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Unified Model of the rf Plasma Sheath

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Unified Model of the rf Plasma Sheath

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Abstract

By developing an approximation to the first integral of the Poisson equation, one can obtain solutions for the voltage-current characteristics of a radio-frequency (rf) plasma sheath that are valid over the whole range of inertial response of the ions to an imposed rf voltage or current-specified conditions. The theory adequately reproduces the time-dependent voltage-current characteristics of the two extreme cases corresponding to the Lieberman rf sheath theory and the Metze-Ernie-Oskam theory. Contained within the approximation is a time constant which controls the amount of ion response to the rf electric field. A prescription is given for determining this ion relaxation time constant, which also determines the time-dependent ion impact energy on the electrode surface.

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1. Introduction

Lieberman's¹ theory of an rf-driven plasma sheath leads to an analytic solution for the anharmonic voltage required to produce a prescribed sinusoidal rf displacement current in the electrons. The model assumes that the electrons are cold in the sense that the electron density distribution is equal to the ion density inside the sharp electron boundary, and is zero towards the wall or electrode. There is no dc current except for the ion flow to the wall, and all the rf current is due to the oscillation of the electron boundary within the sheath, which is a displacement current. The time-independent ion density distribution within the sheath is computed self-consistently assuming that the ions see only the time average of the potential. In other words, the ions are completely inertial, responding only to the average field. Lieberman's solution is one of the few exact new solutions obtained to any model problem in plasma physics. The limitations of this model are mainly due to a lack of electron particle current to the wall -- as a result there is no satisfactory dc limit.

Another model for an rf plasma sheath is that of Metze, Ernie, and Oskam,² (denoted MEO) which assumes that the electrons and the ions both respond instantaneously to an imposed time variation of the sheath potential. This is an *adiabatic* model of the particle dynamics in which the ions as well as the electrons are inertialess. This model includes electron particle current to the wall, as well as ion current and electron displacement currents. This model becomes accurate as the applied fields approach the dc limit.

It is obvious that a model that connects these two limiting theories would be of importance in the simulation of plasma sheath dynamics. Such a model is developed here and appears to be fairly robust. As a computational model formulated as an ordinary differential equation in time connecting the overall sheath potential and total plasma current, it is capable of incorporating boundary conditions expressed as either time-dependent potential or current. The model is a generalization of the MEO model, with a controlled amount of inertial response of the ions added into the theory by means of an approximate first integral of the Poisson equation and a time damping procedure for simulation of the average ion response.

2. Basic Foundations of the Sheath Model

Consider a collisionless one-dimensional region of a time-dependent electropositive plasma sheath between the bulk (or presheath) region and the wall (or electrode). There is a time-and-space-dependent potential V and field E :

$$\begin{aligned} V(x,t), \\ E(x,t) = -\frac{\partial}{\partial x} V(x,t) = -V'(x,t). \end{aligned} \quad (1)$$

The electrons and ions both respond to this field. The field itself must satisfy Poisson's equation at all times:

$$V''(x,t) = -\frac{e}{\epsilon_0} (n_i(x,t) - n_e(x,t)). \quad (2)$$

Only positive ions and electrons are present in the sheath, of densities n_i and n_e .

2.1 The Electrons

The electrons are assumed to be in an equilibrium Boltzmann distribution at temperature T_e , responding instantaneously, *i.e.* adiabatically, to the field, producing an electron density n_e :

$$n_e(x,t) = n_e(x_0,t) \exp \left[\frac{e}{kT_e} (V(x,t) - V(x_0,t)) \right] \quad (3)$$

where x_0 is some reference point within the plasma. For typical sheath parameters one estimates that the electrons should respond adiabatically up to and beyond the GHz region of applied rf frequencies. The electron particle current per area, j_e , is approximated from kinetic theory as the current to the wall, which is located in the positive x direction at $x = x_w$,

$$j_e(t) = -\frac{1}{4} e u_T n_e(x_w, t), \quad (4)$$

where u_T is the thermal velocity of the electrons:

$$u_T = (8kT_e / \pi m_e)^{1/2}. \quad (5)$$

This description of the electrons is quite standard and is the same as that used by MEO in their analysis.² The assumption of an adiabatic Boltzmann distribution is expected to be quite good for the range of applied frequencies used in most applications.

2.2 The Ions

The ions do not always respond fast enough to the applied time-dependent field to justify the type of solution assumed for the electrons in Eq.(3). The higher-frequency rf fields that are used in plasma processing, combined with moderate-mass ions, lead to a restricted "following" of the field by the ions and a large effect on the ion energy distribution that impacts the wall or electrode surface. Let the ion density be written as the adiabatic solution to a damped or "averaged" potential $\bar{V}$ which is distinct from the actual potential, and even independent of it in a transient situation. The purpose of this potential will be to serve as an algebraic representation of the ion density and velocity, which are the physical quantities that have inertia and do not instantaneously respond to the electric field within the sheath. In the damped potential, the monoenergetic ion density n_i is assumed related to the ion velocity u_i by the continuity equation, using a reference point x_o for the ion density and velocity:

$$n_i(x, t) = n_i(x_o, t) u_i(x_o, t) / u_i(x, t). \quad (6)$$

The ions are also assumed to obey the energy conservation equation,

$$\frac{1}{2} m_i u_i^2(x, t) + e \bar{V}(x, t) = \frac{1}{2} m_i u_i^2(x_o, t) + e \bar{V}(x_o, t), \quad (7)$$

where m_i is the ion mass. One should understand that the writing of Eqs.(6) and (7) is no more than a restatement of the assumption that the ion motion is being

described as an adiabatic solution in a potential which is distinct from the instantaneous potential.

This damped potential is constructed from the actual potential by a relaxation relation:

$$\frac{\partial}{\partial t} \bar{V}(x,t) = \dot{\bar{V}}(x,t) = -(\bar{V}(x,t) - V(x,t)) / \tau_{ave}. \quad (8)$$

A means of determining the time constant, τ_{ave} , which controls the amount of "averaging" involved in $\bar{V}$, will be given in Section 5. Note that, for τ_{ave} much less than the applied rf period τ_{rf} , $\bar{V}$ follows V closely and the ions

instantaneously respond to the same potential as the electrons. The rf period is just the reciprocal of the frequency: $\tau_{rf} = 1 / \nu_{rf}$. For τ_{ave} much greater than τ_{rf} , the ions respond to a potential that approaches a true time average of the actual potential. It is the parameter τ_{ave} which controls all the physics of the ion relaxation in the time-dependent field; the analysis in Section 5 will show that it is close to the ion fall time through the sheath.

For simplicity the following notation will be used at the reference position:

$$\begin{aligned} n_e^0 &= n_e(x_0, t), \quad n_i^0 = n_i(x_0, t), \quad u_i^0 = u_i(x_0, t), \\ V^0 &= V(x_0, t), \quad \bar{V}^0 = \bar{V}(x_0, t). \end{aligned} \quad (9)$$

All of the assumptions thus far can be summarized by the substitution of Eqs. (3), (6), and (7) into Eq. (2).

3. Self-Consistent Equations for the Sheath

3.1 Formulation Thus Far

The electron distribution in Eq.(3) is substituted into the Poisson Eq.(2), giving

$$V'' = -\frac{e}{\epsilon_o} \left(n_i - n_e^o \exp\left(\frac{e}{kT_e}(V - V^o)\right) \right), \quad (10)$$

where the ion density is given by combining Eqs.(6) and (7):

$$n_i = n_i^o \left(1 - \frac{2e}{m_i (u_i^o)^2} (\bar{V} - \bar{V}^o) \right)^{-1/2}. \quad (11)$$

Eqs.(10) and (11) both contain time as a parameter. For a fixed t , as a second-order ordinary differential equation in coordinate space, the solution for V to Eq.(10) depends on several quantities, namely an unknown function n_i (because of the dependence on $\bar{V}$), several constants related to the reference point x_o , and the function and slope of the dependent variable V at the reference point. One notes that the potential in which the ions "move" is not yet self-consistent with the solution of Poisson's equation. This self-consistency will occur upon invoking Eq.(8), which determines $\bar{V}(x, t)$ in terms of $V(x, t')$, $t' \leq t$, in a set of coupled partial differential equations. For a steady-state, time-independent sheath, one knows that the damped potential should be exactly equal to the actual potential, and Eqs.(10) and (11) reduce to the standard equation of the sheath as discussed by Chen,³ and developed in much earlier plasma work. Having said this, one then argues for the imposition of physically reasonable conditions at the plasma reference position that result in proper behavior of the sheath solution.

3.2 Including Sheaths into a Circuit Description

Two things must be obtainable from a sheath model in the context that is being pursued here. First of all, we need the current-voltage properties of the

sheath so that one can write down the functional relation of the total current to the bulk plasma properties and the sheath voltage in order to include the sheath in the circuit equations for the whole system. Secondly, we need to be able to derive important kinetic properties of the sheath particles, such as the ion energies that fall through the potential drop. This will be discussed later, although it is obvious now that we are committed to using the time-dependent kinetic energy of the mono-energetic ion fluid as predicted by the damped potential, given by Eq.(7). For the moment, consider the plasma, the plasma sheaths, and external circuitry with an analysis similar to Kirchhoff's rules for writing down the circuit equations. What is needed from the sheath itself will be written down here.

The total current per area j_{tot} through the sheath is expressed as

$$\begin{aligned} j_{tot} &= j_e + j_i + j_d , \\ j_i &= e u_i^o n_i^o , \\ j_d &= \epsilon_o \dot{E} . \end{aligned} \tag{12}$$

One can show that the above expression for the total current is constant across the sheath if the full time-dependent continuity and Poisson's equations are being solved consistently. Here we are using approximations to the dynamics, so the expression is not independent of x . The adiabatic Boltzmann approximation for the electron particle current j_e is given by Eq.(4) in terms of the electron density at the wall, which is related by a potential-drop (Boltzmann) factor in Eq.(3) to the bulk or reference electron density. The continuity Eq.(6) for the ions equates the ion current j_i to the value at the reference position. The electron displacement current j_d should be evaluated at the wall position to be consistent with both the point of determination of j_e and the later connection to be developed to the total sheath potential.

The displacement current is expressed as the partial time derivative of the field at the wall. Only the wall potential or potential difference between the wall and the bulk plasma is a quantity accessible in the circuit equations for the system. Thus the sheath model must supply the relation of E to V . In the case of a dc sheath, or the MEO sheath model which is based on adiabatic response of the dc model to a time-varying applied potential, one can write down the *first integral* of the Poisson equation, $E = E(V)$ and evaluate

$$\dot{E}(x,t) = \frac{dE}{dV} \dot{V}(x,t), \quad (13)$$

which completes the determination of current as a function of voltage.

3.3 The First Integral of the Poisson Equation

The utility of the first integral of the Poisson equation is seen in References 2 and 3 in their analyses of the adiabatic following rf model (MEO) and the pure dc situation. The procedure here is slightly different because one cannot write down an exact first integral of the Poisson equation with the ions evolving in a distinct potential. The situation calls for an approximation, which is developed in the following.

We begin with Eqs.(10) and (11), which we restrict somewhat by incorporating some specifications at the reference point x_o . We set

$$\begin{aligned} V^o &= 0, \quad \bar{V}^o = 0, \\ n_e^o &= n_i^o = n_o, \\ u_i^o &= M u_B = M(kT_e / m_i)^{1/2}, \end{aligned} \quad (14)$$

where the ion "injection" velocity u_i^o is set to a factor $M \geq 1$ times the Bohm velocity.³ Setting $M=1$ ensures that the *dc solution* meets physically acceptable conditions with a *vanishing* field within the bulk.³ Eqs.(10) and (11) are now written:

$$V'' = -\frac{en_o}{\epsilon_o} \left((n_i / n_o) - \exp(eV / kT_e) \right), \quad (15)$$

$$n_i / n_o = \left(1 - 2e\bar{V} / M^2 kT_e \right)^{-1/2}. \quad (16)$$

The first integral is normally found by multiplying Eq.(15) by V' and doing the indefinite integral from the reference point x_o to x . This gives:

$$\frac{1}{2} E^2(x, t) - \frac{1}{2} E_o^2 = -I(x, t) + \frac{n_o kT_e}{\epsilon_o} \left(\exp(eV(x, t) / kT_e) - 1 \right), \quad (17)$$

where $I(x, t)$ is the following indefinite integral:

$$I(x,t) = \frac{en_o}{\epsilon_o} \int_{x_o}^x dx' \frac{n_i}{n_o} V' = \frac{en_o}{\epsilon_o} \int_{x_o}^x dx' V' \left(1 - 2e\bar{V} / M^2 kT_e\right)^{-1/2}. \quad (18)$$

In Eq.(17) the constant of integration is the electric field at the reference point, E_o , which is assumed to be independent of time also. Eq.(18) has also been written using Eq.(16) to express the ion density in terms of the damped potential. One notes that Eq.(18) cannot be integrated until the damped potential is related to the actual potential as in the dc case.

For notational convenience, scale the potentials (note that these potentials are negative after having fixed the zero of the potential in Eq.(14)) with the electron temperature via :

$$\chi = eV / kT_e, \quad \bar{\chi} = e\bar{V} / kT_e, \quad (19)$$

so that $I(x,t)$ can be written:

$$I = \frac{n_o kT_e}{\epsilon_o} \mathcal{I}, \quad (20)$$

$$\mathcal{I} = \int_0^\chi d\chi' \left(1 - 2\bar{\chi} / M^2\right)^{-1/2}.$$

There are three interesting cases for which we can approximate the $\mathcal{I}$ integral.

The first is the dc case, or the MEO case, in which the "ion potential" $\bar{\chi}$ is identical to the actual potential χ . This gives

$$\mathcal{I} = \mathcal{I}(\chi) = M^2 \left(1 - \left(1 - 2\chi / M^2\right)^{1/2}\right). \quad (21)$$

This result could now be introduced into Eq.(17) to complete the determination of the first integral $E(V)$ and its needed properties to compute the displacement part of the current as discussed in Section 3.2.

The second case occurs when $\bar{\chi}$ is nearly constant or slowly varying compared to χ as a function of the space coordinate. This may be argued to yield

$$I(x,t) \approx I(\chi, \bar{\chi}) = \chi(x,t) \left(1 - 2\bar{\chi}(x,t) / M^2\right)^{-1/2}. \quad (22)$$

This is a useful result, valid for χ and $\bar{\chi}$ functionally independent of each other, as it shows that the integral should be proportional to χ under these conditions.

The third case occurs when $\bar{\chi}$ and χ are assumed to be related by a function only of the time variable, as might be the case for some types of damping behavior:

$$\bar{\chi}(x,t) \approx \alpha(t) \chi(x,t). \quad (23)$$

The substitution of Eq.(23) into Eq.(20) leads to the result:

$$I(x,t) \approx I(\chi, \bar{\chi}) = M^2 \frac{\chi(x,t)}{\bar{\chi}(x,t)} \left(1 - \left(1 - 2\bar{\chi}(x,t) / M^2\right)^{1/2}\right). \quad (24)$$

The above three cases can be compared keeping the following in mind: the approximation to be used here should reduce exactly to the MEO or dc case as the damped potential approaches the actual potential ($\bar{\chi} \rightarrow \chi$). One sees that this is true, as Eq.(24) reduces to the result in Eq.(21) as $\bar{\chi} \rightarrow \chi$. Thus we are assured that the approximation contained in Eq.(24) would generate a sheath model that reduces to the MEO adiabatic limit when the applied time variations are sufficiently slow. The second case which produces the result in Eq.(22) is not so different from the third case result in Eq.(24) as it might seem: in the limit that $\bar{\chi} \rightarrow 0$ the two agree, and in the limit that $\bar{\chi} \rightarrow -\infty$ they differ only by a factor of two. Putting all this in perspective, it seems that Eq.(24) is the approximation of choice. To reiterate, Eq.(24) reduces correctly to the adiabatic response limit and it also contains the direct proportionality to the instantaneous potential required by the asymptotic-type approximation of the second case.

Substituting Eq.(24) into Eq.(17), recovering the outside factor in Eq.(20), gives:

$$\frac{1}{2} E^2(x,t) - \frac{1}{2} E_o^2 = \frac{n_o k T_e}{\epsilon_o} \left(M^2 \frac{\chi}{\bar{\chi}} \left(\left(1 - 2\bar{\chi} / M^2\right)^{1/2} - 1 \right) + e^{\chi} - 1 \right). \quad (25)$$

This is the approximation to the first integral of the Poisson equation that we will be using for the case of a damped potential. The time derivative of E necessitates the evaluation of both partial derivatives, $\partial E / \partial \chi$ and $\partial E / \partial \bar{\chi}$.

4. Final Working Equations and Numerical Testing

In this section the collection of constitutive equations for the current-voltage properties of the unified rf sheath model are written down and discussed from an applications point of view. The ability of this unified model to simulate the results of the earlier sheath models is presented by detailed calculations.

4.1 Constitutive Equations of the Unified Sheath Model

As previously introduced in Eq.(14), the potential and damped potential will be set to zero at the reference point, which is in the plasma bulk or the sheath-presheath boundary. This is a matter of setting the zero, not a restriction of any kind. The electron particle current through the sheath is obtained by combining Eqs.(3) and (4):

$$j_e(t) = -\frac{1}{4} e u_T n_o \exp(eV(t) / kT_e), \quad (26)$$

where the electron thermal velocity is given in Eq.(5) and $V(t)$ is the potential difference between the reference point and the wall. The negative sign is present because of the flow of electrons toward the positive x direction. The displacement current is evaluated as written in Eq.(12), knowing that the approximate relation in Eq.(25) is available:

$$j_d(t) = \epsilon_o \dot{E} = \epsilon_o \left(\frac{\partial E}{\partial V} \dot{V}(t) + \frac{\partial E}{\partial \bar{V}} \dot{\bar{V}}(t) \right), \quad (27)$$

where $\bar{V}(t)$ and its time derivative are computed as in Eq.(8), at the wall position. For completeness Eq.(25) is rewritten in unscaled form, along with the two required partial derivatives needed in the displacement current:

$$\frac{1}{2} E^2 - \frac{1}{2} E_o^2 = \frac{n_o kT_e}{\epsilon_o} \left(M^2 \frac{V}{\bar{V}} \left(\left(1 - 2 \frac{e\bar{V}}{M^2 kT_e} \right)^{1/2} - 1 \right) + \exp\left(\frac{eV}{kT_e} \right) - 1 \right) \quad (28)$$

$$E \frac{\partial E}{\partial V} = \frac{en_o}{\epsilon_o} \left\{ M^2 \frac{kT_e}{e\bar{V}} \left(\left(1 - \frac{2e\bar{V}}{M^2 kT_e} \right)^{1/2} - 1 \right) + \exp\left(\frac{eV}{kT_e} \right) \right\}, \quad (29)$$

and,

$$E \frac{\partial E}{\partial \bar{V}} = \frac{en_o}{\epsilon_o} \frac{M^2 V}{\bar{V}^2} \left\{ 1 - \left(1 - \frac{2e\bar{V}}{M^2 kT_e} \right)^{1/2} - \frac{e\bar{V}}{M^2 kT_e} \left(1 - \frac{2e\bar{V}}{M^2 kT_e} \right)^{-1/2} \right\}. \quad (30)$$

Eqs.(27) through (30) furnish an evaluation of the displacement current in terms of the quantities $\dot{V}$, $\ddot{V}$, and $E(V, \bar{V})$. Of course Eq.(8) constructs the "averaged" or damped potential in terms of the instantaneous potential. Additional quantities to be specified include M and E_o , which should be chosen as small as is consistent with the existence of a physically acceptable solution,³ but more will be said about M and E_o later.

The ions flow through the sheath with a constant flux in the damped potential. This is a consequence of the assumptions made in writing Eqs.(6) and (7). Thus the ion flux is constant and determined by the presheath or bulk density n_o at the reference point and the factor M times the Bohm velocity. The same decision must be made here as in the dc sheath case: how does one connect to the bulk plasma and what quantities are to be adjusted to achieve a good match? This refers to the consistency (or lack thereof) of the Bohm velocity with the field at the implied reference point. Only a more accurate kinetic analysis can resolve this problem. Hopefully the choices will make only small changes in the sheath thickness³ and not greatly affect the voltage-current relation of the rf sheath.

The main development of this report is Eq.(28), and it can be implemented in several ways. The next subsections will give examples and checks of this equation. In the Appendix we present a detailed description of the utilization of the sheath model in both potential-controlled and current-controlled modes.

4.2 Testing of the Unified Model in the Adiabatic Limit

In this example we compare the present theory to the MEO description of the same sheath problem. The two results should compare well, with the

differences being due to the somewhat different means of connecting from sheath to bulk plasma. The atomic and plasma parameters are chosen to correspond to the Lieberman parameters used in the example presented with his rf model.¹ The specified and derived quantities are:

$$\begin{aligned}
 v_{rf} &= 13.56 \text{ MHz} , \\
 kT_e / e &= 3 \text{ eV} , \\
 m_i &= 69 \text{ amu (CF}_3^+), \\
 n_o &= 1.53 \times 10^{16} / \text{m}^3 , \\
 u_B &= 2.05 \times 10^3 \text{ m/s} , \\
 j_i &= 5.0 \text{ A/m}^2 .
 \end{aligned} \tag{31}$$

In order to compare to the MEO theory, the relaxation time constant, τ_{ave} , is set to one thousandth of the rf period to ensure that very little damping takes place in the potential. Thus $\bar{V}(t)$ is very nearly equal to $V(t)$ at all times.

The isolated plasma sheath problem can be solved numerically with either a specified applied rf voltage or run under current control to find the voltage that produces a specified current through the sheath. It matters not to the numerical algorithm presented here or to the MEO model which method is chosen, but since the Lieberman solution (called ML1) is restricted to current control, we adopt that method. The components of the total current per area through the sheath are:

$$\begin{aligned}
 j_{tot}(t) &= j_{ac} \sin(2\pi v_{rf} t) + j_{dc} , \\
 j_{ac} &= 85.2 \text{ A/m}^2 , \\
 j_{dc} &= 0 .
 \end{aligned} \tag{32}$$

In Fig. 1 the total potential across the sheath is plotted for two rf cycles at the end of an integration of 80 rf cycles as determined by the unified model. The transients have all disappeared by this time. The integration time step was 0.001 ns, but this was not optimized except to check that the integration was accurate for this time increment. The field at the bulk, E_o , could be set to zero for this case and the factor M could be left at unity. Setting $\tau_{ave} = \tau_{rf} / 1000$ kept the damped potential (not shown) within one percent of the instantaneous potential.

Fig.1

Fig.2

In Fig. 2 the MEO prediction of the potential for the above sheath problem is presented. One sees that the results are close to those in Fig. 1 as expected. Thus one can be confident that the approximation of Eq.(28) reduces to the MEO model when appropriate.

4.3 Testing of the Unified Model in the Heavy Ion Limit

The other extreme of ion dynamics is contained in the ML1 (Lieberman) model, which assumes that the ions respond to the full time average of the potential. This is simulated in the unified model by setting the characteristic time for damping to be much larger than the rf period. In this particular test we use $\tau_{ave} = 10 \tau_{rf}$. This problem is also run under "current control" with the parameters given in Eqs.(31) and (32), except that the total dc current is increased to 90% of the ion current. The ML1 model has a zero electron current to the electrode, implying that the total dc current is the same as the ion current. If the unified model is constrained to have the total dc current equal to the ion current, all of the electrons are required to be held away from the electrode with a sheath potential that is larger than the ML1 potential. We find that a compromise situation of 10% electron dc current gives reasonable results.

The results of the unified model for the instantaneous potential and the damped potential are shown in Fig. 3 at the end of an 80 rf cycle period of integration. The potentials predicted by the ML1 model is given in Fig. 4 for the same parameters. The agreement is fairly good, but not precise. In this particular calculation we left the M constant fixed at 1, but had to increase E_o in order to find an acceptable solution. The field at the sheath boundary, E_o , had to be increased to 45 V/cm . This was accomplished by incrementing E_o^2 as necessary in the expression for $E(V, \bar{V})$, Eq.(25), to maintain a real root for E as the equations were being integrated. An alternative procedure is to set $E_o = 0$ and to increase M as the integration proceeds in order to find an injection velocity that will make the solution well behaved. In this case, M had to be increased to 1.2 . These results are shown in Fig. 5 and they agree better with the ML1 analytic result in Fig. 4 than the first procedure.

One notices the difference in the potentials between this inertial case and the inertialess case of the previous section.

Fig.3

Fig.4

Fig.5

5. The Potential Damping Time Constant

Crucial to the use of this unified sheath model is a derivation of the proper time constant, τ_{ave} , to be used in the construction of the damped potential by means of Eq.(8). Of course all that is used in practice is the damping of the total potential across the sheath, that is:

$$\dot{\bar{V}}(t) = -(\bar{V}(t) - V(t)) / \tau_{ave}. \quad (33)$$

A formal prescription for τ_{ave} can be derived as follows. Begin with the exact fluid momentum equation for the ion fluid velocity, using a simplified notation with ϕ denoting the potential:

$$u u' + \frac{e}{m_i} \phi' + \dot{u} = 0. \quad (34)$$

Eq.(34) has an exact first integral, of the form,

$$\frac{1}{2} u^2 + \frac{e}{m_i} \phi + \int_{x_0}^x dx' \dot{u} = C_2, \quad (35)$$

where C_2 is the constant of integration (energy). We would like to express Eq.(35) in the form of the averaged potential, in order to correspond to Eq.(7),

$$\frac{1}{2} u^2 + \frac{e}{m_i} \bar{\phi} = C_2, \quad (36)$$

which identifies,

$$\bar{\phi} = \phi + \frac{m_i}{e} \int_{x_0}^x dx' \dot{u}. \quad (37)$$

We combine this with the time derivative of Eq.(36), namely,

$$u\dot{u} + \frac{e}{m_i} \dot{\bar{\phi}} = 0, \quad (38)$$

to give

$$\bar{\phi} = \phi - \int_{x_o}^x dx' \frac{1}{u} \dot{\bar{\phi}}. \quad (39)$$

This may also be expressed as

$$\bar{\phi} = \phi - \int_{x_o}^x dx' (2C_2 - 2e\bar{\phi}/m_i)^{-1/2} \dot{\bar{\phi}} \quad (40)$$

using Eq.(36). Thus far this analysis can be considered exact. Now by comparing Eq.(39) or Eq.(40) with Eq.(33) we find that the desired approximation for the damping equation is

$$\int_{x_o}^x dx' \frac{1}{u} \dot{\bar{\phi}} = \int_{x_o}^x dx' (2C_2 - 2e\bar{\phi}/m_i)^{-1/2} \dot{\bar{\phi}} \approx \tau_{ave} \dot{\bar{\phi}}. \quad (41)$$

The upper limit will be replaced by the wall position. Eq.(41) is rearranged to solve for τ_{ave} , with arguments explicit:

$$\tau_{ave} = \left(\int_{x_o}^{x_w} dx' \dot{\bar{\phi}}(x', t) / u(x', t) \right) / \dot{\bar{\phi}}(x_w, t). \quad (42)$$

Because $\bar{\phi}$ has a faster variation with x than does u , it makes sense to invoke an asymptotic approximation for the integral in Eq.(42) based on the upper limit. If the space dependence of $\bar{\phi}$ is quadratic, we have

$$\tau_{ave} \approx (x_w - x_o) / 3u_w, \quad (43)$$

where u_w is a mean ion velocity at the wall and $x_w - x_0$ is a mean sheath thickness. Obviously we have concluded that the damping time constant is nearly the ion fall time through the sheath. If the space dependence of the potential were exponential, $\bar{\phi} \propto \exp(x / d)$, we would find:

$$\tau_{ave} \approx d / u_w, \quad (44)$$

a similar result. Because $d \approx -\bar{\phi} / E$ and $u_w \approx (-2e\bar{\phi} / m_i)^{1/2}$, we have an immediate estimate of the time constant at hand. Alternatively one can use other estimates of the ion fall time.⁴

6. Discussion and Conclusion

This work has developed a unified model of the sheath voltage-current relation that is capable of incorporating the effects of ion inertia. Tests show that the model reproduces the two extreme limits as represented by the Lieberman and the Metze-Ernie-Oskam models. The model requires the integration of a single additional ordinary differential equation to construct the time relaxation of the potential difference across the sheath. The principle new result is contained in Eq.(28), which is the approximate first integral of the Poisson equation containing the Boltzmann electrons in the potential and fluid ions in a distinct damped potential. The damped potential depends upon the time constant τ_{ave} . The analysis given in Section 5 shows that τ_{ave} is close to being the ion fall time through the sheath. Estimates of this time constant are presented and referenced.⁴

The time-dependent energy of the ions as they impact the wall or electrode must be determined. To be consistent with the use of the damped potential, one should equate the time-dependent energy of the ions at the electrode to the time-dependent damped potential drop across the sheath. Thus the ion energies in eV are depicted in Figs. 1 and 2 as the instantaneous potential across the sheath, and in Figs. 3, 4, and 5 as the damped potential across the sheath.

The precise means of connecting the sheath to the bulk or presheath may be worth revising. We have investigated two procedures of connecting the sheath model to the bulk or presheath, either modifying the field at the boundary, or increasing the ion velocity above the Bohm velocity within the equations of the unified model. Both seem acceptable and agree with each other to reasonable accuracy.

The amount of electron heating by the Fermi mechanism is sometimes regarded as part of a sheath model.¹ Generally this amount of power is quite low compared to other power flows within the plasma.^{1,5} Because of this we do not investigate any relation of this sheath model to the electron heating theories.

Appendix

The implementation of the time-dependent differential equations for the inclusion of the sheath model into a more global simulation could be confusing. In this Appendix we present the explicit form of the numerical method for two cases, a sheath under prescribed potential control and a sheath under current control. As the potential control is somewhat simpler, it is given first. In both cases we recommend setting the field at the plasma-presheath boundary equal to zero and incrementing the Mach number, M , as this procedure appeared to give better agreement in the comparison to the ML1 model in Section 4.3.

Potential Control

The operational equations will be written down in the order that they are used in a single step in an Eulerian integration scheme for the advance from time t to time $t + \Delta t$. One begins with the prescribed time-dependent voltage between the plasma presheath and the electrode, which might be specialized to a particular ac and dc decomposition:

$$V = V_{ac} \sin(\omega t) + V_{dc}. \quad (A1)$$

The time dependence of all dependent variables will be suppressed. One can next evaluate the electron particle current according to Eq.(26), namely:

$$j_e = j_e(V). \quad (A2)$$

The damped potential, $\bar{V}$, is available as one of the dependent variables. Having the potential and damped potential, we now use Eq.(28) to evaluate the electric field at the electrode:

$$E = E(V, \bar{V}) \quad (A3)$$

The sign conventions in this report require a positive field. In this step one must check for the existence of a real solution to Eq.(28). Either E_o or M may need to be incremented to correct for an imaginary root. If we leave E_o fixed at zero, we can increment M until the root for E becomes real. This value of M is retained for the remainder of the computation and the incrementation is extended if necessary.

The partial derivatives of the field with respect to the potential and average potential can now be found from Eqs.(29) and (30):

$$\begin{aligned}\partial E / \partial V &= \frac{\partial E}{\partial V}(V, \bar{V}, E), \\ \partial E / \partial \bar{V} &= \frac{\partial E}{\partial \bar{V}}(V, \bar{V}, E).\end{aligned}\tag{A4}$$

We are now ready to find the total current once the time derivatives of the potential and damped potentials are determined. From Eq.(A1) or its equivalent we find the potential derivative and from Eq.(8) we find the derivative of the damped potential drop:

$$\begin{aligned}\dot{V} &= V_{ac} \omega \cos(\omega t), \\ \dot{\bar{V}} &= -(\bar{V} - V) / \tau_{ave}.\end{aligned}\tag{A5}$$

This now determines the total current per area as found by combining Eqs.(12) and (27):

$$j_{tot} = j_e + j_i + \epsilon_o \left(\frac{\partial E}{\partial V} \dot{V} + \frac{\partial E}{\partial \bar{V}} \dot{\bar{V}} \right),\tag{A6}$$

where j_i is fixed by the density at the presheath-sheath boundary and the ion velocity. The final step is the numerical incrementation of the differential equation for the damped potential,

$$\bar{V} = \bar{V} + \Delta t \dot{\bar{V}}.\tag{A7}$$

All other dependent variables are evaluated directly. This cycle is repeated in order to integrate the sheath model.

Current Control

Again, we write down the equations exactly as they would appear within a single time step of the numerical solution for the advance of the potentials with a

prescribed current through the sheath. In this case the total current per area through the sheath is specified, which might take the form of:

$$j_{tot} = j_{ac} \sin(\omega t) + j_{dc}. \quad (A8)$$

Both the potential and the damped potential are known at time t as dependent variables in the sheath model, from which we evaluate the electron particle current from Eq.(26),

$$j_e = j_e(V), \quad (A9)$$

and the field from Eq.(28),

$$E = E(V, \bar{V}). \quad (A10)$$

The same precautions apply here as discussed in the potential control section above about the existence of a real solution for the field. This can require the increase of M (or of E_o) prior to a successful evaluation of the field. This incremented value of M is retained as a constant for the remainder of the numerical integration unless it needs to be further increased. We are now able to evaluate the partials of the field with respect to the potentials from Eqs. (29) and (30) as written in Eq.(A4) above:

$$\begin{aligned} \partial E / \partial V &= \frac{\partial E}{\partial V}(V, \bar{V}, E), \\ \partial E / \partial \bar{V} &= \frac{\partial E}{\partial \bar{V}}(V, \bar{V}, E). \end{aligned} \quad (A11)$$

The time derivative of the damped potential is given by Eq.(8):

$$\dot{\bar{V}} = -(\bar{V} - V) / \tau_{ave}. \quad (A12)$$

Because current control requires us to solve Eq.(A6), which arises from Eqs.(12) and (27), "backwards" for the time derivative of V in terms of the current, we have:

$$\frac{dV}{dt} = \left(\frac{\partial E}{\partial V} \right)^{-1} \left((j_{tot} - j_i - j_e) / \epsilon_0 - \frac{\partial E}{\partial \bar{V}} \dot{\bar{V}} \right), \quad (\text{A13})$$

all in terms of quantities known from the preceding steps. We now increment the potentials and continue with the integration:

$$\begin{aligned} V &= V + \Delta t \dot{V}, \\ \bar{V} &= \bar{V} + \Delta t \dot{\bar{V}}. \end{aligned} \quad (\text{A14})$$

The ion current is known (at least insofar as the sheath model is concerned) in terms of the density and the ion velocity.

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