Pacific Northwest National Laboratory

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T-200 Series Tank Concentration Predictions

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Executive Summary

Of the 177 tanks containing nuclear waste at the Hanford site, 68 have not been characterized based on sampling information. It may be possible to characterize 34 of the 68 unsampled tanks by using information from other sources, such as the Tank Characterization Database (TCD) and the Historical Tank Content Estimates (HTCE). This report lists predicted concentrations of 17 analytes along with the associated estimates of uncertainty for the T-200 series tanks (T-201, T-202, T-203, T-204), based on historical data and sample results from tanks other than the T-200 tanks.

The main benefits of reduced sampling are reduced cost, reduced time to achieve acceptable characterization, and reduced exposure of sampling personnel. However, in order to characterize a tank on the basis of reduced sampling, the predicted tank averages and the associated uncertainties must be technically credible and useful.

The objective of this report is to describe our approach to predict (without a sample) or estimate (from a single sample) the concentrations of the 17 analytes for the T-200 tanks. Sampling of these tanks and laboratory analysis of the samples is currently under way (sampling is scheduled for 03/27/97 through 04/21/97). This report briefly describes the statistical prediction techniques. The reader should consult a companion document for a detailed description (Ferryman et al. 1997).

For this analysis, we assume that tanks with similar process histories contain similar wastes. Further, we assume that concentrations in unsampled tanks can be predicted using samples from tanks containing similar wastes.

The first step in our analysis is to group all single-shell tanks containing similar waste types. In this analysis, the T-200 tanks grouped with the sampled B-200 tanks: B-201, B-202, B-203, B-204 (two cores taken from each). Both the T-200 and the B-200 tanks contain mostly "224" waste. Next, analysis of variance is used to predict the mean concentration of each analyte in each T-200 tank and derive the uncertainty in the prediction. Prediction intervals are then constructed using the mean concentration prediction and its uncertainty estimate, assuming the normal distribution. Finally, a form of cross-validation is performed to compare the prediction intervals with the data from sampled tanks.

Based on the grouping and waste assumptions, the results show that the uncertainties for the predicted concentrations are smaller than the spread of all analyte data in the TCD. Also, if data are available from a single core sample, the uncertainties for all analytes are significantly reduced, resulting in narrower prediction intervals.

A goal of this research was to use a model that mimics as closely as possible the physical, chemical, and radiological properties of each tank's waste. The present ANOVA model assumes that the sampling/measurement variability (including within-tank spatial variability) is similar across all tanks. Using this model, the variability estimate is quite stable due to the large amount of data used to estimate it. Due to the physical and chemical nature of the waste in these tanks, it may be desirable to assume that the

sampling/measurement variability is different from group to group (as was done with the tank-to-tank variability for different groups). The difficulty in fitting this model is that some tank groups have few samples. The measurement variability estimated for tanks with few samples would potentially be imprecise (unstable).

Follow-on research in this area should investigate these ANOVA models and assess each model's appropriateness for Hanford tank data. The confidence intervals in this report are based on the assumption that the model is correct, as is the case for all statistical research. If the model is not correct, the confidence intervals cannot be interpreted in a probabilistic framework.

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1.0 Introduction

The objective of this report is to describe the statistical method used to predict concentrations for the T-200 series tanks (T-201, T-202, T-203, T-204) and the results of those predictions. Sample data from the Tank Characterization Database (TCD) for 42 tanks were analyzed. The statistical technique primarily used data for sampled tanks within an identified tank group to predict the concentrations and variabilities for the non-sampled tanks within that group. According to Remund and Simpson (1996), the T-200 tanks and the B-200 tanks (B-201, B-202, B-203, B-204) contain very similar wastes, mostly 224 waste. It is assumed (based on the HTCE tank grouping study) that 224 waste from the T-Plant has a chemical composition similar to 224 waste from the B-Plant. The data for tanks outside of the group are used primarily to estimate the random (measurement) error. The analysis was conducted for each of 17 analytes (aluminum, bismuth, calcium, chromium, cesium-137, iron, fluoride, water, lanthanum, manganese, sodium, nickel, nitrate, phosphate, carbonate, total organic carbon, and uranium).

2.0 Analysis Methodology

A companion document is being produced which estimates/predicts concentrations for all 149 single shell tanks (Ferryman et al., 1997). A detailed description of the statistical prediction methods is contained in that document; only a brief discussion is presented here.

For this analysis, we assumed that tanks with similar process histories contain similar wastes. Further, we assumed that concentrations in unsampled tanks can be predicted using samples from tanks containing similar wastes. Therefore, the waste tanks were first categorized into groups, based on their process waste stream histories. The grouped data was analyzed using analysis of variance (ANOVA) to predict the average concentration and variability for each tank and analyte. Historically, ANOVA has been the primary statistical method used to analyze waste sample results for characterization purposes (Jensen and Liebetrau 1988, Heasler et al. 1994, Simpson et al. 1996). The ANOVA model used is the final model described in the exploratory study by Ferryman et al. (1997).

2.1 Tank Grouping

The grouping was selected by a multivariate cluster analysis based on historical information, the Historical Tank Content Estimates (HTCE). The 149 single shell tanks were grouped into 17 distinct groups, using the HTCE tank concentrations estimated by Agnew (1996). Details of the grouping analysis have been described by Remund et al. (1995) and Ferryman et al. (1997). According to historical records and the cluster analysis results, the T-200 tanks and the B-200 tanks (B-201, B-202, B-203, B-204) contain very similar wastes, mostly 224 waste, and thus were grouped together.

2.2 Analysis of Variance

Univariate ANOVA was used to predict the concentrations in the unsampled T-200 tanks. Two prediction intervals were constructed for each of the 17 analytes: 1) the average tank concentration and 2) a single core composite concentration. The predicted concentration is the same for both the core composite and the tank averages. However, the interval for the core composite concentration is wider than that for the average tank concentration prediction interval. The width of the core composite concentration prediction interval is equal to the width of the average tank concentration prediction interval plus an additional component of uncertainty related to spatial variability within a tank.

In this study, the mean concentrations and components of variability were estimated from core composite data averaged over primary and duplicate samples and over the different laboratory dilution methods (fusion and acid). The average concentration for a given group is used as the prediction for any tank within that group that has not been sampled. The uncertainty estimate for such a prediction is derived from differences observed between sampled tank results within that group and sampling/measurement variability pooled over all tank sampling results. The ANOVA model used to obtain tank predictions for sampled and unsampled tanks along with the associated uncertainties is

$$y_{ijk} = \mu + G_i + T_{ij} + E_{ijk} \quad (1)$$

where

- y_{ijk} is the concentration of a specific analyte in the k th core of tank j in group I ,
- μ is the mean concentration of the analyte across all tanks,
- G_i is the effect of group I (assumed fixed),
- T_{ij} is the tank effect of tank j in group I , and is assumed identically and independently distributed as a $N(0, \sigma_{T,i}^2)$ distribution, and
- E_{ijk} is the within-tank random error term and is assumed identically and independently distributed as a $N(0, \sigma_E^2)$ distribution.

The ANOVA model allows for a non-constant tank-to-tank variability term ($\sigma_{T,i}^2$). In other words, the model allows tank-to-tank concentration differences within a group to vary from group to group. In order to implement this model, however, simplifications were made to estimate $\sigma_{T,i}^2$. The REML (Restricted Maximum Likelihood) method normally iterates until optimal estimates of all variance components are obtained. However, the current optimization procedure does not work well when the number of unknown variance terms is large (18, in this case; 17 groups plus the random error term, E). As an alternative, the tank-to-tank sample variance within a group was used as the estimate of $\sigma_{T,i}^2$ for that group.

2.3 Model Validation

In general, model validation is done by comparing results from a model (concentration predictions from the ANOVA model) to independent sample data. Two methods are being used to validate the ANOVA model. The first is a cross validation method to see how well the existing sample data predicts concentrations for those tanks that have been sampled. The other validation method will be applied when an actual core is sampled from one of the T-200 tanks. The new sample results will be compared to the predicted results.

2.3.1 Cross Validation

Cross validation demonstrates how well the model predicts the contents of a sampled tank. The cross validation is done by removing the sample-based data for a single tank from the data set. The analysis is then rerun to predict the core composite concentration in the tank for which the data was removed. This is done for each sampled tank, one tank at a time. In our analysis, 93.0% of the observed core composite concentrations fell within the predicted interval (90% prediction interval) and 95.2% fell below the 95% upper bound. For 9 of the 17 analytes, lower bounds were truncated to 0 (no physical meaning below 0), so the percentage of cores falling within the prediction intervals is larger than the 90% confidence limit. Table 1 summarizes the cross validation effort, listing each analyte for which sample data exists, the total number of sampled cores, the number of cores that fell below the upper prediction bound, the percentage of cores falling below the upper bound, the number of cores falling within the 90% prediction interval, and the percentage of cores that fell within that interval.

Table 1. Cross Validation Results for the ANOVA

Analyte	# Cores	Upper Bound	Upper Bound	Interval	Interval
Al ^(a)	101	89	88%	88	87%
Bi	93	89	96%	87	94%
Ca	101	96	95%	96	95%
Cr	94	89	95%	84	89%
¹³⁷ Cs	91	89	98%	88	97%
Fe	101	95	94%	92	91%
F	94	90	96%	89	95%
H ₂ O	101	98	97%	86	84%
La	94	91	97%	91	97%
Mn	100	95	95%	95	95%
Na	101	98	97%	92	91%
Ni	89	84	94%	83	93%
NO ₃	99	95	96%	95	96%
PO ₄	100	94	94%	94	94%
CO ₃	91	87	96%	87	96%
TOC	94	89	95%	88	94%
U	100	97	97%	94	94%
Total	1644	1565	95%	1529	93%
(a) Al results contain questionable data from Tank U-110.					

The percentage of core concentrations that fell below the upper prediction bound ranged from 88% to 98%, with an average of 95.2%. The distribution of the original data and the residuals from the ANOVA indicate non-normality for some analytes. However, because the cross validation showed agreement percentages in the 95% range, we consider the 95% upper bound to be nominally correct and the prediction intervals robust to departures from normality.

2.3.2 Future Core Observation

One of the objectives of this study was to predict the concentrations for a core composite to be sampled in the future. Given the limited opportunities for sampling, a prediction of the analyte concentrations can help to guide sample selection and laboratory analysis to maximize the acquisition of new knowledge.

The core composite predicted concentrations and prediction intervals for the T-200 tanks are shown in Figure 1. Also included on the plots are the observed data from the B-200 tanks (within-group data), the estimated concentrations and 90% confidence intervals for each analyte in each B-200 tank, and a boxplot for the entire data set (TCD) which was used in the analysis (42 tanks). The symbols in the figures are as follows:

- [95% lower confidence bound for the B-200 tanks and prediction bound for the T-200 tanks,
-] 95% upper confidence bound for the B-200 tanks and prediction bound for the T-200 tanks,
- estimated/predicted concentration ($\mu\text{g/g}$ or $\mu\text{Ci/g}$ for Cs or wgt. % for water), and
- o observed core composite average (from TCD).

For those tanks where no lower bound is plotted, the lower bound was calculated to be less than or equal to zero, and thus has no physical meaning. The TCD data is presented at the bottom of each figure as a box-plot. The box within the box-plot represents 50% of the data (25th percentile to 75th percentile). The point inside the box represents the median value. The horizontal line on each side of the box represents two standard deviations from the mean, while the points outside the lines represent data values that fall outside the two standard deviation range ($\bar{x} \pm 2\hat{\sigma}$).

Figure 1. Concentration Predictions and 90% Prediction Intervals for a Future Core Extracted from a T-200 Tank, Along with the TCD Data Used in the Analysis

Figure 1. (contd)

3.0 T-200 Tank Concentration Predictions

The primary objective of this analysis was to characterize an unsampled tank by predicting analyte concentrations. The results of the analysis and a discussion of the results are contained in the following sections.

3.1 Results

Figure 2 displays the predicted mean analyte concentrations and the associated prediction intervals for each T-200 tank. These prediction intervals are narrower than the prediction intervals for core composite observations (Figure 1), because the latter include the within-tank variability. Table 2 presents, in tabular form, the results illustrated in Figure 1 (Columns 5, 6, and 7) and Figure 2 (Columns 2, 3, and 4). The prediction intervals on the mean concentrations shown in Table 2, Columns 2, 3, and 4, are the ones that should be used to represent our best estimate of tank contents and the corresponding confidence interval. These estimates (and intervals) are given since we are interested in making inferences about average tank concentrations and not for a single core sample.

The two types of prediction intervals in Table 2 have very different uses. The prediction intervals for the average tank concentration are used for tank characterization purposes such as the calculation of waste inventories for each tank. The prediction intervals for future core composite samples are the appropriate interval to compare with the averaged results for each core taken from the T-200 series tanks.

3.2 Discussion

From Figure 2, some interesting observations can be made about the prediction intervals and the data used for each analyte:

- Al, Cr, Cs, F, Na, Ni, NO₃, PO₄, CO₃, and U - Intervals for predicted concentrations in the T-200 tanks (T-201, T-202, T-203, T-204) are very similar to the intervals for the estimated concentrations in the B-200 tanks (B-201, B-202, B-203, B-204), because the within-group variability ($\hat{\sigma}_{T,i}$) is much less than the within-tank variability ($\hat{\sigma}_E$), as also shown in Table 3, Columns 4 and 5. This implies that sampling a core from the T-200 tanks will only *slightly* reduce the prediction interval width for these analytes in the sampled tank (it may significantly shift the location (mean) of the interval).
- Bi, La, and Mn - Intervals for predicted concentrations in the T-200 tanks are much larger than the intervals for the estimated concentrations in the B-200 tanks, because the within-group variability ($\hat{\sigma}_{T,i}$) is much larger than the within-tank variability ($\hat{\sigma}_E$), see Table 3. This within-group variability is large mainly because Tank B-201 has much higher concentrations of these three analytes than the other B-200 tanks. The large within-group variability implies that sampling a single core from each of the T-200 series tanks will *drastically* reduce the interval width for these analytes in the sampled tank (it may also significantly shift the location of the interval).

Figure 2. Average Tank Concentration Predictions and 90% Prediction Intervals for the T-200 Tanks, Along with the TCD Data Used in the Analysis

Figure 2. (contd)

Table 2. Predicted Analyte Concentrations and 95% Prediction Bounds from the ANOVA Results for an Unsampled T-200 series Tank

Analyte	Average Tank Concentration			Core Composite Sample		
	95% Lower Bound	Prediction	95% Upper Bound	95% Lower Bound	Prediction	95% Upper Bound
Al, $\mu\text{g/g}$	0	1,656	13,154	0	1,656	33,608
Bi, $\mu\text{g/g}$	4,923	55,683	106,443	4,282	55,683	107,084
Ca, $\mu\text{g/g}$	0	4,678	15,220	0	4,678	15,945
Cr, $\mu\text{g/g}$	2,359	3,077	3,795	1,676	3,077	4,478
^{137}Cs , $\mu\text{Ci/g}$	0	0	69	0	0	170
Fe, $\mu\text{g/g}$	0	7,702	18,081	0	7,702	20,537
F, $\mu\text{g/g}$	5,335	6,246	7,157	3,708	6,246	8,784
H ₂ O, wgt. %	49	71	92	46	71	95
La, $\mu\text{g/g}$	8,248	12,256	16,264	8,234	12,256	16,279
Mn, $\mu\text{g/g}$	7,037	16,344	25,650	6,740	16,344	25,947
Na, $\mu\text{g/g}$	17,515	33,166	48,817	1,115	33,166	65,216
Ni, $\mu\text{g/g}$	0	276	1,050	0	276	2,483
NO ₃ , $\mu\text{g/g}$	19,495	53,176	86,857	0	53,176	149,926
PO ₄ , $\mu\text{g/g}$	0	10,055	20,183	0	10,055	29,376
CO ₃ , $\mu\text{g/g}$	0	8,631	31,907	0	8,631	68,956
TOC, $\mu\text{g/g}$	0	2,791	10,434	0	2,791	11,269
U, $\mu\text{g/g}$	0	209	5,701	0	209	16,680

- Ca, Fe, H₂O, and TOC - Intervals for predicted concentrations in the T-200 tanks are somewhat larger than the intervals for the estimated concentrations in the B-200 tanks, because the within-group variability ($\hat{\sigma}_{T,i}$) is somewhat larger than the within-tank variability ($\hat{\sigma}_E$), see Table 3. This within-group variability is large mainly because Tanks B-201 and/or B-202 have much different concentrations of these four analytes than the other B-200 tanks. The large within-group variability implies that sampling a core from each of the T-200 series tanks will *noticeably* reduce the interval width for these analytes in the sampled tank (it may also significantly shift the location of the interval).

The next three paragraphs compare the sampled concentrations in the B-200 tanks and the predicted concentrations in the T-200 tanks to the rest of the TCD data used in the analysis. This comparison shows how our analysis compares to an alternative approach; to use the average concentration and spread of the entire TCD data base for predicting the concentrations in the T-200 tanks.

Table 3. Potential Error Reduction by Sampling One Core from the T-200 Tanks (assuming the one core is near the median of previous samples)

Analyte	$\hat{\sigma}_{\text{mean}}$ No Core	$\hat{\sigma}_{\text{mean}}$ One Core	$\hat{\sigma}_{T,i}$ Group	$\hat{\sigma}_E$ Random	Total $\hat{\sigma}_{\text{core}}$ No Core	Total $\hat{\sigma}_{\text{core}}$ One Core	% Decrease One Core
Al, $\mu\text{g/g}$	6,990	6,522	2,499	18,122	19,423	19,260	7
Bi, $\mu\text{g/g}$	30,857	4,857	27,556	4,918	31,246	6,912	84
Ca, $\mu\text{g/g}$	6,408	2,261	5,681	2,416	6,848	3,309	65
Cr, $\mu\text{g/g}$	436	374	315	731	851	821	14
^{137}Cs , $\mu\text{Ci/g}$	42	31	0.40	94	103	99	6
Fe, $\mu\text{g/g}$	6,309	3,712	5,454	4,589	7,801	5,902	41
F, $\mu\text{g/g}$	553	517	195	1,439	1,542	1,529	7
H ₂ O, wgt. %	12	6	11	7	14	9	50
La, $\mu\text{g/g}$	2,436	207	2,178	207	2,445	293	92
Mn, $\mu\text{g/g}$	5,657	1,397	5,040	1,441	5,838	2,007	75
Na, $\mu\text{g/g}$	9,514	8,303	6,596	17,002	19,483	18,921	13
Ni, $\mu\text{g/g}$	470	440	138	1,256	1,341	1,331	6
NO ₃ , $\mu\text{g/g}$	20,474	19,190	5,601	55,135	58,814	58,379	6
PO ₄ , $\mu\text{g/g}$	2,011	1,713	1,473	3,268	3,837	3,690	15
CO ₃ , $\mu\text{g/g}$	2,829	2,490	1,077	6,766	7,334	7,210	7
TOC, $\mu\text{g/g}$	4,646	2,009	4,082	2,229	5,153	3,001	57
U, $\mu\text{g/g}$	3,338	3,147	77	9,439	10,012	9,950	6

- Al, Na, NO₃, and U - The sampled (in the B-200 tanks) and predicted (in the T-200 tanks) concentrations for these analytes are much lower than the rest of the TCD data used in the analyses. For Na, even the upper prediction bound is lower than most of the TCD data used in the analyses.
- Bi, Cr, H₂O, La, and Mn - The sampled data and the predicted concentrations for these analytes are higher than for most of the TCD data used in our analyses.
- Ca, Cs, Fe, F, Ni, PO₄, CO₃, and TOC - The sampled data and the predicted concentrations for these analytes are similar to the rest of the TCD data used in our analyses.

In Table 3, Columns 2, 3, 4 and 5 list standard error terms for each analyte estimated from the ANOVA, including: average tank predictions (no core), average tank estimations (hypothetical if one core is sampled), the within-group variability, and the within-tank variability (random measurement error). The next two columns list the total standard error, with and without sampling, for a future core prediction (derived from the other error terms, $\hat{\sigma}_{\text{core}} = \sqrt{\hat{\sigma}_{\text{mean}}^2 + \hat{\sigma}_E^2}$). The last column in Table 3 lists the percent reduction in the variability for the mean tank concentration estimate that would be achieved by taking a single sample (as compared to no core sample).

4.0 Anticipated Sampling Activity and Implications

It is anticipated that all of the T-200 tanks will have one core sample taken and analyzed (instead of the standard two cores). This section discusses the benefits of that approach, beyond the obvious significance of saving money. An example of a hypothetical core sample analysis is also included.

4.1 Sampling

The object of sampling a core is to estimate the analyte concentrations and the variabilities associated with those concentrations. Assuming that the core composite sample yields concentrations that are relatively close to the predicted concentrations, the confidence intervals after sampling will be smaller than the predicted intervals. However, for 10 of the 17 analytes (Al, Cr, Cs, F, Na, Ni, NO₃, PO₄, CO₃, and U), collecting a core sample may have only a very minor effect on the interval width. This occurs, as mentioned earlier, because the random error term of the variability (within-tank variability) is larger than the tank-to-tank variability, as illustrated in Table 3.

For the remaining 7 analytes (Bi, Ca, Fe, H₂O, La, Mn, and TOC), collecting a single core sample may cause a major reduction in the interval width. This occurs, as mentioned earlier, because the tank-to-tank variability is larger than the within-tank variability, as illustrated in Table 3. Collecting a second core sample from each of these tanks, however, results in only a minor reduction in the interval width from the one-sample case (because the tank-to-tank variability term makes a negligible contribution to the estimate uncertainty when a core is sampled).

An analysis was performed to study the change in the tank-to-tank standard error as a function of a sampled value relative to the mean. As mentioned previously, a single core sample from a tank will affect only the variability estimate for the within-group term, $\hat{\sigma}_{T,i}$, and not the random error term (the width of the interval may not change much; however, its location may change considerably). This analysis showed that the value of the new sample does not have a significant effect on the confidence interval width for the concentration estimate (the maximum standard error increase for any analyte was only 2.4% for a $\hat{\sigma}_{T,i}$ that increased by 10%, and 18.8% for a $\hat{\sigma}_{T,i}$ that increased by 100%).

4.2 Example

To compare a future sampled core composite to the intervals listed in Table 2, one must first average the primary and the duplicate sample results. We illustrate this with an example. Suppose the ICP results for bismuth (Bi, acid or fusion digest) for a T-201 core composite sample were:

38200 $\mu\text{g/g}$ (primary result) and
41500 $\mu\text{g/g}$ (duplicate result).

The average of the results is 39850 $\mu\text{g/g}$. This average is compared with the Bi confidence interval in Table 2 (core composite sample), noting that it is contained within the interval (4282, 107084). This

illustrates a measure of agreement between the new T-201 core composite sample and previous sampling information from the B-200 series tanks. To continue the example, another ANOVA run is made, this time including the new core composite sample from T-201. Figure 3 shows how the mean and width of the Bi confidence interval for Tank T-201 are relocated and reduced when one core sample is taken (which is also illustrated in Table 3 by the 84% reduction of the $\hat{\sigma}_{\text{mean}}$ term from the no sample case to the one sample case). The symbols in Figure 3 have the same definitions as in Figures 1 and 2. This figure illustrates one of the strengths of the ANOVA analysis, which is that when a single core is taken, the confidence interval width shrinks. The confidence interval shrinks in this case because the within group tank-to-tank variability is no longer a significant contributor to the estimate uncertainty (i.e., since a sample has been taken from the tank the estimate no longer needs to account for tank-to-tank variability within the group).

Figure 3. Average Tank Concentration Predictions for a Hypothetical Sample Core Extracted from Tank T-201. Note the following:

1. Modest reduction of prediction intervals for the non-sampled T-200 tanks
2. Substantial reduction in interval from a non-sampled T-201 (top plot) to the one sample core from T-201 (bottom plot)
3. Minor reduction in the intervals for the B-200 tanks

5.0 Conclusion

The predicted concentrations for the T-200 tanks for our 17 analytes are listed in Table 2 and shown in Figure 2. Sample data from the TCD for 42 available sampled single-shell tanks was used in this analysis. All 149 single-shell tanks were first divided into 17 distinct groups (based on similar anticipated wastes), with the T-200 tanks and the B-200 tanks being grouped together. The ANOVA model used the B-200 data to predict concentrations in non-sampled T-200 tanks. To construct the prediction intervals, the bounds were set to $\bar{x} \pm 1.645\hat{\sigma}$ (restricting the lower bound to be ≥ 0). An investigation of the distribution of the data and the residuals (predicted - observed) showed that some of the analytes may not be normally distributed. However, cross validation showed that prediction intervals and upper bounds are nominally justified (95.2% of the sampled cores fell below the upper 95% prediction bound).

Two prediction intervals were calculated. The first interval predicts the concentrations of core composites to be taken from one of the T-200 tanks. The data from analysis of the T-200 series waste samples (average of the core composite primary and duplicate values) should be compared to this prediction interval (Figure 1 and right-hand columns in Table 2). The second interval is for the average tank concentration (Figure 2 and Columns 2, 3, and 4 in Table 2). These intervals encompass the best present estimates of core composite concentrations in the non-sampled tanks.

This analysis is based on the assumption that the 224 waste from the T-Plant has a chemical composition similar to 224 waste from the B-Plant. If a noticeable difference exists for certain analytes in the waste streams from the two plants, the chance that a T-200 core composite sample from a T-200 tank will fall within the 90% prediction interval will be less than 90%. However, an advantage of this technique is its robustness. A single core sample from one T-200 tank will give an indication of the mean concentration of the 224 waste from the T-Plant. The predictions and location of the prediction intervals for all T-200 tanks can then be adjusted based on this sample.

A goal of this research was to use a model that mimics as closely as possible the physical, chemical and radiological properties of each tank's waste. The present ANOVA model assumes that the sampling/measurement variability (including within-tank spatial variability) is similar across all tanks. Using this model, the variability estimate is quite stable, due to the large amount of data used to estimate it. Due to the physical and chemical nature of the waste in these tanks, it may be desirable to assume that the sampling/measurement variability is different from group to group (as was done with the tank-to-tank variability for different groups). The difficulty in fitting this model is that some tank groups have few samples. The measurement variability estimated for tanks with few samples would potentially be imprecise (unstable).

Follow-on research in this area should investigate these ANOVA models and assess each model's appropriateness for Hanford tank data. The confidence intervals in this report are based on the assumption that the model is correct, as is the case for all statistical research. If the model is not correct, the confidence intervals cannot be interpreted in a probabilistic framework.

6.0 References

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