{\Phi} \left(\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right)$$

$$C_4 = \frac{J_{x0}}{\Phi \sin \theta \cos \theta (\sin \theta + \cos \theta)}$$

$$A_5 = \frac{\frac{C_3}{C_2} + \frac{J_{x0}}{2\Phi \cos \theta} (\sin \theta - \cos \theta) - \frac{C_1 J_{x0} \tanh[C_1(x_m - x_1)]}{2 \sin \theta}}{\sqrt{C_2} I_0(2\sqrt{C_2 x_1}) + C_1 I_1(2\sqrt{C_2 x_1}) \cdot \tanh[C_1(x_m - x_1)] \cdot \frac{x_m}{x_1}}$$

$$A_2 = \frac{1}{\cosh[C_1(x_m - x_1)]} \left(\frac{A_5 I_1(2\sqrt{C_2 x_1})}{\sqrt{x_1}} + \frac{J_{x0}}{\sin \theta (\sin \theta + \cos \theta)} \right)$$

$$A_7 = -\frac{A_2 C_1 \sinh[C_1(x_m - x_1)]}{2 \cos \theta} + \frac{J_{x0}}{\Phi \cos \theta} \left(\frac{1}{2 \cos \theta} - \frac{x_1}{\sin \theta (\sin \theta + \cos \theta)} \right) + \frac{A_5 I_0 (2 \sqrt{C_2 x_1})}{\Phi \sqrt{C_2} \cos \theta}$$

$$A_4 = \frac{A_5 \sqrt{x_1} I_1 (2 \sqrt{C_2 x_1})}{\Phi C_2 \cos \theta} + \frac{J_{x0} x_1^2}{2 \Phi \sin \theta \cos \theta (\sin \theta + \cos \theta)} - \frac{1}{2 \cos \theta} \left(A_2 \cosh[C_1(x_m - x_1)] + \frac{J_{x0} x_1}{\Phi \cos \theta} \right) + A_7 x_1$$

An important feature of the solution is that all terms are multiplied by J_{x0} . For this reason it can be selected to normalize the solution. A standard normalization is to set the average velocity equal to one. The average velocity is defined as

$$\langle w \rangle = \frac{2}{A} \int_0^{x_m} \int_{\text{left}}^{\text{right}} w \, dy \, dx = \frac{2}{A} \int_0^{x_m} \int_{\text{left}}^{\text{right}} - \left(\frac{\partial \phi}{\partial x} + J_{x0} \right) dy \, dx$$

where A is the cross-sectional area of the duct (in our non-dimensional case this is one), and where the nondimensionalized form of equation 7 was used to substitute for the velocity. Using the nondimensionalized equation 6 and equations 12,13,14, and 15; the above integral can be computed and is given as

$$\langle w \rangle = -2J_{x0} - \frac{2g_1(x=x_m)}{\cos \theta} + \frac{f_1(x=x_m)}{\cos^2 \theta} + \frac{1}{\sin \theta \cos \theta} \left(A_5 \left(\frac{\tan \theta}{2x_m} - \cos \theta \right) \frac{I_2(2\sqrt{C_2 x_1})}{\sqrt{C_2}} + x_1^2 \cdot \left(A_7 + \frac{C_3}{C_2 \tan \theta} + \frac{J_{x0}}{6\Phi x_m \cos \theta} \right) \right) + \frac{2 \tan \theta}{\cos \theta} \left(\frac{A_2}{C_1} \sinh[C_1(x_m - x_1)] - J_{x0} \tan \theta (x_m - x_1) \right) \quad (16)$$

The normalization factor is easily found by evaluating $\langle w \rangle$ with $J_{x0} = 1$. Then all subsequent velocity solutions should be multiplied by this factor, $nf = \frac{1}{\langle w \rangle}$.

Thus we have constructed a solution to the core flow equations that will yield the current, potential, and velocity profiles throughout the bottom half duct. The top half can be found through symmetry giving a complete solution throughout the entire duct..

III. COMPARISON TO FULL SOLUTION

As stated earlier the core flow approximation must be checked against the full solution in order to find its regime of applicability. The full solution, fully developed code MH2D will be used as the basis of comparison. It has proved to be accurate in a large number of geometries and for a variety of parameter ranges. A short code that evaluates the the core flow solution constructed from the nondimensional form of equation 6 and the current and potential given in equations 12,13,14, and 15 will be used to generate the core approximation profiles. Both are plotted using "DISSPLA" graphics⁴.

Shown in figure 2 is a typical set of potential distributions in the fluid, where the last grid point is on the inner surface of the wall. The distributions shown here, and for the full solution as well (figure 3), are normalized by setting the nondimensional pressure gradient equal to one. A cosmetic change of the x and y axis of the core solution plots, in order to match that of the full solution, is implemented in the core program. Also the data is shifted upward so that the potential in the center of the duct is zero. This does not alter the solution, but does allow for an easier visual comparison.

At 45° the potential has the same parabolic distribution in both walls that is predicted by the full solution. The symmetry at 45° causes the same amount of the J_{x0} current to be collected in each wall and thus J_y is almost zero since there is no potential difference between the walls. As the angle is reduced the core solution still accurately models the potential. When the thickness of the classical side layer ($\sim Ha^{-1/2}$) becomes greater than the region I thickness, this is expected to break down. At zero angle the first grid point of the core flow solution will predict a parabolic potential distribution in the right wall consistent with the collection of the J_{x0} current, but the next grid point will jump discontinuously to a constant (in y) potential in the fluid. This response is the expected result that the potential

Figure 2. Core Solution Potential Profiles for $\Phi = 0.005$ and $\theta =$
A) 1°, B) 5°, C) 10°, D) 30°, and E) 45°.

Figure 3. Full Solution Potential Profiles for $Ha = 5000$, $\Phi = 0.005$, and $\theta =$ A) 1° , B) 5° , C) 10° , D) 30° , and E) 45° .

does not change along field lines for a core flow approximation with the magnetic field parallel to the right wall. The full solution predicts the potential to change smoothly from the parabolic wall shape to the constant fluid shape over the thickness of the side layer. Since the thickness of the side layer is generally small (for fusion) then it is difficult to distinguish between the two unless a fine grid is used close to the wall.

A typical set of velocity plots is shown in figure 4 and 5. From these plots we see the full solution is closely modelled by the core approximation when the angle of the field is relatively large. As the angle gets small, though, the electromagnetic shear layer is significantly influenced by the viscous forces tending to form a side layer. This side layer has a finite width and a parabolic shape that can not be modelled by the core flow approximation. The angle at which the shear layer becomes a side layer, and thus the angle at which the core flow solution becomes an increasingly bad representation of velocity, is estimated through the visual comparison of many sets of plots and is given in Table III as a function of Ha and Φ . The presence of a discontinuity in the velocity profile of the core solution is allowed due to the lack of viscosity and thus the lack of derivatives of $\underline{V}$ in the basic set of equations. This discontinuity, resulting directly from the boundary condition implementing continuity of current through the corner at $x = x_1$ (bc8), runs across the plotting grid and causes a sawtooth pattern to appear. Due to the finite number of grid points and the fact that the grid lines do not coincide with the discontinuity line (except at $\theta = 0$), this sawtooth pattern always appears to some extent, but it is not a feature of the solution, which is smooth.

The general trends of the angle of agreement can be discerned from Table III. As is well known, the core flow approximation is the correct full solution at infinite Hartmann number. Thus the trend for better agreement between the solutions with an increase in Ha is not surprising. In the general area Φ of order (0.01), increases in Φ also tend to enhance agreement. This seems to be largely due to an increased interaction of the field with the fluid.

Figure 4. Core Solution Velocity Profiles for $\Phi = 0.005$ and $\theta =$
A) 1° , B) 5° , C) 10° , D) 30° , and E) 45° .

Figure 5. Full Solution Velocity Profiles for $Ha = 5000$, $\Phi = 0.005$, and $\theta =$ A) 1° , B) 5° , C) 10° , D) 30° , and E) 45° .

TABLE III. - Approximate Angles of Agreement

Φ	Ha = 100	Ha = 1000	Ha $\geq$ 5000
0.05	angles very close to 45° only	at angle ~10° the peak velocity predicted by the core solution is within 25% of that predicted by the full solution. Greater than 10° even better.	same as Ha = 1000 but angle $\geq$ 8°
0.01	same as 0.05	angles of ~20° or greater give core solution peak velocities within 25% of full solution peak	same as Ha = 1000 but angles $\geq$ 10°

Although at small angles the velocity representation becomes increasingly worse, the flow quantity in the side layer takes on the parabolic shape consistent with the potential distributions shown in figure 2. Defining the flow quantity as

$$q(y) = \int_{\text{bottom wall}}^{x=1} V(x,y) dx$$

this function has been computed and plotted in figure 6. From this we see that the flow quantity in region I mimics the flow quantity associated with a parabolic side layer even at angles much smaller than those given for agreement of velocity profiles. *We conclude that the thickness and shape of the velocity profile at small angles is dictated by the viscosity, and thus can not be modeled by the core solution; but the flow quantity is determined by the electromagnetic forces and is predicted by a core solution.* This fact greatly adds to the usefulness of the core solution at small angles for applications where the flow quantity and not the complete velocity profile is all that is required.

Figure 6. Flow Quantity in Region I

IV. CONCLUSIONS

A core flow approximation method was developed from the basic set of core flow MHD equations and the thin wall boundary condition in an effort to describe a liquid metal flowing in a square duct in the presence of an obliquely incident magnetic field. The solution of the LM velocity profile was compared to that of the full solution algorithm MH2D. A parametric area of agreement for the velocity profiles was ascertained and described in Table III. This area, however, was obtained through a visual comparison of the surface plots of velocity generated for each solution and can only be considered a rough estimate. It does however provide a general guideline upon which a choice between the core flow approximation and the full solution can be made. Comparison of the potentials and the flow quantities show that the area of agreement for these quantities goes to even smaller angles, which may justify use of the core flow approximation in cases where the the correct velocity profile is not as essential as the flow quantity.

Since the direct evaluation involved in the core flow solution is considerably faster and not subject to the convergence and grid problems associated with numerical full solution algorithms in fusion parameter ranges, its use is certainly desirable whenever possible. A more detailed and quantitative comparison is necessary to further quantify the area of agreement, as well as an expansion of the core solution to more advanced duct geometries. We leave that to someone else.

A special thanks to Dr. Kathy McCarthy, formerly of UCLA, for her many discussions and helpful suggestions concerning this material; and to Hidetoshi Hashizume for the use of his code MH2D. This work was made possible by the generous aide of all the nice people at Oak Ridge Associated Universities/DOE, through a Magnetic Fusion Energy Technology Fellowship.

V. REFERENCES

1. J. Eggleston and M. Tillack, *MH2D User's Manual*; UCLA-FNT-45, 1991.
2. M. Tillack, A. Ying, and H. Hashizume, *The Effect of Magnetic Field Alignment on Heat Transfer in Liquid Metal Blanket Channels*; UCLA-FNT-24, 1989.
3. J. Shercliff, The Theory of Electromagnetic Flow Measurement; Cambridge University Press, 1962.
4. DISSPLA User's Manual; ISSCO Graphics Company.

Appendix I - Details of Equation Formulation.

The thin wall boundary condition $t\sigma_w \frac{\partial^2 \phi_w}{\partial s^2} = \frac{J \cdot \underline{n}}{t}$, is written at each wall and in each region using the definitions of $\underline{n}$ and $\partial/\partial s$ given in Table II. These four equations are

$$\begin{aligned}
 \text{Region I} \quad & t_L \sigma_{wL} \cos^2 \theta \frac{d^2 \phi_L}{dx^2} = J_{x0} \sin \theta + J_y \cos \theta \\
 & t_{RI} \sigma_{wRI} \sin^2 \theta \frac{d^2 \phi_{RI}}{dx^2} = J_{x0} \cos \theta - J_y \sin \theta \\
 \text{Region II} \quad & t_L \sigma_{wL} \cos^2 \theta \frac{d^2 \phi_L}{dx^2} = J_{x0} \sin \theta + J_y \cos \theta \\
 & t_{RII} \sigma_{wRII} \cos^2 \theta \frac{d^2 \phi_{RII}}{dx^2} = -J_{x0} \sin \theta - J_y \cos \theta
 \end{aligned}$$

where the pairs $t \cdot \sigma$ refer to the thickness and conductivity of the wall described by the subscripts. In order to retain the symmetry property at the midplane, the thickness and conductivity of opposite facing walls must be equal. These equations can be expanded by making the substitutions

$$\begin{aligned}
 \phi_L &= g(x) \\
 J_y &= f(x) \\
 \phi_{RI} &= g(x) - \delta_{L,RI} f(x) / \sigma_f \\
 \phi_{RII} &= g(x) - \delta_{L,RII} f(x) / \sigma_f
 \end{aligned}$$

where the $\delta_{L,R}$ denotes the distance from the left wall to the right wall in each separate region. For the case of the square duct, these δ 's are given in Table II. For a rectangular duct, however, they are easily computed from the geometry of the problem. Application of these substitutions and a little rearranging for the case where all the wall conductivities and thicknesses are equal will give the dimensional form of the equations presented 8, 9, 10, and 11. For a square duct the choice of the characteristic length is easy. For a rectangular duct some thought is required to select the appropriate length. This work has not yet been done and looks to greatly complicate the equations. The potential for expansion of this work, though, is clearly

*demonstrated.

M97053590

Report Number (14) DOE/OR/00033--T786
UCLA-FNT--46

Publ. Date (11) 199103
Sponsor Code (18) DOE/ER, XF
JC Category (19) UC-420, DOE/ER

DOE