Application of Equivalent Elastic Methods in Three-Dimensional Finite
Element Structural Analysis

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RECEIVED**FFR 10 1998****OSTI****ABSTRACT**

This paper describes use of equivalent solid [EQS] modeling to obtain efficient solutions to perforated material problems using three-dimensional finite element analysis [3D-FEA] programs. It is shown that the accuracy of EQS methods in 3D-FEA depends on providing sufficient equivalent elastic properties to allow the EQS material to respond according to the elastic symmetry of the pattern. Peak stresses and ligament stresses are calculated from the EQS stresses by an appropriate 3D-FEA submodel approach. The method is demonstrated on the problem of a transversely pressurized simply supported plate with a central divider lane separating two perforated regions with circular penetrations arranged in a square pattern. A 3D-FEA solution for a model that incorporates each penetration explicitly is used for comparison with results from an EQS solution for the plate. Results for deflection and stresses from the EQS solution are within 3% of results from the explicit 3D-FE model. A solution to the sample problem is also provided using the procedures in the ASME B&PV Code. The ASME B&PV Code formulas for plate deflection were shown to overestimate the stiffening effects of the divider lane and the outer stiffening ring.

NOMENCLATURE

p Transverse pressure
w Transverse deflection
 R_o Outside radius of plate
 r_o Radius to center of outermost hole
 R^* Radius of EQS region
EQS Equivalent Solid Plate
 P, P_1, P_2, P^* Pitch
h Minimum ligament width
t Thickness of plate
 D^*, D_R Plate and ring rigidities
 E^*, E_{p^*} Effective moduli
 ν Poisson's ratio
 ν^* Effective ratio
r Radius of plate
 $\{\epsilon_{ij}\}$ Strain vector
 $\{\sigma_{ij}\}$ Stress vector
 θ Angle defining penetration pattern
 $[a_{ij}]$ Matrix of effective elastic constants
 ϕ Azimuthal angle for penetration

INTRODUCTION

3D-FEA programs using solid elements are rapidly replacing plate, shell and 2D solid element programs for routine structural design analyses of pressure vessel and piping equipment. The advantage of 3D-FEA technology is that it reduces the need to invoke engineering approximations to

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obtain solutions to complex geometries and loadings that are required when using plate and shell programs. In addition, with concurrent engineering software in use for automatic 3D solid modeling, less engineering time may actually be required to produce 3D-FEA solutions than the traditional 2D solutions.

However, there are situations where 3D-FEA modeling is too complex even for the most optimistic 3D modeler. One of these situations is the perforated region of a heat-exchanger tubesheet where as many as 1500 penetrations may be present. Explicit 3D-FEA modeling of each penetration will result in models of enormous size that are intractable for all but the largest of supercomputers.

This paper deals with adopting the well-known EQS method to work compatibly with 3D-FEA to enable efficient use of computing facilities with no loss in accuracy. Using a 3D-FEA program a unified approach is presented for obtaining the EQS properties for any general penetration pattern. Local stresses are then calculated using a 3D-FEA submodel approach.

LITERATURE

The ASME Boiler and Pressure Vessel Code (ASME B&PV Code, 1995) Sects. VIII and III include EQS plate properties for flat plates with triangular patterns while Sect. VIII, Div. 1 now contains procedures for both triangular and square patterns. Many publications exist on this topic in the open literature such as Meijers (1966), O'Donnell (1975), Slot (1979), Slot and O'Donnell (1971) and Osweiller (1989) which provide data for EQS plate properties for standard triangular, square, rectangular, and rhombic patterns. The paper by Jones (1979) provides practical methods for implementation of EQS methods for plates with square penetration patterns into 2D axisymmetric finite element programs. Gardner (1969) and Singh and Soler (1984) provide excellent summaries of perforated plate analyses

based on EQS design-by-formula methods for use in the design of heat exchanger equipment.

The procedures discussed by Osweiller (1989) systemize this methodology a step further and these works establish the basis for the European Unfired Pressure Vessel Standard (EUPVS), the procedures in the Committee on European Normalization (CEN), and the French pressure vessel design standard CODAP. In fact the CEN/CODAP procedures have been used extensively as the basis for recent proposals to update the perforated plate rules in Sections VIII and III of the ASME B&PV Code to include plates with square penetration patterns.

Although these papers and others like them provide methods for many practical cases that occur in design, they cannot cover all required situations. Some of these include plates with non-circular holes, plates where hole diameters are different for alternating rows, plates with holes that are not of uniform diameter through the thickness, plates with holes which are not drilled perpendicular to the mid-plane of the plate, and arrays of penetrations in shells rather than flat plates. Even though the repeating pattern does not comply with the classical patterns treated in the literature, the EQS method is still valid. The 3D-EQS-FEA method described here can be applied to all of these general patterns.

EQUIVALENT SOLID MODEL

The fundamental assumption underlying the EQS method is that a repeated array of penetrations can be treated as a continuum. This concept treats the perforated region as an EQS region having the same nominal dimensions as the actual perforated region (i.e., thickness and diameter) but whose elastic constants have been adjusted to account for the increased flexibility of the material due to the perforations. The adjusted elastic constants are called effective elastic constants. The EQS method requires that the effective constants be such that for a given set of boundary tractions, the

boundary deformations of the EQS material be identical to boundary deformations for the actual perforated material. Consequently, the EQS material and the actual perforated material possess the same elastic strain energy.

Consider an infinitely large array of penetrations which are arranged in a periodic pattern as shown in Figure 1. Assume that the perforated region is represented as an EQS elastic body which is

Figure 1: Array of Penetrations

continuous and in a state of stress due to the influence of external forces. Assume further that this EQS continuous body satisfies the general form of Hooke's law, is in a state of equilibrium, deforms isothermally, and has components of strain which are linear functions of the components of stress. The form of Hooke's law for an EQS material in Cartesian coordinates (x_1, x_2, x_3) for the general case of a homogeneous anisotropic body is:

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ & & a_{33} & a_{34} & a_{35} & a_{36} \\ & & & a_{44} & a_{45} & a_{46} \\ & & & & a_{55} & a_{56} \\ & & & & & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} \quad \text{Eq.(1)}$$

The a_{ij} terms are called the EQS effective elastic constants. Note that the shear strain components are engineering shear strain components equal to twice the corresponding tensor components. From energy considerations, the a_{ij} coefficients in Eq. (1) are symmetric ($a_{ij} = a_{ji}$). Since the perforations do not affect the thermal expansion of the material, the coefficient of thermal expansion of the actual material is used in the EQS material region.

In general there are 21 effective elastic constants required to define an EQS material. The number of non-zero constants is related to the number of elastic symmetry planes that are present in the pattern.

Lekhnitskii (1963) provides a thorough treatment on the number of independent non-zero elastic constants required to define a general Hooke's law for EQS materials with elastic symmetry. The discussion that follows differs from Lekhnitskii (1963) in that it leads to specifications of boundary value problems that can be solved using 3D-FEA to determine those constants.

NUMBER OF EFFECTIVE ELASTIC CONSTANTS

If a large piece of material were subjected to uniformly distributed surface normal displacements or surface tractions at infinity, the periodic planes of geometric symmetry are also planes of loading symmetry. Symmetric loading planes remain plane, stretch or contract uniformly, and have zero shear stresses acting on them. Likewise, if this large piece of material were subjected to uniformly distributed surface shear displacements or surface shear tractions at infinity, the periodic planes of geometric symmetry would be planes of loading anti-symmetry. Anti-symmetry loading planes warp uniformly, remain inextensible (neither stretch or contract), and have zero normal stresses acting on them.

Consider two coordinate systems. The first is the

original x_1, x_2, x_3 system. Let a second system, x_1', x_2', x_3' , be related to the original system by a rotation to the planes of elastic symmetry. Since the elastic response to a set of loads must be identical for each of the two systems, the elastic strain energy due to these loads must be the same in both systems.

In the original x_1, x_2, x_3 coordinate system, the elastic strain energy, V , can be expressed as:

$$V = a_{11}\sigma_{11}^2/2 + a_{12}\sigma_{11}\sigma_{22} + \dots + a_{66}\tau_{31}^2/2 \quad \text{Eq.(2)}$$

Since a_{ij} is invariant with coordinate system, V can also be written in the x_1', x_2', x_3' coordinate system as:

$$V = a_{11}\sigma'^2_{11}/2 + a_{12}\sigma'_{11}\sigma'_{22} + \dots + a_{66}\tau'^2_{31}/2 \quad \text{Eq.(3)}$$

The stress (and strain) components in the x_1, x_2, x_3 system can be related to those in the x_1', x_2', x_3' system through the direction cosines between the coordinate axes of the two systems. These expressions can then be used to formulate the elastic strain energy V in Eq. (3) in terms of the stress components and the direction cosines relative to the original x_1, x_2, x_3 system. Equating the coefficients for the terms $\sigma_{11}^2, \sigma_{11}\sigma_{22}, \dots$ it can be concluded that certain of the a_{ij} coefficients are zero while others are related through the direction cosines. Knowing the rotations to the planes of elastic symmetry allows the number of independent elastic constants to be reduced to fewer than 21.

Using Lekhnitskii (1963), if there are sets of equivalent directions in a body which can be superimposed by a rotation through an angle θ_n about a certain axis, then this axis is an axis of elastic symmetry of a body for which the order of elastic symmetry is $n = 360^\circ/\theta_n$.

Assume that in the body under consideration,

there does exist a plane of elastic symmetry and that there is a line which is normal to that symmetry plane. Assigning the x_3 -axis to be coincident with this line makes the plane of elastic symmetry the x_1 - x_2 plane.

Since the x_3 -axis is normal to the x_1 - x_2 plane, coordinate transformation of the system in the x_1 - x_2 plane is significantly simplified. If the angle of rotation about the x_3 -axis, θ_n , which is measured from the x_1 axis to the x_1' axis, producing a positive rotation about the x_3 axis, then stress components in the x_1', x_2', x_3' system are related to those in the x_1, x_2, x_3 system through the angle, θ_n , by:

$$\begin{aligned} \sigma'_{11} &= \sigma_{11}\cos^2\theta_n + \sigma_{22}\sin^2\theta_n + 2\tau_{12}\sin\theta_n\cos\theta_n \\ \sigma'_{22} &= \sigma_{11}\sin^2\theta_n + \sigma_{22}\cos^2\theta_n - 2\tau_{12}\sin\theta_n\cos\theta_n \\ \sigma'_{33} &= \sigma_{33} \\ \tau'_{12} &= \tau_{12}(\cos^2\theta_n - \sin^2\theta_n) + (\sigma_{22} - \sigma_{11})\sin\theta_n\cos\theta_n \\ \tau'_{23} &= \tau_{23} \\ \tau'_{31} &= \tau_{31} \end{aligned} \quad \text{Eq.(4)}$$

Utilizing Eqs. (2), (3), and (4), it is possible to examine more closely how the nature of the material law is affected by the order of elastic symmetry, n . This will be demonstrated by evaluating the material law for several values of n .

Case 1: Elastic Symmetry of the Second Order ($n = 2$)

An elastic symmetry of order two, $\theta_n = 180^\circ$, is typified by an array of penetrations whose center lines form rectangles, i.e. the array has rectangular pitch. By substituting $\theta_n = 180^\circ$ into Eq. (4), Hooke's law becomes:

$$\epsilon_{11} = a_{11}\sigma_{11} + a_{12}\sigma_{22} + a_{13}\sigma_{33} + a_{14}\tau_{12}$$

$$\epsilon_{22} = a_{12}\sigma_{11} + a_{22}\sigma_{22} + a_{23}\sigma_{33} + a_{24}\tau_{12}$$

$$\epsilon_{33} = a_{13}\sigma_{11} + a_{23}\sigma_{22} + a_{33}\sigma_{33} + a_{34}\tau_{12}$$

$$\gamma_{12} = a_{14}\sigma_{11} + a_{24}\sigma_{22} + a_{34}\sigma_{33} + a_{44}\tau_{12}$$

$$\gamma_{23} = a_{55}\tau_{23} + a_{56}\tau_{31}$$

$$\gamma_{31} = a_{56}\tau_{23} + a_{66}\tau_{31}$$

Eq.(5)

and the number of independent elastic constants is reduced from 21 to 13.

The effective elastic constants a_{15} , a_{16} , a_{25} , a_{26} , a_{35} , a_{36} , a_{45} , a_{46} are all 0. Here we are merely restricting the x_1 - x_2 plane to possess elastic symmetry with respect to the x_3 axis.

Note that the order of elastic symmetry does not change if the rows of holes alternate in diameter (D_1 not equal to D_2 in Figure 1) for a staggered pattern of penetrations or if the pattern consists of an unequal spacing (P_1 not equal to P_2). All that is required for symmetry of order two is that $\theta_n = 180^\circ$.

Case 2: Elastic Symmetry of the Third Order ($n = 3$)

The rhombic pattern of holes has $\theta_n = 120^\circ$ and has elastic symmetry of order three. The number of independent elastic constants is seven for elastic symmetry of order three.

The relationships for the elastic constants are:

$$a_{14} = a_{24} = a_{34} = a_{56} = a_{35} = a_{36} = 0,$$

$$a_{22} = a_{11}, \quad a_{12} = a_{21}, \quad a_{23} = a_{13}, \quad a_{33} = a_{66} = a_{55}$$

$$a_{15} = -a_{25} = a_{46}, \quad -a_{16} = a_{26} = a_{45}$$

$$a_{44} = 2(a_{11} - a_{12})$$

Eq.(6)

Case 3: Elastic Symmetry of the Fourth Order ($n = 4$)

If the distance between the penetrations for a rectangular pattern is the same in both directions, then the pattern forms a square pitch shown in Figure 2 for which $\theta_n = 90^\circ$ and the elastic symmetry is order four.

Figure 2: Square Penetration Pattern

For the square pattern, the response of the EQS material to a stress in the x_1 direction is identical to the response of the material to a stress in the x_2 direction. Thus, Eq. (5) is simplified because the only way such a response can be obtained is if $a_{14} = a_{24} = 0.0$. In addition, a_{34} must be zero since a stress in the x_3 direction generates the same response in both the x_1 and x_2 directions. Therefore, for fourth order, there are six independent elastic constants.

Hooke's law for fourth order is:

$$\epsilon_{11} = a_{11}\sigma_{11} + a_{12}\sigma_{22} + a_{13}\sigma_{33}$$

$$\epsilon_{22} = a_{12}\sigma_{11} + a_{11}\sigma_{22} + a_{13}\sigma_{33}$$

$$\epsilon_{33} = a_{13}\sigma_{11} + a_{13}\sigma_{22} + a_{33}\sigma_{33}$$

$$\gamma_{12} = a_{44}\tau_{12}$$

Eq.(7)

$$\gamma_{23} = a_{66} \tau_{23}$$

$$\gamma_{31} = a_{66} \tau_{31}$$

Case 4: Elastic Symmetry of the Sixth Order
($n = 6$)

An elastic symmetry of order six, $\theta_n = 60^\circ$, is typified by an array of penetrations whose center lines form equilateral triangles. There are five independent elastic constants for the sixth order elastic symmetry if the $x_1 - x_2$ plane also possesses elastic symmetry with respect to the x_3 axis.

For this case, the non-zero effective elastic constants are:

$$a_{11}=a_{22}, a_{12}, a_{13}=a_{23}, a_{33}, a_{55}=a_{66}$$

$$a_{44} = 2(a_{11}-a_{12})$$

Eq.(8)

CALCULATION OF ELASTIC CONSTANTS

The EQS effective elastic constants are calculated by obtaining sufficient independent solutions that will give allow determination of each of the a_{ij} constants. A convenient way to do this is to set up boundary value problems that correspond to a non-zero value for one of the EQS stresses with the remaining five being zero. Solutions to these problems are called *fundamental solutions*. Using the fundamental solutions along with Hooke's law for the particular pattern enables direct determination of each constant.

The first three of these fundamental solutions correspond to cases where all of the planes of geometric symmetry are planes of loading symmetry. The last three of these fundamental solutions correspond to cases where one set of parallel planes of geometric symmetry are sets of planes of loading symmetry while the other two sets of parallel planes of geometric symmetry are sets of planes of loading anti-symmetry.

Consider the elementary or unit cell of the repeating pattern of order 4 shown in Figure 3.

Figure 3: Unit Cell

The unit cell is bounded by planes of geometric symmetry such that the repeating geometry pattern can be generated by replicating the unit cell with translations in the coordinate directions. Note that the term unit cell does not mean that the cell has unit dimensions. It is simply the fundamental unit or building block which can define the repeating pattern by coordinate transformation.

Using Figure 3 to define the reference surfaces, the boundary conditions for the six fundamental solutions are summarized in Table 1. 3D-FEA solutions can be obtained for each of the six conditions. For nodes that are on the intersection of two or more planes, prescribed displacement boundary conditions take precedence over prescribed force boundary conditions.

Table 1: Boundary Conditions for Unit Cell Fundamental Solutions

Solution	Strain Component Equal to 1.0	Non-Zero Stress Component	Unit Cell Surface					
			$x_1 = 0$	$x_1 = l_1$	$x_2 = 0$	$x_2 = l_2$	$x_3 = 0$	$x_3 = l_3$
1	ϵ_{11}	σ_{11}	$\delta_1 = 0$ $F_2 = 0$ $F_3 = 0$	$\delta_1 = l_1$ $F_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = c^*$ $F_3 = 0$	$F_1 = 0$ $F_2 = 0$ $F_3 = 0^{**}$	$F_1 = 0$ $F_2 = 0$ $F_3 = 0$
2	ϵ_{22}	σ_{22}	$\delta_1 = 0$ $F_2 = 0$ $F_3 = 0$	$\delta_1 = c^*$ $F_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = l_2$ $F_3 = 0$	$F_1 = 0$ $F_2 = 0$ $F_3 = 0^{**}$	$F_1 = 0$ $F_2 = 0$ $F_3 = 0$
3	ϵ_{33}	σ_{33}	$\delta_1 = 0$ $F_2 = 0$ $F_3 = 0$	$\delta_1 = c^*$ $F_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = c^*$ $F_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = l_3$
4	γ_{12}	τ_{12}	$F_1 = 0$ $\delta_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = l_1/2$ $F_3 = 0$	$\delta_1 = 0$ $F_2 = 0$ $F_3 = 0$	$\delta_1 = l_2/2$ $F_2 = 0$ $F_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = c^*$
5	γ_{23}	τ_{23}	$\delta_1 = 0$ $F_2 = 0$ $F_3 = 0$	$\delta_1 = c^*$ $F_2 = 0$ $F_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = l_2/2$	$F_1 = 0$ $\delta_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = l_3/2$ $F_3 = 0$
6	γ_{31}	τ_{31}	$F_1 = 0$ $F_2 = 0$ $\delta_3 = 0$	$F_1 = 0$ $F_2 = 0$ $\delta_3 = l_1/2$	$F_1 = 0$ $\delta_2 = 0$ $F_3 = 0$	$F_1 = 0$ $\delta_2 = c^*$ $F_3 = 0$	$\delta_1 = 0$ $F_2 = 0$ $F_3 = 0$	$\delta_1 = l_3/2$ $F_2 = 0$ $F_3 = 0$

$c^* =$ Displacement constrained to be uniform such that the total normal force on the surface is zero.
 $**\delta_3 =$ Set to zero for one node at the center of cell.

The a_{ij} ($i=j$) constants can be calculated directly from the six fundamental solutions because they, together with Equation (1) give sufficient independent equations to determine a_{ij} directly. However, it is convenient to use an energy formulation to obtain the a_{ij} terms for the case $i \neq j$.

Let W_{JK} denote the work done by the set of boundary forces J acting through the corresponding displacements produced by the second set of forces K .

If J and K represent fundamental solutions J and K specified in Table 1, then

$$W_{JK} = \{\sigma_{ij}\}_J^T \{\epsilon_{ij}\}_K \quad \text{Eq.(9)}$$

where $\{\sigma_{ij}\}_J$ is the EQS stress vector for fundamental solution J and $\{\epsilon_{ij}\}_K$ is the EQS strain vector for fundamental solution K .

From Eq. (1), $\{\epsilon_{ij}\}_K = [a_{ij}]\{\sigma_{ij}\}_K$. Also, the work done by the $\{\sigma_{ij}\}_J$ boundary forces for Fundamental Solution J is

$$W_{JJ} = \{\sigma_{ij}\}_J^T \{\epsilon_{ij}\}_J \quad \text{Eq.(10)}$$

and, likewise

$$W_{KK} = \{\sigma_{ij}\}_K^T \{\epsilon_{ij}\}_K \quad \text{Eq.(11)}$$

Combining Eqs. (9) through (11) and using the fact that EQS strains of unity are used for $\{\epsilon_{ij}\}_J$ and $\{\epsilon_{ij}\}_K$, the a_{ij} coefficients can be calculated from the formula

$$a_{ij} = [W_{JK}/(W_{JJ}W_{KK})] \quad \text{Eq.(12)}$$

CALCULATION OF PEAK STRESS

The most convenient way to compute the actual stresses and strains is by applying deformations from the EQS model onto the boundaries of a 3D-FEA submodel of the region of interest. The submodel includes a detailed representation of the penetration of interest and other geometric features but with the boundaries of the submodel at least one pitch dimension removed from the penetration center. The submodel can include local effects such as mis-drilled holes, auxiliary holes, holes adjacent to solid material, etc. Displacements taken from the EQS model are interpolated to the boundaries of the submodel to compute the stress and strain distribution in the submodel.

Stress multipliers can also be used to obtain actual stresses and strains from the EQS stresses and strains. The stress multipliers are obtained from the individual fundamental solutions and then the total stress obtained by superposition of the contributions from each of the six EQS stress components. The stress multiplier method is used in most design-by-formula methods such as The ASME B&PV Code. The assumption made by the stress multiplier method is that EQS strains imposed on the unit cell do not vary significantly from one cell to the next since the unit cell solutions represent an infinite array of holes in a uniform stress field. The advantage of the

submodel method is that this assumption is not necessary due to the consideration of adjacent penetrations and materials required for the submodel construction.

TRANSVERSE PRESSURE ON A CIRCULAR PLATE WITH SQUARE PENETRATION PATTERN

In this section, the 3D EQS method is verified by comparing results of a FE model that explicitly models each penetration with the results of the EQS solution. Consider a circular plate with a diametral divider lane that separates the plate into two perforated regions with circular holes arranged in a square array as shown in Figure 4. The pitch of the penetration pattern (P) is one inch (25.4 mm) and the penetration diameter is 0.95P. The outside radius of the plate is 10P and the thickness is 2P. The divider lane is 2P wide from penetration center to penetration center. The material is assumed to have a modulus (E) of 30×10^6 psi (207000 MPa) and Poisson's ratio of 0.3.

Figure 4: Transversely Loaded Plate

Table 2: EQS Plate Properties for Square Penetration Pattern

EQS Plate Properties for Square Penetration Pattern						
h/P	E_p^*/E	ν_p^*	G_p^*/G	G_z^*/G	E_z^*/E	ν_z^*
0.05	0.12670	0.03560	0.00970	0.1260	0.29118	0.3
0.136	0.23836	0.08662	0.04914	0.24348	0.41370	0.3

The plate is subjected to a uniform transverse pressure and is simply supported at its outer diameter. A 3D-FEA solution is obtained for a model that explicitly contains each penetration and the divider lane. This solution serves as a benchmark for the EQS solution. The EQS solution is based on a 3D-FEA of a model that incorporates the complete orthotropic EQS properties for the square penetration region and uses isotropic material properties to represent the divider lane.

A 3D-FEA of the unit cell, Figure 5, was developed and the EQS constants were calculated using the procedure recommended here. The results of the calculations are shown in Table 2. The constants are within 2% of the results obtained by Slot (1972) using generalized plane strain assumptions to simulate the 3D effects near the free surface. The effect of thickness on the EQS constants was investigated by changing the free surface boundary at the top and bottom of the plate to be identical to the generalized plane strain conditions considered by Slot (1972). Identical results were obtained thus verifying the method proposed here. If the plate thickness to pitch ratio were smaller, it is expected that the 3D effects would be larger.

One-quarter segment of the plate bounded by two diametral planes of symmetry and simply supported at the outer radius. A solid model rendering of the plate is shown in Figure 6. Each of the 27 complete penetrations and 6 half penetrations modeled explicitly. The perforated

region is surrounded by a solid rim and solid divider lane. This model used 132,570 twenty node hexagonal elements with ten equally spaced elements through the thickness. This model has 1,832,355 degrees of freedom. A plan view of the explicit 3D-FEA model is shown in Figure 7.

The EQS 3D-FEA model developed for the plate is shown in Figure 8. This model has a total of 19,656 twenty node hexagonal elements with eight equally spaced elements through the thickness, and 265,056 degrees of freedom. The orthotropic equivalent solid material region was surrounded by a solid rim and divider lane-similar to the explicit model.

Both models are meshed with PATRAN (1990) and solved with an in-house developed general purpose 3D-FEA program that includes twenty node hexagonal elements in its element library. This 3D-FEA program was used because it has very large capacity and an extremely efficient iterative equation solver which facilitates solving problems of the size discussed here.

Figure 5: Unit Cell for Square Pattern

Figure 7: Top-View of 3D-FEA Model

Figure 6: Three-Dimensional Solid Model

Figure 8: 3D-EQS-FEA Model

Figure 9 shows a comparison between the transverse deflections calculated by the 3D-EQS-FEA with those calculated by the explicit 3D-FEA. The transverse deflection at the center of the plate is virtually the same for both models. Results from both the explicit 3D-FEA and the 3D-EQS-FEA show that the circumferential variation in the transverse deflection at radii near the center of the perforated region is less than 6%. This is due to the stiffening effect of the divider lane.

Figure 9: Comparison of Calculated Transverse Deflections

The deformation of internal penetrations (i.e., penetrations which are fully within the penetration pattern) is affected by surrounding penetrations on all sides while the outer stiffening ring and divider lane have significant influence on the deformation of outer penetrations (i.e., penetrations which are adjacent to the solid stiffening ring or solid divider lane). The penetrations are identified in the plan view of the plate shown in Figure 7. The differences in response between an inner and an outer penetration are shown in the ligament distortion plots. When the EQS biaxial stress ratio is nearly one for an inner penetration such as

penetration 3-2, the penetration distorts nearly uniformly as seen in Figure 10. On the other hand, significant warping is observed for an outer penetration such as 4-6 shown in Figure 11 due to the in-plane shear stress that builds up at the interface between the perforated and non-perforated material.

Figure 10: Distortion Plot for Penetration 3-2

Figure 11: Distortion Plot for Penetration 4-6

Azimuthal stresses on the penetrations are calculated from EQS results using the submodel approach with 3D-FEA as outlined earlier. Displacements are taken from the 3D-EQS-FEA model and interpolated onto the boundaries of the submodel along with surface pressure. A FEA of the submodel then produces the azimuthal stress on the penetration surface.

The submodel boundaries used to calculate azimuthal stresses for penetrations 3-2 and 4-6 are set one pitch length surrounding boundary of the penetrations. Figure 12 shows the comparison between the azimuthal stress on penetration 3-2 calculated by the explicit 3D-FEA model and the 3D-EQS-FE submodel approach. The azimuthal stresses are essentially identical for both models. Figure 13 provides the comparison for an outer boundary penetration 4-6. Again, the two methods produce essentially identical stresses.

Figure 12: Azimuthal Stresses for Penetration 3-2.

Figure 13: Azimuthal Stresses for Penetration 4-6

ASME PROCEDURE

In this section, a solution is developed for the sample problem that is consistent with the methods in Section VIII Div. 1 of the ASME B&PV Code (hereafter, the Code). Unfortunately, the Code does not provide a direct formula for either transverse deflection or radial stress versus radius of the plate so equations must be derived consistent with the assumptions inherent in the Code procedure. The intent is to verify the code procedure by direct comparison with the results of the explicit 3D-FEA of the plate.

The Code accounts for the additional plate stiffness due to the divider lane by way of an equivalent pitch, P^* . EQS properties are assigned to the plate using an effective ligament efficiency h/P^* . The effective ligament efficiency accounting for the divider lane is

$$h/P^* = \frac{h}{P} \sqrt{1 - 2r_0 U_L / \pi R^2} \quad \text{Eq.(13)}$$

where U_L is the distance between the center of the

diametral lane penetrations, r_0 is the radius to the center of the outermost penetration, and R^* is the radius to the outside edge of the outermost penetration. For this example, penetration 4-6 is the outermost penetration at $r_0=6.4P$. For $U_L=2P$ and $R^*=6.875P$, $h/P^*=0.136$. The EQS properties for an effective ligament efficiency of 0.136 are given in Table 2.

The Code formulas for plate deflection and stress are based on the work of Gardner (1969) as modified by Singh & Soler (1984) to deal with the divider lane. The formula is based on homogenizing the EQS region to be compliant with axisymmetric bending given by the differential equation

$$\nabla^4 w = p/D^* \quad \text{Eq.(14)}$$

where $D^* = E^* t^3 / [12(1 - \nu^{*2})]$, w is the transverse deflection, and p is the pressure. Gardner (1969) treated the outer unperforated rim as a ring. The solution to Eq. (14) in polar coordinates is obtained by matching the deflection and slope at the interface between the ring and the EQS plate for which Eq. (14) applies. Doing that gives the deflection at the mid-surface of the EQS plate as

$$w = \frac{pR^{*4}}{64D^*} \left\{ \lambda^4 - 2 \left(\frac{3 + \nu^*}{1 + \nu^*} \right) \lambda^2 + \left(\frac{5 + \nu^*}{1 + \nu^*} \right) + \frac{4A(1 - \lambda^2)}{1 + \nu^*} + 8(K - 1)B \right\} \quad \text{Eq.(15)}$$

where $\lambda = r/R^*$, $K = R_0/R^*$, $D_R = Et^3/12$

$$\xi = [E(1 - \nu^*) \ln K] / E^* \text{ and}$$

$$A = \frac{2(K - 1)(K^2 + 1) - \xi}{\xi + 1}$$

$$B = \frac{2(K - 1)(K^2 + 1) + 1}{(1 + \nu^*)(\xi + 1)}$$

The maximum transverse deflection w/t calculated by Eq. (15) occurs at $\lambda = 0$. Substituting $\lambda = 0$ and EQS properties for h/P^* of 0.136 given in Table 2 gives a value for w/t from Eq. (15) that is 10% less than that calculated by the 3D-FEA model.

Somewhat different EQS properties are given in Sect. VIII Div. 1 of the Code. These properties include adjustments for thickness effects and are due to Osweiller (1989). For $t/P = 2.0$, $E^*/E = 0.171$ and $\nu^* = 0.34$. Using these properties, the deflection of the plate is within 10% of the 3D-FEA results. These results suggest that the combined effects of the adjustment in Eq. (13) for the divider lane and treating the outer stiffening ring as a ring makes the plate overly stiff.

A comparison of the effects of these assumptions on EQS stresses is possible by differentiating Eq. (15) to compute bending moments and then EQS stresses. Accordingly, the radial σ_r stress is

$$\sigma_r/p = \frac{3(3 + \nu^*)}{8} [C - \lambda^2] \left(\frac{R^*}{t} \right)^2 \quad \text{Eq.(16)}$$

$$C = [4(K - 1)(K^2 + 1) + 2 + (1 + \nu^*)(\xi + 1)] / [(\xi + 1)(3 + \nu^*)]$$

The largest value of stress from Eq. (16) is for $\lambda = 0$. The EQS stress calculated by Eq. (16) for $\lambda = 0$ is about 40% higher than the stress from the 3D-EQS-FEA model and is not particularly sensitive to the EQS properties.

CONCLUSION

In this paper, a unified EQS modeling approach has been developed for 3D-FEA that can be applied to any geometric feature that recurs periodically and for which some line or plane of orthogonality can be defined. Several examples are presented to demonstrate the number of elastic constants required to adequately define an EQS material. A general procedure is defined for calculating the EQS material constants using 3D-FEA computer programs. The method is demonstrated for the case of a plate with a square penetration pattern and a central divider lane. Stress and deflection results from two and three-dimensional equivalent solid models are compared to results of an explicit 3D-FE model showing that there is an advantage to using the

3D-EQS method because the correct flexibility is incorporated into the analysis giving more accurate results. Very accurate results were obtained from the 3D-EQS-FE model at a savings of a factor of about 30 on computing time. The ASME B&PV Code formulas for plate deflection were shown to overestimate the stiffening effects of the divider lane and the outer stiffening ring. The thickness effects on EQS properties included in Sect. VIII Div. 1 seem to improve the comparison with 3D-FEA results.

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REFERENCES

- ASME Boiler and Pressure Vessel Code, Sections III and VIII 1995, The American Society of Mechanical Engineers, New York, New York.
- Singh, K. P. and Soler, A. I., 1984, Mechanical Design of Heat Exchangers and Pressure Vessel Components, ARCTURUS Publishers, First Edition.
- Mieijers, P., 1970, "Plates with a Doubly-Periodic Pattern of Circular Holes Loaded in Plane Stress of Bending," First Int'l Conf. on Pressure Vessel Technology, reprinted in Pressure Vessels and Piping: Design and Analysis: A Decade of Progress, New York, New York, Chapter 4, pgs. 1061-1079.
- Slot, T and O'Donnell, W. J., 1971, "Effective Elastic Constants for Thick Perforated Plates with Square and Triangular Penetration Patterns," Transaction of ASME, J. of Engr. for Industry, November, pgs. 935-942.
- O'Donnell, W. J., 1972, "Perforated Plates and Shells," Pressure Vessels and Piping: Design and Analysis A Decade of Progress, New York, New York, Chapter 4, pgs. 1041-1101.
- Lempriere, B. M., 1968, "Poisson's Ratio in Orthotropic Materials," AIAA Journal, November, Vol. 6, No. 11, pgs. 2226-2227.
- Lekhnitskii, S. G., 1963, Theory of Elasticity of an Anisotropic Elastic Body, Holden-Day, Inc., San Francisco, California.
- Jones, D. P., 1979, "Axisymmetric Finite Element Analysis of Plates Containing Penetrations Arranged in a Square Pattern with Experimental Qualification," presented at the 1979 ASME PVP Conference, paper number 79-PVP-79.
- Osweiller, F., 1989, "Evolution and Synthesis of the Effective Elastic Constants Concept for the Design of Tubesheets," ASME J of Pressure Vessel Technology, Vol. 111, August, pgs. 209-217.
- Paliwal, D. N. and Saxena, R. M., 1993, "Design of Tubesheet for U-Tube Heat Exchangers," ASME J of Pressure Vessel Technology, February, Vol. 115, pgs. 59-67.
- PATRAN Plus User Manual Volumes I & II, Release 2.5, PDA Engineering, October 1990.
- Gardner, K.A., 1969, "TubeSheet Design: A Basis for Standardization," Vol. I-49, pp. 621-648, Proc. Int. Conf. on Pressure Vessel Tech., Delft, The Netherlands, ASME Publication.

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