SALVAGING TRANSIENT DATA WITH OVERLOADS AND ZERO OFFSETS

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We are sometimes presented with data with serious flaws, like overloads, zero shifts, and impulsive noise, including much of the available pyrotechnic data. Obviously, these data not be used if at all possible. However, we are sometimes forced to use these data as the only data available. Methods to salvage these data are discussed. Using the methods requires judgment, and the results must be accepted with the understanding that the answers are credible, not necessarily correct. None of the methods will recover information lost due to overloads or non-linearities of the data system. The best that can be accomplished is the recovery of data, after the data system has recovered from the overload. Several correction methods are discussed: High pass filtering of the data, correction with a two forms of an exponential function, and a correction with the form $t \exp(-at)$. Examples showing the results of the methods will be given using flawed pyrotechnic data.

INTRODUCTION

When presented with flawed data, the best procedure is to reject the data and collect a more valid data set. Other authors (Galef, 1985; IES, 1997) have shown the risks involved if the data are accepted. Sometimes it is not possible to gather more data. The data were gathered as a one time experiment that can not be repeated, or that additional experiments would be prohibitively expensive. In this paper we discuss acceleration data, typically pyrotechnic data, but the methods could be applied to other data sets.

For all the methods, the corrected data can under-predict the environment as data are lost from the overload or nonlinear response of the instrumentation system. An additional risk of the first three methods is that if the error model does not fit the data, unrealistic corrections will result. The goal is the prevention of unrealistic behavior at the lower frequencies. The advantage of the corrections is that unrealistic responses in the shock response spectrum (SRS) at the lower frequencies are avoided. Since the shock response spectrum is commonly used for test specification, the unrealistic responses at the lower frequencies can result in severe over-tests in later applications of the specification.

Sometimes additional information is known about the environment which can suggest corrections which will lead to at least a credible data set. The correction methods require assumptions about the characteristics of a credible data set. A model of the acceleration error is chosen, and the error model is fitted to the acceleration, velocity, or displacement waveforms to force the waveform to fit the characteristics of a credible data set. A common assumption is that the initial and final acceleration, velocity, and/or displacement are all zero or are known. Another assumption often used is the acceleration and velocity waveforms should both be oscillatory. A set of models for the errors is
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provided, and the model which best suites the data is selected. Three models of the errors discussed are: 1) a
correction of the form \( t \exp(-\alpha t) \), 2) an overload with an exponential recovery, followed by a zero offset, and 3) the
errors are essentially low frequency and a suitable high pass filter will correct the data set.

A companion paper [Smallwood, 1997] discusses the first method for the correction of some aliasing errors, and in
some cases the same procedures can be used to salvage data with other flaws. The companion paper also shows that
almost all numerical integration methods used to derive the velocity and displacement waveforms will have similar
effects on the results. A simple rectangular rule will be used for integration in this paper.

The second method uses an exponential window (to model the overload and recovery) combined with a rectangular
window (to model the zero shift) to model the error. A set of parameters (amplitude and decay rate of the exponential
window, the amplitude of the rectangular window, and the location in time of the start of the error) are optimized to
minimize the mean square error between the correction and the original data set. The correction is then subtracted
from the original data set to give the corrected data set. A variation of this method will follow the exponential
correction with a single pole high pass filter for reasons to be described later.

Another method is to simply high pass filter the data to remove the flaws. Guidelines for choosing the high pass filter
parameters are discussed. If the cutoff frequency is set too high useful data are lost, but if the cutoff frequency is too
low unrealistic velocity and displacement waveforms persist. If the rate of the filter roll-off rate is too high phase
shifts and overshoot affect the results, but if the roll-off is too low unrealistic velocity and displacement waveforms
persist.

The acceleration of the data will be called \( a(t) \). The data will be sampled at the points \( iT, \ i = 0: N - 1 \), where \( T \)
is the sample interval, resulting in the data set \( \{a\} \). The velocity will be computed by numerically integrating the
acceleration data set with a rectangular rule to give the velocity and displacement data sets \( \{v\} \) and \( \{d\} \). Corrections
to the data sets will be \( \{\tilde{a}\}, \{\tilde{v}\}, \) and \( \{\tilde{d}\} \) respectively. The corrected data sets will be \( \{\tilde{a}\}, \{\tilde{v}\}, \) and
\( \{\tilde{d}\} \) respectively.

**CORRECTIONS OF THE FORM \( t \exp(-\alpha t) \)**

Corrections of this form are discussed in detail in the companion paper [Smallwood, 1997]. Briefly the correction is
of the form

\[
\begin{align*}
\tilde{a}(t) &= A_1 t \exp(\alpha t) \quad \text{for } t < 0 \text{ and } \alpha > 0 \\
\tilde{a}(t) &= A_2 t \exp(-\alpha t) + A_3 (1 - \exp(-\alpha t)) \quad \text{for } 0 \leq t \leq NT \text{ and } \alpha > 0
\end{align*}
\]

This will be called a TEXP correction. A delay can be easily introduced by substituting \( t - \nu \) for \( t \) in Eq. 1. The
acceleration correction is numerically integrated resulting in the velocity and displacement corrections \( \{\tilde{v}\} \) and \( \{\tilde{d}\} \).
The constants \( A_1, A_2, \) and \( A_3 \) are chosen to match the acceleration, velocity, and displacement at the Nth sample. The
decay rate and the delay at this point are still free parameters. To optimize the choice of the decay rate and delay they
are chosen to minimize the mean square error in the velocity or displacement

\[
e = \sum_{i=0}^{N-1} (v_i - \tilde{v}_i)^2 \quad \text{or} \quad e = \sum_{i=0}^{N-1} (d_i - \tilde{d}_i)^2
\]

The correction is has frequency components that decline as \( 1/f^2 \). Hence the correction is composed mostly of low
frequency components. Fitting the correction to the velocity or displacement emphasizes the low frequencies and will
assure that the low frequency content of the correction closely matches the low frequency content in both magnitude
and phase of the original waveform. The correction is subtracted from the original waveform to give the corrected waveform

\[ \{\ddot{a}\} = \{a\} - \{\hat{a}\} \] (3)

The subtraction of the correction from the original waveform will act like a parametrically designed high pass filter.

**THE EXPONENTIAL CORRECTION**

A common problem in pyroshock measurements is the overload of the instrumentation system caused by high amplitude acceleration responses frequently caused by accelerometer ringing. Valid data cannot be recorded until the system recovers. Sometimes this recovery can be modeled as an exponential decay. In some cases the exponential recovery is followed by a zero shift of the system. It cannot be claimed that valid measurements can be made after such a serious insult to the instrumentation system has occurred. But sometimes an exponential correction followed by a zero shift can lead to credible data. The correction will be of the form

\[
\ddot{a}(t) = A_1 \exp(-\alpha(t - v)) + A_2 \quad \text{for } v \leq t < NT, \quad \alpha > 0 \\
\ddot{a}(t) = 0 \quad \text{elsewhere}
\] (4)

This will be called an EXP (exponential with step) correction. As for the previous correction the constants \(A_1\) and \(A_2\) are picked to match the acceleration and velocity at the \(N\)th sample. For the exponential correction a free parameter is not available to match the displacement. Since the displacement cannot be corrected, the decay rate and delay are picked to minimize the mean square error in velocity

\[
e = \sum_{i=0}^{N-1} (v_i - \hat{v}_i)^2
\] (5)

If a correction for the displacement is desired a function of the form

\[
\ddot{a}(t) = A_3 \exp(\alpha t) \quad \text{for } t < 0 \quad \text{and } \alpha > 0 \\
\ddot{a}(t) = A_1 \exp(-\alpha t) + A_2 \quad \text{for } 0 \leq t < NT \quad \text{and } \alpha > 0 \\
\ddot{a}(t) = 0 \quad \text{elsewhere}
\] (6)

can be used. As before a delay can be introduced by substituting \(t-v\) for \(t\). In this case either the displacement or velocity error Eq. (2) can be used to chose the decay rate and delay. This will be called a DEXP (double exponential with step) correction.

These (EXP and DEXP) corrections are not continuous, and the corrections will have a frequency content that declines as \(1/f\). The functions are useful for accelerations which have errors that look like step functions.

**CORRECTION WITH A HIGH PASS FILTER**

The acceleration, velocity, and displacement can also be corrected with a high pass filter. This will be called a HP correction. To minimize overshoot, the order of the filter should be chosen as low as possible. The gain of an \(n\)th order high pass filter will decline proportional to \(f^n\) as the frequency approaches zero. A first order filter is required to assure the velocity will return to zero. A second order filter is required to assure both the velocity and displacement will return to zero. Also, to minimize distortion in acceleration waveform a filter with a "smooth" gain in the pass band is desirable.
A simple high-pass filter is sometimes used. The data are smoothed with a simple running average. The running average is designed with an equal number of samples before and after the current sample to avoid the introduction of a delay in the smoothed signal. The running average is essentially a FIR (finite impulse response) low pass filter with a \( \sin(x)/x \) filter shape. The smoothed signal is then subtracted from the original data, leaving the high pass filtered signal. The number of points in the running average dictates the cutoff frequency of the filter.

A second order Butterworth filter was used in this paper. The filter cutoff frequency must still be chosen. Judgment is always required to pick an appropriate cutoff frequency. A high pass filter will introduce transients with a duration on the order of magnitude equal to the period of the cutoff frequency. If a very low cutoff frequency is chosen the transients will effect the corrected waveform for a long time, which is usually not desirable. We would like the corrections to be localized in time. If the cutoff frequency is too high the filter will remove desirable frequency components.

A variation of the exponential correction (Eq. 3) is to follow the correction with a first order high pass filter to correct the displacement. This will be called the EHP (exponential followed by a high pass filter) method.

**EXAMPLE**

The calculations in this paper were performed in MATLAB™. The following functions were used: butter was used to design the high pass filters, filter was used to filter the waveforms, fmin was used to minimize a function of a single variable, fmins was used to minimize a function of several variables.

The example will be two pyroshocks recorded on a structure which was cut with prima cord. The acceleration labeled \( x \) (Fig. 1) was about 15 inches (40 cm) from the cut. The acceleration labeled \( h \) (Fig. 2) was about 7 inches (18 cm) from the cut. Both accelerometers were Endevco Model 7270/20k accelerometers mounted with the Bateman, et al (1989) mechanical shock isolation system. The \( x \) data were not filtered with an analog anti-aliasing filter. The \( h \) data were filtered with a 20 kHz analog filter. Both accelerometers were sampled at 200,000 samples/second (a sample period of 5 \( \mu \)s). The waveforms were 6000 samples (0.03 s) long. The acceleration, velocity, and displacement (derived by numerically integrating the acceleration using the rectangular rule) are shown as Figs. 1 and 2. It is easily seen that the displacement would rapidly increase because of the velocity offsets obscuring the true displacement. The magnitude of the FFT is plotted as Fig. 3. As can be seen the acceleration labeled \( x \) is almost reasonable, a little offset in the velocity is evident. This could be instrumentation problems or aliasing errors. The spectrum of \( x \) shows evidence of aliasing above about 6 kHz. \( x \) will be called the "good" waveform. The measured values should be close to the actual environment (at least to a frequency of about 10 kHz).

The waveform labeled \( h \) has some problems. A large velocity offset is evident. It appears that the accelerometer zero shifted during the test. Guidelines for a good correction are that the environment as measured by \( x \) should not change much with the correction. The environment as measured by \( h \) should change, particularly at frequencies below 1 kHz. However, the corrected \( h \) should result in an environment which resembles the environment as measured by \( x \).

Four of the correction methods will be illustrated: 1) EXP (exponential correction, Eq. 3), 2) EHP (exponential followed by a high pass filter), 3) TEXP (Eq. 1), and 4) HP (a 1000 Hz high pass filter). The acceleration, velocity, and displacement of the correction, and of the corrected waveform for each of the illustrated methods are shown as Figs. 4-19.

The EXP correction (Fig. 4) for \( x \) is only a few g's out of several thousand peak g's, but this reduces the final displacement an order of magnitude (Fig. 5). The velocity for the first few milliseconds is not changed very much (a good indication). The correction to \( h \) (Fig. 6) is much larger (a peak of about 2000 g). The velocity correction is over 100 in/s. This indicates much more serious flaws in \( h \) as compared with \( x \). However, the corrected velocity peak for \( h \) (Fig. 7) is about the same as for \( x \). The corrected velocity is oscillatory. Both these are good signs that the correction is reasonable. The corrected displacement for \( h \) (Fig. 7) is still not credible.

The EHP correction (Fig. 8) does not change the acceleration or velocity of \( x \) much from the EXP correction, but does produce a credible displacement. However, the acceleration correction (a couple of hundred g's) (Fig. 9) indicates that maybe a little too much of the high frequency oscillatory acceleration is being removed. Similar
statements can be made about the corrections to $h$ (Figs 10 and 11). The indicated displacement (Figs. 9 and 11) is now milli-inches instead of inches for both $x$ and $h$.

The TEXP correction produces a correction which has an unusual shape, but with modest amplitudes. The correction for $x$ (Fig. 12) has a peak of only a few g's, while the correction for $h$ (Fig. 14) has a peak amplitude of about 50 g's. The corrected velocity for $x$ (Fig. 13) seems reasonable, however the corrected displacement for $x$ still shows significant low frequency oscillation. The corrected velocity for $h$ (Fig. 15) seems less credible and the corrected displacement for $h$ is an order of magnitude larger than the corrected displacement for $x$. In summary, this correction method seems to pick a decay rate which is too small, and places the correction too early in time. This tends to leave an excessive amount of low frequency energy in the corrected waveform. The results are not as satisfying as the other methods for these examples.

The high-pass (HP) correction for both $x$ and $h$ has peaks of 100s of g's. The corrected velocity and displacement for $x$ are both credible. The corrections for $h$ are a little less credible, indicating the more serious problems with $h$. These defects are evident in the corrected waveforms (Fig. 19).

The maxi-max absolute acceleration shock response spectra (SRS) with 5% of critical damping for the original waveforms are compared with the corrected waveforms in Figs. 20-23. As can be seen, each of the methods does not change the SRS of $x$ very much. The SRS of $h$ is changed a greatly below 1 kHz. As expected EXP leaves a residual velocity error, which causes the displacement to fail to return to zero. The SRS of the corrected $h$ generally follows the SRS of the original $h$ above a few thousand Hertz, and follows the characteristics of the SRS of $x$ below this frequency. The correction for $h$ changes the SRS of $h$ at the low frequencies, but produces an SRS very similar to the SRS of $x$. These were the desired goals. The exception is the TEXP correction, which follows the original $h$ SRS to about 100 Hz before rolling off, suggesting the correction left too much of the low frequency energy in the corrected waveform.

The FFT of the corrected waveforms are not shown, but the corrections have little effect above about 1 kHz.

The magnitude of the acceleration correction relative to the original data is one indication of the validity of the corrections. As long as the corrections are small the corrections are credible. As the corrections become larger confidence in the corrections decreases. At some point, which requires judgment, the data are so corrupted that corrections are not reasonable and the data must be discarded.

CONCLUSIONS

If used with care, the corrections discussed can produce credible data sets from data with minor flaws. The choice of the correction method will depend on the data set. The corrected data sets may underestimate the environment at high frequencies, but should result in reasonable estimates in the mid frequencies (1-10 kHz, for an SRS with an upper usable frequency of 10 kHz). The corrections prevent the gross overestimates of the SRS at the low frequencies (below 1 kHz, for a typical 10 kHz data set) which are common in flawed pyrotechnic data. The intent is not to hide the data flaws. The original data set must always be preserved. The corrected data sets should never be placed in a data bank without references to the original data set, and a clear explanation of the correction method used.

REFERENCES


IES, 1997, Handbook for Dynamic Data Acquisition and Analysis, IES Design, Test, and Evaluation Division Recommended Practice 012.1, IES-RP-DTE012.1, Institute of Environmental Sciences, Mount Prospect, IL.


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Figure 1 Acceleration, velocity, and displacement of $x$

Figure 2 Acceleration, velocity, and displacement of $h$

Figure 3 FFT of $x$ and $h$

Figure 4 EXP correction of $x$

Figure 5 EXP corrected $x'$

Figure 6 $x$ corrected with EXP
Figure 7: $h$ corrected with EXP

Figure 8: EHP correction for $x$

Figure 9: $x$ corrected with EHP

Figure 10: EHP correction for $h$

Figure 11: $h$ corrected with EHP

Figure 12: TEXP correction for $x$
Figure 13 x corrected with TEXP

Figure 14 TEXP correction for h

Figure 15 h corrected with TEXP

Figure 16 HP correction for x

Figure 17 x corrected with HP

Figure 18 HP correction for h
Figure 19: $h$ corrected with HP

Figure 20: EXP corrected SRS of $x$ and $h$ compared with original SRS

Figure 21: EHP corrected SRS of $x$ and $h$ compared with original SRS

Figure 22: TEXP corrected SRS of $x$ and $h$ compared with original SRS

Figure 23: HP corrected SRS of $x$ and $h$ compared with original SRS