Collective flow as a probe of heavy-ion reaction dynamics

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Collective flow of nuclear matter probes the dynamics of heavy-ion reactions and can provide information about the nuclear-matter equation of state. In particular, the incident energy dependence of collective flow may be a sensitive means to deduce the existence of a Quark Gluon Plasma phase in the equation of state. Collective flow measurements from 30 A MeV to 200 A GeV incident energies are briefly reviewed. Preliminary results on collective flow from the WA98 experiment at the CERN SPS are presented.

1. INTRODUCTION

The possibility that nuclear matter might exhibit non-random collective motion in relativistic heavy-ion collisions was first suggested more than twenty years ago in connection with hydrodynamic calculations of nuclear collisions [1]. It was more than ten years before collective flow was first unambiguously identified at the Bevalac [2,3]. In the years since then collective flow effects have been studied in nuclear collisions at incident energies from 30 A MeV to 200 A GeV.

Collective motion in the final state is governed by pressure gradients in the compressed and heated nuclear matter. In a hydrodynamic description of the nuclear collision, collective flow should therefore provide information about the stiffness of the nuclear equation of state, i.e. about how the pressure varies with nuclear density. In microscopic descriptions of the nuclear collision using transport models (e.g. BUU), such as applied at low incident energies where a hydrodynamic description may not be valid, the single particle phase space distributions evolve under the influences of the Coulomb field and nuclear mean field, as well as nucleon-nucleon collisions. Collective flow at these energies is therefore expected to be sensitive to the density and momentum dependence of the nuclear mean field, as well as the density dependence of the nucleon-nucleon cross section.

An important feature of collective flow is that it develops over the entire history of the nuclear collision. It is therefore an hadronic observable which may be sensitive to the initial conditions of the hot dense matter produced in relativistic nuclear collisions. In the case that the nuclear matter equation of state (EoS) exhibits a transition from a hadron phase to a Quark Gluon Plasma (QGP) phase, consisting of deconfined quarks and gluons, it is expected that the EoS should exhibit a corresponding softening due to the increased number of degrees of freedom. A QGP phase would result in decreased pressure gradients, and hence decreased collective flow. Thus a detailed study of the incident energy dependence of collective flow might provide evidence for QGP formation.
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Figure 1. Contour plots of baryon density in the reaction plane in Au + Au collisions at 400 A MeV. The results are from transport model calculations from Ref. [4].

Collective flow can be decomposed into its component along the beam axis and the component transverse to the beam axis. The longitudinal component dominates at high incident energies and is difficult to interpret. While it may contain information on stopping and nuclear compression with subsequent expansion, the longitudinal motion might simply reflect the incoming momentum due to nuclear transparency. We will not discuss longitudinal flow further in this report.

The transverse flow can be further characterized as to whether it is isotropic in the transverse direction. Isotropic transverse flow is usually termed radial flow while non-isotropic transverse flow may be generally termed directed flow, although in the past the term directed flow has been used synonymously to mean a net flow direction, as with sideways flow. Transverse collective flow may be characterized by a (transverse) radial flow velocity. Examples of these two types of transverse flow may be seen in Fig. 1 which shows the time evolution of a Au + Au collision at 400 A MeV for three different impact parameters. For central collisions with impact parameter b=0 the system is observed to expand radially in the transverse direction after the initial compression phase. At
large impact parameters, on the other hand, a clear sidewards deflection of the projectile and target are observed. Experimentally one would like to quantify such behaviour and compare it to calculations to extract information about the nuclear compressibility and nuclear EoS.

2. ANALYSIS METHOD

By definition, the distinguishing characteristic of collective flow is that within a local volume element the matter moves with the same flow velocity, \( \beta_x \). As a result, different particle species in the volume element will on average carry different amounts of collective energy according to their mass, \( 1/2m\beta_x^2 \). This is in contrast to random thermal energy which is shared equally independent of mass, \( 3/2T \). Therefore, for a simple thermal system undergoing collective expansion the average energies, or transverse momentum spectra may be analyzed simultaneously for different mass species to extract the temperature, \( T \), and radial flow velocity, \( \beta_r \).

The study of directed flow requires that one first define a preferred direction and then determine if there is preferred motion relative to that direction for some subset of emitted particles. Generally, one chooses the reaction plane, or more particularly the projectile fragment direction, as the preferred direction, but it could as well be the direction of a single particle, or some subset of particles.

Although there are many ways in which the directed flow can be quantified, the most commonly used method is that proposed by Danielewicz and Odynev [5]. In this method the projectile transverse flow direction is first determined by calculating the so-called directivity, \( \bar{Q} \), defined as the sum of transverse momentum vectors, \( \vec{p}_T \) of all \( M \) identified particles, but with opposite signs in the forward and backward rapidity regions.

\[
\bar{Q}_i = \sum_{\nu \neq i} M \omega_\nu(y) \vec{p}_{T,\nu}
\]

with \( \omega_\nu(y) = \begin{cases} 1 & y > y_{cm} + \delta \\ -1 & y < y_{cm} - \delta \end{cases} \) \tag{1}

Here \( \delta \) is chosen to avoid the mid-rapidity region and thereby give maximum sensitivity to the event plane information.

One then calculates the average transverse momentum (per nucleon) in the \( \bar{Q}_i \) direction in different rapidity regions.

\[
< P_x(y)/A > = \frac{1}{N(y)} \left[ \sum_i (\vec{p}_i/A \cdot \bar{Q}_i) \right] \tag{2}
\]

When the projected average transverse momentum is plotted as a function of rapidity one obtains the so-called directed flow or S-curve plots as shown in Fig. 2. As expected, the projected average transverse momentum is observed to correlate with the projectile flow direction in the forward rapidity region and anti-correlate in the backward rapidity region.
Figure 2. Average transverse momentum per nucleon projected onto the projectile fragment direction versus \( \eta/\eta_{\text{Beam}} \) for Au + Au collisions at various incident energies. The average increases with energy. From the EOS collaboration Ref. [6].

Fig. 2 also demonstrates that the magnitude of the projected average transverse momentum increases as a function of incident energy from 0.25 to 1.15 A GeV, as measured by the EOS collaboration [6]. The information in such S-curve plots is normally further distilled by calculating the slope of the curve in the vicinity of mid-rapidity to obtain a quantity which has become known as "Flow", \( F \)

\[
F = \left. \frac{d \langle P_x(y)/A \rangle}{dy} \right|_{y=0_{\text{cm}}}
\]

The directed flow could equivalently be characterized in other ways such as by the maximum or average \( \langle P_x(y)/A \rangle \) in the forward rapidity region, but the use of \( F \) has been shown to have least sensitivity to detector bias and has become the conventional quantity determined experimentally.

The directed flow analysis discussed above relies on particle identification and momentum measurement over a large acceptance. In many experiments, especially at high incident energies such as at the AGS and SPS one may have only multiplicity or calorimetry information over a large acceptance. In this case one can generalize the directivity, \( \hat{Q} \) to define the preferred direction.

\[
\hat{Q} = \sum_{\nu=1}^{M} \omega_{\nu}(y) \hat{u}_{T_{\nu}}
\]

where \( \hat{u}_T \) is a unit vector giving the transverse direction of the hit or calorimeter cell, and the weight is taken as \( \omega_{\nu}(y) = p_{T_{\nu}}, E_{T_{\nu}}, \) or 1, depending on whether momentum, transverse energy, or multiplicity is measured.
When viewed as a vector in the complex plane, the above expression is seen as the first term in a Fourier decomposition which can be generalized to consider higher Fourier terms as [7]:

$$Q_n = \sum_{\nu=1}^{M} \omega_\nu(y)e^{in\phi} \quad (5)$$

Each vector $Q_n$ is characterized by a direction, $\Psi_n$ corresponding to the direction or axis of flow and a magnitude $v_n$ where

$$\Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{ImQ_n}{ReQ_n} \right) \quad \text{and} \quad v_n = \frac{|Q_n|}{Q_0} \quad (6)$$

The first Fourier coefficient is closely related to the directed flow analysis described above. The second Fourier coefficient is related to a sphericity or event shape analysis, but in the transverse dimensions. It can be shown that $v_2 = \frac{f_1-f_2}{f_1+f_2}$ where $f_1$ and $f_2$ give the magnitude of the axes of the ellipse describing the event shape. In the case of no net directed flow, such as at mid-rapidity for a symmetric system, one expects $v_1 = 0$. Even in such a case, effects such as squeeze-out may give non-zero values of $v_2$ if the matter is flowing non-isotropically.

3. Results: $< 200$ A MeV

At low incident energies the attractive nuclear mean field dominates the interaction between the two heavy-ions. Upon overcoming the repulsive Coulomb field, the nuclei clutch due to the short range nuclear mean field and begin to rotate as a di-nuclear complex. At large impact parameters and moderately high incident energies the centrifugal forces overcome the nuclear mean field and the nuclei separate again after a brief rotation through negative angle with energy dissipated through nucleon-nucleon collisions and exchange. This is the well-known deeply inelastic process. In directed flow studies at these energies this collective motion has been termed rotational flow. As the incident energy increases the importance of the repulsive nucleon-nucleon collisions increases resulting in decreasing rotational flow angles. At the so-called balance energy, which is the incident energy at which the attractive and repulsive forces balance one another, the nuclear deflection changes sign from negative to positive deflection angles.

The systematic dependencies of the balance energy have been studied extensively. Details of the observed systematics are discussed elsewhere in these proceedings [8]. The balance energy has been shown to decrease with increasing mass of the colliding nuclei [9], but increase with increasing impact parameter [10]. Most recently, the balance energy has also been shown to increase with the N/Z ratio of the system [11]. The balance energy has been shown to be sensitive to the interplay between the attractive mean field and the repulsive nucleon-nucleon collisions in the overlap zone. Hence the increasing size of the collision zone, and therefore increased repulsion, for increasing system mass or decreasing impact parameter explains the general trends noted above. The fact that the n-p cross section is much larger than the n-n or p-p cross sections also explains the
decreased repulsion as the N/Z ratio varies from 1. Detailed comparison with transport model calculations have shown that the results clearly favor momentum-dependent mean field interactions and suggest a preference for reduced in-medium nucleon-nucleon cross sections [8,12]. The calculated results generally show little sensitivity to the stiffness of the EoS as incorporated via the density dependence of the mean field.

![Graph](image)

Figure 3. The excitation function of scaled flow values for different projectile-target systems (from Ref. [14]).

4. Results: < 20 A GeV

As the incident energy is increased above the balance energy, the directed flow is observed to further increase [13]. With the original flow measurements of the Plastic Ball and Streamer Chamber collaborations at the Bevalac, and recent results from the EOS collaboration at the Bevelac and at BNL in the E895 experiment, and from the FOPI collaboration at SIS, there is now a sizeable body of data which clearly demonstrates a systematic increase of directed flow with increasing incident energy up to incident energies of about 1 A GeV. Such a compilation of directed flow results taken from Ref. [14] is shown in Fig. 3.

The directed flow results for the various systems shown in Fig. 3 have been scaled by 

\[(A_1^{1/3} + A_2^{1/3})^{-1}\]

as suggested by model calculations [14]. The scaled results exhibit a universal behaviour with the scaled directed flow attaining a maximum value of about 40 MeV/c at around 1 A GeV incident energy. It remains at this value for higher Bevalac or SIS energies, with an indication that the directed flow will decrease with further increase of incident energy.
Figure 4. The excitation function of the transverse directed flow as calculated for 3+1 dimensional hydrodynamics for Au+Au collisions at b=3fm (from Ref. [17]). (Note that the directed flow is quantified as the average $<P_z>$ in the forward rapidity region rather than the usual flow variable.)

First results of flow measurements from the AGS for Au + Au collisions at 11.4 A GeV from the E877 collaboration demonstrated that directed flow is small but observable at this incident energy [15]. It was shown via a Fourier analysis, as described above, that the flow directions of the transverse energy in the forward and backward rapidity regions were anti-correlated, as expected for directed flow. A more detailed recent preliminary analysis [16] in which a conventional S-curve flow analysis was performed indicates scaled flow values of around 10 MeV/c, clearly demonstrating the reduction of sideways directed flow at high incident energy.

As mentioned above, the directed flow is expected to be sensitive to the pressure gradients produced in the nuclear collision, which in turn should be sensitive to the nuclear EoS. In the case that the nuclear matter undergoes a phase transition to a QGP phase, the nuclear EoS is expected to become very soft resulting in decreased pressure gradients and hence decreased flow. Evidence for such behaviour is observed in the excitation function of transverse directed flow calculated for 3+1 dimensional hydrodynamics for Au + Au collisions [17] shown in Fig. 4. These calculations predict a dramatic local minimum in the excitation function of directed flow at an incident energy corresponding to the threshold for QGP formation, which was assumed to occur at a transition temperature of $T_C = 160$ MeV in these calculations. By contrast, in the event that there is no phase transition, the
directed flow decreases smoothly without structure for increasing incident energy. The overall decrease in the directed flow is simply attributed to the fact that with increasing incident energy the spectator matter leaves the reaction zone before it can be deflected by the expanding participant matter. While the calculations of Fig. 4 are rather simplistic, the experimental observation of a local minimum in the directed flow excitation function would clearly be very convincing evidence that the system has passed through the QGP phase transition. If the transition temperature is low this may occur already at AGS energies, as seen in Fig. 4. The search for such a QGP signature is a major goal of the experimental program of the E895 experiment [18].

5. Results: SPS Energies

While the directed flow measurements are sensitive to the pressure buildup in the overlap matter, this sensitivity is indirect since it is largely due to the deflection of the spectator matter by the hot expanding participant matter. Information about the density and pressures attained might be better deduced by studying the collective flow of the participant matter itself. At low incident energies, this is complicated by the fact that the compression and re-expansion happens on a timescale which is short in comparison with the spectator transit time. The result is that the spectator matter forms two confining walls which allow the participant matter to expand freely only in the out-of-plane directions. The result is the well-known out-of-plane squeeze-out effect [19]. Recent results from the EOS collaboration [20] in which a radial flow analysis has been performed to extract the temperature and radial flow velocity as a function of angle with respect to the event plane have shown very nicely how the magnitude of the squeeze-out effect decreases with increasing centrality, as there is less confining spectator matter. At the same time the magnitudes of both the temperature and radial flow velocity increase with centrality [20].

As the incident energy increases the spectator matter leaves the vicinity of the interaction zone more quickly, allowing less time for it to be influenced by the expansion of the participant matter, while at the same time causing less obstruction of the expanding participant matter. Thus, as the deflection of the spectator matter decreases with increasing energy an undistorted view of the flow of the produced particles should become more feasible. Due to the asymmetry of the geometry of the overlap region for non-central collisions it is expected that the density gradient and hence pressure gradient will be greatest in the reaction plane. Therefore one expects a preferential flow of the produced particles in the reaction plane [21]. The competition between the preferred in-plane flow and the time-varying in-plane obstruction by the spectator matter will strongly influence the mid-rapidity flow pattern and may provide exquisite sensitivity to the development of the pressure gradients [22].

Unlike directed flow, the radial flow velocity of produced particles and participants at mid-rapidity is observed to show no decrease with increasing incident energy. Recently, NA44 has extracted a radial flow velocity of 0.4c by a simultaneous fit to the $\pi$, $K$, and $p$ transverse mass spectra at mid-rapidity for central collisions of Pb + Pb at 158 A GeV [23]. This is slightly larger than observed at energies below 2 A GeV [13]. Unfortunately,
there exists a large fit ambiguity in the flow velocity due to an anti-correlation between
the fitted temperature and flow velocity, with a large range of temperatures and velocities
giving acceptable fit results. On the other hand, the transverse momentum dependence
of HBT radii suggest that the radial flow velocities in 158 A GeV Pb + Pb collisions [24]
may be even larger.

Figure 5. Distribution of relative angles $\Psi$ between proton and light-particle ($A > 1$)
flow directions measured in the target rapidity region with the Plastic Ball of WA98 for
peripheral 158 A GeV Pb + Pb collisions.

Figure 6. Distribution of relative angles $\Psi$ between proton and light-particle ($A > 1$)
flow directions measured in the target rapidity region with the Plastic Ball of WA98 for
intermediate centrality 158 A GeV Pb + Pb collisions.

In order to study the flow pattern at mid-rapidity it is first necessary to establish the
event-plane, which requires a measureable amount of sidewards directed flow. For 200 A
GeV oxygen- and sulphur-induced reactions, WA80 has shown that the pions of the target
rapidity region flow in the same direction, which is interpreted to result from absorption
in the target spectator matter [25]. Preliminary results from the Plastic Ball detector of
WA98 studying identified particles emitted in the target rapidity region ($-1.6 < \eta < 0.5$)
indicate that the target rapidity flow direction can be determined in 158 A GeV Pb +
Pb collisions. In this preliminary analysis the transverse energy vector (Eq. (4)) is calcu-
lated separately for protons and for light-particles ($A > 1$). The relative angle between
the proton and light-particle flow directions, $\Psi = \Psi_p - \Psi_{A>1}$ is then studied as a func-
tion of centrality, as determined by the total transverse energy measured in the MIRAC
calorimeter of WA98. The distribution of relative flow angles is shown in Figs. 5 and 6
for peripheral and intermediate centralities, respectively. A clear correlation between
the proton and light-particle transverse energy directions is observed for intermediate cen-
trality, which indicates that the target baryons tend to move in the same direction. The
relative angle distributions have been fitted to the form
\[ \frac{dN}{d\Psi} = C \cdot (1 + A_1 \cdot \cos \Psi) \]  
(7)

The strength of the correlation $A_1$ has been extracted and is plotted in Fig. 7 as a function of the total transverse energy. As expected, the strength of the correlation is greatest at intermediate centrality, where the event-plane is best defined. In peripheral collisions the target spectators receive very little transverse kick, and in very central collisions the system approaches azimuthal symmetry. Recently, NA49 has presented preliminary results for 158 A GeV Pb + Pb collisions which show a similar trend in studies of the correlation between the transverse energy flow planes in the forward and backward rapidity regions [26].

It is most interesting to study the emission pattern of produced particles near mid-rapidity as a function of the event-plane, or target flow direction. In Fig. 8 very preliminary WA98 results are shown comparing the transverse momenta spectra of negative tracks measured in the WA98 negative track spectrometer, for intermediate centrality 158 A GeV Pb + Pb collisions, under the condition that the proton transverse energy flow direction, measured in the Plastic Ball detector, is in the same, opposite, or out-of-plane direction relative to the negative track direction in the tracking arm. A similar plot is shown in Fig. 9 for photons measured in the WA98 photon spectrometer. Both detectors have coverage in the rapidity region $y \approx 2 - 3$ just behind mid-rapidity. Ratios of spectra are shown since effects of efficiency and acceptance should cancel to allow preliminary conclusions. It should be noted that the negative tracks consist dominantly of $\pi^{-}$ and that the photons result dominantly from $\pi^{0}$ decay. Both measurements show similar behaviour as they should, presumably reflecting that of the pions, while the photon results show weaker trends presumably reflecting the weakened correlation due to the $\pi^{0}$ decay.

The results show that the pion yield is suppressed for emission in the same direction as the target spectators as compared to out-of-plane or opposite emission directions. This suggests that there still exists significant absorption by the target matter near mid-rapidity even at SPS energies. However, the pion momenta are boosted when emitted in the direction away from the target spectators as compared to emission in the same or out-of-plane directions. This may reflect the expected in-plane expansion of the participant matter without obstruction of emission by the spectator matter when emitted.
Figure 8. Comparison of transverse momenta spectra of negative tracks for different coincidence requirements with the target flow direction.

Figure 9. Comparison of transverse momenta spectra of photons for different coincidence requirements with the target flow direction.

in the direction away from the target spectators [21,22]. It suggests that at SPS energies we may have a first opportunity to study quantitatively the undistorted flow pattern of the participant matter.

6. Conclusions

A brief summary of collective flow studies from MSU energies to SPS energies has been presented. There has been a great deal of activity over this entire range of incident energies with a large number of interesting recent results, some of which have been discussed here and elsewhere in these proceedings. While flow measurements have provided evidence for the momentum dependence of the nuclear mean field, and for the in-medium dependence of the nucleon-nucleon cross sections, the expected sensitivity to the stiffness of the
equation of state of nuclear matter has yet to be convincingly demonstrated. It is hoped that this sensitivity will soon become apparent, in particular in results from the AGS and SPS where the highest densities are attained and where the results are most recent and still very preliminary. A thorough and systematic analysis of collective flow phenomena might provide evidence for the transition to a Quark-Gluon Plasma phase in the nuclear equation of state.

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REFERENCES

8. See G.D. Westfall, these proceedings.
13. See W. Reisdorf, these proceedings.
18. See H. Liu, E895 Collaboration, these proceedings.
24. See R. Stock, NA49 Collaboration, these proceedings.