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1991 Annual Report
to
The National Science Foundation

December 1, 1991

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I. Summary of Research

Research

The Center research program has a number of different aspects, most of which involve longer term commitments as suggested below.

- **Developent of research software,** both the sponsorship of new programs, and the continued development of existing software. This past year, continuing work was done on the Evolver program of Ken Brakke and the hyperbolic manifolds programs of Jeff Weeks. Under Richard McGehee a major new development was initiated to create four dimensional interactive visualization software focused on "seeing" structures like invariant tori for sympletic maps.

- **Teams of individuals visually exploring mathematical structures** For example, Hsu (a postdoc at MSRI), Kusner (asst. prof. at U. of Mass.), and John Sullivan (Center postdoc) tested various conjectures relating to the Willmore problem by means of the Evolver program. The problem is to find compact surfaces of a given genus in space which minimize the integral of squared mean curvature.

- **Workshops.** The Center will sponsor two major workshops during the current academic year:

  Computational Crystal Growers Workshop, organized by Jean Taylor, February 24-29, 1992 (a request for partial support from DOE is pending). Computational models are playing a central role in the analysis of crystal growth, whether the approach is from mathematics, materials science, or physics. Indeed, computational methods have led to theoretical advances and vice versa. The intent of this workshop is to get together many of the people, from each of the three disciplines, that are actively working on various computational models for various types of crystal growth, and thereby to facilitate the exchange of ideas, algorithms, and results.

  It is planned that there be no scheduled talks, apart from an initial introductory round of five-minute talks the first day and a showing of videotapes. Rather, the facilities (space, equipment, and staff) and the layout of the Geometry Center will be used to promote one-on-one and small group interactions. We hope that people will be able to demonstrate their programs in action, either on the workstations or on videotape, and to discuss issues of modeling, programming, etc.

  The people invited have devised a broad range of strategies to model crystal growth. They include direct mappings of manifolds (including graphs of functions over regions in the plane), the crystalline variant of direct mapping, Hamilton-Jacobi formulations, phase-field formulations, the use of characteristics, a new variational formulation, and Monte Carlo simulations on lattices. It has been found in purely theoretical studies that combining various methods leads to proofs inaccessible by a single method alone (e.g., Tom Ilmanen's new regularity proof for motion by mean curvature). We hope there can be similar fruitful interaction among the various computational schemes. At the minimum, we can hope to find which techniques are most promising for which problems.

Frederick J. Almgren

During the past year I have continued to work in my long term program of work in the geometric calculus of variations. Theoretically this work is in the context of geometric measure theory. The associated development of algorithms lies within the field of computational optimal geometry.

Theoretically I have major progress on theorems guaranteeing the existence of geometric evolutions such as would model the growth of a crystal as it freezes from its melt or its vapor. These are the first such general results and required the development of significant new methods of geometric analysis (the isotropic and completely uniform case, however, was treated slightly earlier by S. Luckhaus). Collaborators in this work are Robert Almgren, Jean Taylor, and Li He Wang.

The computational work I have done has been in association with the Minimal Surface Team of the Center whose members this year were F. Almgren, R. Almgren, K. Brakke, A. Roosen, J. Sullivan, J. Taylor. This past year the team convened the week of July 22 at the Five Colleges Summer Intitute, Mount Holyoke College. The following three weeks were spent at the Center in Minneapolis. R. Almgren and A. Roosen, in particular, made major progress in developing code to model crystal growth. My work and that of J. Taylor made contributions to these schemes.

I spent the three weeks following that at the Aspen Center for Physics participating in a session with metalurgists and physicists who were working in geometry and physics of solidification processes. One of the reasons for doing this is to make more likely that when we achieve our long term theoretical and computational goals they will be as useful as possible as tools for scientific research.

During the past year I have been supervising the Ph.D. work of John Steinke and Alice Underwood and the undergraduate research of Ivan Blank (and to a lesser extent David Ben-Zvi). The projects of Underwood, Blank, and Ben-Zvi are directly related to activities of the center.

James W. Cannon

(1) Cannon, colleague Walter Parry of Eastern Michigan University, and Ph.D. student Eric Swenson are working on the algorithmic recognition of hyperbolic groups by means of their recursive patterns at infinity. This program requires the solution, or approximate solution, of a combinatorial version of the Riemann Mapping Theorem. Parry developed a first algorithm which Cannon has implemented on the Sun. The algorithm is interesting but provably runs slowly on certain classes of rather simple examples. Parry, Cannon, and Swenson together have developed a modified algorithm currently being implemented by Cannon and Swenson. The new algorithm is expected, on average, to run many times faster than the old algorithm. In particular, it provably runs well on the simple examples where the other algorithm falters.

Bernard Chazelle

Most of my research this year has concentrated on the derandomization of probabilistic geometric algorithms.

My best result in that direction has been the discovery of an optimal convex hull algorithm in any fixed dimension (which had been an open problem for many years). Although the algorithm is deterministic, it is best understood as a cooling process in statistical thermodynamics. Actually, I have been pursuing the analogy between algorithm design and statistical physics in the hope of shedding light on the design of so-called "potential" functions, which constitutes one of the least understood aspects of amortized algorithms analysis.

Another research accomplishment has been the design of an optimal strategy for cutting hyperplanes to provide for efficient divide-and-conquer strategies. This extends the classical theory of epsilon-nets to geometric range spaces.

Miscellaneous results include new algorithms for geometric optimization based on parametric searching, randomized algorithms for two-dimensional computer graphics (which have the advantage of being extremely simple to implement and analyze).

With my graduate student, L. Palios, we have investigated the decomposition of low-dimensional piecewise-linear manifolds. Our latest result provides an optimal algorithm for partitioning the boundary of a 3-polytope into convex-like pieces.

With another graduate student, Hervé Brönnimann, we have derived new lower bounds for polytope range searching, which almost match the existing upper bounds.

David Dobkin

Dobkin has been working on the organization and development of software for doing computational geometry in 2 and 3 dimensions. A library of procedures is beginning to develop which will be easily usable by others. Difficult issues of building common interfaces, handling degenerate data in robust fashion, developing algorithms which behave efficiently in practice as well as theory continue to be tackled. We are also exploring user interface issues in developing a programming environment in which such algorithms can be implemented in the future.

David B.A. Epstein

At a recent international conference on group theory, a well-known group theorist gave an hour-long purely theoretical lecture proving the existence of certain canonical forms for group elements in a particular group, and deriving the growth function of the group. Subsequent to the lecture, we ran our "automata" program on the same group. After five minutes it produced all the results that had been presented in the lecture, together with proofs of correctness. Both sides were relieved to find that their results agreed with each other.

In the journal *Experimental Mathematics*, many of the philosophical principles underlying the Geometry Center play a central role. The Geometry Center has been a vital element in establishing this journal, and the journal would not have existed without it. Many people associated to the Geometry Center are involved with the journal: Epstein (Chief Editor), Silvio Levy (Editor), Thurston and Marden of the potential we can show that things like the Brohlin measure exist more specifically, there exist unique invariant (k,k)-currents, which can be obtained by appropriate pull-backs and averaging. Even in the case of rational functions, this yields a much easier proof of existence of the Brohlin measure. In the non-homogeneous case, we try to understand the support of dd^c (potential function), which turns out to be homeomorphic to the cone over the Julia set of the endomorphiusm of P^{n-1}, by a homeomorphism which conjugates the mapping to the associated homogeneous mapping. This is the only analog there is of the Bottcher coordinate in higher dimensions. There is a further application of these results to Newton's method in several variables, which is still filled with open problems.

In preparation with B. West are Volumes Two and Three of Differential Equations, A Dynamical Systems Approach. Also in preparation is a book to be called Three Jewels by Thurston.

Benoît B. Mandelbrot

Benoît B. Mandelbrot's work continues to be largely concerned with certain singular measures called multifractals. The part of this work that is connected with The Geometry Center is the study of harmonic measure for the Laplacian potential around the two dimensional fractal aggregates called DLA. The importance of this measure in physics is that—in combination with an element of randomness—it governs the growth of DLA. Using the Center's Cray, Drs. Mandelbrot and Evertsz have evaluated this potential, and (thanks to high quality graphics executed at Yale) have made a number of empirical observations that have escaped earlier investigations. This work is done in close connection with Peter W. Jones of Yale and offers a fresh instance of the fruitfulness of graphics in spanning the range from "pure" mathematics to very concrete physics.

David Mumford

My work has been on the development of mathematical models for identification of structures in visual signals. The key problem is that these must be robust with respect to the characteristic distortions of visual signals: the combination of smoothly varying deformations with unexpected breaks and discontinuities in which multiple models conflict.

Charles S. Peskin and David M. McQueen

FIBER ARCHITECTURE OF THE AORTIC VALVE:
We have combined geometry, mechanics, and computation (and also borrowed a little bit of fluid dynamics) to put together a theory which leads to a derivation from first principles of the complicated fiber architecture of the aortic heart valve leaflets. The key hypothesis of the theory is that the uniform pressure load on the closed aortic valve is supported by a one-parameter family of fibers under tension. We have derived an equation of equilibrium for such a structure, a partial differential equation from which all mechanical variables can be eliminated by the proper choice of coordinates. The result is a purely geometric equation, which (with suitable boundary conditions) determines the shape of the leaflets and the arrangement of the fibers.

Surprisingly, this equation takes the form of an evolution equation in which a single fiber moves to sweep out the leaflet surface. In this interpretation, the fiber moves

Jean Taylor

I supervised work of Andy Roosen on simulations of dendritic crystal growth and Ostwald ripening and wrote a paper with him describing the theoretical framework and the experimental results. The ability to do these computations, and the ability to attract a student as capable as Roosen to work on these problems, is a direct result of having the superb computer equipment that I've received as a result of the Geometry Center.

I completed two long papers, to appear as Overviews in Acta Metallurgica. The figures were generated on equipment provided by The Geometry Center. I finished a short (6 page) Scripta Met paper with Frank Morgan and wrote two short survey articles. I somewhat revised Motion of Curves by Crystalline Curvature in response to referees suggestions. This involved adding several figures, all obtained by the use of equipment obtained through The Geometry Center.

I investigated properties of initially polyhedral 2D surfaces evolving under weighted mean curvature in R3 and expanded some of my computer programs to compute such growth. I began to investigate the effect of edge energies.

I worked on revisions to preprint on Motion of Multiple-Phase Junctions under Prescribed Phase Boundary Velocities.

I also worked on use of crystalline method to compute stationary (but not minimizing) crystalline minimal surfaces.

I worked out the structure of a proposed paper (probably for Scripta Met) on the difference between evolution of soap bubble clusters via diffusion and motion by mean curvature, with application to the shapes of small crystals seen in materials. This paper will be joint with Rob Kusner and Tom Ilmanen.

Major changes in my research directions included putting diffusion in the crystalline motion problems, expanding the motion problems to 3D, and learning more about phase field model.

My future plans include further extending code to 3D, using current code to model different growth laws, and modeling eutectic growth. In addition, I plan on finishing my paper on triple junctions and characteristics; writing a short paper on limiting shapes of motion by mean curvature versus soap froth evolution; and working with Gelfand on use of characteristics for nonconvex mobility functions.

II. Budget

The Geometry Center
Statement of Unobligated Funds

The Geometry Center expects to carryover approximately $224,000 in NSF funding.

Plans for the use of the funds are specified in the proposed budget for the period February 1, 1992-January 31, 1993. They include the hiring of additional staff, including additional postdoctoral fellows. (The search committee for these positions will meet in mid-December.)

III. Updated STC Data Base

CENTER DATA REPORT
12/12/91

Grant Number: 89-20161
Center Director: Albert Marden
Institution Name: University of Minnesota
Title: Computation and Visualization of Geometric Structs
NSF Coordinator: Al Thaler

Fiscal Year: 92

Number of Faculty Participants: 20
Number of Non-Faculty Participants: 3
Number of Visitors: 56
Number of Post Doctorates: 1
Number of Graduate Students: 14
Number of Undergraduate Students: 10
Number of High School Teachers: 12
Ethnicity Summary
American Indian, Alaskan Native: 0
Asian or Pacific Islanders: 2
Black, not Hispanic Origin: 1
Hispanic: 3
White, not Hispanic Origin: 102
Unknown: 8

Nationality Summary
US Citizens: 101
Foreign Nationals: 13

Gender Summary
Number of Males: 99
Number of Females: 17

Sources and Amounts of Support for the STC (Per 1000s)
Received Support: $0
Commited Support: $0
Pending Support: $653
Total: $653

Faculty Funding Summary (Other Than STC / Per 1000s)
Funding from NSF: $835
Funding from DOD: $0
Funding from DOE: $0
Funding from EPA: $0
Funding from NASA: $0
Funding from NIH: $0
Funding from NOAA: $0
Funding from USDA: $0
Funding from Other Federal: $0
Funding from Other: $537

Total: $1372

Faculty Reported STC Funding (Per 1000s)
Total: $272

IV. Description of Outreach Activities

Outreach Activities

Education

Last Summer, 1991 we organized the first summer program under Center auspices. There were two parts. The first was a two week intensive course, Geometry and the Imagination, created and taught by John Conway, Peter Doyle, Jane Gilman, and Bill Thurston, two of whom are Center faculty members. Earlier versions had been taught twice before at Princeton. The 60 students were a mix of high school and college students, and high school and college teachers. The course was an extraordinary experiment in providing a course that challenged everyone, from the uninitiated to the mathematically sophisticated. The science reporter of the largest daily in Minnesota, the Star Tribune, wrote, "This was perhaps the most unusual, energetic and mind-expanding math class ever taught in Minnesota." It made heavy use of physical objects (e.g., polyhedra, mirrors) to develop intuition. The course notes have been issued as Center Report 30.

The course has inspired other mathematicians to teach courses at their universities likewise aimed at bringing a conceptual understanding of modern geometry to students without specific technical backgrounds. The methods used in the course to introduce curvature have been particularly influential in this development.

The other part of the summer program was a research and training program for talented senior high school and college students. There were 16 students present for up to 12 weeks. There was a special summer faculty to help each student find a worthwhile project, and to work with them through the summer. The summer faculty are called 'coaches' because of the requirement both of depth and breadth in science, and the ability to motivate. This is a very demanding job and it is difficult to find mathematicians who are willing and able to do it. The coaches for the past summer were Stan Wagon (Macalester), Jeff Ondich (Carleton) and Anthony Iano-Fletcher (Warwick). The first week the students arrived, they had a tutorial by Wagon on the use of Mathematica, and one by Maxwell on the use of the animation package Softimage. Student completed projects included a professional quality program to display knots and links developed in collaboration with staff, a study of sunrises and sunsets on the planet Mercury (up to three a day), a hyperbolic flight simulator developed in collaboration with staff, a classification of four dimensional archimedian solids with John Conway, and the logo for the newly formed European Mathematical Congress with Peter Doyle and Max Karoubi.

At the end of the summer, each student was required to write a report describing his or her activities. These are gathered together in Center Report 33.

Thus, the goals of our summer courses are to a) provide strong, attractive models for teaching aspects of geometry to college students without specific math background, b) provide background for high school math teachers and to assist them in developing materials which can be used in their classrooms, and c) demonstrate the advantages of team teaching that brings together different viewpoints, expertise, and personalities. The purpose of our summer long student program is to provide an intellectually rich, non-competative, non-intimidating experience based a personal exploration in depth into a topic in mathematics or computer science. This too provides an alternative to the normal academic process in mathematics education. The goal is not necessarily to produce more math majors or research mathematicians, rather, to give students a deeper appreciation of mathematics and the role of computing.

Scientifically, it is the first example, at least in mathematics, of how the powerful technology of computer graphics can be used to visualize, optically accurately, spaces with noneuclidean metrics. It also reveals how very difficult it is to communicate mathematics, specifically to write a script to be accessible to a general audience, even a general mathematical audience. Writing the script was one of the most difficult parts of the video. Another difficult part is the artistic input which is animating the scenes to be attractive and inviting for the viewer. Of course the computer graphics itself was completely original. "Not Knot" won professional recognition in the graphics world: it was chosen for the Siggraph 91 Electronic Theater, and it won the Nicograph 1991 award in Japan for the category of scientific computer graphics.

Initially, it was hoped that the video would be accessible and have something to communicate to everyone, from high schoolers to research mathematicians. Our experience is that that can happen, but not without extra help from a lecturer. The scenes of a video go by quickly, and even with every word of the script carefully chosen there is a lot to absorb. This was one of the reasons a 50 page supplement was written. The supplement adds commentary on representative frames from the movie, answering the questions that were asked over many public showings to many different groups, and providing capsule summaries of relevant topics, like knot theory and hyperbolic geometry. We believe the supplement greatly increases the value of the animation. The topics in the summer course were related to the video, which was shown at the beginning and at the end of the course, so the students could compare their understanding.

"Not Knot," as well as Taylor's video, has opened the door to using video as a major output device for communication to research mathematicians, and to a larger scientific, educational and public audience. The production of a number of other Center sponsored videos is underway.

Conferences hosted at The Geometry Center

November 2, 1991. Minnesota Math Mobilization (M^3)/Twin Cities Urban Math Collaborative/The Geometry Center hosted a conference on geometry programs in Minnesota, 1991.

Helaman Ferguson, speaker
65 registered teachers
postponed due to snow

November 10-11, 1991. Mathematicians and Education Reform (MER) Network. A meeting of three Regional Geometric Initiative and The Geometry Center (DIMACS wanted to but was unable to attend). Attended by NSF program director and AMS representative (Allyn Jackson). There was scientific/education discussion of programs and vertically integrated education model.

V. Management and Planning

ORGANIZATION CHART
THE GEOMETRY CENTER
November 1991

Executive Committee
Almgren, F.
Epstein, D.
Hanrahan, P.
Keynes, H., Education Director
Marden, A., Director
McGehee, R., Science Advisor
Thurston, W., Codirector

External Advisory Board
Hyman Bass (Prof. of Math, Columbia Univ.)
James Blinn
Frederick Gehring (Prof. of Math, Univ. of Michigan), Chair
Daniel Gorenstein (Director DIMACS & Prof. of Math, Rutgers)
John Guckenheimer (Dir. of Research Programs, Cornell Theory Center and Professor of Math, Cornell)
Deb Hughes Hallett (Prof. of the Practice in the Teaching of Math, Harvard)
Maria Klawe (Head, Dept. of Computer Science, Univ. of BritishColumbia)
Jill Mesirov (Dir., Mathematical Sciences Research, Thinking Machines Corp.)
Alan H. Schoenfeld (Chairman, Educ. in Math, Science & Technology, Graduate School of Education, UC-Berkeley)

Director
A. Marden

Codirector
W. Thurston

Education Director
H. Keynes

Science Advisor
R. McGehee

Geometry Computing Group Faculty

Almgren, F.	Cannon, J.	Chazelle, B.
Conway, J.	Dobkin, D.	Douady, A.
Epstein, D.	Freedman, M.	Hanrahan, P.
Hubbard, J.	Keynes, H.	Mandelbrot, B.
Marden, A.	Milnor, J.	Mumford, D.
Peskin, C.	Taylor, J.	Thurston, W.
Wilks, A.		

Staff

Administrative
Vail, A.

Technical
Bertilson, S.
Gunn, C.
Levy, Si.
Levy, St.
Munzner, T.
Phillips, M.

Directors Report

The Geometry Center is up and running well beyond the scale of the Geometry Supercomputer Project that spawned it. The Center mission of research and education supported by advanced computing, graphics and video technology opens many opportunities and possiblities for having an impact in each of these areas.

The Center is driven by science and is supporting some of the finest mathematics and computer science research produced by Center faculty and associates. At the same time, it is part of the Center mission to inspire increasing numbers of geometers to get involved in activities that the Center can support. In effect, the Center is a new kind of mathematics institute. The Center wants to sponsor small teams of one or more mathematicians to carry out significant scientific/computational/visualization projects. The kinds of workshops we want to promote involve a) computational and theoretical issues with associated code development for modelling important phenomena, b) the 'experimental' approach to studying structures that have resisted theoretical explanation, and c) the development of large scale research software. The Center wants to promote research that will benefit from the Center lab and staff expertise.

As the Center is forging new paradigms within the mathematics and computer science communities rather than following well paved roads, its management is quite challenging. An initial idea must be carefully analyzed as to its appropriateness and feasibility. Implementation of a novel program can require great effort. The key factor is to discover and then recruit for active involvement those who are exceptionally motivated, skilled and talented. The first 10 months in the life of the Center include some notable successes in terms of expanding, fruitful research and educational programs. Less noticeable in the written record are the explorations of possible programs for the future. For me as Director, it has been an extremely intense, consuming period, and exhilarating as well.

Research. Through the 'center without walls' aspect of The Geometry Center, there is a strong ongoing research program. Its fruits are fundamental for Center activities in education and graphics. We need to do more to bring more mathematicians into the computing/graphics world. We need to encourage more mathematicians to take the risk of developing software to explore, to see what has never been seen and from that to draw inspiration for further research advance. This is one reason behind the Center involvement in starting the journal *Experimental Mathematics*. We want to strengthen our visitor program and increase the number of postdocs and long term visitors on sabbaticals or other outside support. A promising development is the possibility of sharing postdocs with the mathematics and computer science departments.

Education. Beyond the current programs, how can the Center connect to K-12 student and teacher education? How can mathematics research and visualization at the cutting edge connect to the schools? We are actively analyzing the situation with the assistance of high school teacher Arnie Cutler. Ideas include curriculum development involving narrated videos with text supplement, highly structured instructional labs with 3D graphics workstations, and "road shows" with 3D graphics equipment temporarily brought into classrooms with a carefully organized teaching module. In particular, we have had meetings at the School of Education, which is initiating a new program for undergraduate math majors leading to certification and a masters in education. There are ways the Center might have an input in that program. Our efforts seem to be seeding new possibilities in math education within the math department as well.

VI. International Contacts/Visits

Center faculty members Douady and epstein are based in France and England respectively. Epstein runs a major SERC-funded computational activity at the University of Warwick and there are close scientific relationships to that.

Professor Michael Herman of Ecole Normale Paris will be visiting the Center during March and April, 1992. He is one of the top international experts in dynamical systems and he will be helping direct the workshop on 4-D.

The Symposium 'Geometry and Computers' will be in Tokyo, January 20-22, 1992. This was organized jointly by The Geometry Center and Professor Sadayoshi Kojima of the Tokyo Institute of Technology and several members of The Geometry Center will be attending. It is funded entirely by Japanese sources. Professor Kojima is trying to establish in Japan a center similar to ours.

Preliminary discussion have been held with Professor B. Dahlberg, who is scientific director of Volvo, concerning a possible training program.

Anthony Iano-Fletcher, University of Warwick, England, visited the Center in April and again in June-July. He assisted undergraduate summer research students with their projects. Students gave Dr. Iano-Fletcher excellent reviews during their exit interviews.

Maria Iano-Fletcher, University of Leicester, England, visited the Center in April and also in June. She worked with high school students on research projects. She was well-liked by the students and was a positive influence on their work. She has been awarded a grant by the UK Higher Education Initiative to conduct a course along similar lines at Leicester University, England, during the coming academic year.

Three students from the University of Leicester, England, (Harris, Kay and Hooker) attended the two-week summer course, "Geometry and the Imagination."

APPENDICES

Publications: Center Supported

Cannon, J.W.

J. W. Cannon, The theory of negatively curved spaces and groups (Trieste lectures), in Ergodic Theory, Symbolic Dynamics and Hyperbolic Spaces, Tim Bedford, Michael Keane and Caroline Series, eds., Oxford University Press, Oxford, New York, and Tokyo, 1991, pp. 315-369.

J. W. Cannon and Daryl Cooper, A characterization of cocompact hyperbolic and finite-volume hyperbolic groups in dimension three, Trans. Amer. Math. Soc., 14 pages, to appear.

J. W. Cannon, Mathematics in Marble and Bronze: the Sculpture of Helaman Rolfe Pratt Ferguson, The Mathematical Intelligencer 13 (1991), 30-39.

Walter Parry and J. W. Cannon, Notes on 2-layer valence-3 tilings of quadrilaterals, manuscript, Summer 1991.

Walter Parry and J. W. Cannon, Properties of optimal weight functions on tilings of quadrilaterals, in preparation, Fall 1991.

Dobkin, David

D. Dobkin, "Computational Geometry and Computer Graphics — A symbiotic pair", Proceedings of the IEEE, special issue on computational geometry, 1992, to appear.

Epstein, David B.A.

"Word processing algorithms, rewrite rules and group theory", in "The Mathematical Revolution Inspired by Computing", edited by J.H.Johnson and M.J.Loomes, OUP (1991).

"Combable groups", Advances in Diff. Geom. and Topology, Cala Gonone 1988, Rend. Sem. Fac. Sci. Univ. Cagliari, Suppl. Vol. 58. pp. 423-429.

"The use of Knuth-Bendix methods to solve the word problem in automatic groups", with D.F.Holt and S.E.Rees (to appear), Journal for Symbolic Computation.

Freedman, Michael

M.H. Freedman and Z.-X. He, "Divergence-free fields: Energy and asymptotic crossing number", Annals of Math. 134 (1991). 189-229.

S. deMichelis and M. H. Freedman, "Uncountably many exotic R4's in standard 4-space", to appear in Journal of Differential Geometry.

Mathematical Theories of Shape: do they model perception?,in *Proc. Conference 1570, Soc. Photo-optical \& Ind. Engineers*, 1991, pp. 2-10.

A Bayesian treatment of the stereo correspondence problem using half-occluded regions (with P. Belhumeur), submitted to 1992 Computer Vision and Pattern Rec. Conference.

Texture segmentation by minimizing vector-valued energy functionals: the coupled membrane model (with T.-S.Lee, A.Yuille), submitted to 1992 European Conf. on Computer Vision.

Peskin Charles S. and McQueen, David M.

Mechanical equilibrium and the fractal fiber architecture of the aortic heart valve leaflets. Submitted to SCIENCE, Nov. 1, 1991.

Taylor, Jean

Crystalline Geometric Crystal Growth, in Proceedings of the Centre for Mathematics and its Applications, Aus. Nat. Univ. vol 26 (1991), 231-234.

Motion by Crystalline Curvature, in Computing Optimal Geometries, Jean E. Taylor, ed., AMS Selected Lectures in Mathematics (1991), 63-65.

The Geometry Center was also instrumental in the publication of Computing Optimal Geometries (which was edited by Jean Taylor).

Publications: Use of Center Facilities

Brakke, Kenneth

Surface Evolver Manual, Version 1.65, GCG Research Report 31

Cannon, James

J. W. Cannon, The combinatorial Riemann mapping theorem, submitted to Acta Math., 80 pages. (Also to appear as a Center preprint.)

Epstein, David B.A.

Epstein, David B.A. and Gunn, Charles, Supplement to Not Knot. Jones and Bartlett. 1991.

Word Processing in Groups, book by Epstein, Cannon, Levy, Thurston, Mike Paterson and Derek Holt (all but the last two are Center members), in press (Jones and Bartlett), 335 pages

Gunn, Charles

"Visualizing Hyperbolic Space", in the Proceedings of the Eurographics Workshop on Computer Graphics and Mathematics, October, 1991.

"Remarks on Mathematical Courseware", in the Proceedings of the Conference on Computer Graphics and Education '91, Barcelona.

"Computers in Mathematics: Snapshots of a Possible Future", Mathematica Journal, Fall, 1990, Vol. 1, Issue 2, pp 24-26.

Levy, Silvio

Automatic Generation of Hyperbolic Tilings, by Silvio Levy, article accepted for publication in Leonardo, summer 1992 (date tentative)

Three-Dimensional Topology and Geometry, vol. I, by Thurston, edited by Levy, Epstein, and others, to appear in early 1992, Princeton University Press, 400 pages

The Mathematica Journal, edited by Silvio Levy, published by Addison-Wesley, four issues so far.

Experimental Mathematics, journal to be published by Jones & Bartlett, to appear in early 1992, David Epstein (Editor-in-Chief), Silvio Levy (Editor), various members of the Center on the editorial board.

A Beginner's Book of TeX, by Raymond Seroul and Silvio Levy, Springer-Verlag, 1991, 282 pages.

Appendix II. External Advisory Committee Meetings

The Center has not yet held an External Advisory Committee meeting. A March 1992 meeting is currently being planned.

External Advisory Committee members include:

Hyman Bass (Prof. of Math, Columbia Univ.)
James Blinn
Frederick Gehring (Prof. of Math, Univ. of Michigan), Chair
Daniel Gorenstein (Director DIMACS & Prof. of Math, Rutgers)
John Guckenheimer (Dir. of Research Programs, Cornell Theory Center and Professor of Math, Cornell)
Deb Hughes Hallett (Prof. of the Practice in the Teaching of Math, Harvard)
Maria Klawe (Head, Dept. of Computer Science, Univ. of BritishColumbia)
Jill Mesirov (Dir., Mathematical Sciences Research, Thinking Machines Corp.)
Alan H. Schoenfeld (Chairman, Educ. in Math, Science & Technology, Graduate School of Education, UC-Berkeley)

Appendix III. Inventions, Patent Applications, Patents

The Center has no inventions, patent applications, nor patents to disclose at this time.

Appendix IV. Awards/Prizes

The Center was awarded a Nicograph 1991 award in the category of scientific computer graphics for its videotape "Not Knot."