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Adaptive Optics Instrument for Long Range Imaging

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FINAL REPORT

Project ST-560
June 1998

**ADAPTIVE OPTICS INSTRUMENT FOR LONG
RANGE IMAGING**

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ABSTRACT

The science and history of imaging through a turbulent atmosphere is reviewed in detail. Traditional methods for reducing the effects of turbulence are presented. A simplified method for turbulence reduction called the Sheared Coherent Interferometric Photography (SCIP) method is presented. Implementation of SCIP is discussed along with experimental results. Limitations in the use of this method are discussed along with recommendations for future improvements.

ADAPTIVE OPTICS INSTRUMENT FOR LONG RANGE IMAGING ST-560 FINAL REPORT

Submitted by
Thomas M. Crawford

INTRODUCTION

Anyone who has viewed the twinkling stars in the sky or has observed the dancing of distant lights in a desert scene, has seen the effects of atmospheric optical turbulence. If they have observed the same starry sky or desert scene through a pair of binoculars or a small telescope, they have, probably without realizing it, seen the need for adaptive optics imaging instrumentation.

If someone does a careful qualitative examination of various distant objects (stars, street lights, etc.) through a pair of binoculars, he can see three atmospheric optical effects in action. First, he can see intensity variations such as we observe in twinkling stars. Second, he can see beam wander taking place such as in the dancing of distant street lights or the waviness of a scene when viewed over desert terrain. Finally, if he tries to focus a distant scene into a sharp image, he will find that it is impossible to produce a clear, focused image. The third case is even true if a high speed camera is used to eliminate the first two time varying effects mentioned above. All three of these effects are the result of optical turbulence introduced between the distant scene and our eye by the atmosphere.

These same three effects have wreaked havoc with astronomers since Isaac Newton's time. In 1730, Newton in writing his treatise, **Opticks**, noted the problems that the atmosphere caused with optical imaging, and he saw no solution. He wrote:

If the Theory of making Telescopes could at length be fully brought into Practice, yet there would be certain Bounds beyond which Telescopes could not perform. For the Air through which we look upon the Stars, is in perpetual Tremor; as may be seen by the tremulous Motion of Shadows cast from high Towers, and by the twinkling of the fixed Stars. But these Stars do not twinkle when viewed through Telescopes which have large apertures. For the Rays of Light which pass through divers parts of the aperture, tremble each of them apart, and by means of their various and sometimes contrary Tremors, fall at one and the same time upon different points in the bottom of the Eye, and their trembling Motions are too quick and confused to be perceived severally. And all these illuminated Points constitute one broad lucid Point, composed of those many trembling Points confusedly and insensibly mixed with one another by very short and swift Tremors, and thereby cause the Star to appear broader than it is, and without any trembling of the whole. Long Telescopes may cause Objects to appear brighter and larger than short ones can do, but they cannot be so formed as to take away the confusion of the Rays which arises from the Tremors of the Atmosphere. The only Remedy is a most serene and quiet Air, such as may perhaps be found on the tops of the highest Mountains above the grosser Clouds.

Babcock in 1953 appears to be the first person to propose adaptive optical methods to compensate for the distortions caused by atmospheric optical turbulence affecting telescope images. He proposed using deformable optical elements driven by a wavefront sensor which characterized the distorting effects of the atmosphere. Since 1953, the field of adaptive optics has blossomed into a scientific/engineering discipline of its own. It is driven by the available state-of-the-art technology, so it is a highly evolving field. Various review articles and books have been written, and at least two short courses exist which describe the state of adaptive optics technology at the present time. Much work is being done in the adaptive optics field currently, with most of it dealing with imaging of satellites and astronomical bodies through the atmosphere over our heads.

Optical imaging and optical beam propagation through horizontal and slant paths in the atmosphere is a much more difficult problem than imaging astronomical bodies overhead. It turns out that the optical aberrations produced by the atmosphere when looking straight up, for example, at satellites and astronomical bodies, are equivalent to the optical aberrations produced by only about 300 meters of atmosphere when imaging along a horizontal path. This poses a significant problem when high resolution imaging over long horizontal or slant paths is desired. The atmospheric boundary layer around an aircraft can degrade high resolution images even more, as can observing distant objects through the large thermal gradients in the atmosphere near the ground.

Since atmospheric optical turbulence poses a formidable problem when imaging over long distance horizontal and slant path situations the Department of Energy Office of Arms Control and Non-Proliferation has funded the Idaho National Engineering Laboratory to research and develop an adaptive optics instrument for long range imaging. We are presently engaged in the engineering research required to develop an instrument for imaging objects at distances greater than 10 kilometers in horizontal and slant path situations. The purpose of this paper is to outline the theory of atmospheric optical turbulence as it applies to our problem, to discuss the various possible methods for imaging over long distance horizontal and slant paths, and to discuss the reasons that sheared coherent interferometric photography (SCIP) is being considered for horizontal and slant path imaging situations.

OPTICAL PROPAGATION THROUGH ATMOSPHERIC OPTICAL TURBULENCE

The theory behind propagating light through the atmosphere is complicated, and draws from diverse fields such as optical physics, meteorological/atmospheric physics, electromagnetics, and statistical physics. Beland has written a good review article on propagation through atmospheric optical turbulence, Lei and Tiziani outline the effects of aircraft boundary layer turbulence on airborne imaging, and Tyson's book provides an elementary treatment of optical turbulence in the generation of optical aberrations. I will outline some of the important highlights of this theory in this section.

We live in a dynamic optical medium - our atmosphere. Temperature, pressure, and humidity differences cause random atmospheric density fluctuations. Since the index of refraction of a gas

is a linear function of its density (at least for pressures up to a few atmospheres), the atmosphere is not a homogeneous optical medium. As an example, a pocket of air having a temperature difference with respect to the surrounding air of only about 1°C will have a resultant difference of about 10^{-6} in its index of refraction. Though this is a small difference, it will cause a small change in the phase of a portion of a wavefront that happens to be passing through it. This effect will be cumulative as the wavefront passes through similar air pockets. An alternate way of viewing this situation is to consider these air pockets to be weak lenses which divert light rays slightly. Again as rays pass through more and more of these lenses, the effects accumulate.

The wavefront phase changes which accumulate and aberrate the wavefront are further complicated by the constant motion of our atmosphere. Up and down drafts, other air currents, and winds from micro- and macro-meteorological effects randomly transport these air pockets around as well as mix them with surrounding air. Because of this, a second wavefront passing through the atmosphere along the same path at a later time will not see exactly the same atmosphere as the previous wavefront did, and the aberrations which it accumulates will be different from those of the previous wavefront. This means that any multiple path interferences, imaging aberrations, and ray directions caused by wavefront aberrations will be time dependent, producing the time dependent scintillations (twinkling), image blur and fluctuations, and beam walk that we see in trans-atmospheric imaging situations.

The above mentioned air pockets of high- and low-density air are transported about by random air currents which can best be described using statistical means. Kolmogorov studied the mean-square velocity difference between two different points in space separated by a displacement vector $\mathbf{r}$. For doing this, he defined an atmospheric structure tensor $\mathbf{D}_{ij}$ as

$$\mathbf{D}_{ij} = \langle [v_i(\mathbf{r}_1 + \mathbf{r}) - v_i(\mathbf{r}_1)] [v_j(\mathbf{r}_1 + \mathbf{r}) - v_j(\mathbf{r}_1)] \rangle \quad (1)$$

where v_i and v_j are the x, y, and z components of the velocity difference between two separate air pockets, $\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$, and the brackets, $\langle \rangle$, represent an ensemble average. To visualize the geometry of the situation, see figure 1.

To simplify the theoretical description of atmospheric turbulence, three reasonable assumptions about the atmosphere are usually made. First, the atmosphere is considered to be locally homogeneous ($\mathbf{v}$ depends on $\mathbf{r}$); second, the atmosphere is considered to be locally isotropic ($\mathbf{v}$ depends only on the magnitude of $\mathbf{r}$); and third, the turbulence is considered incompressible ($\nabla \cdot \mathbf{v} = 0$). Using these assumptions, the atmospheric structure tensor can now be written as the structure function

$$D_v = \langle [v_r(\mathbf{r}_1 + \mathbf{r}) - v_r(\mathbf{r}_1)]^2 \rangle \quad (2)$$

In deriving Kolmogorov theory, the basic physical concept is the cascade of energy from large scale sizes to small scale sizes. Basically, the source of energy for the atmospheric motions come from large scale wind shears and convection which form eddy currents. This energy is progressively handed down to smaller and smaller eddy currents until it is dissipated on the small scale by viscosity effects. Two distance scales therefore become important in this theory: the outer scale, ζ_o , and the inner scale, ζ_o . Depending on the situation, the outer scale is regarded as

(1) the largest scale size where local homogeneity and isotropy are reasonable, (2) the scale size of the energy source, or (3) the dimension of the flow as a whole. It can vary from a few meters to a few hundred meters, mainly depending on the altitude of the air being observed. The inner scale is the size of the smallest eddy currents below which the viscosity of the air dissipates the energy quickly. It again is altitude dependent and can range from a few millimeters near the ground to a few centimeters or more in the troposphere and stratosphere.

If r falls within the range of the inner and outer scales (also called the inertial subrange), then equation (2) takes on a $2/3$ law dependence in r

$$D_v = C_v^2 r^{2/3} \quad \text{for } \xi_0 < r < \zeta_0 \quad (3)$$

where C_v^2 , the velocity structure constant, is a measure of the amount of energy in the turbulence.

For atmospheric optics applications, the atmospheric velocity structure function, D_v , is not the most convenient parameter. Instead, the atmospheric parameter that is of interest is a similar parameter called the atmospheric refractive index structure function, D_n . Since D_n and D_v are defined in analogous ways, it is possible to define an equation similar to equation (3) relating the refractive index structure function to a refractive index structure constant, C_n^2 ,

$$D_n = C_n^2 r^{2/3} \quad \text{for } \xi_0 < r < \zeta_0 \quad (4)$$

where C_n^2 is a measure of the strength of the optical turbulence present within the atmosphere.

In spite of its name, the refractive index structure constant is not a constant at all. As would be intuitively expected, C_n^2 varies both seasonally and with time of day. It also varies with the geographic location, weather conditions, and altitude, and it is easily perturbed by artificial means like aircraft motion, etc. Various measurements of C_n^2 have been made by different groups, and Beland provides a good review of this information. In order to provide a general feel for the magnitude and variation of C_n^2 , some of this data will be presented graphically.

Figure 2 shows data taken by Beland, et. al., of the variability of C_n^2 over a short period of time at three closely located positions. The data consists of measurements made by three separate instruments at a height of 2 meters above a flat, grassy terrain on a clear, sunny day. The three instruments were positioned in a straight line and separated by about 30 meters. As can be seen by this data, the refractive index structure constant varies over more than an order-of-magnitude in a short period of time, though at least for the period of time and location in question, it seems to vary about some average value. It would be expected that C_n^2 would vary highly with different types of terrain and different altitudes above the ground.

Figure 3 shows data taken by Walters and Kunkel of the variability of average C_n^2 values as a function of time-of-day. It shows 15 minute average measurements of C_n^2 along with its standard deviation for an elevation of 9 meters above a desert floor (cloudless conditions). This data is plotted over a 24 hour period. The data exhibits a diurnal cycle of C_n^2 with a peak in the daytime, near constant on average at night, and minima near sunrise and sunset. These minima occur at times when the air temperature and ground temperature are roughly equal (1/2 hour

before sunrise and 1/2 hour after sunset). The daytime behavior shows the dominance of convection, rising and falling with solar irradiance. The nighttime behavior seems to be much more variable and fluctuates about a mean value.

Both figures 2 and 3 show behavior as a function of time near the ground. Figure 4 shows a series of typical measurements of C_n^2 as a function of altitude above the New Mexico desert. This data was taken by Brown, et. al. Figure 4a was taken on a clear, calm, sunny summer day; figure 4b was taken on a clear, calm, sunny winter day; and figure 4c was taken on a clear calm night. As can be seen from this data, C_n^2 can vary by many orders of magnitude with altitude, time-of-day, and time-of-year. Similar variations can be expected for different geographical locations around the world. Typical values for C_n^2 range between $10^{-22} \text{ m}^{-2/3}$ and $10^{-13} \text{ m}^{-2/3}$ with $10^{-15} \text{ m}^{-2/3}$ typically being given as its average value.

Due to the variable nature of the atmosphere as shown in the above examples, no accurate theoretical models exist for predicting C_n^2 under all types of atmospheric conditions. Various empirical models for predicting C_n^2 as a function of altitude have been made from various measurements. Two classes of models have been developed and a brief explanation of these will be given in the next few paragraphs.

The first class of models are derived from large amounts of averaged data. The most widely used of these are the SLC (submarine laser communications studies) models developed by Miller and Zieske. The SLC models were derived by curve-fitting a piecewise-continuous polynomial to a variety of measurements (e.g., scintillometer data, acoustic sounder, aircraft, and others). The SLC model represents median values of C_n^2 above the AMOS observatory on Mt. Haleakala, Maui, Hawaii. This observatory is situated on a mountain 3 km above the surrounding land and ocean, and it is derived for a subtropical atmosphere with a high tropopause. For these reasons, it is not completely typical of the atmosphere in many places in the world, but it is still a highly used model. Table 1 outlines the SLC nighttime and daytime models. A graph of these models is presented in figure 5.

Two other important empirical models for profiling C_n^2 with altitude have been developed by the Air Force Geophysical Laboratory (see Beland). The first, a refinement of the SLC nighttime model based on a series of high-resolution balloon measurements above the AMOS Observatory, is called the AFGL AMOS Night Model. The second, a similar model characterizing the air above the New Mexico Desert on summer nights, is called the CLEAR I Night Model. Both of these models are defined by the following equation

$$\text{LOG}_{10}(C_n^2) = A + B z + C z^2 + D \exp(-[(z - E)/F]^2/2) \quad (5)$$

where z is the altitude above mean sea level (in km) and where the coefficients for the AMOS and CLEAR I models are defined in tables 2 and 3 respectively. It should also be noted that both models begin 14 meters above ground level (AMOS Observatory is 3.038 km in elevation while the New Mexico site is 1.216 km in elevation). Graphs of these two models are shown in figures 6 and 7 respectively.

The second class of models for profiling C_n^2 with altitude are also empirical in nature, but they

attempt to incorporate some dependencies on winds and meteorology to model the complexity of actual C_n^2 measurements. The first such model historically was the Hufnagel model. It is a one parameter model which tries to take into account the upper atmospheric wind profile. It was developed for altitudes from 3 to 24 km above the surface, so it does not model the boundary layer near the ground. At the suggestion of Valley, Ulrich extended the model to include a boundary layer term. The new model, called the Hufnagel-Valley (HV) model, added an additional exponential boundary layer term along with a second parameter that could also be adjusted. The most widespread form of the HV model is

$$C_n^2(h) = 8.2 \times 10^{-26} W^2 h^2 e^{-h} + 2.7 \times 10^{-16} e^{-h/1.5} + A e^{-h/0.1} \quad h > 3 \text{ km} \quad (6)$$

where

$$W^2 = (1/15) \cdot \int_5^{20} v^2(h) dh$$

attempts to model the wind profile in a range from 5 to 20 km above the ground and parameter, A, can also be adjusted to fit the particular situation. Usually, the two parameters, W^2 and A, are adjusted to provide values for the Fried coherence length, r_o , and the isoplanatic angle, θ_o , which agree with real measurements. (r_o and θ_o are both defined later in this paper.) This seems to me to be a backward way of doing things, but it is basically designing a model of the atmosphere using the easiest observables available. A common set of parameters in widespread use is called the HV 5/7 model. The parameters, A and W, are chosen to provide a Fried coherence length of 5 cm and an isoplanatic angle of 7 mrad, which agrees fairly well with many atmospheric conditions. The values for the HV 5/7 model are $A = 1.7 \times 10^{-4}$ and $W = 21$. The HV 5/7 model characterizes the atmosphere between 3 and 24 km above the surface of the ground, and no attempt is made to include the boundary layer. It is thus intended for either day or night use. A graph of the HV 5/7 model's C_n^2 altitude profile is shown in figure 8. It should be stated that the HV model is unrealistic in many ways. Its main appeal is that it provides a model consistent with moderate values for r_o and θ_o , and in most applications, these integrated values are more useful than the C_n^2 altitude profile. More information about these and other atmospheric turbulence models is given in the above articles, in Tyson's book, and Beland's review article.

The refractive index structure constant discussed above provides a measure of the amount of optical turbulence present at any point in the atmosphere. Large C_n^2 values imply large optical turbulence while small C_n^2 values imply an optically quiet atmosphere. Unfortunately, the refractive index structure constant is not physically intuitive to most people. It has funny units and it does not seem to relate to everyday life. Fortunately, a related alternative observable has been adopted by the adaptive optics community for describing the optical turbulence present along a propagation path in the atmosphere. This quantity, called the Fried's coherence length, is defined for plane wave radiation as

$$r_o = 0.185 \cdot \left\{ \lambda^{-2} \int_0^L C_n^2(s) ds \right\}^{-3/5} \quad (7)$$

where L = total path length
 s = variable along path

Note that in much of the literature, the above equation appears in a slightly different form. We have chosen to write equation (7) as a function of the distance along the optical path instead as a function of altitude. This makes the equation more general so that it applies to horizontal and slant paths as well as vertical paths. This particular form is given by Goodman's book. If on the other hand, we are interested in using one of the above altitude dependent models for C_n^2 , we can rewrite the above equation as

$$r_o = 0.185 \times \{ \lambda^{-2} \text{SEC } \beta \int_0^L C_n^2(h) dh \}^{-3/5} \quad (8)$$

where β = zenith angle
 h = altitude

The above two equations are written for plane wavefronts traveling through the atmosphere. However in most imaging situations, spherical wavefronts more closely approximate light traveling from a point in the atmosphere to an imaging system. In this case, we need a weighting factor included in the above equations to take into account the spherical traveling waves, allowing us to rewrite equation (7) as

$$r_{o \text{ sph}} = 0.185 \times \{ \lambda^{-2} \int_0^L C_n^2(s) (s/L)^{5/3} ds \}^{-3/5} \quad (9)$$

for the case where the object being imaged is located at the origin and the imaging system is located at a distance L away. For the opposite case where the imaging system is located at the origin and the object being imaged is located at distance L away, equation (9) becomes

$$r_{o \text{ sph}} = 0.185 \times \{ \lambda^{-2} \int_0^L C_n^2(s) (1 - s/L)^{5/3} ds \}^{-3/5} \quad (10)$$

Equation (9) would be used in a space to ground imaging situation while equation (10) would be used in a ground to space imaging situation. In both of these cases, we could add the SEC β term for cases where C_n^2 is given as a function of altitude.

An interesting fact about equations (9) and (10) is that they show that heavy turbulence is more detrimental to imaging when it is located near the imaging system than when it is located near the object being imaged. This is similar to viewing someone through a shower door. A person near the door cannot see a person on the other side who is located a relatively far distance away, but the person who is far away can see the first person fairly well. Similarly, we can read through a frosted piece of glass easily if the frosted piece of glass is placed directly on the paper we are reading. For this reason, it is easier to image the ground from space at high resolution

than it is to image an object in space from the ground. Spying from space is easier than observing the spying satellite.

Fried's coherence length, sometimes called the seeing cell size, is a convenient quantity for characterizing optical turbulence because it quantifies the effects of the turbulence on a beam propagating through the atmosphere. This is the problem about which we are really concerned in most optical applications. The physical significance of Fried's coherence length is that it is the parameter which describes the average resolution of a telescope looking through atmospheric turbulence. r_0 is basically the maximum diameter of a diffraction limited optical system that can be used in the atmosphere. The result is that when looking through turbulence with a large telescope, the telescope acts like a large light bucket with a resolution approximately equal to that of an r_0 sized telescope. Figure 9 shows the normalized response calculated by Fried of the resolution of a telescope as a function of diameter as its diameter is increased from a diameter, $D_0 \ll r_0$, to a diameter, $D_0 \gg r_0$.

As with C_n^2 , r_0 depends on season, time of day, weather, geography, altitude, etc. r_0 typically ranges from 5 to 20 cm at visible wavelengths for most astronomical applications, for example, looking nearly straight up through small zenith angles. Horizontal and slant path observations have much smaller r_0 values. For example, if someone views objects several kilometers away through a pair of binoculars (<5 cm aperture diameters), he can see that there is significant blurring and distortion from atmospheric optical turbulence. In fact in desert areas, significant blurring and distortion can occur even when viewing distant scenes with our naked eye (roughly 1 mm to 5 mm aperture diameters), which implies a very small value for r_0 .

Another useful variable for classifying atmospheric optical turbulence is called the isoplanatic angle, θ_0 . Basically, the isoplanatic angle is the maximum angle between two separate rays traveling through the atmosphere where the rays can be considered to be traveling through the same optical turbulence. Consider the situation shown in figure 10 where we illuminate a point on the scene with a focused laser beam and then use this illuminated point as an artificial laser guidestar. We use the guidestar to characterize the atmospheric optical turbulence occurring between the scene and the imaging equipment and then use adaptive optics methods to correct for the optical aberrations introduced by the atmosphere. Physical intuition tells us that if we view a region too far away from the laser guidestar, then the light coming from this region will travel through a different set of atmospheric optical turbulence than the light coming from the laser guidestar and any adaptive optics corrections will not work. There is, however, a region in the vicinity of the laser guidestar, where for all practical purposes, the light from it travels through the same atmosphere as the light coming from the laser guidestar itself. The isoplanatic angle is defined as the maximum angle away from the central region around the laser guidestar that we can view and still have the incoming rays of light traveling through the same optical turbulence as those from the guidestar. The isoplanatic angle is defined by the following equation

$$\theta_0 = 0.0581 \times \left\{ \lambda^{-2} \int_0^L C_n^2(s) s^{5/3} ds \right\}^{-3/5} \quad (11)$$

For astronomical applications, θ_0 typically has values in the 1 to 10 μrad range. Here again, these values will be much less for horizontal viewing than for vertical viewing. Also, as with r_0 , we could add the zenith angle dependence if C_n^2 is given as a function of altitude.

So far, the theory presented has dealt with the spatial characteristics of atmospheric optical turbulence. Since the atmosphere is dynamic in nature and therefore changes with time, temporal characteristics are also important for describing optical turbulence's effect on optical systems and for specifying the required bandwidths for adaptive optics systems.

In the case of atmospheric optical turbulence, we need to be concerned with two time scales. The first time scale that is important deals with how fast the atmosphere moves across the path of interest (advection). This time scale can be estimated as $\zeta_0/v_\perp$, where $v_\perp$ is the mean wind speed perpendicular to the direction of observation. For typical atmospheric values ($\zeta_0 \approx 10$ m and $v_\perp \approx 10$ m/sec), the advection time scale is roughly 1 second.

The second time scale of concern results from the dynamics of the turbulence itself. It is the result of the motion of the eddy currents as they affect the optical path. Similar to the above argument, we can estimate this second time scale at a point in space as being on the order of $\zeta_0/\Delta v_\perp$, where in this case, $\Delta v_\perp$ is the change in the wind speed's perpendicular component due to the eddy currents present in the atmosphere. If we estimate $\Delta v_\perp$ to be roughly 10% of the mean wind speed, then this time scale will be roughly 10 seconds.

Since the second time scale (that from the temporal dynamics of the turbulence) is much larger than the advection time scale, we may neglect it in comparison to the mean flow of the atmosphere. This is the essence of the frozen turbulence hypothesis. Basically the internal dynamics of the eddy currents are neglected and the turbulent eddies are simply treated as frozen in time and transported across the observation path by the wind. The reasonableness of the frozen turbulence hypothesis has been established by a variety of measurements, and it is even qualitatively observable when examining distant scenes across the desert through a small telescope or set of binoculars.

It should be noted that an important limitation to the frozen turbulence hypothesis occurs when the transverse wind speed is small compared to the eddy current motion. This could occur when the wind is blowing in a direction nearly parallel (or anti-parallel) to the observation path. Mathematically, the frozen turbulence hypothesis is valid if

$$\zeta_0 \gg (\lambda L)^{1/2} \quad (12)$$

where L is the path length. This condition applies even at very low wind speeds. It assumes that the propagation is along a homogeneous path so that ζ_0 is unambiguously defined. This is not really true for near vertical propagations, though one could assume an average ζ_0 . Substitution of typical values for ζ_0 (≈ 10 m) at visible/IR wavelengths shows that the conditions in equation (12) are almost always met even over long horizontal distances.

The above two time scales are useful to show that the frozen turbulence hypothesis is valid in most cases. These time scales really only relate to the optical effects at a particular point in

space. Another temporal characteristic is also important in the field of adaptive optics. This characteristic is the coherence time, τ_o , of the atmosphere. τ_o is the time constant of the atmosphere and is the result of the wind blowing the various air pockets across the beam path and thereby changing the phase of the beam with respect to time. Basically, when the wind blows the atmosphere a distance of roughly r_o across an optical beam, its phase will change and the wavefronts traveling through the atmosphere at that point in time will become uncorrelated with those wavefronts that crossed the same point a time τ_o before. The coherence time will be roughly on the order of $r_o/v_{\perp}$. Zuev and Lukin provide a better estimate

$$\tau_o \approx .53 (r_o/v_{\perp}) (D/r_o)^{1/6} \quad (13)$$

where D is the diameter of the beam, and where the wind velocity is considered constant along the beam's path. Tyson goes a step further and provides an expression for when neither the wind velocity nor the refractive index structure constant are constant along the path of the beam. In this case

$$\tau_o = 0.0581 \times \left\{ \lambda^{-2} \int_0^L C_n^2(s) v_{\perp}^{5/3}(s) ds \right\}^{-3/5} \quad (14)$$

The above atmospheric coherence time or time constant provides an estimate as to how fast the atmosphere between two points is changing. Basically, if we are looking at a distant street light or a star through the atmosphere, this time constant provides an estimate as to how fast the street light or star will dance or twinkle with time. If we desire to take a snap shot of a distant scene, this time constant provides the dividing line between a short exposure and a time averaged exposure. For example, if the amount of light being received from the scene is of sufficient intensity, then a short exposure ($t \ll \tau_o$) will remove the dancing (fluctuations in image tilt) and will provide higher resolution images of the scene. The short exposures will not however do anything for the image blur caused by the frozen time atmospheric aberrations. On the other hand, long exposures ($t \gg \tau_o$) will have both the image blur and the added problem of the time averaged image dancing. Finally, the atmospheric coherence time provides an estimate of how fast a conventional adaptive optics system will need to be changed in order to correct for atmospheric temporal effects.

ATMOSPHERIC OPTICS CALCULATIONS

It is informative to calculate the various spatial and temporal characteristics of the atmosphere for both vertical and horizontal optical paths. These are useful for estimating the magnitude of the problems associated with imaging over long horizontal and slant paths. They are also necessary for evaluating the different active and adaptive optics methods for imaging through atmospheric turbulence. For performing these calculations, we used the CLEAR I and HV 5/7 models and the experimental data provided in the literature to estimate C_n^2 along the optical paths of interest. The CLEAR I and HV 5/7 models were chosen over some of the other models because they provide a good estimate of the amount of optical turbulence present in the atmosphere over the New Mexico desert. Since our experiments will be performed in a desert

environment (the Idaho National Engineering Laboratory), we assumed that these would be the closest models to our particular situation. Furthermore, the atmospheric optical turbulence over deserts is probably typical or greater than the amount of optical turbulence which would be expected in most atmospheric imaging situations. Similarly, when we chose to use certain numbers from the experimental data in the literature, we used the data that would most closely approximate the desert environment.

We calculated values for r_o , r_{osph} , θ_o , and τ_o using equations (7), (9), (10), (11), (13), and (14). Modified versions of both the CLEAR I and the HV 5/7 models of $C_n^2(h)$ were used for looking both up and down through the atmosphere. We modified these models slightly by adding a typical boundary layer for the desert. On the HV 5/7 model, we assumed $C_n^2 = 5 \times 10^{-13} \text{ meters}^{-2/3}$ for the first 14 meters above the ground, then assumed $C_n^2 = 1 \times 10^{-15} \text{ meters}^{-2/3}$ until an altitude of about 2500 meters above sea level was obtained, then finally we assumed the standard HV 5/7 model until an altitude of 31000 meters above sea level was reached. On the CLEAR I model, we assumed $C_n^2 = 5 \times 10^{-13} \text{ meters}^{-2/3}$ for the first 14 meters above the ground, then we assumed the CLEAR I model until an altitude of 30000 meters above sea level was obtained, then $C_n^2 = 1 \times 10^{-20} \text{ meters}^{-2/3}$ for altitudes between 30000 meters and 31000 meters. On both models, C_n^2 was assumed zero for altitudes above about 31000 meters, and we assumed that the ground was at an elevation of 1216 meters. For viewing along horizontal paths through the atmosphere, we assumed two different cases, $C_n^2 = 1 \times 10^{-13} \text{ meters}^{-2/3}$ and $5 \times 10^{-13} \text{ meters}^{-2/3}$, which appear to be typical for horizontal viewing at a distance of several meters to several tens of meters above the ground. Tables 4 and 5 show the results of our calculations using the various models. These will be discussed below.

Table 4 shows the results of our calculations for imaging along vertical paths, looking both up and down, for two different path lengths between the object and the image. These two path lengths approximate what we would see either looking up at or down from an aircraft or satellite respectively. We can glean several important pieces of information from this table.

First, we can see that even though the HV 5/7 and the CLEAR I models provide different C_n^2 altitude profiles, the values for r_o , θ_o , and τ_o when looking vertically approximate each other very closely. For our purposes, it should make very little difference which model we choose to adopt for future calculations.

Second, we can see that the plane wave values for r_o as well as the values for θ_o and τ_o do not depend on the distance between the source and the imaging instrument. It should also be noted, though it is not shown in the table, that the plane wave values for r_o were the same whether one looks up or down.

Third, when looking upward, the spherical wave values for r_o do not vary significantly from the plane wave values for r_o . This is not really surprising since in each of these cases, the spherical wavefronts radiating from a point source will approximate plane wavefronts after traveling long distances, and it is at this point that the wavefronts are traveling through the most dense portion of the atmosphere, that is, near the ground where our imaging system is located. It is also noted that larger path lengths produced wavefronts that are more planar, and that the spherical r_o values for the 300,000 meter path equal the plane wave r_o values.

Fourth, we notice that the situation for imaging vertically downward is completely different. The spherical wave r_o values for these cases are much larger than those obtained for either the plane wave or the upward looking spherical wave cases, and the situation gets better, the farther away from the distorting medium that we get. This is expected since most of the optical distortion occurs to the wavefronts as they pass through the layers of atmosphere near the ground, and afterwards they travel while radiating spherically outward with little more distortion. By the time they reach the observer, large portions of the wavefront are therefore coherent with respect to each other making the Fried's coherence length very large. This illustrates the shower door analogy mentioned earlier.

Table 5 shows the results of similar calculations for imaging along horizontal paths through the atmosphere where C_n^2 is considered constant along the whole path. This would be a good approximation for imaging distant objects from a height of several meters above the ground when the distance between the scene and the imaging equipment is up to several tens of kilometers away. For the larger distances, it is probably less realistic since the imaging equipment would need to be located at either a high point on the earth or in an aircraft. (Note that an aircraft presents further optical distortions due to the aerodynamic boundary layer around the surface of the aircraft.) Furthermore, C_n^2 would not be constant over these long paths due to the variations in terrain and the resultant differences in temperature and air currents over it. The calculations are informative though in that they give us a good feeling as to the difficulty of imaging over horizontal paths, and certainly long paths will be much more difficult than shorter paths to image through.

From table 5 we can see horizontal imaging is much more difficult than vertical imaging, either up or down. Even for the lower value of C_n^2 , we can see that imaging over a distance of only about 300 meters produces r_o values worse than imaging objects directly over our heads. We can also see the reason that distant street lights in the desert dance around so much; the value for Fried's coherence length quickly approaches or gets smaller than the pupil diameter of eye.

These small r_o values, especially for imaging over distances from several kilometers to several tens of kilometers, also present some formidable problems for imaging using conventional adaptive optics. In conventional adaptive optics, the wavefront sensor will need to sample the approaching wavefront at several points within an area of r_o diameter. This means that the number of individual sensors on the wavefront sensor gets very large when r_o gets small. Furthermore, the number of actuators on the deformable or rubber mirror will need to increase by the same magnitude in order to correct the distorted wavefronts. We need at least one actuator (and probably more) to correct each area of incident wavefront that is roughly r_o^2 in area. Finally the speed of the wavefront correction calculations will need to be several orders of magnitude faster than those needed for astronomical viewing. This can be seen by comparing the values for τ_o in the different cases. All of these problems drive the cost of an already expensive system to prohibitive values, along with making the system difficult or impossible to build. It is for these reasons that alternative methods to conventional adaptive optics systems are being considered.

The values for θ_o in table 5 also show us another problem which arises in conventional laser guidestar/adaptive optics systems. θ_o basically shows the maximum angle over which we can

probe the atmosphere using a single laser guidestar/wavefront sensor system to correct for wavefront distortion. Even for the lower C_n^2 value, and viewing over only 1 kilometer of distance, we would need multiple laser guidestar beams fanning out in both x- and y-directions at angles on the order of 5 mrad in order to probe and characterize the atmosphere. The 5 mrad angle only covers an area on the target of 5 millimeters in diameter at a 1 kilometer distance, so basically we will need to flood the target with thousands of laser guidestars to observe any reasonable area on the target. This situation becomes even worse over larger distances and therefore presents a technically unfeasible solution for correcting atmospheric optical turbulence. We are again forced to look for other solutions to the problem of correcting for atmospheric optical turbulence.

To summarize, the theory of atmospheric optical turbulence and the resultant calculations indicate that looking horizontally through the atmosphere over large distances is orders of magnitude more difficult than astronomical viewing over much larger distances. In the next section, we will outline the technology of conventional laser guidestar/deformable mirror adaptive optics systems, showing in more detail the reason they will not work for horizontal and slant path imaging situations.

CORRECTION OF ATMOSPHERIC OPTICAL TURBULENCE USING CONVENTIONAL ADAPTIVE OPTICS TECHNIQUES

Ever since Babcock's proposal in 1953, various methods have been developed and are continuing to be developed to correct for the wavefront aberrations introduced by atmospheric optical turbulence. This document will not attempt to survey the technologies which are used for imaging through atmospheric optical turbulence as this has been done quite well in the previously mentioned books and review articles. Time will be spent, however, outlining the components of a conventional adaptive optics system so that a general understanding of the system's components can be appreciated, and so that reasoning can be presented for the need to pursue non-conventional active optics approaches to imaging through atmospheric optical turbulence. Sheared coherent interferometric photography (SCIP), a non-conventional speckle/holographic approach to imaging through atmospheric optical turbulence will be outlined in a following section.

Consider the situation where an astronomer would like to view a dim, distant galaxy through a large telescope. Unfortunately, as mentioned before, the atmospheric optical turbulence in the lower part of the atmosphere produces various wavefront aberrations which cause time dependent image dancing, image blurring, and intensity fluctuations. The first two effects prevent the clear resolution of the galaxy. In fact with an uncorrected telescope system, the astronomer cannot image the galaxy any better than many amateur astronomers can from their own backyards. If the atmosphere overhead has a typical r_0 value of 10 cm, then a decent pair of large aperture binoculars will resolve just as much as an astronomer's 200 inch telescope. The astronomer's only advantage over an amateur astronomer is the large telescope's light collection ability. It is possible however for an astronomer to use one of several conventional adaptive optics methods to correct for atmospheric distortion and produce high resolution images.

Figure 11 shows a method of adaptive optics which can be used if a bright star is located within the atmosphere's isoplanatic angle, θ_0 , of the dim galaxy being imaged. Light from both the star and the galaxy are collected by the telescope and imaged onto the imaging plane. Along the way through the optical system, the aberrated wavefronts from the star and the galaxy pass through a system of adaptive optics components. A beam splitter is then used to sample the wavefronts of the bright star and project these onto a realtime wavefront sensor.

Since the star is a distant point source, its spherical wavefronts are essentially planar by the time they reach the top of our atmosphere, but the optical turbulence in our atmosphere distorts these wavefronts so that they are no longer planar upon reaching the telescope. The wavefront sensor measures these distortions, a computer calculates what needs to be done to remove the aberrations, and an adaptive optics system adjusts for these distortions so that future wavefronts can be corrected. All of this must be done within the atmospheric time constant, τ_0 . The wavefront sensor, the computer, and the adaptive optics components therefore make up a feedback loop which is continuously updating itself for incoming wavefronts. In the process of removing the atmospheric distortion present in the bright star's wavefronts, the system also corrects the wavefronts coming from the dim galaxy so that highly resolved images of it can be produced.

Unfortunately, many dim objects and satellites which astronomers and others would like to view do not have bright stars in close angular proximity to them. In this case however, it is possible to produce an artificial guidestar as shown in figure 12. An artificial guidestar (or laser guidestar) is produced using a laser beam which is focussed to a point about 20 km above the earth's surface. The atmosphere naturally scatters light in all directions by Rayleigh scattering, or if the laser is tuned to the proper wavelength, it can be fluorescently scattered from the atmosphere's sodium layer in the stratosphere. In either case, some of this scattered light is backscattered from the guidestar and collected by the telescope on the ground. Using this artificial star in the upper atmosphere, the astronomer can utilize an adaptive optics system similar to that outlined in the previous paragraph to remove the optical aberrations present in the incoming light and thereby significantly improve the telescope's image producing ability. The only real difference is that the laser guidestar can be placed much closer to the dim galaxy or other object being imaged, and in fact, it can be placed right on top of the dim galaxy providing there is an efficient way of discriminating between the laser guidestar's narrow bandwidth light from the galaxy's broad band light. This is the method being developed by Fugate and others at the Air Force Phillips Laboratory's Star Fire Optical Range in Albuquerque, NM. INEEL scientists have also considered using this method for long distance horizontal and slant path imaging through atmospheric optical turbulence. In this case though, the guidestar would be backscattered directly off the scene being viewed instead of using Rayleigh or fluorescent scattering from the atmosphere. Figure 13 shows a schematic diagram of this situation.

There are many critical components for successfully realizing adaptive optics imaging through atmospheric turbulence. The subsystems and components making up the above described feedback loop each have characteristics which depend greatly on the amount of turbulence one is trying to look through. Several of the important components will be described in more detail in the following paragraphs. These include the wavefront sensor, the deformable or rubber mirror system, and the computer system.

The most common wavefront sensor used in conventional adaptive optics imaging systems is the Hartman (or Shack-Hartman) wavefront sensor. (For a schematic diagram, see figure 14.) The Hartman wavefront sensor is a simple device which measures the slopes of wavefronts falling onto it. Light waves entering an array of microlenses making up the sensor are focussed to an array of points on a detector array. If the incoming wavefronts are planar and parallel to the microlens array, then each of the focal points will fall on the detector array at their normal positions. If however, the plane waves are tilted slightly, then each of these focal points will be biased in one direction on the detector array, and their displacement will be related to the tilt of the wavefront. Finally, if the wavefronts entering the microlens array are aberrated, then each of the focal points on the detector array will be displaced to a position related to the slope of that particular portion of the wavefront. In each of these cases, the distance that a focus falls away from its normal position is proportional to the slope of the wavefront at that point. Using this slope information, the actual shape of the incoming wavefronts can be determined so that an adaptive optics system can be used to remove the aberrations.

The surface density of microlenslets in the microlens array, σ_m , and the surface density of detectors in the detector array, σ_d , are both proportional to the square of the smallest atmospheric r_o that we desire to image through. These surface densities will also be dependent on the magnifying power, MP, of the telescope or other instrument doing the imaging. This can be seen by the fact that as a result of the definition of r_o , wavefronts entering a telescope from distant objects can be considered aberration free within any area of diameter r_o . In other words, the size of the aberrations on a planar wavefront caused by the atmosphere between the object being viewed and the telescope will be on the order of or larger than r_o . When one of these distorted wavefronts travels through the telescope system, the approximate size of the wavefront aberrations becomes

$$r_t = r_o / \text{MP} \quad (15)$$

If we assume that the Hartman wavefront sensor needs to sample the wavefront α times along each orthogonal direction within each aberration along the wavefront, then the minimum microlens surface density will be

$$\sigma_m = (\alpha/r_t)^2 \quad (16)$$

or

$$\sigma_m = \alpha^2 \text{MP}^2 / r_o^2 \quad (17)$$

Furthermore if we assume that each microlenslet needs a 4 detector quadcell behind it, then the minimum detector array density will be

$$\sigma_d = 4 \times \sigma_m = 4 \times \alpha^2 \text{MP}^2 / r_o^2 \quad (18)$$

We can calculate some typical numbers for these surface densities. Assuming a typical magnifying power of 100 and a typical vertical viewing r_o value of 5 cm (see table 4), and if we

want to characterize each distortion along a wavefront 10 times in each direction, then the microlenslet array surface density will need to be at least 400 lenslets per mm^2 , and the detector array surface density will need to be at least 1600 detectors per mm^2 . These surface densities fall within the range of manufacturing capability.

On the other hand, if we decide to image horizontally near the ground over distances from 5 to 10 kilometers in a moderately distorting atmosphere, then r_o will be on the order of 0.5 cm (see table 5). If we again set $MP = 100$ and $\alpha = 10$ as in the previous example, then we will need a minimum of 40,000 lenslets per mm^2 and 160,000 detectors per mm^2 in the microlens array and detector array respectively. These numbers begin to get very unrealistic, especially when considering the microlenslets are only a few wavelengths in diameter themselves, and these numbers get worse very quickly when trying to image through highly distorting atmospheres over long distances, or at higher magnifying powers.

The second critical component in the adaptive optic feedback loop is the adaptive optics system used to correct the wavefront distortions. In conventional adaptive optics systems, this is usually done by using a deformable or rubber mirror. Several of these mirrors can be used in series to remove different orders of aberrations such as tilt, astigmatism, focus error, etc. Systems can be designed using mirrors on either the input or output side of the telescope, or systems can be designed which flex the primary mirror of the telescope itself. In reality, these mirrors are not rubber at all but consist of fairly standard mirrors having an array of piezoelectric transducers attached to the back of their substrates. These piezoelectric transducers push and pull on the mirror's substrate, generating stresses in the mirror and causing it to flex slightly at different locations on the mirror. In this way, the mirror can be deformed slightly from its planar form and shaped into the conjugate of the wavefront distortions. When the aberrated wavefronts reflect off the mirror, the aberrations cancel with their conjugate on the mirror, and the wavefronts are thereby corrected. For the mirrors to remove the majority of aberrations present in the wavefronts, they do not have to be pushed very far out of their planar positions. For astronomical imaging, these motions only need to be on the order of a few wavelengths, and this motion is quite possible using piezoelectric transducers.

In order for a deformable mirror to remove even the lowest aberration orders, the mirror will need to be flexed to approximately the conjugate shape of the aberrated wavefront. There needs to be at least one piezoelectric transducer controlling each r_o^2 sized area on the input side of the telescope, or each r_t^2 sized area on the output side of the telescope. In analytical form, this can be written as

$$\# \text{ of PZT's} \approx D_A^2 / r_o^2 \gg D_m^2 / r_t^2 \quad (19)$$

where D_A and D_m are the diameters of the telescope's input or output apertures respectively.

As before, some simple calculations can be used to illustrate the problems associated with using conventional deformable mirror technology to remove wavefront aberrations while imaging horizontally through the atmosphere. For the situation where we are doing astronomical imaging ($r_o = 5$ cm) through a 14 inch (36 cm) diameter telescope operating with a magnifying power of 100, a deformable mirror on the input side of the telescope will need to have roughly 50

piezoelectric transducers flexing the mirror, and these will need to be spaced roughly 5 cm apart in both directions along the mirror. A deformable mirror on the output side of the telescope would still need 50 transducers, but these would need to be spaced 0.5 mm apart in both directions (not an easy task). On the other hand, if we are imaging 5 to 10 km horizontally through a moderately distorting atmosphere using the same telescope system, a deformable mirror on the input side of the telescope would need 5000 piezoelectric transducer spaced 5 mm apart in both directions, or a deformable mirror on the output side of the telescope would need 5000 piezoelectric transducers spaced .05 mm apart in both directions. As before these situations get worse if we image over longer atmospheric paths, through highly distorting atmospheres, or with greater telescope magnifying powers.

Finally, the problem of reading and calculating the positions of the foci present at the detector plane of the Hartman wavefront sensor, and then using this information to control the voltages placed on the deformable mirror's piezoelectric transducers needs to be mentioned. Computer codes exist for doing this task quickly when the number of piezoelectric transducers is reasonable. This task has been done for much larger telescopes being used for astronomical viewing. A 2 meter astronomical telescope viewing through an $r_0 = 5$ cm atmosphere would require about 1600 piezoelectric transducers. Adaptive optics telescope systems like this have been used to view objects in space, and they have had control times of several hundred hertz. However, in the situation outlined above, we need to control 5000 piezoelectric transducers at a rate of several kHz ($t_0 \gg 10^{-3}$ seconds). Assuming a 5000 step computer code cycling through at a rate of 10,000 times per second so that we can control the 5000 piezoelectric transducers faster than the atmosphere is changing will require a computer operating at 50 MHz. This is not impossible with today's PC technology, but a 5000 step computer code is probably a gross underestimation of the need. Certainly this aspect of the problem is getting better with the newer computer technology.

In summary then, conventional adaptive optics imaging systems have been built for astronomical imaging purposes using Hartman wavefront sensors and piezoelectric transducer flexed mirrors. These systems focus on either an astronomical guidestar or an artificially generated laser guidestar in order to probe and characterize the optical turbulence present in the atmosphere. They then remove the optical aberrations by flexing a rubber mirror to the conjugate of the aberrations. From the above calculations, we can conclude that conventional adaptive optics technology is not feasible for removing the effects of atmospheric optical turbulence induced when imaging over long horizontal paths in the atmosphere. The reasons for this are that the Hartman wavefront sensor requires too great of a microlenslet surface density and too great of a detector surface density to be reasonably feasible in either cost or technology. Secondly, the number of piezoelectric transducers mounted on the back of a deformable mirror is too large and the spacing between adjacent piezoelectric transducers is too small for this method to be cost effective or technically feasible. For these reasons, we need to consider non-conventional methods of removing atmospheric distortion while imaging over long horizontal paths. The next section will discuss the SCIP method of imaging through atmospheric optical turbulence.

CORRECTION OF ATMOSPHERIC OPTICAL TURBULENCE USING THE SCIP IMAGING TECHNIQUE

The majority of this project was devoted to studying a non-conventional active optics technique for removing the effects of atmospheric optical turbulence called sheared coherent interferometric photography (SCIP). In principle, SCIP is similar to various holographic and speckle imaging techniques, but it also has the potential of removing the effects of atmospheric optical turbulence and other distortions during the imaging process. SCIP has several advantages over conventional wavefront sensor/rubber mirror adaptive optics techniques. It is much simpler in concept since it has no moving mechanical parts and the system inherently removes the optical distortions automatically. It is compatible with much simpler, cheaper, and less perfect telescope systems, and it is much less expensive to implement due to its simplicity. It also has eye safety advantages since the laser beacon is spread over a larger area than is necessary for laser guidestars used in conventional adaptive optics systems.

SCIP, like other speckle and holographic imaging techniques, is an active, coherent imaging technique. Since SCIP is related in principle to speckle and holographic imaging, its principles will be easier to understand if we first outline the more generic speckle and holographic imaging techniques.

Speckle and Holographic imaging techniques:

Consider the situation shown in figure 15 where we actively illuminate a distant scene using a laser beacon system. Laser light scattered from the scene will form a speckle field in space. If we collect light from this speckle field using a lens, and focus this light onto its image plane, we know from experience that we will form an image of the object.

If we look at the situation in figure 15 in more detail, the speckle field of the illuminated object is just a complex diffraction pattern of the object. At large distances from the object, this speckle field is the Fraunhofer diffraction pattern of the object, which is proportional to the two-dimensional Fourier transform, $F\{\}$, of the optical field being scattered or reflected from the object. Mathematically this diffraction pattern has the form

$$U(x_f, y_f) = \frac{\exp(jkz) \exp[j(k/2z)(x_f^2 + y_f^2)]}{j\lambda z} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_o, y_o) \exp[-j(2\pi/\lambda z)(x_f x_o + y_f y_o)] dx_o dy_o \quad (20)$$

where $j=(-1)^{1/2}$, $k=2\pi/\lambda$ is the wavenumber, (x_o, y_o) and (x_f, y_f) are the (x, y) coordinates in the object and speckle field planes respectively, $U(x_o, y_o)$ and $U(x_f, y_f)$ represent the complex electric field amplitudes in the object and speckle field planes respectively, and z is the distance from the object to the speckle field plane.

Since the speckle field at large distances from a coherently illuminated object is proportional to the two-dimensional Fourier transform of the field amplitude at the object, it is a simple matter to synthetically generate an image of the object by measuring the speckle field's complex amplitude (i.e., amplitude and phase) as a function of position in the far-field speckle plane, and taking the inverse two-dimensional Fourier transform of this data. (In actuality the two-dimensional Fourier transform of this data will create an image, but the image will be inverted.) This is exactly what a lens does; it is an analog computer which takes the two-dimensional Fourier transform of what is measured at its front focal plane, and projects this "calculation" onto its back focal plane.

Speckle and holographic imaging techniques use the above Fourier transform relationship between the object plane and speckle field plane, and between the speckle field plane and the image plane, to image distant objects. Figure 16 shows a simple example of speckle/holographic imaging. In this situation, we again coherently illuminate the scene with a laser beacon system, and collect the speckle field on some type of detector array. Now the detector array measures the square of the electric field at any point, (x_o, y_o) , on its surface, but it is not capable of measuring the phase. For this reason a reference beam from the laser beacon is also projected onto the detector array, thereby forming a hologram across the detector array's surface. Counting the number of fringe shifts which occur across the surface of the hologram in either coordinate direction allows one to determine the relative phase of the wavefront at any point, (x_o, y_o) , on the detector plane. We can thereby determine the field amplitude (the square root of intensity) and phase at any point on the detector array's surface, and if the two-dimensional Fourier transform of this data is calculated, we generate an image of the scene. The Fourier transform calculation behaves like a synthetic lens.

Three points should be made about the above speckle imaging technique. First, the resolution of the speckle imaging system is dependent on the area and geometry of the detector array in exactly the same way as it is for a conventional imaging system. The resolution of the system will be dependent on the diameter of the system. Large detector arrays have been built using arrays of photomultiplier tubes, but this soon gets prohibitively expensive. On the other hand, small detector arrays (like CCD or CID arrays) will not have good resolution because of their small synthetic aperture. We can remedy this situation however by using a large telescope to condense and then project the speckle field pattern onto the small array. Basically this is what is normally done when one looks through a telescope. The telescope reduces the area covered by the speckle or diffraction field pattern and projects it onto the front surface of the eye. The lens of the eye then calculates the two-dimensional Fourier transform of the speckle field and projects this information onto the retina of the eye, thereby forming an inverted image on the retina.

The second point concerns the density of detector elements in the detector array. Due to the properties of the Fourier transform, large objects will form small speckle cells in the speckle field. These small speckle cells are modulated by a speckle field envelope created by the finer

details in the scene. Large, sparse detector arrays will be able to detect the fine details of a scene to high resolution, but sparse detector arrays may not be able to detect the grosser details of the scene correctly. These two facts will directly affect the quality and resolution of the synthetic image which is formed by taking the Fourier transform. The detector spacing within the array therefore needs to be small enough so that several detectors can sample information from one speckle cell. (This is the opposite effect of using a detector array in the image plane of a camera. In the latter case, the size of the scene being imaged is proportional to the dimensions of the detector array, and the resolution of the finer details of the scene depends on the detector element spacing within the array.) In order for a detector array to be used in speckle imaging and record enough information so that the whole object can be imaged, the detector spacing, d_s , needs to be less than the speckle cell size of the speckle field pattern projected onto it, or

$$d_s \approx \lambda z / 2w \quad (21)$$

where w is the smaller of the object's size or the illuminating laser beam's width. If a telescope is being used to condense and project the speckle field onto a small detector array, then

$$d_s \approx (1/MP)(\lambda z / 2w) \quad (22)$$

where MP is the magnifying power of the telescope.

The final point that should be made about speckle or holographic imaging techniques concerns imaging through distorting media such as the atmosphere. Just as with conventional imaging systems, wavefronts traveling from the object will be distorted and aberrated when they travel through atmospheric optical turbulence or other distortions. Using speckle/holographic imaging methods to measure the amplitude and phase of these aberrated wavefronts will produce an aberrated synthetic image upon taking the two-dimensional Fourier transform. Speckle and holographic techniques by themselves therefore do not buy us anything when imaging through atmospheric optical turbulence. Holographic techniques do exist however where optical turbulence and distortion can be made to cancel and automatically correct the wavefront before its amplitude and phase are measured. One of these methods will now be discussed.

Figure 17 shows a clever technique for making holograms of an object through an aberrating medium. In this situation, the object is again illuminated using a laser beacon system. The wavefronts of light scattered from the object pass through the aberrating medium which is located between the object and the detector plane. Unlike the previous example however, the reference beam is not a plane wave by the time it reaches the detector plane. Instead it is made to pass through the same aberrating medium as the object's wavefronts. The result of this technique is that ideally the reference wavefronts have exactly the same aberrations on them that the object's wavefronts possess. When both the object and reference wavefronts fall onto the detector plane, the aberrations caused on both wavefronts by the distorting medium cancel and form an unaberrated hologram of the object on the detector plane's surface. Since the resulting holographic field has been corrected, the two-dimensional Fourier transform of this holographic information will produce an unaberrated image of the scene. This technique even works for time dependent distorting media such as are present in the atmosphere provided there is no significant time delay between the reference and object beams. A technique similar to this is being

developed by the Air Force Phillips Laboratory for imaging through aircraft boundary layers, where a reference mirror can be positioned just on the other side of the aircraft boundary layer. (In their case, they are using an optical phase conjugator for the mirror so that other differences also exist in their technique.) Unfortunately this technique will not work for long distance atmospheric surveillance situations since both the object and reference beams must essentially pass through the same atmosphere (i.e., within the isoplanatic angle, θ_0 , of the atmosphere). This requires that the reference mirror be placed within the isoplanatic angle of the object being imaged. In most practical surveillance situations, this requirement cannot be met since the targets are uncooperative.

SCIP Imaging

So far we have outlined some simple examples of speckle and holographic imaging techniques. We have also described a simple holographic method of imaging through optical turbulence and other distorting media. Unfortunately, in their present form, speckle and holographic imaging techniques cannot be used for long distance atmospheric imaging surveillance. As was seen above, the need for the reference beam to travel through the same atmospheric optical turbulence as the object beam makes these techniques either impractical or impossible to implement. The SCIP technique however is able to circumvent this problem. I will now describe the SCIP technique and outline its theory of operation.

Figure 18 shows a schematic diagram of the SCIP method of imaging. In the SCIP technique, three separate, mutually coherent laser beams having basically the same wavelength but slightly differing frequencies are used to illuminate a scene from slightly different angles. This is usually done by splitting a single laser beam into a reference beam carrying 50% of the power and two secondary beams, each carrying 25% of the power. The two secondary beams are sheared slightly from the main beam in orthogonal directions, one in the x-direction and one in the y-direction, so that the three beams radiate an "L" shaped pattern.

The amount of shear introduced between the primary beam and the two secondary beams needs to be less than about 1/2 to 2/3 of a speckle cell size. One of the properties of speckle fields is that a slight shift in the angle of the source beam will cause an equivalent shift in the angle of the resultant speckle pattern. Since the two secondary beams are separated from the primary beam by a distance, s , in the x- and y-directions, this means that their resulting speckle fields will be sheared by the same distance, s , from the primary speckle field in their respective directions. Since s is less than 1/2 to 2/3 of the size of a speckle cell, then these three speckle fields will overlap.

As was mentioned above, the three beams are also each shifted slightly in frequency with respect to each other. Each of the two secondary beams are shifted in frequency by a different amount from the primary beam so that we can keep track of which beam produces which speckle pattern.

The frequency shifts are on the order of a few tens of Hz, which is much smaller than an optical frequency of 10^{15} Hz. The frequency shifts cause the three overlapping speckle fields to beat with each other. Since the secondary beams are shifted in frequency from the primary beam by different amounts, the resulting beat frequencies, $\delta\nu_x = \nu_0 - \nu_x$ and $\delta\nu_y = \nu_0 - \nu_y$, are unique, where ν_0 is the fundamental laser frequency and ν_x and ν_y are the respective frequencies of the x-

sheared and y -sheared beams respectively. This provides an easy way to discriminate between the x -sheared and y -sheared speckle patterns. Furthermore, the phase of the beat note at any point of the speckle pattern represents the phase difference between the reference beam and the particular sheared beam. It turns out that this is enough information to determine the phase of the unaberrated reference beam to within a constant at any point within the speckle field.

SCIP's receiving system consists of a detector array similar to that used in the speckle/holographic techniques mentioned above. Again, the detector spacing and the size of the detector array need to be determined by the size of the object being imaged, the wavelength of the light being used, the distance between the object and the receiver planes, and the desired resolution of the system. All this is done using exactly the same formulas outlined above in the speckle/holographic imaging section. A telescope can also be used in a similar way to condense the speckle fields and project them onto a smaller, denser detector array. The SCIP technique is basically another method of speckle/holographic imaging.

The difference between the SCIP technique and the speckle/ holographic techniques mentioned previously is that the previous speckle and holographic techniques use planar reference beams to measure the phase changes across the speckle pattern while the SCIP technique scatters the reference beam as well as the two secondary beams off of the target itself. Since the shear is so small, all three SCIP beams travel through the same atmosphere. Since they all travel through the same atmospheric optical turbulence, any wavefront aberrations which accumulate during their propagation up to and returning from the target, will cancel in the resulting speckle pattern.

Outline of one-dimensional SCIP Theory

With a few minor changes, an outline is presented for a simple, one-dimensional theory developed by Idell for the SCIP technique. Figure 19 shows a sketch of the geometry used for developing this theory. Twin illuminators sheared by a separation distance of s (for the imaging dimension being considered) illuminate a target using slightly different frequencies. (Note - $\delta\nu$ is so small that $\delta\lambda/\lambda$ is virtually zero.) Assuming that the two beams have the same amplitude, the optical field at the target will have the form:

$$E(x,t) = E_{ox} \exp[j(\omega_0 t - k\rho(x,z))] + E_{ox} \exp[j(\omega_x t - k\rho(x-s,z))] \quad (23)$$

where $\omega_0 = 2\pi\nu_0$ and $\omega_x = 2\pi\nu_x$ are the angular frequencies of the reference and x -sheared beams respectively (note - $\omega_0 \approx \omega_x$ to within a few hertz), $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, t is time, s is the shear between the two beams, and $\rho(x,z)$ and $\rho(x-s,z)$ are given by:

$$\begin{aligned} \rho(x,z) &= (x^2 + z^2)^{1/2} \approx z + x^2/(2z) \\ \rho(x-s,z) &= ((x-s)^2 + z^2)^{1/2} \approx z + x^2/(2z) - xs/z \end{aligned} \quad (24)$$

where $s \approx x \ll z$ is assumed. Plugging these approximations into equation (23) gives:

$$E(x,t) = E_{ox} \exp[j\omega_0 t] \exp[-jk(z + x^2/(2z))] + E_{ox} \exp[j\omega_x t] \exp[-jk(z + x^2/(2z))] \exp[jkxs/z]$$

(25)

Now since the intensity, I , is proportional to $E E^*$, these two beams form an illumination pattern at the target of the form:

$$I(x,t) = I_0 \{1 + \cos(\delta\omega x - kxs/z)\} \quad (26)$$

$$\text{where } \delta\omega x = \omega_0 - \omega x.$$

From equation (26), we can therefore see that the two beams form a scanning fringe pattern on the scene which probes the scene for us. The period, P , of this fringe pattern can be calculated using:

$$(2\pi/P)x = kxs/z$$

$$\text{where } k = 2\pi/\lambda$$

so

$$P = \lambda z/s \quad (27)$$

Using this relation along with the fact that $\delta\omega = 2\pi\delta\nu$, then equation (26) for the two-beam illumination pattern on the target can be rewritten as:

$$I(x,t) = I_0 \{1 + \cos[2\pi(\delta\nu x t - x/P)]\} \quad (28)$$

We can now define $v_x = \delta\nu x P$ which allows us to rewrite (28) as:

$$I(x,t) = I_0 \{1 + \cos[(2\pi/P)(v_x t - x)]\} \quad (29)$$

where v_x is basically the speed at which the two-beam fringe pattern scans the target which we are observing. From equation (29), we can now recalculate the electric field at the target by taking its square root:

$$E(x,t) = \sqrt{2} E_0 \{(1/2) \cdot (1 + \cos[(2\pi/P)(v_x t - x)])\}^{1/2}$$

which simplifies to

$$E(x,t) = \sqrt{2} E_0 \cos[(\pi/P)(v_x t - x)] \quad (30)$$

This equation is simpler than the two-beam field equation that we started with in equation (25), and we have thrown away some non-useful phase information. In essence we just redefined the phase of the wavefront to be zero at a different point. This equation will however provide us with enough information to calculate the field and intensity of the speckle pattern at the detector plane.

To sum up the progress thus far, equation (30) represents the electric field distribution which the two sheared laser beams produce at the target. It is a moving fringe pattern which scans the target.

So far, we have not said anything about the target's reflecting or scattering properties. Let us therefore assume that the target possesses the following complex reflection or scattering coefficient properties:

$$r_o(x) = |r_o(x)| e^{j\phi(x)} \quad (31)$$

where $\phi(x)$ is the phase which the target adds to the wavefront upon reflection.

The far field speckle pattern produced by reflecting or scattering light off of the target is just the Fraunhofer diffraction pattern produced by equation (20) when $U(x_o, y_o)$ is just the product of equations (30) and (31). Therefore

$$U(x_o, y_o) = \sqrt{2} r_o(x_o) E_o \cos[(\pi/P)(v_x t - x_o)] \quad (32)$$

which can be rewritten as

$$U(x_o, y_o) = \sqrt{2} r_o(x_o) E_o \frac{\exp[j(\pi/P)(v_x t - x_o)] + \exp[-j(\pi/P)(v_x t - x_o)]}{2} \quad (33)$$

so the far field or Fraunhofer speckle pattern produced on the detector plane by the two-sheared-beam illumination of the target will have the form

$$U_s(x_f, y_f, t) = \frac{\sqrt{2} E_o \exp[jkz] \exp[j(k/2z)(x_f^2 + y_f^2)]}{2j\lambda z} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_o(x_o) \{ \exp[j(\pi/P)(v_x t - x_o)] + \exp[-j(\pi/P)(v_x t - x_o)] \} \exp[-j(2\pi/lz)(x_f x_o + y_f y_o)] dx_o dy_o \quad (34)$$

Note that since we are dealing only with a one-dimensional reflection and illumination situation, we can ignore the y-integration since it will at the most be just a constant multiplier of our final answer.

After a little bit of algebra, we can simplify the above equation to the following form:

$$U(x_f, t) = C \{ \exp[j\pi v_x t/P] F_+ + \exp[-j\pi v_x t/P] F_- \} \quad (35)$$

where $F_{\pm}$ are the Fourier transforms of the complex target reflectance at the spatial frequencies $(x_o/\lambda z) \pm (1/2P)$, or

$$F_{\pm} = \int_{-\infty}^{\infty} r_o(x_o) \exp[-j(2\pi/\lambda z)[x_f \pm (\lambda z/2P)] x_o] dx_o \quad (36)$$

where

$$C = \frac{\sqrt{2} E_o \exp[jkz] \exp[j(k/2z)(x_f^2 + y_f^2)]}{2j\lambda z} \quad (37)$$

Equation (35) shows us that the time evolving far field speckle pattern which is falling onto the detector plane is really two counter propagating traveling speckle patterns which "sort of form" a standing wave speckle pattern in the far field. Using the above equations, we can now calculate the speckle pattern's intensity distribution on the detector plane by again squaring the complex field:

$$\begin{aligned} I_s(x_f, t) &= |U_s(x_f, t)|^2 = C C^* | \exp[j\pi v_x t/P] F_+ + \exp[-j\pi v_x t/P] F_- |^2 \\ (38) \quad &= (I_o/2\lambda^2 z^2) \{ |F_+|^2 + |F_-|^2 + \exp[j2\pi v_x t/P] F_+ F_-^* + \exp[-j2\pi v_x t/P] F_+^* F_- \} \end{aligned}$$

Furthermore, if we define

$$\mathcal{F}_{\pm} = A_{\pm}(x_f \pm (\lambda z/2P)) \exp[j\phi_{\pm}(x_f \pm (\lambda z/2P))] = A_{\pm} \exp[j\phi_{\pm}] \quad (39)$$

where $A_{\pm}$ and $\phi_{\pm}$ are the amplitude and phase of $\mathcal{F}_{\pm}$ respectively. Using these definitions, we can now put the speckle field's intensity distribution (equation (38)) in the form:

$$I_s(x_f, t) = (I_o/2\lambda^2 z^2) \{ A_+^2 + A_-^2 + 2A_+ A_- \cos[(2\pi v_x t/P) + \Delta\phi_s] \} \quad (40)$$

$$\text{where } \Delta\phi_s(x_f) = \phi_+(x_f + (\lambda z/2P)) - \phi_-(x_f - (\lambda z/2P)) \quad (41)$$

is the phase difference between the two separate speckle patterns at any point in the far-field speckle plane. Notice that $\Delta\phi_s$ is differenced in distance by the phase shear distance, $s_p = \lambda z/P$. If we compare this with equation (27), it is obvious that $s_p = s$, or as was stated before the phase shear distance in the speckle plane is the same as the shear distance between the two original illuminators. This is true whenever the distance from the source plane to the target is the same as the distance between the target and the speckle plane. Basically, the target acted like a mirror in that a change in the angle of incidence will change the angle of the speckle field by an equivalent amount. Figure 20 shows the relationship between the absolute phase and the phase differences

found in equation (41).

Now the speckle cell size on the detector plane is related to the size of the target being observed according to the following approximation:

$$L_{cell} \approx \lambda z/W \quad (42)$$

where W is the size of the target being illuminated and observed. In order that there be no ambiguity in the final image, the Nyquist criteria must be satisfied and the speckle plane must be sampled at least twice per speckle cell size. This means that the detector spacing, X_d , must have a value of

$$X_d \leq \lambda z/2W \quad (43)$$

and if $P \approx W$, then

$$X_d \leq \lambda z/2W = s_p/2 = s/2 \quad (44)$$

provides the relationship between the size of the target, the detector spacing in the speckle plane, and the shear of either the speckle field or of the original sources. Since the speckle size is $s = \lambda z/W$, intensity samples taken at $X_d = \lambda z/2W$ will probably have noticeable field amplitude fluctuations. A larger separation of the illuminators will result in a bigger shear. Thus the phase at the speckle phase would be sampled even less often and result in aliasing. The imaging system will therefore require that the magnitude of the shear be less than z/W .

Furthermore, as will be shown in the next section, the moving speckle pattern is sampled four (or possibly more) times in one fringe period in this algorithm. This means that each time the fringe pattern illuminating the target has shifted a distance of $P/4 = \lambda z/4s$, the detector plane will sample the speckle pattern. The frequency with which this measurement cycle repeats depends on the rate that the speckle pattern slips over the detector array. An intensity measurement should be made before the speckle pattern moves one speckle size, $s = \lambda z/W$. This consideration determines practical data acquisition requirements.

Speckle Plane Phase Determination

So far in this paper, we have derived an expression for the far-field, beating speckle pattern produced by illuminating a distant target having scattering reflection properties, $r_o(x_o)$, with two mutually coherent sheared laser beams which have been shifted in frequency slightly with respect to each other. We have seen that in the far-field speckle plane, the only measurable observable of the speckle field is the intensity, $I_s(x_f, t)$, given by equation (40). We have however noticed that $I_s(x_f, t)$ does contain information about the speckle pattern's amplitude and phase, and since this information is needed for deriving a synthetic image of the scene, we will now describe a method for extracting this information from the beating speckle pattern. This section will describe phase information extraction while the following section will deal with amplitude information extraction.

In order to develop a method of extracting phase information from multiple measurements of $I_S(x_f, t)$, we will first re-write equation (40) as:

$$B(x_f, t) = \frac{I_S(x_f, t)}{I_0/2\lambda^2 z^2} = \{A_+^2 + A_-^2 + 2A_+A_- \cos[(2\pi v_x t/P) + \Delta\phi_S]\} \quad (45)$$

As we can see, equation (45) contains three unknowns, A_+ , A_- , and $\Delta\phi_S$, which we would like to determine. In principle, we can calculate these using data from three separate time measurements, but for simplicity, we will take intensity measurements at four different times throughout a cycle at every point, x_f . We will choose the four observation times to be $\pi/2$ apart in phase, which means that $2\pi v_x t/P$ will take on values of 0, $\pi/2$, π or $3\pi/2$. Doing this, we will then make the following four measurements:

$$\begin{aligned} B_0 &= A_+^2 + A_-^2 + 2A_+A_- \cos[\Delta\phi_S] \\ B_{\pi/2} &= A_+^2 + A_-^2 + 2A_+A_- \cos[\Delta\phi_S + \pi/2] \\ B_\pi &= A_+^2 + A_-^2 + 2A_+A_- \cos[\Delta\phi_S + \pi] \\ B_{3\pi/2} &= A_+^2 + A_-^2 + 2A_+A_- \cos[\Delta\phi_S + 3\pi/2] \end{aligned}$$

Which will simplify to:

$$\begin{aligned} B_0 &= A_+^2 + A_-^2 + 2A_+A_- \cos[\Delta\phi_S] \\ B_{\pi/2} &= A_+^2 + A_-^2 - 2A_+A_- \sin[\Delta\phi_S] \\ B_\pi &= A_+^2 + A_-^2 - 2A_+A_- \cos[\Delta\phi_S] \\ B_{3\pi/2} &= A_+^2 + A_-^2 + 2A_+A_- \sin[\Delta\phi_S] \end{aligned} \quad (46)$$

Using these relations, it is now a simple manner to calculate $\Delta\phi_S$:

$$\tan[\Delta\phi_S] = -[B_{\pi/2} - B_{3\pi/2}]/[B_0 - B_\pi] \quad (47)$$

$$\begin{aligned} \text{where } 0 \leq \Delta\phi_S < \pi & \text{ for } B_{3\pi/2} - B_{\pi/2} \geq 0 \\ \Delta\phi_S &= \pi/2 \text{ for } B_0 - B_\pi = 0 \\ \text{and } \pi \leq \Delta\phi_S < 2\pi & \text{ for } B_{3\pi/2} - B_{\pi/2} \leq 0 \\ \Delta\phi_S &= 3\pi/2 \text{ for } B_0 - B_\pi = 0 \end{aligned}$$

Equation (47) provides us with a method of measuring the change in the phase between the two beating speckle patterns at any point in the speckle or detector plane. We now need to come up with a way of measuring the absolute phase using this information.

Referring back to equation (41)

$$\Delta\phi_S(x_f) = \phi(\xi_x + (1/2P)) - \phi(\xi_x - (1/2P)) \quad (41)$$

which defines $\Delta\phi_s(\xi_x)$, and also to the sketch shown in figure 20, we can see that the relation $\Delta\phi_s(\xi_x)/(1/P) = P \Delta\phi_s(\xi_x)$ approximates the slope at a point on the phase in spatial frequency space. We can therefore make the following approximation:

$$\begin{aligned} d\phi_s/d\xi_x &\approx d\phi_x/d\xi_x = P Df_s(x_x) \\ &= (\lambda z/s) \Delta\phi_s(\xi_x) \end{aligned} \quad (48)$$

where $d\phi_x/d\xi_x$ is the approximate derivative, $d\phi_s/d\xi_x$. By simple integration, we can therefore determine the phase, $\phi_s(\xi_x)$, at any point in the detector plane to within a constant of integration. Integration gives us

$$\begin{aligned} \phi_s(\xi_x) - \phi_0 &= \int [d\phi_x/d\xi_x] d\xi_x \\ &= \int (\lambda z/s) \Delta\phi_s(\xi_x) d\xi_x \end{aligned} \quad (49)$$

If we now remember that $\Delta\phi_s(\xi_x)$ was measured by each detector in the detector plane, we have enough information to numerically integrate equation (49). In our one-dimensional SCIP scenario, we will assume a linear array of detectors which have a spacing between detectors of X_d . We are also assuming that our array of detectors will be densely packed, that is that $X_d \ll \lambda z/2P$. In spatial frequency space, this means that $d\xi_x \approx X_d/\lambda z$ which provides us with the interval over which we will integrate. Therefore

$$\begin{aligned} \phi_{sn}(x_x) - \phi_0 &\approx \sum_{i=1}^n (\lambda z/s) \Delta\phi_{si}(\xi_x) (X_d/\lambda z) \\ &\approx (X_d/s) \sum_{i=1}^n \Delta\phi_{si}(\xi_x) \end{aligned} \quad (50)$$

where $\Delta\phi_{si}(\xi_x)$ is the value calculated for $\Delta\phi_s(\xi_x)$ at the i^{th} detector in the linear array, and the upper summation limit, n , is the index of the n^{th} detector at which we would like to know the value of $\phi_{sn}(\xi_x)$. The constant of integration, ϕ_0 , is arbitrary and will be considered to be zero in the rest of our calculations.

It should also be stated that the simple numerical integration outlined in equation (50) is used to illustrate how the phase $\phi_{sn}(\xi_x)$ is determined for each point n along our detector array. In actuality, we will probably use a more accurate numerical integration method such as Taylor or Simpson integration to minimize the integration errors during the summation. These methods are very familiar to most engineers and scientists. The above method does illustrate the mathematics behind calculating $\phi_{sn}(\xi_x)$ however. In the next section, we will illustrate how the amplitude of the speckle pattern is determined.

Speckle Plane Amplitude Determination

Now that we have a way to determine a relative phase at any point in the speckle field, the next step in our imaging process is to determine the field amplitude value, $A_s(\xi_x)$, at any point in our speckle plane. To do this, consider figure 21 where $A_s(\xi_x)$ and its relation to the two terms, $A_-(\xi_x - (1/2P))$ and $A_+(\xi_x + (1/2P))$, are shown graphically. From this, we see that we can approximate the field amplitude as

$$A_s(\xi_x) = (1/2)\{A_+(\xi_x + (1/2P)) + A_-(\xi_x - (1/2P))\} = (A_+ + A_-)/2 \quad (51)$$

If we square this expression, then we see that

$$A_s^2(x_x) = (A_+^2 + A_-^2 + 2A_+A_-)/4 \quad (52)$$

Now using the expressions from equations (46), it is easy to show that

$$A_+^2 + A_-^2 = [B_0 + B_{\pi/2} + B_{\pi} + B_{3\pi/2}]/4 \quad (53)$$

$$\text{and} \quad 2A_+A_- = [(B_0 - B_{\pi})^2 + (B_{\pi/2} - B_{3\pi/2})^2]^{1/2}/2 \quad (54)$$

After inserting these values into equation (52), we see that

$$A_s(x_x) = \{[B_0 + B_{\pi/2} + B_{\pi} + B_{3\pi/2}] + 2[(B_0 - B_{\pi})^2 + (B_{\pi/2} - B_{3\pi/2})^2]^{1/2}\}/4 \quad (55)$$

is a good approximation for the one-dimensional speckle plane field amplitude. We now know enough about the speckle field to generate an image of our scene.

SCIP Image Generation

Equations (50) and (55) relate a new speckle field amplitude, $A_s(\xi_x)$, and phase, $\phi_{sn}(\xi_x)$, to the measurable speckle field observables B_0 , $B_{\pi/2}$, B_{π} and $B_{3\pi/2}$. This new speckle field would be that which would be generated by a single laser illuminating and being scattered off the target and then being observed in the far-field. The one difference is that this new speckle field has all atmospheric effects removed. Using this information, we now have enough information to generate a synthetic image of our scene using the SCIP technique. To do this, let's define

$$F_s(\xi_x) = A_s(\xi_x) \exp[j\phi_s(\xi_x)] \quad (56)$$

where in an analogous fashion to equation (36), we can consider $F_s(\xi_x)$ to be a good approximation of the Fourier transform of the scene's complex amplitude reflection coefficient, $r_o(x_o)$. We can therefore write

$$F_s(\xi_x) = \int_{-\infty}^{\infty} r_o(x_o) \exp[-j(2\pi/\lambda z) \xi_x x_o] dx_o \quad (57)$$

Therefore by taking the inverse Fourier transform of $F_s(\xi_x)$

$$r_o(x_o) = \int_{-\infty}^{\infty} A_s(\xi_x) \exp[j\phi_s(\xi_x)] \exp[j(2\pi/\lambda z) \xi_x x_o] d\xi_x \quad (58)$$

we regenerate the scene's complex reflection coefficient, and thereby learn everything about the scene that there is to know. Therefore, the inverse Fourier transform of $F_s(x_x)$ basically calculates or generates a synthetic image of the scene. This is how the SCIP method produces an undistorted image of the interrogated object.

ONE-DIMENSIONAL EXPERIMENTAL SCIP ALGORITHM VERIFICATION

As part of our research during this last year, the above algorithms were tested in a crude, one-dimensional laboratory experiment using a diffraction pattern created by two pinholes in an aperture. The layout depicted schematically in Figure 22 was set up on an optical table. A two-path interferometer was used to project two, slightly sheared wavefronts so that they illuminated the back of the aperture being imaged. This was done by recombining the two beams and then tilting one of the mirrors so that a fringe pattern appeared on the scene. The fringe pattern was adjusted so that its period was about 1.2 to 1.5 times the pinhole spacing. These fringes were then made to walk across the aperture by moving one of the mirrors in the interferometer and thereby changing the path length of one of the legs of the interferometer.

The scene being imaged was an aperture constructed of aluminum foil with two pin holes spaced 0.7 inches apart in the x-direction. Since this experiment was to test the above algorithm in only one dimension, we were unconcerned with the geometry of our scene in the y-direction, and we assumed the diffraction pattern produced by the two pin holes along the x-axis approximated that of two infinitely long slits. This is not absolutely true, but it was close enough to test our algorithm. Finally, the far-field or Fraunhofer diffraction pattern of the scene was produced on a CCD array by placing a lens directly behind the aperture and placing the CCD array a distance equivalent to the focal length of the lens away from the lens. This lens was not used for imaging, but only for producing a far-field speckle or diffraction pattern of the object.

Data for conducting this experiment was taken by digitizing the beating SCIP speckle pattern falling on the CCD array at four different moments in time. Only values obtained along the x-axis of the diffraction pattern were used. In this way, four one-dimensional intensity profiles approximating those of a double slit diffraction or speckle pattern were recorded. The four separate diffraction patterns were recorded by recording a pattern, translating one of the legs in the interferometer until the SCIP fringe pattern moved by 1/4 of its period and then recording the next pattern. In this way, each recorded far-field speckle or diffraction pattern had a phase distribution which was $\pi/2$ different from the previous recording. The values of B_o , $B_{\pi/2}$, B_{π} and

$B_{3\pi/2}$ at each point along the axis could then be read directly off of their respective diffraction patterns.

Four graphs of the one-dimensional intensity profiles obtained in our experiments are shown in figure 23. Though it may not be apparent in these graphs, it is interesting to note that the peaks in the diffraction pattern will walk to the right by $1/4$ of a peak width until the pattern repeats when the illuminating SCIP fringe pattern move one full period and therefore itself repeats. What is more apparent from these graphs is that the valleys in the diffraction pattern are deep in some of the graphs, and are filled in in some of the graphs. The reason for this is that as the SCIP fringe pattern is walked over the two pinholes, at times one of the pinholes is illuminated while the other pinhole falls at a minimum in the fringe intensity pattern, at times the second pinhole is illuminated while the first pinhole falls at a minimum in the fringe pattern, and at times both pinholes are illuminated equally. Of course other stages of illumination between the two pinholes also exist. Therefore as the SCIP fringe pattern scans the two pinholes, the far-field speckle or diffraction pattern will evolve between a two-pinhole diffraction pattern and a one-pinhole diffraction pattern. And since the pinhole being illuminated continues to shift back and forth, the peaks in the diffraction pattern will appear to walk.

Figure 24 shows a graph of the one-dimensional intensity distribution of our one-dimensional "image" which we calculated from the above data using the above describe algorithm. As can be seen in this graph, we were able to image our two "slits" or pinholes using the algorithm remarkably well. Though we did not have very good measurements of our detector spacing in our CCD array, and we were unsure that we had translated our SCIP fringe pattern by exactly $1/4$ fringe, we did find that pinhole spacing in our one-dimensional "image" corresponded closely with our original object. The random noise peaks in the image are probably results from a combination of the above two factors, the fact that we assumed two slits instead of two pinholes, and the fact that the image is a "speckled image" due to the use of coherent illumination. The speckle effect in the image can be done away with by averaging many similarly generated images. This is similar to what non-coherent optical instruments do by averaging a large number of wavelengths. The other effects will be minimized by more careful measurements and by either imaging true one-dimensional objects or by going to the two-dimensional SCIP techniques.

TWO-DIMENSIONAL ALGORITHM MODELING

After performing the above one-dimensional SCIP algorithm development and experimental verification, we began working to extend the algorithm into two-dimensions. This section describes the theory used to develop and justify the SCIP algorithm as deployed in the computer imaging code. Also mentioned are models that were used to test and debug the algorithms. These models are described here because they provide insight into understanding the interference pattern that the SCIP algorithm seeks to invert.

Interference Pattern at the Target

Three sheared coherent beams are assumed to illuminate the target. One of the beams is the

main, or reference, beam. One beam is offset (sheared) from the reference beam in the x-direction. The third beam is offset from the reference beam in the y-direction (see figure 25). The complex amplitude of these three interfering beams is expressed as

$$E = E_0 \exp[j(\mathbf{k}_0 \cdot \mathbf{r}_0 - \omega_0 t)] + E_1 \exp[j(\mathbf{k}_1 \cdot \mathbf{r}_1 - \omega_1 t)] + E_2 \exp[j(\mathbf{k}_2 \cdot \mathbf{r}_2 - \omega_2 t)] \quad (59)$$

where E_i , $\mathbf{k}_i$, and ω_i are the amplitude, wave vector, and frequency of the i^{th} beam respectively. The index $i=0$ represents the reference beam, $i=1$ is for the x-sheared beam, and $i=2$ is for the y-sheared beam. The vector $\mathbf{r}_i$ is the displacement vector from the beam's source to a point in space, typically on the target's surface. In this expression, $\omega_0 \approx \omega_1 \approx \omega_2$

If the coordinate origin is located at the source of the reference beam, then the distance to the target is given by

$$\begin{aligned} r_0 &= |\mathbf{r}_0| = \{R^2 + x'^2 + y'^2\}^{1/2} \\ r_1 &= |\mathbf{r}_1| = \{R^2 + (x' + s_x)^2 + y'^2\}^{1/2} \\ r_2 &= |\mathbf{r}_2| = \{R^2 + x'^2 + (y' + s_y)^2\}^{1/2} \end{aligned} \quad (60)$$

for each source respectively. In these expressions, R is the source-target range, x' and y' are target coordinates, s_x and s_y are the beam shears in the x and y directions.

For $R^2 \gg x'^2 + y'^2$, equations 60 become

$$\begin{aligned} k_0 r_0 &\approx k \{R + (x'^2 + y'^2)/2R\} \\ k_1 r_1 &\approx k \{R + (x'^2 + y'^2)/2R\} + 2\pi \{s_x / \lambda R\} x' \\ k_2 r_2 &\approx k \{R + (x'^2 + y'^2)/2R\} + 2\pi \{s_y / \lambda R\} y' \end{aligned} \quad (61)$$

Therefore, the superposition of the beams at the target can be adequately approximated as

$$E = E_0 \exp[j(-\omega_0 t)] + E_1 \exp[j((2\pi s_x x' / \lambda R) - \omega_1 t)] + E_2 \exp[j((2\pi s_y y' / \lambda R) - \omega_2 t)] \quad (62)$$

Image plane pattern

If the image plane is a distance R away from the source then the Fraunhofer diffraction field amplitude, $A(u,v)$, at the plane where the detector array is located is

$$A(u,v) = A(x,y) = \iint \Gamma(x',y') E(x',y',t) e^{-j2\pi u x'} e^{-j2\pi v y'} dx' dy' \quad (63)$$

where $u = x/\lambda R$ and $v = y/\lambda R$. Equation (63) is also the spatial, two-dimensional Fourier transform of the target. We use $\Gamma(x',y')$ to represent a reflection function at the target. The parameters u and v are the spatial frequencies in the image plane with x and y the coordinates in the detector plane.

Substituting equation (62) for $E(x',y',t)$ in equation (63), and assuming that we properly account

for the polarization of the light so we can ignore its vector nature, we get

$$\begin{aligned}
A(u,v) = & E_0 \exp(-\omega_0 t) \iint \Gamma(x',y') \exp(-j2\pi u x') \exp(-j2\pi v y') dx' dy' \\
& + E_1 \exp(-\omega_1 t) \iint \Gamma(x',y') E_1 \exp(-j2\pi (u - s_x/\lambda R) x') \exp(-j2\pi v y') dx' dy' \\
& + E_2 \exp(-\omega_2 t) \iint \Gamma(x',y') E_2 \exp(-j2\pi u x') \exp(-j2\pi (v - s_y/\lambda R) y') dx' dy'
\end{aligned} \quad (64)$$

For convenience we can write this Fourier transformed amplitude as

$$\begin{aligned}
A(u,v) = & E_0 F\{u,v\} \exp(-\omega_0 t) + E_1 F\{u - s_x/\lambda R, v\} \exp(-\omega_1 t) + E_2 F\{u, v - s_y/\lambda R\} \exp(-\omega_2 t) \\
= & E_0 F\{u,v\} \exp(-\omega_0 t) + E_1 F\{u_{(.)}, v\} \exp(-\omega_1 t) + E_2 F\{u, v_{(.)}\} \exp(-\omega_2 t) \\
= & E_0 F_0 \exp(-\omega_0 t) + E_1 F_1 \exp(-\omega_1 t) + E_2 F_2 \exp(-\omega_2 t)
\end{aligned} \quad (65)$$

where $F\{u,v\} = \iint \Gamma(x',y') \exp(-j2\pi u x') \exp(-j2\pi v y') dx' dy'$ is the Fourier transform and we have also defined $u_{(.)} = u - s_x/\lambda R$, and $v_{(.)} = v - s_y/\lambda R$ as indicated.

Now optical detectors see intensity which is proportional to the square of the complex field so that $I(x,y,t) = |A(x,y,t)|^2$. After performing a little algebra, we see that the CCD array will detect the following intensity distribution:

$$\begin{aligned}
I(x,y,t) = A(x,y,t) A^*(x,y,t) = & E_0^2 |F_0|^2 + E_1^2 |F_1|^2 + E_2^2 |F_2|^2 \\
& + 2 \operatorname{Re} \{E_0 E_1^* F_0 F_1^* \exp(j \delta \omega_1 t)\} \\
& + 2 \operatorname{Re} \{E_0 E_2^* F_0 F_2^* \exp(j \delta \omega_2 t)\} \\
& + 2 \operatorname{Re} \{E_1 E_2^* F_1 F_2^* \exp(j \delta \omega_3 t)\}
\end{aligned} \quad (66)$$

where $\delta \omega_1 = \omega_1 - \omega_0$, $\delta \omega_2 = \omega_2 - \omega_0$, and $\delta \omega_3 = \omega_2 - \omega_1$. Now if we write $F_0 = A_0 \exp(j \phi_0)$, $F_1 = A_1 \exp(j \phi_1)$, and $F_2 = A_2 \exp(j \phi_2)$, then the intensity becomes

$$\begin{aligned}
I = & E_0^2 A_0^2 + E_1^2 A_0^2 + E_2^2 A_0^2 \\
& + 2 E_0 E_1 A_0 A_1 \cos(\Delta \phi_1 + \delta \omega_1 t) \\
& + 2 E_0 E_2 A_0 A_2 \cos(\Delta \phi_2 + \delta \omega_2 t) \\
& + 2 E_1 E_2 A_1 A_2 \cos(\Delta \phi_3 + \delta \omega_3 t)
\end{aligned} \quad (67)$$

where

$$\begin{aligned}
\Delta \phi_1 = & \phi_0 - \phi_1 = \phi(u,v) - \phi(u - s_x/\lambda R, v) \\
\Delta \phi_2 = & \phi_0 - \phi_2 = \phi(u,v) - \phi(u, v - s_y/\lambda R) \\
\Delta \phi_3 = & \phi_1 - \phi_2 = \phi(u - s_x/\lambda R, v) - \phi(u, v - s_y/\lambda R)
\end{aligned}$$

The signal represented by equation (67) is the time dependent intensity that is measured by each element of the CCD array.

The SCIP Algorithm

The target's image must be extracted from the intensity pattern of equation (67), detected as a function of time. If $A_0(x,y)$ and $\phi(x,y)$ were known then $F_0 = A_0 \exp(j\phi_0)$ would be available. Using $F_0(x,y)$, we could calculate the inverse Fourier transform

$$f_0(x,y) = \iint F_0(x,y) \exp(-j(xx' + yy')/\lambda R) dx dy \quad (68)$$

which would then allow us to calculate the target's image, $I_0 = |f_0|^2$.

As was done in the one-dimensional case, we can use the time varying signal given in equation (67), and accumulate a Fourier transform in time at each detector array element and obtain the DC component and each of the three beat frequency components:

$$\begin{aligned} B_0 &= \int I(t) dt \\ B_1 &= \int I(t) \exp(j \delta \omega_1 t) dt \\ B_2 &= \int I(t) \exp(j \delta \omega_2 t) dt \\ B_3 &= \int I(t) \exp(j (\delta \omega_2 t - \delta \omega_1 t)) dt \end{aligned} \quad (69)$$

In terms of these B_i 's, the original signal is given approximately by

$$\begin{aligned} I(t) \approx & B_0 + [B_1 \exp(j \delta \omega_1 t) + B_1^* \exp(-j \delta \omega_1 t)] \\ & + [B_2 \exp(j \delta \omega_2 t) + B_2^* \exp(-j \delta \omega_2 t)] \\ & + [B_3 \exp(j (\delta \omega_2 t - \delta \omega_1 t)) + B_3^* \exp(-j (\delta \omega_2 t - \delta \omega_1 t))] \end{aligned} \quad (70)$$

If we define $B_1 = |B_1| \exp(j\Delta\phi_1)$, $B_2 = |B_2| \exp(j\Delta\phi_2)$, and $B_3 = |B_3| \exp(j\Delta\phi_3)$, then

$$\begin{aligned} I(t) \approx & B_0 + 2 |B_1| \cos(\Delta\Phi_1 + \delta\omega_1 t) \\ & + 2 |B_2| \cos(\Delta\Phi_2 + \delta\omega_2 t) \\ & + 2 |B_3| \cos(\Delta\Phi_3 + \delta\omega_3 t) \end{aligned} \quad (71)$$

Which when we compare this with equation (67), we can arrive at

$$\begin{aligned} B_0 &\approx E_0^2 A_0^2 + E_1^2 A_1^2 + E_2^2 A_2^2 \\ |B_1| &\approx E_0 E_1 A_0 A_1, \quad |B_2| \approx E_0 E_2 A_0 A_2, \quad |B_3| \approx E_1 E_2 A_1 A_2 \\ \Delta\Phi_1 &= \Delta\phi_1, \quad \Delta\Phi_2 = \Delta\phi_2, \quad \Delta\Phi_3 = \Delta\phi_3 \end{aligned} \quad (72)$$

So that for example we can approximate

$$A_0 \approx (1/2) (1/N_T)^{1/2} \{ [|B_1| + 2(|B_1| + |B_1| + |B_1|)] \}^{1/2}$$

$$+ [|B_i| - 2(|B_i| + |B_i| + |B_i|)]^{1/2} \} \quad (73)$$

at each CCD element. Note that this expression is only approximate due to the discreteness of the available data.

The quantities $\Delta\Phi_i$ are the phase differences between neighboring grid points on the CCD array. They are related to the absolute phase by the expressions in equations (67) as

$$\begin{aligned} \phi_0(x + s_x, y) &= \phi_0(x, y) + \Delta\Phi_1(x + s_x, y) \\ \phi_0(x, y + s_y) &= \phi_0(x, y) + \Delta\Phi_1(x, y + s_y) \end{aligned} \quad (74)$$

where x, y is the position of the detector element.

Discretization of the Detector Plane

Since the CCD is a discretized detector, it is necessary to move the formulation from the continuous to the discrete domain. Let δx and δy be the detector spacings in the x and y directions respectively. Let N_x and N_y be the number of elements in the x and y directions respectively. Assume the CCD elements are located at (x_i, y_i) where i and j index the array element. The element locations are arbitrarily assigned as

$$\begin{aligned} x_i &= (i - N_x/2) \delta x \\ y_j &= (j - N_y/2) \delta y \end{aligned} \quad (75)$$

A discrete Fourier transform (DFT) in time is taken at each of the detector elements making up the array. The sampling interval $\delta t = \delta\omega_1/2n\pi$ where n is the number of sample points in a beat wavelength determines the resolution of the DFT. The discretized versions of equations (69) are

$$\begin{aligned} B_0 &= \delta t \sum_{k=0}^{N_T-1} I_k \, dt \\ B_1 &= \delta t \sum_{k=0}^{N_T-1} I_k \exp(j \delta\omega_1 t_k) \\ B_2 &= \delta t \sum_{k=0}^{N_T-1} I_k \exp(j \delta\omega_2 t_k) \\ B_3 &= \delta t \sum_{k=0}^{N_T-1} I_k \exp(j \delta\omega_3 t_k) \end{aligned} \quad (76)$$

where $t_k = k \delta t$ and $I_k = I(t_k)$. The phase of the image must be determined self consistently. Note that at an interior point of the array we must satisfy the following four equations for $\phi(x_i, y_j)$

simultaneously.

$$\begin{aligned}
\phi_0(x_i+s_x, y_j) &= \phi(x_i, y_j) + \Delta\Phi_1(x_i, y_j) \\
\phi_0(x_i, y_j+s_y) &= \phi(x_i, y_j) + \Delta\Phi_2(x_i, y_j) \\
\phi(x_i, y_j) &= \phi(x_i-s_x, y_j) + \Delta\Phi_1(x_i-s_x, y_j) \\
\phi(x_i, y_j) &= \phi(x_i, y_j-s_y) + \Delta\Phi_2(x_i, y_j-s_y)
\end{aligned} \tag{77}$$

These four equations apply to each of the $(N_x-2)(N_y-2)$ interior points, three of these apply for each of the $2(N_x+N_y-4)$ edge points and two apply for each of the four corner points. In the algorithm we can arbitrarily specify only one of the $N_x N_y$ absolute phases.

There are many more equations than unknowns. Since the absolute phase is arbitrary, one can find any solution that satisfies these equations. The equations may be combined in any combination that results in a linear equation and the tools of linear algebra brought to bear in the solution for the phase function. One choice of a combination results from adding the equations together giving

$$\begin{aligned}
4\phi(x_i, y_j) &= \phi_0(x_i-s_x, y_j) + \phi_0(x_i, y_j-s_y) + \phi_0(x_i+s_x, y_j) + \phi_0(x_i, y_j+s_y) \\
&\quad + \Delta\Phi_1(x_i-s_x, y_j) + \Delta\Phi_2(x_i, y_j-s_y) - \Delta\Phi_1(x_i, y_j) - \Delta\Phi_2(x_i, y_j)
\end{aligned} \tag{78}$$

where the edge elements will depend on its three neighboring elements and the corner pieces will depend on its two neighbors. The terms that should be ignored in the expression for edge and corner elements can be inferred by examining figure 26.

Equation (78) determining $\phi_{ij} = \phi(x_i, y_j)$ can be thought of as the average of the adjacent elements around ϕ_{ij} plus a term that is a combination of the phase differences.

$$\phi_0(x_i-s_x, y_j) + \phi_0(x_i, y_j-s_y) + \phi_0(x_i+s_x, y_j) + \phi_0(x_i, y_j+s_y) - 4\phi(x_i, y_j) = f_{ij} \tag{79}$$

$$\text{where } f_{ij} = -[\Delta\Phi_1(x_i-s_x, y_j) + \Delta\Phi_2(x_i, y_j-s_y) - \Delta\Phi_1(x_i, y_j) - \Delta\Phi_2(x_i, y_j)].$$

There is now one equation for each grid point. Linear algebra may now be implemented to solve this coupled system of equations for the absolute phase ϕ_{ij} .

TWO-DIMENSIONAL COMPUTER MODELING

A major portion of this project was devoted to developing the two-dimensional SCIP computer code, testing and debugging it, and getting it to work for simulated computer data. The above two-dimensional SCIP algorithm was programmed using C++ into a code for testing. This code was then tested with simulated data to determine whether the algorithm worked, before trying it with laboratory data. Figure 27 shows a computer simulated target which was used in our algorithm testing. It is similar to an Air Force bar chart. Using computer modeling, the evolving Fraunhofer SCIP diffraction pattern was calculated. One frame of this evolving speckle diffraction pattern is shown in figure 28. This evolving diffraction pattern simulates what a CCD array would see in the far field. It simulates the two-dimensional SCIP algorithm data which a CCD array would see for calculating the image of the target.

Figures 29, 30, and 31 provide three examples of calculated SCIP images from the simulated data for three values of N_t : $N_t = 64$, $N_t = 128$, and $N_t = 256$ respectively. As can be seen from these "images", the two-dimensional SCIP algorithm appears to work for computer simulated data. The image quality is very dependent on the value of N_t . Lower values of N_t produce lower resolutions, and have "ghosts" which are artifacts of the discrete number of points used in taking the inverse Fourier transform.

TWO-DIMENSIONAL LABORATORY LABORATORY EXPERIMENTS

After getting the two-dimensional SCIP computer code working with simulated data, the remainder of this project was used to try to get this code to work for laboratory simulated data. Unfortunately, we were never successful in getting this code to work on laboratory generated data.

Figure 32 shows a schematic diagram of the two-dimensional laboratory setup. A New Focus tunable, external cavity diode laser was used in these experiments. Light from this laser was split into three parts - a reference beam, an x-sheared beam, and a y-sheared beam - then expanded and sent to illuminate a small target. Electro-optical phase modulators were used to frequency modulate these beams. The reference beam was frequency modulated by 10.000 KHz, the x-sheared beam was frequency modulated by 10.002 KHz, and the y-sheared beam was frequency modulated by 10.003 KHz. This meant that the FM sidebands for the reference beam and the x-sheared beam beat on the target at 2 Hz, and the reference beam and the y-sheared beam beat on the target at 3 Hz. There was also a beat between the x- and y-sheared beams of 1 Hz. These beats could be observed as a scanning fringe pattern in two orthogonal directions at the target plane. These fringe patterns were made to be as parallel and perpendicular as possible.

Several different CCD arrays (8-bit resolution) were used to gather data from the evolving far field speckle pattern from the target. This data was then digitized using standard frame grabbing techniques and input into the two-dimensional SCIP code for calculating the SCIP image of the object.

Due to the low 8-bit resolution of the CCD cameras used, no recognizable images were calculated using laboratory data, no matter how simple the target. Various efforts were made to determine why the SCIP code could not handle this data even though it worked well with simulated data. Our efforts included trying different cameras with better digitizing depths, trying different methods of determining detector spacing on the CCD array, trying different methods of measuring the important variables needed in the calculations. We also talked with Dr. David Voelz at the Air Force Phillips Lab in Albuquerque who had worked with one-dimensional SCIP techniques for trying to image orbiting satellites from the ground.

From our two-dimensional SCIP experiments and our conversations with Dr. Voelz, we came to the realization that SCIP is not as robust as was initially believed. Dr. Voelz could not get the technique to work well except when floating his optics on a heavy optical table and controlling both air currents and temperatures. Wavefronts also needed to be controlled to better than $1/20$

of a wave. Experience in this project tends to indicate the same thing. Wavefronts, after passing through all the necessary optics in the laboratory, especially the optical modulators, were much worse than $1/20$ of a wave. Plans were made to set up an experiment for floating our laboratory experiments in an air current controlled environment using a 12-bit CCD array. The angles used in the laboratory also are too large when compared to the wavelength of the laser beams, which means that a smaller target (< 1 mm across) would help in achieving $\lambda < 1/20$ or better wavefront. Program funding was exhausted before these experiments could be completed.

SUMMARY AND CONCLUSION

An important conclusion as a result of this project is that, even though SCIP works in theory, it is currently not a robust technology for performing outdoor imaging and removing atmospheric turbulence. Improvements in optical isolation and CCD camera sensitivity will certainly help to prove the viability of this technique. Unfortunately, current methods difficult to implement and the requirements for precision make this method almost impossible to implement as an imaging tool. However, applications may exist for SCIP technology in ultrasonic or RF imaging.

Table 1 - SLC models for C_n^2 altitude profiles.

Altitude (meters above ground)		C_n^2 (meters ^{-2/3})
SLC Nighttime Model	$h < 18.5$	8.40×10^{-15}
	$18.5 < h < 110$	$2.87 \times 10^{-12} \times h^{-2}$
	$110 < h < 1500$	2.5×10^{-16}
	$1500 < h < 7200$	$8.87 \times 10^{-7} \times h^{-3}$
	$7200 < h < 20,000$	$2.00 \times 10^{-16} \times h^{-0.5}$
SLC Daytime Model	$h < 18.5$	1.70×10^{-14}
	$18.5 < h < 240$	$3.13 \times 10^{-13} \times h^{-1}$
	$240 < h < 880$	1.30×10^{-15}
	$880 < h < 7200$	$8.87 \times 10^{-7} \times h^{-3}$
	$7200 < h < 20,000$	$2.00 \times 10^{-16} \times h^{-0.5}$

Table 2 - Equation (5) coefficients for the AMOS model of calculating C_n^2 altitude profiles.

	3.052 < z < 5.2 km	5.2 < z < 30 km
A =	-12.412	-17.1273
B =	- 0.4713	- 0.0301
C =	- 0.0906	- 0.0010
D =	last term is zero	+ 0.5061
E =		+15.0866
F =		+ 3.2977

Table 3 - Equation (5) coefficients for the CLEAR I model of calculating C_n^2 altitude profiles.

	1.23 < z < 2.13 km	2.13 < z < 10.34 km	10.34 < z < 30 km
A =	-10.7025	-16.2897	-17.0577
B =	-4.3507	+ 0.0335	- 0.0449
C =	+0.8141	- 0.0134	- 0.0005
D =	last term is zero	last term is zero	+ 0.6181
E =			+15.5617
F =			+ 3.4666

TABLE 4 - CLEAR I and HV 5/7 model calculations for looking vertically (up and down) through the atmosphere ($\lambda = 852 \text{ nm}$, $v_{\perp} = 1 \text{ meter/second}$).

model/path length	r_o pl	r_o up sph	r_o down sph	θ_o	τ_o
L = 30,000 meters					
CLEAR I	4.50 cm	4.53 cm	154 cm	16.1 mrad	$1.41 \times 10^{-2} \text{ sec}$
HV 5/7	4.16	4.20	134	14.0	1.31×10^{-2}
L = 300,000 meters					
CLEAR I	4.50	4.50	1540	16.1	1.41×10^{-2}
HV 5/7	4.16	4.16	1340	14.0	1.31×10^{-2}

TABLE 5 - Calculations for looking horizontally through the atmosphere ($\lambda = 852$ nm, $v_+ = 1$ meter/second).

For constant $C_n^2 = 1 \times 10^{-13}$ meters^{-2/3}:

L	r_o _{pl}	r_o _{sph}	θ_o	τ_o
10 m	15.3 cm	27.3 cm	8640 mrad	4.79×10^{-2} sec
30	7.90	14.2	1490	2.48×10^{-2}
50	5.81	10.4	658	1.83×10^{-2}
100	3.83	6.84	217	1.20×10^{-2}
300	1.98	3.54	37.4	6.23×10^{-3}
500	1.46	2.61	16.5	4.58×10^{-3}
1000	.963	1.72	5.45	3.02×10^{-3}
3000	.498	.889	.939	1.56×10^{-3}
5000	.367	.655	.415	1.15×10^{-3}
10000	.242	.432	.137	7.60×10^{-4}
30000	.125	.223	.0236	3.93×10^{-4}
50000	.0921	.164	.0104	2.89×10^{-4}
100000	.0608	.109	.00344	1.91×10^{-4}

For constant $C_n^2 = 5 \times 10^{-13}$ meters^{-2/3}:

L	r_o _{pl}	r_o _{sph}	θ_o	τ_o
10 m	5.81 cm	10.4 cm	3290 mrad	1.83×10^{-2} sec
30	3.01	5.38	567	9.44×10^{-3}
50	2.21	3.95	250	6.95×10^{-3}
100	1.46	2.61	82.6	4.58×10^{-3}
300	.755	1.35	14.2	2.37×10^{-3}
500	.556	.993	6.29	1.74×10^{-3}
1000	.367	.655	2.07	1.15×10^{-3}
3000	.190	.339	.358	5.96×10^{-4}
5000	.140	.250	.158	4.38×10^{-4}
10000	.0921	.164	.0521	2.89×10^{-4}
30000	.0476	.0850	.00898	1.50×10^{-4}
50000	.0351	.0627	.00397	1.10×10^{-4}
100000	.0231	.0413	.00131	7.27×10^{-5}

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FIGURE 1 - Geometry for calculating the atmospheric structure tensor, D_{ij} .

FIGURE 2 - Five-minute segment of C_n^2 measurements by three separate instruments. The measurements were made by Beland, et. al. [17], at a height of 2 meters above the ground over a flat, grassy terrain on a clear, sunny day. The three instruments were positioned in a straight line and separated by about 30 meters.

FIGURE 5

FIGURE 6

CLEAR 1 model altitude profile.

FIGURE 7

FIGURE 8

FIGURE 9 - Dependence of the normalized resolution on the normalized diameter of the imaging optics for long-exposure imaging as calculated by Fried [23].

FIGURE 10 - In an adaptive optics system, areas imaged outside the isoplanatic angle from the laser guidestar will not be corrected by the adaptive optics.

FIGURE 11 - Schematic of an adaptive optics system for astronomy which uses a natural guidestar for probing the effects of atmospheric optical turbulence.

FIGURE 12 - Schematic of an adaptive optics system for astronomy which uses an artificial laser guide star for probing the effects of atmospheric optical turbulence.

(A)

(B)

FIGURE 14 - Schematic diagram of the Hartman wavefront sensor. The microlenslets focus planer wavefronts at particular positions on the detector array. Aberrated wavefronts cause these focus positions to move, so that the localized slope of the wavefronts can be measured.

FIGURE 24 - Intensity distribution profile of a 1-dimensional synthetic image which was obtained from the data presented in figure 23.

FIGURE 23 - Speckle intensity profiles taken from a 1-dimensional SCIP experiment. 2s1110 is the reference frame, and the other profiles were taken in phase steps of $\pi/2$.

FIGURE 17 - Schematic diagram of a holographic speckle imaging system which is capable of removing the effects of optical turbulence. As in figure 16, a laser beacon illuminates the target as well as a reference mirror so that a reference beam and the speckle pattern will both be projected onto the detector array. In this case however, the reference beam and the speckle wavefront both travel through nearly the same optical turbulence. In this way, the aberrations are present on both the reference wavefront and the speckle wavefront, and they tend to cancel on the detector array. An unaberrated image results when the Fourier transform is taken of the speckle pattern.

FIGURE 15 - Schematic diagram of a simple speckle imaging system. A laser beacon illuminates a target and forms a far field speckle pattern. A lens, which has the ability to perform a 2-dimensional Fourier transform, transforms this speckle pattern into an inverted image on the detector array.

FIGURE 16 - Schematic diagram of a holographic speckle imaging system. A laser beacon illuminates a target and forms a far field speckle pattern. A reference beam is projected onto the detector array along with the speckle pattern so that the individual detectors can measure both amplitude and phase of the speckle pattern. A computer calculates the Fourier transform of the speckle pattern which results in an image of the scene.

FIGURE 19 - Sketch of the geometry used for making the SCIP calculations in the text. Laser beacons are separated by the shear distance, s , which is also the amount of shear present between the speckle patterns. Two slightly different frequencies illuminate the scene, causing a beating intensity pattern on the scene. The period of this intensity pattern is P . The detector array has a spacing of d_s between the separate detectors.

Figure 25 - Shearing geometry of the source beams.

Figure 26 - An array of CCD elements illustrating how each element is specified in terms of its neighboring elements. The grid spacings in the x- and y-directions have been taken to be δx and δy for the sake of illustration.

Image and Speckle: $t = 20$

Figure 27 - Example of a computer generated object for doing SCIP calculations.

Figure 28 - Example of the evolving Fraunhofer SCIP speckle diffraction pattern from which an image is calculated.

Image: $N_t = 64$

Figure 29 - Calculate SCIP image of the target given in figure 27. $N_t = 64$.

Image: $N_t = 128$

Figure 30 - Calculate SCIP image of the target given in figure 27. $N_t = 128$.

Image: $N_t = 256$

Figure 31 - Calculate SCIP image of the target given in figure 27. $N_t = 256$.

Figure 32 - Schematic diagram of the three-dimensional SCIP lab experiment. Laser beam is split into three beams - a reference beam, an x-sheared beam, and a y-sheared beam. All three beams pass through electro-optical phase modulators. The reference beam is frequency modulated at 10.000 KHz; the x-sheared beam is frequency modulated at 10.002 KHz; and the y-sheared beam is frequency modulated at 10.003 KHz. The three beams are recombined and beat together, expanded, and then passed through a transmissive target. The evolving far-field or Fraunhofer speckle diffraction pattern is collected by a CCD array. The experimental setup is designed so that the reference beam contains 50% of the radiation illuminating the target while the x- and y-sheared beam each possess 25% of the illuminating radiation.