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Dispersive Internal Long Wave Models

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Abstract

This is the final report of a three-year, Laboratory Directed Research and Development (LDRD) project at Los Alamos National Laboratory (LANL). This work is a joint analytical and numerical study of internal dispersive water wave propagation in a stratified two-layer fluid, a problem that has important geophysical fluid dynamics applications. Two-layer models can capture the main density-dependent effects because they can support, unlike homogenous fluid models, the observed large amplitude internal wave motion at the interface between layers. We have derived new model equations using multiscale asymptotics in combination with the method we have developed for vertically averaging velocity and vorticity fields across fluid layers within the original Euler equations. We have found new exact conservation laws for layer-mean vorticity that have exact counterparts in the models. With this approach, we have derived a class of equations that retain the full nonlinearity of the original Euler equations while preserving the simplicity of known weakly nonlinear models, thus providing the theoretical foundation for experimental results so far unexplained.

Background and Research Objectives

Computational fluid dynamics is reaching a stage where the complexity of the fluid flow simulations is comparable to that of real experiments. While this progress constitutes a striking confirmation of the validity of the basic motion equations, like the Navier Stokes, and enables the visualization of flows that would be extremely difficult to observe experimentally, sometimes it falls short of enhancing our understanding of the basic mechanisms at play for certain phenomena. This understanding is particularly important in the cases when the fundamental equations of motion are still too complex for the current computational power and simpler model equations have to be used. For instance, the three-dimensional, large-scale-global, ocean-dynamics-circulation code that has been implemented at Los Alamos on the Connection Machine uses a simplified version of the motion equations (the so called Primitive Equations) that enforces hydrostatic balance. This sometimes leads to gravitational instability (heavier fluid over lighter one) often due to breaking of internal gravity waves, which the simpler equations cannot handle adequately.

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To gain some insight on what the physical motion should be, simpler models that target a specific mechanism expected to be important are still useful. In the context of the above example, it is known that dispersion effects can act against the tendency of nonlinearity to make the internal waves break, yet the Primitive Equations are unable to support these dispersive effects.

In almost all aspects of global-ocean-circulation dynamics, the relevant spatial scales lie in thin domains. That is, the horizontal length scales of the main circulations in the ocean are large compared to its depth. Thus, two-dimensional approximations derived via asymptotic methods based on the smallness of the aspect ratio can be expected to be very effective at capturing most of the relevant dynamics. Indeed, even the presence of density stratification, one of the main agents responsible for exciting truly three-dimensional motion, cannot effectively counterbalance the natural tendency towards two-dimensional dynamics. This is because the stratification (the derivative of the density with respect to the vertical coordinate) is weak and stable (light fluid over heavier fluid) almost everywhere in the ocean. Thus the three-dimensional motion supported by density stratification over most of the ocean is restricted to internal gravity waves and shear flows. However, these two types of stratification-induced motion are quite important, as they take a large share of the total kinetic energy of the ocean, and barotropic (vertically integrated) models fail to capture their effect on global circulation.

An unexpectedly refined description of most of the ocean can be obtained by using a two-layer model, which takes into account the sharp density gradient between the homogenous density upper ("mixed") layer of the ocean and the (extremely) weakly stratified deep layer. Again, within each layer, the fluid dynamics is essentially two dimensional. However, such a model can support the observed large amplitude internal wave motion at the interface (the thermocline) between layers, as well as large shear velocities (such as that induced by winds), which cannot be described by the simpler barotropic models. Furthermore this model is the starting point for multilayer models that are actually being implemented for ocean global circulation codes. Since long-time integration of the equations of motion for the ocean are necessary for studies on climate changes, there is a definite need for models that are simple enough to be integrated for long times at sufficiently high resolution, and yet refined enough to be able to capture most of the relevant physics. Needless to say, two-dimensional models afford a tremendous saving of computational power.

The above applications in geophysical ocean dynamics require equations of motion for two-layer fluids that go beyond the previously established models. In particular, the new models have to be able to handle (besides full two-dimensionality) the nonlinearity

and dispersion balance, a wider range of layer depths, interaction with bottom topography and forcing from pressure distributions at the free surface, and velocity shear flows.

Importance to LANL's Science and Technology Base and National R&D Needs

One of the main goals of this research is to provide theoretical support to the community of computational/geophysical fluid dynamicists at the Laboratory involved in large-scale simulations of ocean dynamics. The asymptotic balance between nonlinear steepening and dispersion addressed here (and the surprisingly rich dynamics it generates) can find its way into high resolution multilayer codes for ocean dynamics. Moreover, in order to study mixing of fresh water (or pollutants) in near coastal waters it is crucial to understand how internal waves respond to the reversal (with respect to unity) of the relative layer thickness caused by the bottom reaching all the way up to the free surface. Using the models that we have derived, we can study in one and two dimensions the case of internal waves running up a sloping bottom. In particular, the phenomenon of change of polarity (from a wave of depression to a wave of elevation) and reflection that occurs at a critical thickness ratio of the two-layer system can now be studied with sufficient accuracy.

Scientific Approach and Accomplishments

Our work has carried out an exhaustive study of models for the long-wave dynamics of a two-layer stratified fluid in the presence of bottom topography and surface forcing (see Table 1). We have derived new weakly and fully nonlinear models that describe the dynamics of most relevant situations for two-layer fluids (1-3,7). In particular, we have analyzed both the case in which the two layers can be considered of finite depth and the case of a large (limiting to infinite) depth of one of the layers. By progressively increasing the degree of specialization of our models (and accordingly their simplicity), we have recovered the existing known models as limiting cases. These are extremely narrow in scope, almost always addressing one-dimensional, unidirectional wave propagation with special assumption on layer depths and irrotationality. Furthermore, no attention is paid to the structures of the original theory in deriving the approximation. The asymptotic expansions we have carried out preserve the crucial Hamiltonian structure at each level of approximation. This ensures that the models we have established then possess conservation laws mimicking the ones of the original (and not so easily tractable) Euler equation of motion. In the course of the derivation of the new models, we have further developed the method of vertically averaging across fluid layers to obtain simpler (lower dimensional) equations from the original Euler system. An unexpected by-product of this has been the discovery of a whole new class of conservation

laws for the layer-averaged vorticity (4-6). These new conserved quantities can in turn be used in the models for introducing similar conservation laws that are the approximate counterpart of the averaged vorticity conservation laws. By exploiting the layer-average formalism, we have been able to remove the weak nonlinearity assumption shared by all previously known dynamical models for two-layer fluids. One immediate benefit of this has been the ability to reproduce theoretically experimental data on two-layer solitary waves of large amplitude that have remained unexplained in the literature for more than a decade (2,3).

Publications

1. Camassa R., with W. Choi, "Nonlinear internal waves in a two-layer fluid," *J. Fluid Mech.* **313**, pp. 83-103, (1996).
2. Camassa R., with W. Choi, "Long internal waves of finite amplitude," *Phys. Rev. Lett.* **77** (9), pp. 1759-1763 (1996).
3. Camassa R., with W. Choi, "Fully nonlinear internal waves," submitted to *J. Fluid Mech.* (1997).
4. Camassa, R. and C.D. Levermore, "Layer-mean quantities, local conservation laws, and vorticity," *Phys. Rev. Lett.* **78**, pp. 650-653 (1997).
5. Camassa, R., D.D. Holm and C.D. Levermore, "Long time evolution equations for shallow water with a varying bottom," *Physica D* **98**, pp.258-286 (1996).
6. Camassa, R., D.D. Holm and C.D. Levermore, "Long-Time Effects of Bottom Topography in Shallow Water," to appear in *J. Fluid Mech.* (1997).
7. Choi, W., "On the fission of algebraic solitons," *Proc. R. Soc. London A* **453**, pp. 1753-1762 (1997).

Dispersive Internal Long Wave Models				
Relative layer depths		$h_1 = O(h_2)$	$h_1 \ll h_2$ (finite)	$h_2 \rightarrow \infty$
Fully Nonlinear		Coupled Green-Naghdi System (New)	Completed	Completed
Weakly Nonlinear	uni-dir. (1-D)	KdV, KP Equations uni-GN Eq. (New)	Intermediate-Long-Wave (ILW) Equation	Benjamin-Ono Equation
	bi-dir. (2-D)	Boussinesq System	Completed	Completed
	forced	Yes (KdV, KP,B) No (uni-GN)	Completed (1-D) Completed (2-D)	Yes (1-D) Completed (2-D)

Table 1. The various levels of approximation for stratified two-layer fluid models. Here h_1 and h_2 refer to the thickness at rest of the top and bottom layer, respectively. KdV and KP stand for Korteweg-de Vries and Kadomtsev-Petviashvili equation, respectively. The entries marked "New" refer to models we have recently derived and have not been reported in the literature before, to the best of our knowledge, "Completed" indicates that we have successfully carried out the derivation of a model in this regime during this project. "Yes" indicates that some information on the behavior of solutions in the presence of external forcing exists, "No" indicates the opposite.