The New BNL Partial Wave Analysis Programs *

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1 Introduction

Experiment E852 at Brookhaven National Laboratory is a meson spectroscopy experiment which took data at the Multi-Particle Spectrometer facility of the Alternating Gradient Synchrotron. Upgrades to the spectrometer's data acquisition and trigger electronics allowed over 900 million data events, of numerous topologies, to be recorded to tape in 1995 running alone. One of the primary goals of E852 is identification of states beyond the quark model, *i.e.*, states with gluonic degrees of freedom. Identification of such states involves the measurement of a systems spin-parity.

Such a measurement is usually done using Partial Wave Analysis. Programs to perform such analyses exist, in fact, one was written at BNL and used in previous experiments by some of our group. This program, however, was optimized for a particular final state, and modification to allow analysis of the broad range of final states in E852 would have been difficult. We therefore decided to write a new program, with an eye towards generality that would allow analysis of a large class of reactions.

2 Partial Wave Analysis Formalism

MASTER

2.1 General Description of Pwa

Partial Wave Analysis, or PWA, is a technique used in hadron spectroscopy to extract information about the spin-parity and decay properties of resonances produced in hadronic interactions. Typically, these resonances are produced at accelerator experiments via a variety of production mechanisms. The resonances produced by these methods also appear in the decay products of other well known resonances, such as the J/ψ , and can be studied there. Although the tools we describe here could be used in an investigation that uses any of these production mechanisms, perhaps with slight modification, so far they have been used only in peripheral production experiments, and so we will use this type of production as an illustrative example in this paper.

The general idea is to parameterize the intensity distribution in terms of variables that have physical meaning when interpreted as properties of interme-

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Figure 1: Diagram representing the amplitude $\psi_{\alpha\beta}(\tau)$. The intermediate state X is produced by the exchange of a Reggion R_{ex} between the π beam and the nucleon target.

Figure 2: Decay of the resonance X into x_1 and x_2 .

The initial decay of the X into its children is evaluated in the Gottfried-Jackson frame. This frame is a rest frame of the resonance X with the z axis in the direction of the beam and the y axis perpendicular to the production plane. The quantities used in defining the amplitude are shown schematically in Figure 2. The state X of mass W has spin J with z projection m . It decays into children x_i with spin s_i and helicity λ_i . These children have a breakup momentum p and relative orbital angular momentum l . The decay amplitude can then be written down [Chu71]

$$A_X = \bar{l} \sum_{\lambda} D_{m\lambda}^{J*}(\Omega) (l s \lambda | J \lambda) (s_1 \lambda_1 s_2 - \lambda_2 | s \lambda) F_l(p) a_{ls} A_{x_1} A_{x_2} \quad (6)$$

where $\lambda = \lambda_1 - \lambda_2$ and $\vec{s} = \vec{s}_1 + \vec{s}_2$, i.e., $\vec{s}$ is the total spin of the two children and λ is the component of $\vec{s}$ in the direction defined by x_i 's momentum. The A_{x_i} are the decay amplitudes of each child.

The $\bar{l} \equiv (2l+1)^{1/2}$ factor, along with the two Clebsch-Gordan coefficients, come from the fact that we are using helicity states and must relate the helicity coupling constant to the ls coupling constant a_{ls} , we want to find through the partial wave expansion. [Chu71, section 4.3]

Rotational properties of the helicity states lead to the D function $D_{m\lambda}^{J*}(\Omega)$ where $\Omega = (\theta, \phi, 0)$ are the Euler angles of x_1 in the Gottfried-Jackson frame. The choice of the third angle $\gamma = 0$ defines a phase convention different from that of Jacob and Wick [JW59], who choose $\gamma = -\phi$.

$F_l(p)$ is an angular momentum barrier factor added to give the amplitude the correct behavior near threshold. We used the Blatt-Weisskopf centrifugal-barrier functions as given by von Hippel and Quigg. [vHQ72]

Finally, the a_{ls} is the ls coupling constant which contains the dynamics of

tivistic Breit-Wigner, although it is sometimes necessary to use a more complex parameterization, such as coupled-channel Breit-Wigners [Fla76] or a K -matrix parameterization [CK95].

Total decay amplitudes are made up as a product of these intermediate decays. In order to parameterize the spin-density matrix, which is determined by the fit, in the simplest way, it turns out to be advantageous to transform the states into the reflectivity basis. The reflectivity basis is defined by eigenstates of reflection in the production plane, the details of the transformation can be found in Chung and Trueman. [CT75]

3 Program Description

The BNL partial wave analysis programs were designed using object-oriented methods to produce a general system for doing partial wave analysis. The resulting objects were implemented using C++. Four programs comprise the basis of the system, although numerous support programs and scripts have been developed.

3.1 gamp

The program `gamp` (generate *amplitudes*) is the decay amplitude calculator. The command line usage is available by running the program with no arguments:

```
usage: gamp -wkeyfile -aampfile [-v] -n# -d[#] -Rreaction file1 file2 etc
-wwave.key: wave definition file.      default: wave.key
-aamplitude.amps: file to write amplitudes. default: amps.dat
-v verbose mode: print the particle data table
-n#: maximum number of events. default: 999999
-d[#]: debug printout
-Rreaction: reaction =
pi0pi0
etaeta
etapi0
etapi-
etapi+pi-pi-
etapi+pi-
Pim2Pi0
etaprimepi-
pi_pi-
ppi+pi-
npi+
OmegaEta
OmegaPi0
OmegaPi-
file1 file2 etc:      list of DST files
```

up to the second ":" the decay of the X is given in the format isobar/bachelor, l where l is the orbital angular momentum of the isobar decay. Had this reaction had additional isobar decays, they would be specified in an analogous way between the second and third ":"'s, etc.

The *datafile*, at present, must be in an E852 itape format containing an event summary record (ESR) group. For purposes of analyzing other data, for example LASS data described below, custom conversion programs were written to put the data into the required format.

3.2 int

The program *int* calculates the normalization integrals used in the maximum likelihood fit. It also has command line usage available:

```
usage: int -[wmialuLUa]
-wfile file = keyfile
-mfile file = datafile
-i[a|r] a = accepted (.ani); r = raw (.rni)
-a add amplitudes to integral
-l# # = lower edge of mass region
-u# # = upper edge of mass region
-s# # = size of one mass bin
-L# # = lower edge of t bin
-U# # = upper edge of t bin
-c use compressed amplitude files
-n# Print NI out every N events in bin (verbose mode only)
-v verbose output
```

It needs as input a *keyfile* and amplitudes for all waves in the *keyfile* for the Monte-Carlo *datafile*. This is a simple program that just reads amplitudes from disk and performs the sum in equation 4, writing as output a matrix of complex numbers to *nifiles*. The indices of the matrix correspond to the waves in the *keyfile*. This links a set of normalization integrals to a particular *keyfile* which must be used in the fit. To say this another way, if the set of waves used in the fit is changed, then the normalization integrals must be recalculated.

The normalization integrals are done in mass and momentum transfer bins that correspond to the bins that will be used in the fit. The values of the ranges for these values, as well as the *keyfile* that was used is encoded into the names of the *nifiles*. For example, the file names

```
Kset13_2:1.32-1.36:0.00-0.15:.ani
Kset13_2:1.32-1.36:0.00-0.15:.rni
Kset13_2:1.32-1.36:0.10-1.00:.ani
Kset13_2:1.32-1.36:0.10-1.00:.rni
Kset13_2:1.32-1.36:0.15-0.30:.ani
Kset13_2:1.32-1.36:0.15-0.30:.rni
```

successive fit commands for a range of bins typically takes care of the initial generation of MINUIT parameters before running fit.

For each bin in which the script runs `fit`, an output file is generated containing the fit results. These *logfile*s contain all the information available about the fit results: fitted production amplitudes, normalization integrals, the *keyfile*, and the error matrix from the fit. Any observable may be calculated from this.

3.4 xvf

`xvf`¹ is a windowing post-processor program which uses the *logfile*s produced by `fit` to calculate observables. The appearance of the program is shown in Figure 4. The fit being looked at is a fit of the $\eta\pi$ final state data recorded in the 1994 run of Brookhaven experiment 852. Plotted is the number of events, acceptance corrected, that are associated with the partial waves selected in the left pane. In this example, the 2^{++} amplitudes are selected. Both are amplitudes are for D -wave decays into $\rho^0\pi^-$, one having $m = 0$ and the other having $m = 1$.

`xvf` calculates this predicted number of events using

$$N = \frac{n}{\eta_z} \sum_{\alpha\alpha'} V_\alpha V_{\alpha'}^* \Psi_{\alpha\alpha'} \quad (8)$$

The normalization integral $\Psi_{\alpha\alpha'}$ is calculated using `int`, and $\frac{n}{\eta_z}$ is the number of observed events in a bin divided by the acceptance. The V_α are the production amplitudes found by the fit. In the sum over waves, α takes on all values selected, such as all the 2^{++} waves above.

`xvf` can also display other observables using similar formulae, such as the phase difference between two production amplitudes or the magnitude (abs) of the production amplitudes.

3.5 Support Programs

In addition the four major programs in the suite, there are also various support programs that are used in an analysis. `wv2min`, for example, reads a *keyfile* and produces a list of fit parameters in a format that is understood by MINUIT. This program is generally run from a script such as `do_fit`. `do_fit` is written in Perl and takes care of running `wv2min`, running `fit` in a range of bins specified, possibly using the results of a previous bin to initialize the next fit. Similar scripts exist to run `int`, and, as mentioned above, `gamp`.

`ViewAmps` is a very simple program which is used extensively for diagnostics. It simply reads an *ampfile* and writes the mass, t and amplitudes to standard output.

¹`xvf` was written by Alexander Ostrovidov based on a much cruder program written by one of this papers authors.

4 Internals

In this section we give a brief description of the internal workings of the programs and the design philosophy that was used. We begin with a description of the general strategy adopted to calculate the decay amplitudes in a very flexible way, allowing extension to many final states. An outline of the objects developed to accomplish this is then given.

4.1 General Strategy

The isobar model leads naturally to the idea of a recursive algorithm to calculate the total decay amplitude. Each level of the recursion calculates the contribution to the total due to individual two body decays in the sequence.

4.2 Description of Objects

4.2.1 Matrices, etc.

Essential to any particle physics analysis is the notion of vectors (both three and four dimensional) and matrices of an arbitrary dimension. We choose to write a `threeVector` class first, and a `fourVector` *contains* a `threeVector` (We felt this was a more natural implementation than using inheritance). All the immediately obvious operators and functions were defined for the vectors: addition, subtraction, scalar multiplication, vector multiplication, rotations and boosts (using the `Matrix` class), transformations from a cartesian to a polar representation, etc.

A `Matrix` class provided for arbitrary size (and shape) matrices of real numbers, and once again, most of the typical algebraic operators are supplied. In addition, a `CMatrix` class proved to be very useful for partial wave analysis, where it is often convenient to write the (complex) amplitudes using a matrix notation. It is maybe is interesting to note that while this would be a textbook example of a use for templates, we wrote these classes before the introduction of templates into the C++ language. The `CMatrix` class was written (almost) completely by the UNIX commands:

```
sed 's/double/complex/' Matrix.hxx > CMatrix.hxx
sed 's/double/complex/' Matrix.cxx > CMatrix.cxx
```

4.2.2 Waves and Amplitudes

Partial waves are specified in the code using the `WaveType` class. `WaveType` has public data members for the quantum numbers of the system being studied and its decay products, which are stored as arrays of `ParticleData`:

```
class WaveType {
public:
    int J;           /* Total spin */
    int P;           /* Intrinsic parity */
```


- [CK95] Su Urk Chung and Eberhard Klempt. A primer on K -matrix formalism. BNL Preprint Uni Mainz IP 92-03, Brookhaven National Laboratory, March 1995. Version VII.
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