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"SPECTRAL METHODS & SUM ACCELERATION ALGORITHMS"

Final Report

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Final Report

Spectral Methods & Sum Acceleration Algorithms

Principle Investigator: John P. Boyd

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Date: 5/91-8/94

The principle investigator pursued his investigation of numerical algorithms during the period of the grant. The attached list of publications is so lengthy that it is impossible to describe them in detail without making this report longer than the original proposal. However, we would like to call attention to the four articles on sequence acceleration and fourteen more on spectral methods, which fulfill the goals of the original proposal. We also continued our research on nonlinear waves, and wrote a dozen papers on this, too.

During the period of the grant, the P. I. served as chair or co-chair of four Ph. D.s: Beth Wingate [Oceanography & Scientific Computing], who expects to graduate in December, 1995, with a thesis on spectral elements on triangular subdomains for ocean flows, Dr. Javad Abdollahi-Alibeik [Environmental Engineering], who defended his thesis on nonlinear water waves in January, 1994, Heng-chu Han [Mechanical Engineering], who expects to graduate with a thesis on spectral elements for lubrication flows in May, 1995, and Wan-li Wu [Meteorology], who expects to finish with a thesis on middle atmosphere dynamics and tracer transport in the summer of 1995. Ms. Wingate and Mr. Han were partially supported by the grant.

The P. I. also wrote most of the first draft of the book Weakly Nonlocal Solitary Waves and Other Exponentially Small Phenomena, which is under review at Cambridge University Press. The P. I.'s earlier book on spectral methods has sold out, and the publisher has dropped his engineering line. A proposal to reprint the book is now being considered by Dover Press.

Table . Articles Published or Submitted, 5/91 to 8/94

Sequence Acceleration

1. "Defeating the Runge Phenomenon for Equispaced Polynomial Interpolation via Tikhonov Regularization." Appl. Math. Lett. 5, 57-59 (1992)
2. "A Lag-Averaged Generalization of Euler's Method for Accelerating Series. Appl. Math. & Comput., in press (1995)
3. "Sum-Accelerated Pseudospectral Methods: Finite Differences & Sech-Weighted Differences." Comp. Meths. Appl. Mech. Eng., 116, 1-11 (1994)
4. "Erfc-Log Filter and the Asymptotics of the Euler and Vandeven Sum Accelerations", submitted to Houston J. Mathematics [special ICOSAHOM 95 issue] (1995)

Spectral Methods

5. "Chebyshev and Legendre Spectral Methods in Algebraic Manipulation Languages", J. Symbolic Computing, 16, 377-399 (1993).
6. "A Comparison of Numerical & Analytical Methods for the Reduced Wave Equation with Multiple Spatial Scales", Appl. Numer. Math, 7, 453-479 (1991).
7. "Sum-Accelerated Pseudospectral Methods: The Euler-Accelerated Sinc Algorithm", Appl. Numer. Math., 7, 287-296 (1991).
8. "Spectral Method Solution of the Stokes Equations on Nonstaggered Grids", with M. Schumack and W. W. Schulz. J. Comp. Phys., 94, 30-58 (1991).
9. "The Arctan/Tan & Kepler-Burgers Mapping for Periodic Solutions with a Shock, Front, or Internal Boundary Layer", J. Comp. Phys., 98, 181-193 (1992)
10. "Multipole Expansions & Pseudospectral Cardinal Functions: A New Generalization of the Fast Fourier Transform", J. Comp. Phys., 102, 184-186 (1992).
11. "A Fast Algorithm for Chebyshev, Fourier & Sinc Interpolation Onto an Irregular Grid", J. Comp. Phys., 103, 243-257 (1992).
12. "Taylor Vortices Between Elliptical Cylinders", with M. Schumack and W. W. Schultz, Phys. Fluids, 4, 2578-2581 (1992).
13. "The Rate of Convergence of Fourier Coefficients for Entire Functions of Infinite Order with Application to the Weideman-Cloot Sinh-Mapping for Pseudospectral Computations on an Infinite Interval. J. Comp. Phys., 110, 360-372 (1994).
14. "Numerical computations of a nearly singular nonlinear equation: Weakly nonlocal bound states of solitons for the Fifth-Order Korteweg-deVries equation".
Submitted to J. Comput. Phys. (1993)

Table(continued)

15. "Multiple Precision Pseudospectral Computations of the Radiation Coefficient for Weakly Nonlocal Solitary Waves: Fifth-Order Korteweg-deVries equation". Submitted to Computers in Physics
16. "A staggered spectral finite element method for the shallow water equations" with M. Iskandarani and D. Haidvogel". Submitted to Int. J. Numer. Meths. Fluids (1993).
17. "A Chebyshev polynomial interval-searching method ("Lanczos economization") for solving a nonlinear equation with application to the nonlinear eigenvalue problem", J. Comput. Phys., in press (1994)
18. "A hyperasymptotic perturbative method for computing the radiation coefficient for weakly nonlocal solitary waves", J. Comp. Phys., in press (1995).
19. "Spectral elements methods on triangles for geophysical fluid dynamics problems", with B. A. Wingate, submitted to Houston J. Mathematics [special ICOSAHOM 95 issue] (1995)

Nonlinear Waves

20. "Monopolar & Dipolar Vortex Solitons in Two Space Dimensions", Wave Motion, 57, 223-243 (1991).
21. "Double Cnoidal Waves of the Korteweg-deVries Equation: The Boundary Value Approach", with S. E. Haupt, Physica D50, 117-134 (1991).
22. "Nonlinear equatorial Waves". In Nonlinear Topics of Ocean Physics, Proceedings of the Fermi Summer School, Course 109, ed. by A. R. Osborne and L. Bergamasco. North-Holland, Amsterdam, 51-97 (1991).
23. "Polycnoidal Waves: Spatially Periodic Generalizations of Multiple Solitons." With S. E. Haupt. In Nonlinear Topics of Ocean Physics, ed. by A. R. Osborne and L. Bergamasco. North-Holland, Amsterdam, 827-856 (1991).
24. "Weakly Non-Local Solitary Waves". In Nonlinear Topics of Ocean Physics, ed. by A. R. Osborne and L. Bergamasco. North-Holland, Amsterdam, 527-556 (1991).
25. "The Energy Spectrum of Fronts: The Time Evolution of Shocks in Burger's Equation", J. Atmos. Sci., 49, 128-139 (1992).
26. "Hyperviscous Shock Layers and Diffusion Zones: Monotonicity, Spectral Viscosity, and Pseudospectral Methods for Very High Order Differential Equations." J. Sci. Comput. 9, 81-106 (1994).
27. "Weakly Nonlocal Envelope Solitary Waves: Numerical Calculations for the Klein-Gordon (ϕ^4) Equation. In press, Wave Motion (1995).
28. "Nonlocal Modons on the Beta-Plane". Geophys. Astrophys. Fluid Dyn. in proof, (1995).
29. "The periodic generalization of Cammasa-Holm 'Peakons': An exact superposition of solitary waves." Submitted to ZAMP (1994).

Table (continued)

The Slow Manifold

30. "Eight Definitions of the Slow Manifold: Seiches, Pseudoseiches, and Exponential Smallness." In press, Dyn. Atmos. Oceans (1995)
31. "The Slow Manifold of a Five-Mode Model". J. Atmos. Sci., 51, 1057-1064 (1994).
32. "Time-Marching on the Slow Manifold: The Relationship Between the Nonlinear Galerkin & Implicit Timestepping Algorithms." Appl. Math. Lett., 7, 95-99 (1994).