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Abstract

The electromagnetic fields associated with a nonlinear compressional Alfvén wave propagating perpendicular to an external magnetic field of arbitrary strength are derived. For the strongly magnetized and high phase velocity case relevant for ion acceleration to high energies, we show that the electric field increases proportionally only to the external magnetic field $\mathcal{O}(B_{ext}[\text{in T}] \text{ MV/cm})$ and that the electrostatic potential increases with the square root of the ion-to-electron mass ratio $\sqrt{M_i/m_e}$.

52.35.Mw, 52.75.Di, 29.25.Ni, 29.17.+w

Nonlinear waves in collisionless plasma and the associated ion acceleration have been the subject of much study with history for magnetized plasmas alone for about 40 years [1,2]. A theoretical description of nonlinear waves in a cold, collisionless, quasi-neutral plasma exists since the late 1950's. Tidman and Krall [3] summarize the most common approach to a qualitative solution of the fluid equation. Ohsawa has derived a similar theory in which the background electrons are allowed to reach relativistic energies [4]. The possibility that these waves could accelerate plasma ions in astrophysical settings drew much attention to this subject and caused many scientists, most noticeably Y. Ohsawa [4-7], to investigate this matter. It has been found [8] that shock waves propagating perpendicular (or under an angle $\theta_c \leq \theta \leq \pi/2$, where $\theta_c = \arctan(\sqrt{M_i/m_e} - \sqrt{m_e/M_i})$ with M_i and m_e being the ion and electron mass, respectively) to the external magnetic field can cause some of the ions to become trapped inside the electrostatic potential generated by the charge separation of the plasma ahead of the shock and thus become accelerated perpendicular to the shock front as well as parallel to it ($v_p \times B$ acceleration). Numerical simulations showed ion acceleration to several tens of MeV [6]; however, a strong shock wave was manually set up in the code and was assumed to be generated by some unspecified means in "real life" situations like in the solar corona. Furthermore, the theory to date relies on the quasi-neutral assumption. For magnetosonic wave structures with $V_A/c \ll 1$ it was shown that the charge separation between plasma electrons and ions is negligible small and that the quasi-neutral approximation ($n_e \simeq n_i$) can be used [1]. ($V_A = c/\sqrt{1 + (m_e/M_i)(\omega_{pe}^2/\Omega_{ce}^2)}$ is the Alfvén speed, with ω_{pe} and Ω_{ce} being the electron plasma frequency and the electron cyclotron frequency, respectively.) For the treatment of relativistic ion acceleration, however, we are interested in regimes with $V_A = \mathcal{O}(c)$ in which the quasi-neutral assumption breaks down [6].

In this letter we report on the properties of highly nonlinear compressional Alfvén waves in plasmas of arbitrary magnetization. Thus the present theory applies to both the overdense (and weakly magnetized) and the underdense and strongly magnetized regimes. Unlike electron acceleration [9], it is necessary to have waves that move ions (which dictates a rela-

tively slow phase velocity initially). The compressional waves (both sound and magnetosonic Alfvén waves) do have this property. Unlike sound waves, the phase velocity of the compressional Alfvén waves can be (relatively) easily controlled to increase from the low entry value to a higher sub-relativistic value by changing the plasma density and/or magnetic field. A judicious choice of shock wave strength, density and/or magnetic field gradient will ensure a stable and adiabatic change of the nonlinear wave structure necessary for high energy acceleration of ions

Assuming a 1-dimensional, non-dissipative, steady, nonlinear wave structure propagating along the x direction, we can derive a theory that does not rely on the quasi-neutral assumption and is therefore valid even for Alfvénic waves in highly magnetized plasmas. We carry out all our calculations in the wave frame so that the partial derivatives with respect to time and the spatial coordinates y and z vanish ($\partial_t = \partial_y = \partial_z = 0$) and we are looking for solutions in which $B_x = B_y = E_z \equiv 0$ for all x . Thus Maxwell's equations reduce to

$$\partial_x E_x(x) = 4\pi e [n_i(x) - n_e(x)] \quad \text{and} \quad (1a)$$

$$\partial_x B_z(x) = -\frac{4\pi e}{c} [V_{iy}(x)n_i(x) - V_{ey}(x)n_e(x)], \quad (1b)$$

while the Lorentz force equations can be written as

$$V_{jx} \cdot \partial_x V_{jx} = \frac{q_j}{m_j} \cdot \left(E_x + \frac{V_{jy}}{c} B_z \right), \quad (2a)$$

$$V_{jx} \cdot \partial_x V_{jy} = \frac{q_j}{m_j} \cdot \left(E_y - \frac{V_{jx}}{c} B_z \right), \quad (2b)$$

$$V_{jx} \cdot \partial_x V_{jz} = 0, \quad (2c)$$

where $V_j = V_i$ or V_e denote the velocities, $q_j = e$ or $-e$ the charge, and $m_j = M_i$ or m_e the mass of the ions and electrons, respectively. Lastly we use the continuity equation in the wave frame:

$$\partial_x [n_i(x) V_{ix}(x)] = \partial_x [n_e(x) V_{ex}(x)] = 0. \quad (3)$$

Fixing the boundary values for all the field magnitudes and fluid velocities at $x = -\infty$ to

$$\begin{aligned}
B_z|_{x=-\infty} &= B_1, \quad n_j|_{x=-\infty} = n_1, \quad V_{jx}|_{x=-\infty} = V_1, \\
V_{jy}|_{x=-\infty} &= V_{iz}|_{x=-\infty} = 0, \quad (j = e \text{ or } i)
\end{aligned} \tag{4}$$

leads to vanishing z -velocities ($V_{iz}(x) = V_{ez}(x) \equiv 0$). Furthermore, Faraday's law $\partial_x E_y(x) = 0$ and the absence of forces at $x \rightarrow -\infty$ lead to a constant electric field $E_y(x) \equiv B_1 V_1 / c$ along the y direction in the wave frame. Using (3) and (4), the ion and electron densities can be expressed in terms of their velocities in the x -direction as $n_i(x) = n_1 V_1 / V_{ix}(x)$ and $n_e(x) = n_1 V_1 / V_{ex}(x)$. In the following, we use the normalized quantities as $\epsilon^2 = m_e / M_i$, $s = x \sqrt{(4\pi n_1 e^2) / (m_e c^2)} = x / (c / \omega_{pe})$, $B = B_z / B_1$, $E = E_x / B_1$, and $v_{jl} = V_{jl} / V_1$ ($j = i$ or e , $l = x$ or y). Introducing the Alfvén Mach number $M_A = V_1 / V_A$ (for weakly magnetized plasmas, the Alfvén speed V_A approaches the parameter $V_A = B_1 / \sqrt{4\pi n_1 M_i}$) and the normalized absolute shock speed $\beta_1 = V_1 / c$, we find the relevant differential equations for the fields (1) and fluid velocities (2) in normalized units:

$$\partial_s E = \frac{\epsilon M_A}{\beta_1} \left(\frac{1}{v_{ix}} - \frac{1}{v_{ex}} \right), \tag{5a}$$

$$\partial_s B = -\epsilon M_A \left(\frac{v_{iy}}{v_{ix}} - \frac{v_{ey}}{v_{ex}} \right), \tag{5b}$$

$$\partial_s v_{ix} = \frac{\epsilon}{M_A} \left(\frac{E}{v_{ix} \beta_1} + \frac{v_{iy}}{v_{ix}} B \right), \tag{5c}$$

$$\partial_s v_{iy} = \frac{\epsilon}{M_A} \left(\frac{1}{v_{ix}} - B \right), \tag{5d}$$

$$\partial_s v_{ex} = -\frac{1}{\epsilon M_A} \left(\frac{E}{v_{ex} \beta_1} + \frac{v_{ey}}{v_{ex}} B \right), \tag{5e}$$

$$\partial_s v_{ey} = -\frac{1}{\epsilon M_A} \left(\frac{1}{v_{ex}} - B \right), \tag{5f}$$

which are 6 coupled first order differential equations for 2 fields and 4 fluid velocities. In the steady state wave frame ($\partial_t = 0$), we can use the conservation of momentum in x and y directions as well as the conservation of energy to derive 3 further relation between fields and fluid velocities ($\mathcal{T}_{\alpha\beta}^{EM} = [2E_\alpha E_\beta + 2B_\alpha B_\beta - \delta_{\alpha\beta}(\vec{E} \cdot \vec{E} + \vec{B} \cdot \vec{B})] / (8\pi)$ is the Maxwell stress tensor):

$$\vec{\nabla} \cdot (m_e n_e \vec{V}_e \vec{V}_e + M_i n_i \vec{V}_i \vec{V}_i + \mathcal{T}^{EM}) = 0 \tag{6}$$

and

$$\vec{\nabla} \cdot \left(\frac{m_e n_e V_e^2}{2} \vec{V}_e + \frac{M_i n_i V_i^2}{2} \vec{V}_i + c \frac{\vec{E} \times \vec{B}}{4\pi} \right) = 0 \quad (7)$$

can, together with (5) and (4), be integrated to give

$$B - 1 = \frac{M_A^2}{2} (1 - v_{ix}^2 - v_{iy}^2 + \epsilon^2 [1 - v_{ex}^2 - v_{ey}^2]), \quad (8a)$$

$$B^2 - E^2 - 1 = 2M_A^2 (1 - v_{ix} + \epsilon^2 [1 - v_{ex}]), \quad (8b)$$

and

$$E = \frac{M_A^2}{\beta_1} (v_{iy} + \epsilon^2 v_{ey}) \quad (8c)$$

for the conservation of energy and momentum in the x and y directions, respectively. The generalization of (8) for relativistic, cold fluid equations is trivial.

Using (5f), we find $v_{ex} = [B - \epsilon M_A \partial_s v_{ey}]^{-1}$, so that we can solve Maxwell's equations (5a) and (5b) for $1/v_{ix}$ and v_{iy}/v_{ix} , respectively. Along with the Lorentz force equations for the ions (5c) and (5d) and the boundary conditions (4), we can find integrals for the ion momentum and energy when assuming $v_{ey} = -E/(\beta_1 B)$ (i.e. allowing the electrons to drift across the magnetic field with the $E \times B$ drift velocity) as

$$v_{ix} = 1 - \frac{B^2 - E^2 - 1}{2M_A^2}, \quad (9a)$$

$$v_{iy} = \frac{\beta_1 E}{M_A^2} + \epsilon^2 \frac{E}{\beta_1 B}, \quad (9b)$$

and

$$v_{ix}^2 + v_{iy}^2 = 1 - 2 \frac{B - 1}{M_A^2} - \epsilon^2 \frac{E^2}{\beta_1^2 B^2}. \quad (9c)$$

Comparing (9) to (8), we find that a possible error introduced by the assumption $v_{ey} = -E/(\beta_1 B)$ is at worst of order ϵ^2 as long as $|V_{ey}/c| = |E/B| < 1$.

Eliminating v_{ix} and v_{iy} from (9), (9c) gives a relation between the electric and magnetic fields with the parameters of mass ratio ϵ^2 , absolute wave speed β_1 , and relative speed of the wave $M_A = V_1/V_A$:

$$E^4 + 2 \frac{E^2}{\beta_1^2 B^2} [B^2 \beta_1^2 (1 - B^2 + 2\beta_1^2) + 2M_A^4 \epsilon^2 (1 + \epsilon^2) + 2M_A^2 \beta_1^2 B (B + 2\epsilon^2)] - 4M_A^2 (B - 1)^2 + (B^2 - 1)^2 = 0. \quad (10)$$

The quartic equation for E can be solved in terms of B to give two real and two complex roots as long as B is less than its maximum value, $B_{max} = 2M_A - 1$. Together with (9a), we also find the range of valid Mach numbers to be $1 \leq M_A \leq 2$. For $B > B_{max}$, the fluid velocity v_{ix} [and therefore the density $n_i(s)$, see (3)] becomes less than zero which indicates a breakdown of the validity of the assumption of cold plasma fluids. Eq. (10) is plotted in Fig. 1 for various values of M_A , β_1 , and ϵ . We can now use the ion-momentum equations in x and y (5) together with (9) to derive two relations involving $\partial_s E$ and $\partial_s B$, or, after eliminating $\partial_s E$, one lengthy equation relating $\partial_s B$ to E and B :

$$\begin{aligned} \partial_s B = & - \frac{BE\epsilon M_A}{\beta_1(1 - B^2 + E^2 + 2M_A^2)(\beta_1^2 B^3 + B^2\epsilon^2 M_A^2 - E^2\epsilon^2 M_A^2)} \times \\ & \times \left[2M_A^4\epsilon^2(1 + \epsilon^2) + 2M_A^2\beta_1^2 B(B + 2\epsilon^2) + B^2\beta_1^2(1 - B^2 + 2\beta_1^2 + E^2) \right]. \end{aligned} \quad (11)$$

Replacing E with one of the two real roots of (10), Eq. (11) can be rewritten as $\Phi(B) + (\partial_s B)^2/2 = 0$, where the “potential” $\Phi(B)$ [1] has a well form as plotted in Fig. 1d. In particular $\Phi(1) = 0$ such that $\partial_s B = 0$ at $s = \pm\infty$. In other words the nonlinear wave structure undergoes one oscillation with an infinite period from $B = 1$ to $B = B_{max}$ and back and is thus a solitary pulse. The fields $B(s)$ and $E(s)$ resulting from numerical integration of (11) are plotted in Fig. 2 and 3. Properties of these solitons can be derived by expanding (10) and (11) in appropriate parameters for different regimes.

In the overdense case ($\Omega_{ce}/\omega_{pe} \ll 1$), we can expand (11) for small values of β_1/ϵ ($= M_A\Omega_{ce}/\omega_{pe}$) to obtain the spatial dependence of the magnetic field in the quasi-neutral regime as $\partial_s B = \pm M_A(B - 1)\sqrt{1 + \epsilon^2}\sqrt{4M_A^2 - (B + 1)^2}/(2M_A^2 - B^2 + 1)$, which reproduces Eq. (14) of [2]. Assuming that $B - 1 = \delta B \ll 1$ and $M_A - 1 = \delta M_A \ll 1$ (small amplitude solitons), this can be integrated to give $\delta B = 2\delta M_A \text{sech}^2\left(s\sqrt{\delta M_A/2}\sqrt{1 + \epsilon^2}\right)$, a characteristic profile for low amplitude solitons, in accordance with Eq. (2.39) of [10]. Lastly, expanding E [from (10)] in β_1/ϵ and finding its maximum with respect to B , we can calculate the maximum electric field present in an Alfvénic soliton in an overdense plasma to $|E_{max}| \simeq \frac{\beta_1}{\epsilon} \frac{\sqrt{2}}{3\sqrt{3}} [4(M_A - 1)^{3/2} - (M_A - 1)^{5/2}] + \mathcal{O}[(M_A - 1)^{7/2}]$. The maximum electric field in the overdense regime thus scales with the square of the external magnetic field as it

was found by Ohsawa [5] using the quasi-neutral approximation. Notice that in Eq. (10), $\partial_s B \propto \epsilon/\beta_1$. Since E is proportional to β_1/ϵ , the pulse width in the overdense regime is independent of ϵ and β as suggested in [2] [see also Fig. 3d].

In the underdense regime ($\epsilon \ll \beta_1$), (10) reduces to

$$E = \pm \left[1 - B^2 - 2\beta_1^2 - 2M_A^2 + \left\{ 4M_A^2(B-1)^2 - (B^2 - 1)^2 + (1 - B^2 - 2\beta_1^2 - 2M_A^2)^2 \right\}^{1/2} \right]^{1/2} \quad (12)$$

up to terms of $\mathcal{O}(\epsilon^2/\beta_1^2)$. Thus, in regime of interest for the ion acceleration, the soliton width and the electrostatic potential increase with $c/\omega_{pi} = c/(\epsilon\omega_{pe})$ (rather than c/ω_{pe} as for the overdense regime). For $\beta_1 \ll 1$ (but still $\epsilon \ll \beta_1$), we find the maximum electric field in this parameter regime to be

$$|E_{max}| \simeq \frac{\sqrt{6}}{9} \left[4(M_A - 1)^{3/2} + (M_A - 1)^{5/2}/3 \right] \quad (13)$$

plus terms of $\mathcal{O}[(M_A - 1)^{7/2}]$. Taking only the first term for low Mach numbers, we find $eE_{x,max}c/\omega_{pi} \simeq 1.1M_iV_Ac(M_A - 1)^{3/2}$ or $E_{x,max}[\text{MV/cm}] \simeq 3.3B_{ext}[\text{T}] \cdot (M_A - 1)^{3/2}$. In particular, $E_{x,max}$ increases with the external magnetic field in the underdense regime as compared to a quadratic dependence in the overdense regime. E_{max} is plotted for different M_A and β_1 in Fig. 3d. We can estimate the electric field for parameters that might be achievable in a laboratory with current technology: Suppose we apply a magnetic field of 1 Tesla to a hydrogen plasma of a plasma density of $1.2(1.7) \cdot 10^{12}/\text{cm}^3$ and generate an Alfvénic soliton with a Mach number $M_A = 1.5(1.8)$ ($\beta_1 = 0.1$). Such soliton would generate an electric field exceeding 1.1 (2.3) MV/cm and would be, due to its rather low speed, well suited for the pick up of background ions (and, if necessary, their acceleration to high energies) within a fraction of the length of conventional ion injectors [11].

Our theory breaks down for local electric fields $E(s)$ greater than $B(s)$, since V_{ey} would exceed the speed of light. For a situation in which E exceeds B , the particles would experience a steady acceleration along the x -axis regardless of the direction of the $\vec{v} \times \vec{B}$ -force (so called “unlimited acceleration” [12,7]). In the valid regime of our theory, however, we

find the factor E/B never to exceeded 0.42 (Fig. 1 – 3) such that the option of unlimited acceleration seems not to be realizable.

A detailed proposal of an experimental setup suitable for ion injection including the results of particle simulation codes will be given in a future publication. Here, we just briefly sketch the suggested setup: a powerful laser pulse [$P_{laser} = \mathcal{O}(10^{18} \text{W/cm}^2)$] shone onto a thin, overdense plasma will be reflected off the film surface while transferring momentum into electron and ion motion. Some of the plasma particles will leave the film on the far side where they enter a low density [$n_0 = \mathcal{O}(10^{14}/\text{cm}^3)$], weakly magnetized plasma in which they set up a compressional Alfvén wave. Letting the background density taper off along the x direction to densities of $\mathcal{O}(10^{12}/\text{cm}^3)$ over a length scale that is long compared to the soliton width, the soliton speed and the electrostatic field increase adiabatically as the soliton propagates from the overdense into the underdense regime. Thermal ions trapped in the electrostatic potential of the initially slow soliton gain kinetic energy as the pulse accelerates. However, those ions, as discussed above, eventually become detrapped due to their $v_y \times B_z$ acceleration so that this mechanism is not well suited for high energy ion acceleration. The background ions that make up the Alfvénic soliton, on the other hand, are slowed down to $v_{ix,min} = 2/M_A - 1$ in the wave frame (see (9a) for $B = B_{max} = 2M_A - 1$) and thus accelerated in the lab frame. For Mach numbers $M_A \lesssim 2$, the kinetic energy of these ions in the laboratory frame becomes $M_i c^2 (1/\sqrt{1 - \beta_1^2} - 1)$. Thus, we suggest to accelerate the Alfvén wave to large velocities at high Mach numbers and to decouple these ions from the nonlinear wave to provide a coherent ion source at high energies which is not limited by $v_y \times B_z$ detrapping.

In summary we derived the structure of magnetosonic solitons valid even for the underdense regime. We have shown that the electric field scales with the square of the external magnetic field for the overdense regime, while only linearly proportional to it in the underdense regime. A new method has been proposed that can make use of strongly nonlinear solitons for ion acceleration and is not limited by detrapping. This mechanism should also play a role in strong disturbances and subsequent prompt acceleration of ions from astro-

physical compact objects (such as neutron stars) [13] to yield a large amount of high energy particles and γ and x-rays.

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FIGURES

FIG. 1. Electric versus magnetic fields normalized to B_1 . In **a** - **c**, $M_A = 1.5$, $\epsilon = 1/\sqrt{1836} \simeq 0.023$, and $\beta_1 = 0.1$ *except*: **a** II: $M_A = 1.8$, **a** III: $M_A = 1.2$, **b** II: $\beta_1 = 0.03$, **b** III: $\beta_1 = 0.01$, and **c** II: $\epsilon = 0.01$, **c** III: $\epsilon = 0.1$. The potential $\Phi(B) = -(\partial_s B)^2/2$ for $\epsilon = 1/\sqrt{1836}$, $M_A = 1.5$, and $\beta_1 = 0.1$ is displayed in **d**.

FIG. 2. The magnetic field as a function of s (in c/ω_{pe}) in B_1 . **a** – **c** show the variation with M_A , β_1 , and ϵ as in Fig. 1. In **d**, we graph the electric field (in B_1) as a function of β_1 for $\epsilon = 1/\sqrt{1836}$ and various M_A . While E increases with β_1 in the overdense regime ($\Omega_{ce} < \omega_{pe}$), it's dependence on β_1 becomes weak for the underdense regime.

FIG. 3. The electric field as a function of s (in c/ω_{pe}) in B_1 . **a** – **c** show the variation with M_A , β_1 , and ϵ as in Fig. 1. **d(a)** shows the full pulse width at half maximum Δ_{fwhm} in c/ω_{pe} of the magnetic field for $M_A = 1.5$ and $\beta_1 = 0.1$ as a function of the parameter $\epsilon = \sqrt{m_e/M_i}$, (b) is the best fit assuming $\Delta_{fwhm} \propto 1/\epsilon$. (a) and (b) overlap in the underdense regime ($\epsilon \ll \beta_1$), while the ϵ -dependence of Δ_{fwhm} becomes weak for the overdense ($\epsilon \gg \beta_1$) regime.