OAK RIDGE

## Economic Analysis of Efficient Distribution Transformer Trends

D. J. Downing
B. W. McConnell
P. R. Barnes
S. W. Hadley
J. W. Van Dyke


This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from the Office of Scientific and Technical information, P.O. Box 62, Oak Ridge, TN 37831; prices available from (615) 576-8401, FTS 626-8401.

Available to the public from the National Technical information Service, U.S. Department of Commerce, 5285 Port Royal Rd., Springfield, VA 22161.

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefuiness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade neme, trademark, manufacturer, or otherwise, does not necessarily constifute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

## DISCLAIMER

Portions of this document may be illegible electronic image products. Images are produced from the best available original document.

# ECONOMIC ANALYSIS OF EFFICIENT DISTRIBUTION TRANSFORMER TRENDS 

D. J. Downing<br>B. W. McConnell<br>P. R. Barnes<br>S. W. Hadley<br>J. W. Van Dyke

March 1998

Sponsored by the
U.S. Environmental Protection Agency under DOE Interagency Agreement No. 1824-J072-A1

Prepared by the POWER SYSTEMS TECHNOLOGY PROGRAM

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831
managed by
LOCKHEED MARTIN ENERGY RESEARCH CORP.
for the
U.S. DEPARTMENT OF ENERGY
under contract DE-AC05-96OR222464

## CONTENTS

LIST OF FIGURES ..... v
LIST OF TABLES ..... vii
ACKNOWLEDGMENTS ..... ix
ABBREVIATIONS AND ACRONYMS ..... xi

1. INTRODUCTION ..... 1-1
1.1 BACKGROUND ..... 1-1
1.2 OBJECTIVE AND APPROACH ..... 1-1
1.3 UTILITY RESTRUCTURING ..... 1-2
1.4 SCOPE AND CONTENT ..... 1-2
2. TRANSFORMER PURCHASING TRENDS ..... 2-1
2.1 THE TOTAL OWNING COST METHOD ..... 2-1
2.2 UNCERTAINTY IN THE DETERMINISTIC TOC ..... 2-2
3. UTILITY RESTRUCTURING ..... 3-1
3.1 THE ELECTRICITY MARKET ..... 3-1
3.2 COST OF LOST ENERGY ..... 3-1
3.3 COST RECOVERY AND INCENTIVES FOR EFFICIENCY ..... 3-2
3.4 RESTRUCTURING EFFECTS ON TOC ..... 3-3
3.4.1 Price ..... 3-3
3.4.2 Core Loss ..... 3-3
3.4.2.1 System Cost ..... 3-3
3.4.2.2 Energy Cost ..... 3-4
3.4.2.3 Fixed Charge Rate ..... 3-4
3.4.3 Load Loss ..... 3-6
3.4.3.1 System Cost ..... 3-6
3.4.3.2 Responsibility Factor ..... 3-6
3.4.3.3 Energy Cost ..... 3-6
3.4.3.4 Loss Factor ..... 3-7
3.4.3.5 Equivalent Annual Peak Load ..... 3-7
3.5 SUMMARY OF RESTRUCTURING IMPACTS ..... 3-7
4. UNCERTAINTY ANALYSIS MODEL ..... 4-1
4.1 INTRODUCTION ..... 4-1
4.2 PROBABILITY DISTRIBUTION ..... 4-1
4.3 UNCERTAINTY ANALYSIS ..... 4-6
4.4 SENSITIVITY ANALYSIS ..... 4-9
5. STATISTICAL DISTRIBUTIONS ..... 5-1
6. UTILITY APPLICATIONS ..... 6-1
6.1 INTRODUCTION ..... 6-1
6.2 DESIGNS AND PARAMETER DISTRIBUTIONS ..... 6-1
6.3 RESULTS OF MONTE CARLO SIMULATION ANALYSIS ..... 6-3
6.3.1 15-kVA Pole-Mount ResidentialTransformer ..... 6-3
6.3.2 25-kVA Pole-Mount Residential Transformer ..... 6-4
6.3.3 25-kVA Pad-Mount Residential Transformer ..... 6-4
6.3.4 50-kVA Pad-Mount Residential Transformer ..... 6-5
6.3.5 300-kVA Pad-Mount Commercial Transformer ..... 6-5
6.3.6 $500-\mathrm{kVA}$ Pad-Mount Commercial Transformer ..... 6-5
6.3.7 1000-kVA Pad-Mount Industrial Transformer ..... 6-5
6.4 SENSITIVITY TO THE CHOICE OF DISTRIBUTION ..... 6-6
6.5 CONCLUSION ..... 6-6
7. SUMMARY ..... 7-1
7.1 OVERVIEW ..... 7-1
7.2 IMPACT OF NONDETERMINISTIC APPROACH ..... 7-1
8. REFERENCES ..... 8-1
APPENDIX: DERIVATION OF TOTAL OWNING COST ..... A-1

## FIGURES

2.1 Relative number of units with per unit load greater than or equal to $x$ ..... 2-3
2.2 Variation in total owning cost of a transformer according to variations in input parameters ..... 2-4
3.1 Load duration curve and real-time price of power for sample transformer ..... 3-8
4.1 Two normal distributions for SC, both with a mean of $\$ 240$ but with standard deviations of $\$ 10$ and $\$ 20$ ..... 4-2
4.2 Normal and uniform distributions for SC, both with a mean of $\$ 240$ and with standard deviations of $\$ 17.32$ ..... 4-3
4.3 Normal and triangle distributions for SC, with a mean of $\$ 238$ and standard deviations of $\$ 11.61$ ..... 4-4
4.4 Plot of gamma density for various choices of alpha and beta ..... 4-4
4.5 Plot of uncertainty costs for Designs 1 and 8 ..... 4-9
4.6 Sensitivity of TOC to changes in inputs ..... 4-11
5.1 A factors for several U.S. utilities, 1991 ..... 5-2
5.2 Quantile plot of the A factor data for several U.S. utilities, 1991 ..... 5-3
5.3 Theoretical quantile-quantile plot of the A factor data for several U.S. utilities, 1991 ..... 5-5
5.4 Theoretical quantile-quantile plot for the A factor data, 1991, omitting the extreme values ..... 5-6
5.5 Theoretical quantile-quantile plot for the log of the A factor data, 1991 ..... 5-7
6.1 Results of simulation analysis for 15-kVA pole-mount transformer ..... 6-9
6.2 Results of simulation analysis for $25-\mathrm{kVA}$ pole-mount transformer ..... 6-10
6.3 Results of simulation analysis for $25-\mathrm{kVA}$ pad-mount transformer ..... 6-11
6.4 Results of simulation analysis for $50-\mathrm{kVA}$ pad-mount transformer ..... 6-12
6.5 Results of simulation analysis for $300-\mathrm{kVA}$ pad-mount transformer ..... 6-13
6.6 Results of simulation analysis for $500-\mathrm{kVA}$ pad-mount transformer ..... 6-14
6.7 Results of simulation analysis for $1000-\mathrm{kVA}$ pad-mount transformer ..... 6-15

## TABLES

3.1 Sample calculation of total revenue requirements ..... 3-5
3.2 Changes in fixed charge rate by varying input parameters ..... 3-6
4.1 Distributions, defining parameters, means and variance ..... 4-5
4.2 Input variables used to calculate $A$ and $B$ and their distributions ..... 4-6
4.3 Designs and base TOC values when $\mathrm{A}=\$ 3.00$ and $\mathrm{B}=\$ 1.00$ ..... 4-7
4.4 Summary statistics for simulation analysis for each transformer design ..... 4-8
4.5 Quantiles of the input variables corresponding to various probabilities ..... 4-10
4.6 TOC values as a function of changing input values ..... 4-10
6.1 Residential transformer bids, $\mathrm{A}=\$ 2.00$ and $\mathrm{B}=\$ 0.25$ ..... 6-2
6.2 Commercial transformer bids, $\mathrm{A}=\$ 2.00$ and $\mathrm{B}=\$ 0.60$ ..... 6-2
6.3 Industrial transformer bids, $\mathrm{A}=\$ 2.00$ and $\mathrm{B}=\$ 1.00$ ..... 6-3
6.4 Distribution for economic variables used in computing A and B ..... 6-3
6.5 Comparison of results changing probability distributions ..... 6-8

## ACKNOWLEDGMENTS

We wish to thank Scott Thigpen and Peter South of the U.S. Environmental Protection Agency, Jim VanCoevering of Oak Ridge National Laboratory, and Donald Duckett of the Florida Power Corporation for their support and guidance.

We convey a special thanks to those utilities that supplied detailed system information for the study. These utilities have not been identified in the report because of the sensitivity of the information they provided. We also gratefully acknowledge the many comments and helpful suggestions provided by these utilities.

## ABBREVIATIONS AND ACRONYMS

A
B
BoE
EC
EIA
FCR
HPY
IEEE
ISO
kWh
LL
LM
$\mathrm{L}_{\mathrm{s}} \mathrm{F}$
NL
P.

PBR
PL
PU
RF
RMS
SC
T\&D
TOC
TVA
equivalent first cost of no-load losses
equivalent first cost of load losses
band of equivalence
energy cost
Energy Information Administration
fixed charge rate
hours per year transformer is energized
Institute of Electrical and Electronic Engineers
independent system operator
kilowatt-hour
full-load loss
loss multiplier
ratio of average load losses to peak load losses
no-load, or core, loss
bid price
performance-based rate making
levelized annual peak load
per unit
peak responsibility factor
root mean square
system cost
transmission and distribution
total owning cost
Tennessee Valley Authority

## 1. INTRODUCTION

### 1.1 BACKGROUND

Distribution transformers are used to transform voltage from electric utility power lines to a lower secondary voltage suitable for customer equipment. Utility distribution transformers account for an estimated 61 billion kilowatt-hours ( kWh ) of the energy lost annually in the generation and delivery of electricity (Barnes et al. 1995). Additional transformer losses in nonutility applications are estimated at 79 billion kWh . More than a million new distribution transformers are purchased and installed annually. Distribution transformers are very reliable, efficient devices with no moving parts and average life spans of over 30 years; but because of the large number of units and the long periods of operation, even small changes in efficiency can add up to large quantities of energy saved.

Most electric utilities purchase transformers by selecting the bid that provides the lowest total owning cost (TOC) of the transformer. TOC includes both the initial capital price and the capitalized cost of the transformer losses during its period of operation. The TOC methodology is a deterministic approach that was developed for power transformers during the early 1980s by the Edison Electric Institute in conjunction with the transformer committee of the Institute of Electrical and Electronic Engineers (IEEE). Most utilities now use the TOC methodology for purchasing distribution transformers. IEEE is developing a guide for evaluating distribution transformer loss which applies the TOC methodology.

When the TOC method was developed, careful analyses were undertaken to identify the parameters affecting lifetime loss performance; and mathematically consistent approaches were developed to quantify the impacts of variation in these parameters on transformer purchase decisions. The selection of the transformer design that provides the lowest TOC is called a "hard-evaluation" approach. Recently, many utilities have begun to use a modification of the hard-evaluation approach in selecting and procuring distribution transformers. This modification, called the band of equivalence (BOE) or "soft-evaluation" approach, is used to account for the variability in the TOC input parameters. It treats transformer designs that are within a fixed percentage of the lowest TOC as equivalent. Normally, the lowest-price transformer within the BoE is selected; this approach often results in selection of a less efficient design than would have been chosen using the hard evaluation approach.

This study investigates uncertainty distributions for the TOC parameters and develops a TOC methodology that accounts for uncertainties. It also examines the BoE approach and the percentage of equivalence to determine if this approach can appropriately account for uncertainties. Improper use of the BoE approach or of other methods for incorporating uncertainty could yield transformer designs that result in greater losses than can be justified on a strict economic basis. The greater losses not only increase the basic operating costs and subsequent consumer electricity costs, but also harm the environment through unnecessary energy generation and the associated emissions. To avoid these consequences, a minimum TOC methodology is needed that provides the lowest losses that are economically feasible while accounting for uncertainties. To account for uncertainty properly, the TOC method should be designed to include statistical variations in the various parameters.

### 1.2 OBJECTIVE AND APPROACH

The objective of this project is to develop a way of accurately incorporating uncertainty into the TOC methodology for purchasing distribution transformers. This method will permit utilities and
other purchasers to select the most cost-efficient transformer with the minimum TOC while incorporating uncertainty in variables that are not under the purchaser's control. In addition, the study examines the effects of performance-based rate making, tax issues, and deregulation of electric utilities on purchase decisions using the TOC approach to determine if modifications are necessary. The TOC methodology and other methods utilities and industries use to make purchasing decisions were thoroughly examined. This examination delineated all of the evaluation parameters used in the purchasing decision analysis and characterized them for variation due to future uncertainty. Changes in TOC were reviewed relative to changes in the input variables and relative to the uncertainty in the variables. Those variables with the largest influence and greatest uncertainty were evaluated more closely. This report will be useful to utilities, utility regulators, and other interested parties discussing alternative methods of incorporating uncertainties into the analysis of transformer purchasing decisions.

### 1.3 UTILITY RESTRUCTURING

The electric utility industry is presently undergoing tremendous change. Performance-based rate making (where earnings are based on business efficiency) is being exploited in a number of jurisdictions. Deregulation of generation and deintegration of the utility between generation, transmission, and distribution will also affect the rationale behind purchase decisions. The implications of uncertainty may become more important in this new business environment. Because of the potential financial impact on restructured utilities and the energy efficiency implications of even small changes in the efficiencies of the transformers, statistically valid and mathematically consistent approaches to quantifying the impact of uncertainties on the TOC assumptions and parameters may be very important.

### 1.4 SCOPE AND CONTENT

This report outlines an approach that will account for uncertainty in the development of evaluation factors used to identify transformer designs with the lowest TOC. In Chapter 2, the TOC methodology is described and the most highly variable parameters are discussed. Chapters 3 and 4 develop the model to account for uncertainties as well as statistical distributions for the important parameters. Sample calculations are presented in Chapter 5. Chapter 6 applies the TOC methodology to data provided by two utilities in order to test its validity.

## 2. TRANSFORMER PURCHASING TRENDS

### 2.1 THE TOTAL OWNING COST METHOD

Many evaluation methods have been used to determine the trade-off between costs of energy losses and initial capital costs in transformers and other electrical equipment. These methods include lowest price, simple or discounted payback methods over a specified time period, and total life cycle cost evaluations. The method used will depend upon the purchasing organization's financial sophistication and structure. For cases other than lowest price and simple payback, the need to establish the present value of energy losses and capital requirements over a relatively long time period carries with it a degree of uncertainty. This degree of uncertainty is the central theme of this report. When properly used, the TOC method of analyzing transformer purchasing decisions is generally regarded as the most cost- and resource-efficient method available today. In general, purchasing that is based on the TOC rather than on a specified minimum efficiency at a nominal load enables transformer manufacturers to offer a variety of products, each designed to minimize the TOC using the input loss values provided by the purchasing utility or customer. Using Monte Carlo simulation, this study establishes that selection methods based upon techniques that "soften" the TOC purchasing decision may increase, not decrease, the uncertainty in TOC.

According to the Working Group on the Guide for Distribution Loss Evaluation (IEEE 1997), the equation for the total owning cost of a new transformer is

$$
\mathrm{TOC}=\text { price }+ \text { cost of core loss }+ \text { cost of load loss }
$$

where

```
cost of core loss \(=\mathrm{A}(\$ /\) watt \() \times\) core loss watts \(\times\) T\&D loss multiplier ,
```

cost of load loss $=B(\$ /$ watt $) \times$ load loss watts $\times T \& D$ loss multiplier ,
and

$$
\begin{gathered}
A=\frac{(S C+E C \times H P Y)}{(F C R \times 1000)}, \\
B=\frac{\left[(S C \times R F)+\left(E C \times L_{s} F \times H P Y\right)\right] \times(P L)^{2}}{(F C R \times 1000)}
\end{gathered}
$$

The A and B factors represent the equivalent first cost of the core (no load) losses and the load losses, respectively. These A and B factors include some simplifying variables that translate the cost of the energy lost into the initial year present values. The Working Group's equation also simplifies the analysis by ignoring the long-term asset value of the transformer and the consequences to revenues and expenses over time. $\mathrm{L}_{5} \mathrm{~F}$ is the ratio of average load losses to peak load losses, and PL is the levelized annual peak load. HPY represents the hours per year that the transformer is energized, typically 8760 hours (see the Appendix).

The specific values of the various parameters are dependent upon the application, financial model, and operating model. For example, earlier versions of the TOC methodology used different values for system cost (SC) and energy cost (EC) for the A (base load) and B (peak load) evaluations. Present thinking has settled upon a single value for SC , the levelized avoided (incremental) cost of the generation, transmission, and distribution capacity required to furnish the next kilowatt of load to the transformer coincident with peak demand. EC is now considered to be the levelized avoided (incremental) cost of the next kilowatt-hour purchased or produced by a utility's generating units. The reader is referred both to the latest literature, specifically the proposed IEEE standard on transformer evaluation methodology (IEEE 1997), and to the earlier very detailed work of Nickel and Braunstein (1981). A complete description of the TOC approach to purchasing distribution transformers is presented in the Appendix.

### 2.2 UNCERTAINTY IN THE DETERMINISTIC TOC

Many anecdotes suggest relatively large variation in the input parameters. However, these supposed variations may be more a misunderstanding of the parameter definition than actual variation. For example, SC can be determined from historic trends in generation, transmission, and distribution costs expressed in current dollars and system capacities. Coupling these data with planning models for system expansion should give a reasonable, relatively stable value for SC that can be modeled as a narrow normal distribution. Uncertainty, such as the impact of capacity excess, would be implied by the distribution's spread, or standard deviation. On the other hand, EC may be quite variable over time, and simple growth models and normal distributions may not be adequate. Both SC and EC are levelized, reducing uncertainty by discounting unknown future values; but note that nonuniform, time-dependent changes result in different discounted values for both SC and EC. Modeling such nonuniform expansion problems is very difficult and probably introduces more uncertainty than warranted because additional parameters are introduced.*

The potential uncertainty in the fixed charge rate (FCR) is strongly dependent upon the financial situation of the utility or purchaser, but the historic trends and the interactive nature of the components of FCR should result in a rather stable value modeled as either a constant or narrow normal distribution. The peak responsibility factor (RF) and loss multiplier (LM) are also rather stable and essentially constant; historic projections should provide very accurate estimates. In addition, variation in RF and LM does not impact the TOC as greatly as variations in the other input parameters.

Peak load (PL) is another story and is perhaps the most uncertain of the input parameters. SC and PL are connected indirectly, since system peak enters into system capacity planning. Uncertainty in PL also produces significant variation in TOC. What is really needed is the RMS averaged load, which is approximated by the product $\mathrm{L}_{\mathrm{s}} \mathrm{F} \times \mathrm{PL}^{2}$. Fortunately, variation in $\mathrm{L}_{\mathrm{s}} \mathrm{F}$ does not seem to introduce strong variation in TOC, but the methods used to estimate $\mathrm{L}_{s} \mathrm{~F}$ are not very accurate. For example, the common $85 / 15$ rule for expressing $L_{s} F$ in terms of system load factor ( $\mathrm{LF}=$ average load/ peak load, $\mathrm{L}_{\mathrm{s}} \mathrm{F}=0.15 \times \mathrm{LF}+0.85 \times \mathrm{LF}^{2}$ ) is based on data from one study and is a very insecure foundation because there is no physical reason why $L_{s} F$ should be related to LF in this manner." If a utility wishes to use a relationship of this type, extensive data gathering will be necessary ( $\mathrm{kWh}, \mathrm{kW}$

[^0]and peak kW as functions of time, size, and use are needed to establish the proper fit of $\mathrm{L}_{\mathrm{s}} \mathrm{F}$ and LF ). Such data supply the needed RMS load directly. However, utilities generally gather $\mathrm{kWh}, \mathrm{kW}$, and power factor data for billing purposes, and a strong reason exists for using some expression relating $L F$ and $L_{s}$.

A declining number of utilities do maintain large databases of transformer loads. Several have supplied condensed versions of the data to ORNL. These data suggest that transformers are lightly loaded and that the values used for equivalent peak loads in evaluating B may be overstated. In particular, the assumption that larger transformers have higher average loads appears to be untrue. While the load variation on smaller units is greater, the average peak load is still less than expected and is generally less than that specified on the name plate. An example of relative load distribution is shown in Fig. 2.1.

It is instructive to examine the relative variation of the TOC with respect to variation of the input parameters of $A$ and $B$ under the assumption that the no-load, or core, loss (NL), the load loss at full load (LL), and the bid price ( P ) are constant during the evaluation. This is the approach used in the balance of this report, and it is the correct approach to use when evaluating the effects of uncertainty on the quality of the purchasing decision. An alternative approach and viewpoint are presented in the next paragraph. As shown in Fig. 2.2, TOC is most sensitive to FCR and PL, followed by SC and EC. The effect of varying RF and $L_{s} \mathrm{~F}$ is essentially insignificant. Clearly, the relative variation is dependent upon the magnitude on the initial values chosen, but the relative importance is not significantly affected.

It must also be noted that NL, LL, and $P$ are implicit functions of $A$ and $B$, since suppliers design the transformer to minimize TOC relative to specified $A$ and $B$ inputs. The specific form of the function linking $P$, NL and LL is proprietary; but by limiting variation in the input variable to $A$ and $B$ to small increments ( $<5 \%$ ), it is possible to estimate the effects on TOC of including this implicit variation. In general, including implicit effects increases the impact on TOC by about a factor of 2 for a small variation. Without explicit data from manufacturers, it is not possible to do a detailed study of the effects of these implicit variations. Such data are needed for a study addressing the total


Fig. 2.1. Relative number of units with per unit load greater than or equal to $x$.


Fig. 2.2. Variation in total owning cost of a transformer according to variations in input parameters.
impact of uncertainty on TOC, since $P$ accounts for about $50 \%$ of the TOC. Simply put, since $P$ is functionally dependent upon NL and LL, $P$ is strongly dependent on the manner in which A and B factors input to the design process affect NL and LL.

## 3. UTILITY RESTRUCTURING

### 3.1 THE ELECTRICITY MARKET

The electric utility industry as it has operated over the past 70 years is changing. Historically, most of the industry has been vertically integrated, meaning that one company provided electric services from generation to transmission to distribution to customer service. It has been widely argued that the generation portion is not a natural monopoly and should be separated from the other functions of electric service. Generation would then become a competitive market in which distribution companies, or even retail customers, would purchase their requirements. Transmission would be controlled by a separately regulated independent system operator (ISO). This arrangement would help to maintain the reliability of the system and avoid the problems of market power in which a company could use its transmission lines to limit competition for generation and increase its prices.

Although generation will become deregulated, distribution of electricity will continue to be a regulated business. It is a natural monopoly in that it is most cost-effective to have a single set of distribution lines for a given region, rather than multiple companies each having electric wires to ultimate consumers. Because of the deintegration of the industry, the distribution business will look more like current distribution-only utilities. Currently, roughly $14 \%$ of the country is served by distribution utilities that have no generation of their own (EIA 1994). These companies sign longterm supply contracts with either nearby investor-owned generating utilities or public power suppliers such as the Tennessee Valley Authority (TVA). Their costs are based on the terms of these contracts rather than the costs of specific generating plants. Most of these distribution companies are either publicly owned cooperatives or municipal utilities.

In the future, retail customers may have contracts with different generation providers at different prices. Distribution companies will bill consumers for the use of their wires; these bills will include the costs of losses. Customers could pay for the losses either by having the generation supplier provide extra power, or by simply paying the distribution company to procure the power.

In this environment, utilities as well as commercial and industrial customers will continue to need to purchase transformers. The purpose of this section is to examine the calculations used to evaluate transformer costs and how they may be affected by the restructuring of the industry.

### 3.2 COST OF LOST ENERGY

The appropriate cost of the power lost is the cost of the power that the company would not have bought had it not had the losses. This cost depends on the generation used to provide the power and the contracts the utility has in place for purchasing power. Currently, distribution-only utilities have contracts that include both a demand payment (based on peak demand for a given period) and an energy payment (based on the total energy used over the period.) Rates may vary based on the time the energy is used and the overall quantity. Other charges also enter into the total bill to reflect ancillary services provided by the generation and transmission provider. Utilities can also buy blocks of power on the wholesale market under various terms and conditions.

Under restructuring, a distribution company will continue to need a contract for provision of power, both for itself and for those customers who choose to continue to buy from the company. And unless the utility can physically disconnect the customers, it will also have to provide power to those customers with supply contracts from other companies if their suppliers are not producing at the time of demand. The power the distribution company needs may be purchased through contracts with
generators, power brokers, a spot market, or a combination of these. The utility may use financial mechanisms such as futures contracts or options to hedge on the risks of price changes. These varied markets greatly complicate determining the cost of lost power because it must be determined which supply was used to provide the lost energy. As a simplification, the utility may designate one of its sources, such as the spot market, as the marginal supply.

### 3.3 COST RECOVERY AND INCENTIVES FOR EFFICIENCY

For billing purposes, transformer load losses are combined with other line losses, the energy used by the utility, and losses due to theft. Losses specific to transformers are not measured. The public utility commission determines a percentage markup on bills based on historical values of the total loss and internal use. These factors then increase the rates for customers. Load and no-load transformer losses (explained in this section) are not priced separately, and not at the marginal cost of the power lost.

Why will distribution utilities want to buy more efficient transformers? The utility is allowed to pass on the cost of new capital equipment through its rates and even to earn a reasonable profit on its investment. While this arrangement theoretically makes utilities neutral to investment (and, some would argue, encourages investment), at the actual decision-making level there is a tendency to purchase the item with the lowest first cost. Capital budgets are often set early, and managers attempt to maximize the equipment purchased. The idea behind the TOC equation is to factor in the longterm costs (such as losses) not included in the capital budget but real to the utility nevertheless. The costs must be put on the same basis as the up-front purchase cost, taking into account other costs, such as taxes and return on investment, that the utility actually pays for an asset.

The long-term costs of losses are paid by the utility through the difference in measured power into the system and out through the individual meters. Because transformer losses are not specifically measured, the utility marks up power rates to account for this loss plus other unmeasured uses within the utility and theft. With regulation, if the losses are reduced, then regulators will lower this percentage mark-up and pass the savings on to consumers. There is consequently little incentive for the utility to reduce losses, except that in between rate hearings they are able to keep the savings as extra profits (regulatory lag).

While the companies may continue to be regulated, they might not use the traditional cost-plus form of rate making. Instead, many may use performance-based rate making (PBR). Under PBR, prices are set based on both the utility's costs and certain industry standards for cost or quality. If the utility is able to improve its performance, either by lowering costs or improving services, it is allowed to keep all or a part of the savings. If the utility can lower its power losses below the percentage that is included in rates, it can keep the savings as profits. For example, if the accepted loss rate is $5 \%$ and the utility can lower the losses from transformers (or line losses, internal use, or theft) to $4 \%$, then it can retain the extra $1 \%$ of revenue. Even after rate hearings, if overall prices remain at or below their agreed-upon cap, the utility may continue to receive the extra profits. The philosophy behind this is that letting the utility profit from the cost savings will give it incentives to lower costs more dramatically than under standard rate making. Additional details can be found in A Primer on Incentive Regulation for Electric Utilities (Hill 1995).

### 3.4 RESTRUCTURING EFFECTS ON TOC

As discussed earlier, the TOC equation is used to factor in the long-term costs of a transformer with the up-front cost. It adds to the initial price the cost of the energy lost in the process of energizing the transformer and passing power through, and it factors in the long-term asset costs of taxes and return on investment.

In the following sections, we discuss the other factors in the equation that could be affected by restructuring.

### 3.4.1 Price

The first factor in the TOC equation that could be affected by utility restructuring is the price of the transformer. In reality, the utility does not pay just the initial price of a transformer, because the transformer is a long-term asset on which the utility must pay taxes and from which the utility will earn a return. The long-term cost of the transformer will be higher because of these other factors. Rather than multiply the price by the fixed charge rate (FCR) to put all costs on an equal annualized basis, we divide the other two parts of the TOC equation by the FCR, simplifying the equation. The FCR is described later.

### 3.4.2 Core Loss

Core (or no-load) losses represent the energy lost from the transformer even when no power is transferring through it. This amount is constant for all hours that the transformer is energized.

### 3.4.2.1 System Cost

The second factor in the TOC equation that restructuring would affect is the system cost (SC) the cost to provide the generation and transmission capacity needed to make up the lost power. Historically, this value has represented the annual capacity charge for base load power (because the core loss represents a constant, or base, load). Large coal plants or nuclear plants could have a high levelized capital cost, from $\$ 100$ to over $\$ 400 / \mathrm{kW}$ per year. However, more recent applications of the TOC equation recommend using the average for all generation, because the power lost is on the margin and so comes from all plants on a time-averaged basis. The amount should roughly equal the demand charge that is charged to large customers by the distributor. Alternatively, the marginal cost of capacity over the life of the transformer should be close to the long-run marginal cost of new capacity. Most projections show that future capacity additions will mainly be gas-fired combustion turbines or combined-cycle units. These have capital costs of around $\$ 400 / \mathrm{kW}$, according to the Energy Information Administration (EIA 1996b). Using an FCR of $15 \%$ gives an annualized charge of $\$ 60 / \mathrm{kW}$.

Under restructuring, it is not clear whether there will be any capacity charge. Instead, utilities or customers would purchase power based solely on the cost of energy used (cents $/ \mathrm{kWh}$ ). More recent deliberations, including the restructuring legislation passed in California, recognize that some type of capacity charge may be needed to ensure that sufficient capacity exists to meet peak demands. Several options have been explored to determine the capacity charge. One method is for the ISO to establish a secondary market for capacity. The ISO would determine the amount of additional capacity it needs to have available for system support (e.g., spinning reserve, voltage support) and accept bids from enough plants to meet its capacity needs. Details must be worked out on a number of issues: whether this payment goes to all suppliers or just those that make capacity available but are not called upon to provide energy, how to incorporate other ancillary services, how to ensure the
economically efficient plants are used, determining the time period for each block of the market, and other technical details of the market.

In England, prices are based on an energy pool that plants bid into as well as long-term fixedprice contracts between suppliers and the distribution companies (Thomas 1996). A power pool in the Pennsylvania, New Jersey, and Maryland area - the PJM Interconnection - includes a capacity reservation system in its restructuring proposal through which it calculates at the start of the year a necessary reserve margin, and consequently, a capacity payment to all generators to ensure sufficient capacity is available (Jaffe and Felder 1996). In general, it can be expected that the SC for generation will decline greatly from current values, if not totally disappear.

The transmission component of the SC under restructuring may undergo a similar, although smaller, change. Transmission costs may be bundled into an energy charge, or the distribution company may be assessed a charge based on demand on the transmission system. Since transmission will continue to be regulated, transmission prices will not change as radically as generation prices.

### 3.4.2.2 Energy Cost

The energy cost (EC) is the third factor that could be affected by restructuring. The marginal cost (or price in a deregulated market) of the energy lost will vary over time as demand and supply change. Since the amount lost is constant over time, the cost of the energy lost would be equal to the time-averaged marginal cost of energy, including transmission losses.

### 3.4.2.3 Fixed Charge Rate

Another factor that restructuring could affect is the FCR, which converts annual costs into a present value based on the cost of capital to the firm, length of time of the investment, and tax regulations on the asset. Other assumptions include the tax rates, percentage debt and equity, rates of return, and both book and tax life of the transformer. As an example, assume a utility spends $\$ 1000$ on a new transformer (Table 3.1). In the first year, the utility has expenses of more than $\$ 187$ for the transformer. This includes $\$ 39$ for interest, $\$ 53$ profit to shareholders, $\$ 32$ for income taxes, $\$ 33$ for depreciation, and $\$ 30$ for property taxes. The amount declines over time, reaching zero after year 30 . Using the average cost of capital for the utility, the total cost of the transformer is not $\$ 1000$ but $\$ 3099$. Discounting this using the average cost of capital for the utility gives a net present value of the transformer of $\$ 1351$.

The constant payment that would give the same present value as the actual stream, divided by the original cost, gives the FCR. In our example, an annual payment of $\$ 136.10$ when discounted equals $\$ 1351$. Dividing by $\$ 1000$ gives an FCR of $13.61 \%$. Table 3.2 shows what happens to the FCR if some of the input values are changed. The class life and tax life for distribution equipment is specified in Internal Revenue Service regulations as 30 years and 20 years respectively (Commerce Clearing House 1993). Property tax rates can vary across the country from less than $1 \%$ to over $10 \%$ (U.S. Advisory Commission on Intergovernmental Relations 1992).

Even with industry restructuring, it is expected that distribution companies will continue to be regulated. Consequently, the parameters involved in the FCR will change little because of restructuring. It can be argued that the distribution company could become a less risky investment because it would no longer have power plants in its asset base, which are more risky than the distribution business. However, since many utilities are already distribution-only, there should not be a great change in their financial status.

| le 3.1. Sample calculation of total revenue requirements |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annualized payment of NPV Levelized fixed charge rate |  |  | $\begin{aligned} & \$ 136.1 \\ & 13.61 \% \end{aligned}$ | Source | Capitalization | Component cost | Weighted cost |
|  |  |  |  |  |  |  |  |
| Price |  |  |  |  |  |  |  |
| Life (years) |  | 30 |  | Debt | 50\% | 8.0\% | 4.00\% |
| Tax life |  | 20 |  | Preferred | 10\% | 10.0\% | 1.00\% |
| Income tax rate |  | 38.0\% |  | Common | 40\% | 11.0\% | 4.40\% |
| Property tax rate |  | 3\% |  | Total capital | 100\% |  | 9.40\% |
| Revenue requirement |  |  |  |  |  |  |  |
| Average rate base |  | Interest payment | Return on equity | Income tax | Book depreciation | Property tax | Total |
| 1 | \$981.75 | \$39.27 | \$53.01 | \$32.49 | \$33.33 | \$29.50 | \$187.61 |
| 2 | 933.65 | 37.35 | 50.42 | 30.90 | 33.33 | 28.50 | 180.50 |
| 3 | 887.61 | 35.50 | 47.93 | 29.38 | 33.33 | 27.50 | 173.65 |
| 4 | 843.47 | 33.74 | 45.55 | 27.92 | 33.33 | 26.50 | 167.04 |
| 5 | 801.10 | 32.04 | 43.26 | 26.51 | 33.33 | 25.50 | 160.65 |
| 6 | 760.35 | 30.41 | 41.06 | 25.17 | 33.33 | 24.50 | 154.47 |
| 7 | 721.11 | 28.84 | 38.94 | 23.87 | 33.33 | 23.50 | 148.48 |
| 8 | 683.26 | 27.33 | 36.90 | 22.61 | 33.33 | 22.50 | 142.67 |
| 9 | 645.64 | 25.83 | 34.86 | 21.37 | 33.33 | 21.50 | 136.89 |
| 10 | 608.01 | 24.32 | 32.83 | 20.12 | 33.33 | 20.50 | 131.11 |
| 11 | 570.39 | 22.82 | 30.80 | 18.88 | 33.33 | 19.50 | 125.33 |
| 12 | 532.77 | 21.31 | 28.77 | 17.63 | 33.33 | 18.50 | 119.55 |
| 13 | 495.15 | 19.81 | 26.74 | 16.39 | 33.33 | 17.50 | 113.77 |
| 14 | 457.53 | 18.30 | 24.71 | 15.14 | 33.33 | 16.50 | 107.98 |
| 15 | 419.91 | 16.80 | 22.68 | 13.90 | 33.33 | 15.50 | 102.20 |
| 16 | 382.29 | 15.29 | 20.64 | 12.65 | 33.33 | 14.50 | 96.42 |
| 17 | 344.67 | 13.79 | 18.61 | 11.41 | 33.33 | 13.50 | 90.64 |
| 18 | 307.05 | 12.28 | 16.58 | 10.16 | 33.33 | 12.50 | 84.86 |
| 19 | 269.43 | 10.78 | 14.55 | 8.92 | 33.33 | 11.50 | 79.08 |
| 20 | 231.81 | 9.27 | 12.52 | 7.67 | 33.33 | 10.50 | 73.30 |
| 21 | 202.67 | 8.11 | 10.94 | 6.71 | 33.33 | 9.50 | 68.59 |
| 22 | 182.00 | 7.28 | 9.83 | 6.02 | 33.33 | 8.50 | 64.96 |
| 23 | 161.33 | 6.45 | 8.71 | 5.34 | 33.33 | 7.50 | 61.34 |
| 24 | 140.67 | 5.63 | 7.60 | 4.66 | 33.33 | 6.50 | 57.71 |
| 25 | 120.00 | 4.80 | 6.48 | 3.97 | 33.33 | 5.50 | 54.08 |
| 26 | 99.33 | 3.97 | 5.36 | 3.29 | 33.33 | 4.50 | 50.46 |
| 27 | 78.67 | 3.15 | 4.25 | 2.60 | 33.33 | 3.50 | 46.83 |
| 28 | 58.00 | 2.32 | 3.13 | 1.92 | 33.33 | 2.50 | 43.20 |
| 29 | 37.33 | 1.49 | 2.02 | 1.24 | 33.33 | 1.50 | 39.58 |
| 30 | 16.67 | 0.67 | 0.90 | 0.55 | 33.33 | 0.50 | 35.95 |
|  | Total | \$519 | \$701 | \$429 | \$1,000 | \$450 | \$3,099 |
|  | NPV at 9.4\% | \$255 | \$345 | \$211 | \$331 | \$209 | \$1,351 |

Table 3.2. Changes in fixed charge rate by varying input parameters

| Case | FCR |
| :--- | :---: |
| Base example | $13.6 \%$ |
| Raise property tax rate to $8 \%$ | $17.1 \%$ |
| Raise common equity rate to $15 \%$ | $15.8 \%$ |
| Lower debt proportion to $25 \%$ | $15.4 \%$ |
| Change all three above | $22.8 \%$ |

### 3.4.3 Load Loss

Load loss represents the energy loss that is dependent on the power actually flowing through the transformer. The amount is not just a linear function of the power flow, but varies as the square of the power.

### 3.4.3.1 System Cost

The system cost under load loss should be the same as that used in the core loss calculation. Although some have used the lower capital cost for peaking plants (e.g., gas turbines) as a measure, future distribution-only utilities should use a lower factor only if their supply contracts provide a differential based on time, which is unlikely. They will be paying the same cost per kilowatt regardless of when that demand occurs.

### 3.4.3.2 Responsibility Factor

The peak responsibility factor ( RF ) adjusts the system cost to reflect the proportion of the transformer load that actually contributes to the peak load of the utility as a whole. This factor remains the same with restructuring, although it is less important with the decrease in relative importance of the SC. The importance of the timing of demand on the transformer, versus demand on the system, will be captured, instead, in the energy cost as it varies over time and demand.

### 3.4.3.3 Energy Cost

The TOC equation as currently defined calculates the impact of this disproportionate loss compared with load, but it assumes a constant EC for the loss. Under restructuring, more utilities will go to real-time pricing, under which the price of energy is a function of the overall supply and demand. Market forces will enter into the equation, and prices will be much higher in times of scarcity during peak loads. Since this is also when transformer losses are highest, the equation must more heavily weight the price of those lost kilowatt-hours, not just the quantity lost, through the loss factor ( $\mathrm{L}_{s} \mathrm{~F}$ ). (The pricing of lost energy at the peak is somewhat blurred for two reasons: the peak load on the transformer does not necessarily match the peak for the system, as explained by the RF factor; and prices do not necessarily match peak demands because plant outages raise prices at nonpeak times.)

While discussions of the TOC recognize that the avoided cost of power should be used, they fail to recognize this compounding of losses at the higher loads and prices. Consequently, the weighted
average price should be weighted, not by the energy used, but by the square of the energy used. For example, Fig. 3.1 shows a typical load function on a transformer, where the average load is only $37 \%$ of the peak, and a representative price curve for the same time. The time-weighted average price of power is only $2.26 \phi / \mathrm{kWh}$ and is the value for EC used in the core loss equation. However, in weighting the power purchased by the square of the power lost (hereafter referred to as the "energy ${ }^{2}$ weighted price"), the price paid (EC) becomes $3.29 \phi / \mathrm{kWh}$, or $45 \%$ higher.

This value is very sensitive to the time of the peak load. If we shift the peak prices to when the load is lower (such as between hours 5 and 21 ), the energy ${ }^{2}$ weighted price drops to $2.39 ~ ¢ / \mathrm{kWh}$. This points out the significance of the timing of the load on the transformer in relation to the price of energy in determining its TOC. High-efficiency transformers will be most cost-effective if the peak on the transformer (and consequent energy loss) occurs when prices are high. If the load is more constant for the transformer (i.e., has a higher load factor), the TOC will be less affected by peak prices. If we raise the load factor to $70 \%$ for the example, the energy ${ }^{2}$ weighted price drops to $2.44 \% / \mathrm{kWh}$.

Since the EC represents the levelized cost of the power lost over the life of the transformer, it becomes necessary to project the price over the next 20 to 30 years. The Energy Information Administration's (EIA's) Annual Energy Outlook for 1997 projects a -0.6\% growth in prices (in constant dollars) between 1995 and 2015 (EIA 1996b) even without full restructuring. Under restructuring, prices could fall even more.

### 3.4.3.4 Loss Factor

The loss factor $\left(\mathrm{L}_{\mathrm{s}} \mathrm{F}\right)$ converts the peak load to the actual energy lost based on the square of the actual load profile. As mentioned earlier, it does not factor in the price differential for the energy at different load levels.

### 3.4.3.5 Equivalent Annual Peak Load

The equivalent annual peak load (PL) levelizes the expected growth in peak demand over the life of the transformer. Under restructuring, peak demands may grow less quickly than overall growth in energy sales. If the market changes to real-time pricing, it can be expected that price-sensitive customers will lower their demands during periods of high prices, either by shifting their demand to a lower cost time or by forgoing the use of the power altogether. Either way, the change will lower the growth of the peak and increase the load factor. This factor will also influence the other factors: RF, $\mathrm{SC}, \mathrm{EC}$, and $\mathrm{L}_{\mathrm{s}} \mathrm{F}$.

### 3.5 SUMMARY OF RESTRUCTURING IMPACTS

Restructuring will bring great change to the electric industry as a whole. The major effect on distribution will be the unbundling of the distribution function from the other components, generation and transmission. The costs of power lost through transformers will be based on market prices rather than on the avoided generation from a utility's own plants. Overall costs will go down, and there will be a shift to relying more on the price of energy than on demand. Prices will be more volatile as real-time pricing becomes more prevalent.

The TOC equation will remain the same, but some of the factors will need to be reconsidered. The SC will decline, as it will reflect the cost of capacity needed to maintain reliability, instead of all capacity. The EC for the core losses will be the time-averaged price of energy, while the EC for load losses will need to factor in the different prices when the transformer losses actually occur. The FCR
will remain largely the same, although distribution companies may have different capital costs from the current integrated utilities. Peak growth may slow, but overall demand may increase with lower prices. Such a change would increase the load factor over time and change the values for RF and $\mathrm{L}_{5} \mathrm{~F}$.

In a recent article in Public Utilities Fortnightly, George Pleat (1996) describes the benefits of buying energy-efficient transformers to a distribution-only utility of the future:

Under the assumption that the state utility commission will regulate the stand-alone distribution company through some kind of performance-based or price-capping mechanism, disco [distribution company] management should acquire an incentive to purchase the transformer with the lowest life-cycle cost. Purchasing higher-cost but more-efficient transformers with reasonable payback periods (maybe 10-15 years) should help reduce longrun operating costs, boosting profits for stockholders.

More specifically the incremental cost associated with a more expensive but more efficient transformer is recovered through savings associated with reduced losses (calculated on marginal costs). It would behoove the disco to pursue these types of decisions.

Despite restructuring, distribution companies will continue to be regulated. Where current rate making tends to make a utility neutral to purchasing high-efficiency transformers, performance-based rate making will build in incentives for them to purchase more efficient transformers if they are costeffective. The TOC calculation, when used with the proper values for parameters, should show the lowest-cost, and highest-profit, transformers for the utility to buy.


Fig. 3.1. Load duration curve and real-time price of power for sample transformer.

## 4. UNCERTAINTY ANALYSIS MODEL

### 4.1 INTRODUCTION

In Chapter 2, the equations for determining the equivalent first cost of no-load losses (A) and load losses (B) were given. The parameters that go into these equations were discussed, and the lack of certainty in their exact values was demonstrated. The effect of their uncertainty translates into uncertainty in the values of A and B, which in turn translates into uncertainty with regard to TOC. This uncertainty should be dealt with in some manner in making decisions about which transformer to purchase. Under some conditions (which affect the values of A and B), one transformer will be a better choice, since its TOC (under these conditions) will be less than that of any of the other transformers. A smaller TOC reduces cost and increases profits. The question is, how can a decision maker know the likely values of $A$ and $B$ in order to make the best choice? One answer to that question is use of a technique called Monte Carlo simulation. This technique selects the values of the inputs (that go into determining A and B) from some given probability distribution. It is assumed that the inputs are familiar to the analyst and that he or she has some idea of the possible values they may take on. This is the Monte Carlo part of the technique. Next, A and B are calculated or their values simulated, given the inputs randomly selected from their distributions. This process can be repeated several times to yield a set of values for A and B which can then be used to calculate a set of TOC values for a given transformer with a stated price and no-load (NL) and load loss (LL) values. This set of TOC outcomes are those that one might expect to observe, given the distributions of the input. The power and speed of present-day computers make Monte Carlo simulation a useful tool for the decision maker. The set of TOC values for each transformer generated via the simulation can be compared, and a choice can be made on the basis of some characteristic or decision criterion. For example, one might average the TOC values for each transformer and select the transformer that has the lowest average TOC value. Other statistics that can be used will be discussed later.

### 4.2 PROBABILITY DISTRIBUTIONS

In Monte Carlo simulation, the value of an input, say SC (avoided cost of system capacity), is selected from a given probability distribution. A probability distribution can be thought of as a model that indicates how likely it is that a given value will occur. (For a continuous variable, the probability of taking on any specific value is zero, but we use this loose description for pedagogical reasons.) Since the value of SC is selected from this hypothetical distribution, some care must be used in its selection for the results to be meaningful. The effect of distribution choice will be examined later, but for now we discuss some common distributions that are often used in Monte Carlo simulation analysis.

Probably the most common distribution is the so called normal distribution (or Gaussian distribution, named after its founder Gauss). This is the bell-shaped distribution that may be described by its mean and variance or standard deviation (which is the positive square root of the variance). The mean of the normal distribution indicates the center of the distribution, while the standard deviation describes the spread of the distribution. The smaller the standard deviation, the less spread and more peaked the distribution becomes. The standard deviation indicates how well we know the possible values the input can take on. For the normal distribution, roughly $68 \%$ of the values will be within one standard deviation of the mean and $95 \%$ of the values will lie within two standard deviations of the mean. The impact of this statement can be seen as follows: Suppose that

SC is normally distributed with a mean of $\$ 240 / \mathrm{kW}$. Consider two possible values for the standard deviation, $\$ 10 / \mathrm{kW}$ and $\$ 20 / \mathrm{kW}$. One would expect about $95 \%$ of the values for SC to lie in the interval $[\$ 220 / \mathrm{kW}, \$ 260 / \mathrm{kW}]$ if the standard deviation is $\$ 10 / \mathrm{kW}$, while the $95 \%$ probability interval corresponding to a standard deviation of $\$ 20 / \mathrm{kW}$ is $[\$ 200 / \mathrm{kW}, \$ 280 / \mathrm{kW}]$. The uncertainty in the choice of the standard deviation is multiplied to yield the uncertainty in the values of SC. The results can be seen in Fig. 4.1.

The uniform distribution is a common choice for an input distribution since it can be developed by a knowledge of the range of values that the input parameter can take on. For instance, one might believe that SC will lie in the range from $\$ 210 / \mathrm{kW}$ to $\$ 270 / \mathrm{kW}$. This is all that is needed to form the uniform distribution. The uniform distribution is flat (not bell-shaped like the normal distribution) on this range. Therefore, the probability that SC will be in some interval with a width of, for example, $\$ 10 / \mathrm{kW}$ does not depend on where in the interval those values lie. So the probability that SC will take on values in the interval [ $\$ 210, \$ 220$ ] is the same as that for the interval [ $\$ 235, \$ 245]$. The width in both cases is $\$ 10$, and that is the important characteristic. Thus, location is not important in the uniform distribution. All intervals of equal width are equally likely to occur. Figure 4.2 is a plot of the uniform distribution overlaid by a normal distribution with the same mean. Note that the normal distribution gives a much higher density about the mean value - indicating a much higher probability for values to lie near the mean - while the uniform density is flat. This flatness in the density can also be interpreted as a lack of information or increased uncertainty about where the most likely value will be. If the most likely value is unknown, but the range of values is fairly certain, then the uniform distribution is a good choice.

The triangle distribution is a cross between the uniform and the normal distributions. To specify a triangle distribution requires the range of possible values and a most likely or modal value. If the


Fig. 4.1. Two normal distributions for SC , both with a mean of $\$ 240$ but with standard deviations of $\$ 10$ and $\mathbf{\$ 2 0}$. The vertical bars enclose $95 \%$ of the area.


Fig. 4.2. Normal and uniform distributions for SC, both with a mean of $\$ 240$ and with standard deviations of $\mathbf{\$ 1 7 . 3 2}$. The vertical bars enclose $95 \%$ of the area.
modal value is selected as the middle of the range, then the triangle distribution is symmetric about the modal value and is similar to the normal distribution. If the modal value chosen is nearer to one of the endpoints of the range the distribution is more skewed and deviates from the normal distribution. The triangle distribution is simple since only three parameters are required to describe it, yet it does contain more information since it has a modal value - or a value that is thought most likely to occur. Figure 4.3 is a plot of the triangle distribution overlaid with a normal distribution with the same mean.

Finally, the gamma distribution is defined by two parameters, commonly denoted by $\alpha$ and $\beta$. One reason for the usefulness of this distribution is that it can take many shapes (i.e. symmetric or skewed) and yet have the same mean value. In terms of its parameters, the mean of the gamma distribution is the product of $\alpha$ and $\beta$. The variance is also described by its parameters as $\alpha \beta^{2}$. Figure 4.4 shows the gamma distribution for various choices of $\alpha$ and $\beta$, whose product $\alpha \beta$ is constant (i.e., they have the same mean).

The knowledge one has concerning a given input helps to determine the choice of distribution. For example, if SC is known fairly well and is felt to vary about some central value, the normal distribution might be a good choice. If SC is known to lie in some range and seems most likely to lie about some particular value (not necessarily in the center of the range), the triangle distribution may be the best choice. If very little is known about SC except that it should be greater than some value and less than another, then the uniform distribution appears most satisfactory.

Finally, if a fair amount is known about SC - that it is not symmetric, that it is skewed in one direction, and that it is fairly peaked (i.e., most values lie close to the mean, but there are possibilities that SC could deviate far from the mean) - then the gamma distribution may be a good choice.


Fig. 4.3. Normal and triangle distributions for SC, with a mean of $\$ 238$ and standard deviations of $\$ 11.61$. The vertical bars enclose $95 \%$ of the area.


Fig. 4.4. Plot of gamma density for various choices of alpha and beta.

How can this information on the input distributions be used to determine A and B? Initially look at an approximate value for $A$ and $B$ based on the mean values of our selected distributions. Table 4.1 describes the mean and variance for each distribution using the parameters that define it. Whatever distributions are selected for the inputs, a good approximate mean value for the output (i.e. either A or B) can be obtained by substituting the mean values of the inputs into the formula defining either $A$ or $B$.

Table 4.1. Distributions, defining parameters, means and variance

| Distribution | Defining parameters | Mean | Variance $^{a}$ |
| :--- | :---: | :---: | :---: |
| Normal | $\mu, \sigma^{2}$ | $\mu$ | $\sigma^{2}$ |
| Uniform | $\mathrm{a}, \mathrm{b}$ | $\frac{b-a}{2}$ | $\frac{(b-a)^{2}}{12}$ |
| Triangle | $\mathrm{a}, \mathrm{b},{ }^{b} \mathrm{c}$ | $\frac{a+b+c}{3}$ | $\frac{a^{2}+b^{2}+c^{2}-a b-a c-b c}{18}$ |
| Gamma | $\alpha, \beta$ | $\alpha \beta$ | $\alpha \beta^{2}$ |

${ }^{a}$ Standard deviation equals the positive square root of the variance.
${ }^{b}$ The term $b$ is the modal value in the triangle distribution.

An example will help to make this more concrete. The input variables that go into calculating A and $B$ are given in Table 4.2 along with some selected distributions and defining parameters. The means and variances are also listed in Table 4.2. To obtain an approximate mean value for A , we substitute the mean values for the input variables into the equation for $A$.

Since A is defined by

$$
A=(S C+E C * H P Y) /(F C R * 1000),
$$

then

$$
\tilde{\mu}_{A}=(240+0.03433 * 8760) /(0.175 * 1000)=3.09
$$

Similarly for B , the defining equation is

$$
B=\left(S C * R F+E C * L_{s} F * H P Y\right) * P_{L}^{2} /(F C R * 1000)
$$

and its approximate mean value is

$$
\tilde{\mu}_{B}=(240 * 0.45+0.03433 * 0.1667 * 8760)(1.05)^{2} /(0.175 * 1000)=1.00 .
$$

So the choice of parameters for our input variables can tell us the approximate mean values for A and $B$. If we change any of these parameters, it will have some effect on the approximate mean values for $A$ and $B$. For example, suppose the mean of SC is reduced to 200 with everything else remaining the same. This change in SC causes the approximate mean value of $A$ and $B$ to go to 2.86 and 0.88 , respectively. The point of this exercise is to demonstrate that the approximate mean values of $A$ and $B$ are directly related to the choice of the parameters used to describe the input variables. If
a utility put out a bid with $A=\$ 3$ and $B=\$ 1$, then the parameters in Table 4.2 would be good choices. If the utility selected $A=\$ 5$ and $B=\$ 1$, then the values in Table 4.2 would be inappropriate, since the mean value of A would be approximately $\$ 3.00$.

Table 4.2. Input variables used to calculate $A$ and $B$ and their distributions

| Input variable | Assumed <br> distribution | Parameters | Mean | Variance |
| :--- | :--- | :--- | :--- | :--- |
| SC | Normal | $\mu=240, \sigma^{2}=10$ | 240 | 10 |
| EC | Triangle | $\mathrm{A}=0.01, \mathrm{~B}=0.33, \mathrm{C}=0.6$ | 0.03433 | 0.000104 |
| Cost of capital $^{a}$ | Uniform | $\mathrm{A}=0.08, \mathrm{~B}=0.12$ | 0.10 | 0.000133 |
| Depreciation $^{a}$ | Uniform | $\mathrm{A}=0.025, \mathrm{~B}=0.035$ | 0.03 | 0.00000833 |
| Income taxes $^{a}$ | Uniform | $\mathrm{A}=0.01, \mathrm{~B}=0.02$ | 0.015 | 0.00000833 |
| ${\text { Local taxes, ins., etc. }{ }^{a}}^{\text {Fixed charge rate }}$ | Uniform | $\mathrm{A}=0.02, \mathrm{~B}=0.04$ | 0.03 | 0.0000333 |
| RF | Uniform | $\mathrm{A}=0.1515, \mathrm{~B}=0.1985$ | 0.175 | 0.000184 |
| RF | Uniform | $\mathrm{A}=0.4, \mathrm{~B}=0.5$ | 0.45 | 0.000833 |
| $\mathrm{~L}_{\mathrm{S}} \mathrm{F}$ | Triangle | $\mathrm{A}=0.08, \mathrm{~B}=0.12, \mathrm{C}=0.3$ | 0.16667 | 0.002289 |
| $\mathrm{P}_{\mathrm{L}}$ | Uniform | $\mathrm{A}=1.01, \mathrm{~B}=1.09$ | 1.05 | 0.000533 |

${ }^{a}$ The sum of the input equals the fixed charge rate (FCR). The mean and variance of FCR are 0.175 and 0.000084 , respectively, but its distribution is complicated.

The parameters in Table 4.2 will yield an average value of A equal to roughly $\$ 3$ and of $B$ equal to $\$ 1$; but each time a sample is drawn from each input distribution, these individual values may vary considerably from the average values, which in turn will yield values of A and B different from their mean values. The distribution of ratios and products of random variables are very difficult to determine in most cases. That is one reason Monte Carlo simulation is used, to help determine in a pseudo-random manner what the distribution on A and B look like. Randomly sampling from the given input variable distributions and calculating A and B yields a pair of A and B values. Several of these taken together help describe approximately what the distribution of $A$ and $B$ values are. The larger the number of simulations is, the better picture we have of what the distributions look like.

### 4.3 UNCERTAINTY ANALYSIS

How can the simulated values help in decision making? Although the distribution of values on $A$ and $B$ may be of interest, it is more important to see how these randomly generated pairs can be used to make better decisions. Assume that a utility company requests bids based on $\mathrm{A}=\$ 3$ and $\mathrm{B}=\$ 1$. The utility receives the transformer designs and corresponding prices given in Table 4.3.

The pseudo-efficiency (or loss index) is simply the square root of the product of the LL and NL values. It is a representative value for the efficiency where a smaller value implies greater efficiency. This number is not meant as a measure of the true efficiency of the transformer; it is simply a measure that allows a ranking of the transformer. Based on the information in Table 4.3, Design 1 has the lowest TOC value (\$939), Design 5 is the most efficient transformer, and Design 8 is the

Table 4.3. Designs and base TOC values when $\mathbf{A}=\$ 3.00$ and $B=\$ 1.00$

| Design no. | Price | Load loss | No-load loss | Pseudo- <br> efficiency | TOC |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 492$ | 68 | 243 | 128.5 | $\$ 939$ |
| 2 | $\$ 532$ | 56 | 243 | 116.7 | $\$ 943$ |
| 3 | $\$ 505$ | 69 | 230 | 126.0 | $\$ 942$ |
| 4 | $\$ 511$ | 63 | 243 | 123.7 | $\$ 943$ |
| 5 | $\$ 545$ | 57 | 230 | 114.5 | $\$ 946$ |
| 6 | $\$ 539$ | 50 | 271 | 116.4 | $\$ 960$ |
| 7 | $\$ 518$ | 53 | 269 | 119.4 | $\$ 946$ |
| 8 | $\$ 465$ | 54 | 317 | 130.8 | $\$ 944$ |

lowest-priced unit. If the values of $A$ and $B$ were known with certainty, then Table 4.3 would express all that is needed to make a decision. If the utility's decision process were to select the lowest TOC transformer, then it would choose Design 1. If the utility liked the band of equivalence (BoE) approach and considered all transformers within a $3 \% \mathrm{BoE}$ to be essentially equal, it would likely select Design 8 because it is near the lowest TOC and has a much lower price. A utility might also consider the efficiency of the transformer and select Design 5 because it is also close to the lowest TOC and has the lowest pseudo-efficiency.

The values of $A$ and $B$ are not known for certain, and they will change in the future. Depending on circumstances, the values may go up or down or remain nearly the same. The "goodness" of the utility's decision will be determined by this random process. We can simulate that process and use Monte Carlo methods to determine the values of $A$ and $B$ which might occur which in turn allow the calculation of TOC values for these generated A and B values. These new TOC values can then be compared to see if the original selections were good, where good is measured by the loss or gain in revenue compared with the base case or some other method of comparison.

To run a Monte Carlo simulation analysis, we must choose the appropriate distributions for the inputs. Table 4.2 gives the inputs and their assumed distributions for this example simulation. These values were selected on the basis of experience and not from a detailed analysis of data. If data were available to help select appropriate distributions and their associated parameters (e.g., mean and variance), this empirical method of determining the distribution would be preferred.

It was shown earlier that by substituting the mean values for the inputs into the equations defining $A$ and $B$, then the resulting values for $A$ and $B$ are $\$ 3.09$ and $\$ 1.00$, respectively. These values are close to the values that were stated in requesting the bid. This result gives us some confidence that the distributions are not out of alignment with those that were expected to yield $A=\$ 3.00$ and $B=\$ 1.00$.

Using the input distributions in Table 4.2, 500 simulations were performed to generate 500 values for $A$ and $B$, which were used to generate 500 TOC values. Various statistics can be generated to determine properties concerning the choice of a transformer design and its possible TOC values.

Table 4.4 summarizes some of the quantities one might want to calculate from the simulation. The first column of the table is simply the transformer design number. The second column is the base TOC value given that $\mathrm{A}=\$ 3.00$ and $\mathrm{B}=\$ 1.00$. The average TOC is the average TOC over the 500
simulations. Design 1 remains the minimum TOC, even though the simulation results are $\$ 10.00$ higher than the base case. In fact all the average TOC values are higher than the base TOC values, with the difference ranging from $\$ 8.00$ (Design 6) to $\$ 11.00$ (Design 3), with most increasing by $\$ 9.00$ (Designs 2, 5, 7, and 8).

Table 4.4. Summary statistics for simulation analysis for each transformer design

| Design no. | Base TOC | Average TOC | Average <br> difference from <br> minimum TOC | Fraction of runs <br> in which TOC <br> was minimum <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 939 | 949 | 3.30 | 40.8 |
| 2 | 943 | 952 | 5.78 | 19.2 |
| 3 | 942 | 953 | 6.33 | 1 |
| 4 | 943 | 953 | 6.67 | 0 |
| 5 | 946 | 955 | 8.81 | 6.8 |
| 6 | 960 | 968 | 22.24 | 0 |
| 7 | 946 | 955 | 8.60 | 0.2 |
| 8 | 944 | 953 | 7.10 | 32 |

The average difference from the minimum TOC is obtained by subtracting the minimum TOC value at each simulation step from the TOC values of each transformer. This leaves 500 values for each transformer design. The value will be zero if that transformer achieved the minimum TOC or some positive value otherwise. These 500 values are then averaged to yield the average difference from minimum TOC. This average value might be thought of as the cost of uncertainty. If the values of A and B were known with certainty, it would be trivial to pick the optimal transformer design; but realistically A and B are not known. Therefore, the choice of the optimal design will vary as A and B vary. The average difference from minimum TOC estimates the cost of this uncertainty. Note that Design 1 (the minimum TOC design) is the best with respect to this measure, with a cost of uncertainty of $\$ 3.30$. The BoE choice is Design 8 , with a cost of uncertainty of $\$ 7.10$, more than twice the cost of uncertainty of Design 1. It is interesting to note that Design 5 , the design with the lowest pseudo-efficiency, has a rather high cost of uncertainty of $\$ 8.81$. Design 6 should be avoided since it has the highest cost of uncertainty of $\$ 22.24$. It is also interesting to note that Design 2 has the second smallest cost of uncertainty, $\$ 5.78$.

The last statistic is the fraction of runs in which that particular design had the minimum TOC value. It indicates the percentage of the time that transformer design was the clear winner with respect to TOC. On the basis of this statistic, Design 1 (the lowest base TOC) is the best, having the lowest TOC nearly $41 \%$ of the time. The second best design is Design 8, which has the lowest TOC $32 \%$ of the time. This rate is nearly $9 \%$ less often than Design 1, but the cost of uncertainty of Design 8 is more than twice that of Design 1. These statistics indicate that Design 8 is more variable and that under certain conditions, its TOC value can be quite large. Figure 4.5 is a plot of the differences from minimum TOC for Design 1 and Design 8. The values for Design 8 (vertical axis) range from 0 to 50,
while the values for Design 1 (horizontal axis) range from 0 to 20. This plot is another indicator that Design 1 is better in the sense that it is less variable and, even if it does poorly, it does so to a lesser degree than Design 8 . The large numbers of points on the vertical and horizontal axis indicates the large percentage of times these two transformers had the minimum TOC.

The statistics in Table 4.4 can also be used to calculate other interesting cross-comparisons. For example, we can compare the design having the minimum base TOC with the design having the minimum price (i.e., Design 1 and Design 8). Based on Table 4.4, the average TOC for all simulations for Design 1 is $\$ 949.48$ and for Design 8 is $\$ 953.28$, indicating that on the average Design 1 has a smaller TOC.

Thus simulation analysis provides a method for comparing transformer designs and enabling the user to understand the effects that variation in input parameters has on the TOC values. Several statistics can be calculated. Some are presented in Table 4.4 which provide a basis for making a decision in the face of uncertainty. In addition, the variation in TOC values or the cost of uncertainty values like those presented in Fig. 4.5 can indicate how far from the best choice a design may stray. Statistics such as the minimum, maximum, standard deviation, and coefficient of variation are other measures that provide assistance in assessing the distribution of TOC values obtained through simulation.

### 4.4 SENSITIVITY ANALYSIS

An analysis of the sensitivity of the TOC values to changes in the inputs can be obtained for any given design. In the same manner that the sensitivity results in section 2 were obtained, sensitivities for a given transformer design can also be obtained. The importance of this type of analysis is the ability to determine the effect an input has on determining the TOC. If the effect is slight, then that


Fig. 4.5. Plot of uncertainty costs for Designs 1 and 8.
input has little influence and knowledge about its "true" value is not as important as for an input that has a dramatic effect on the TOC.

This sensitivity analysis was performed for transformer Design 1 . The sensitivity analysis is carried out by setting all inputs at their mean value. Then, selecting one input at a time, the quantiles corresponding to the probabilities $0.05,0.25,0.50,0.75$, and 0.95 were obtained and substituted in the equations defining $A$ and $B$. These values of $A$ and $B$ were then used to calculate the TOC. Recall that the quantile $x_{p}$ corresponding to the probability $p$ is that value of the distribution for which $p \%$ of the distribution lies to the left of $x_{p}$. For a symmetric distribution, then, $x_{0.50}$ is the middle of the distribution. For the normal distribution, the middle of the distribution is the mean. Table 4.5 contains the quantiles corresponding to the probabilities $0.05,0.25,0.50,0.75$, and 0.95 for the six input variables.

Using the quantiles corresponding to $p=0.50$ for the base values rather than the mean will have little effect unless the distributions are highly skewed (i.e., nonsymmetric). Hence, we used the $x_{0.50}$ value to be the base value and then, for one input variable at a time, cycled through each of the quantiles to obtain $A$ and $B$ values and determine TOC. Table 4.6 lists the TOC values thus obtained, and Fig. 4.6 is a plot of those values versus the probability or percentile values. It is obvious from Fig. 4.6 that FCR has a large effect on the TOC, as does EC; the other inputs have a rather small effect on TOC, except possibly $\mathrm{L}_{s} \mathrm{~F}$ at its larger values. These results in Fig. 4.6 are slightly different from what is represented in Fig. 2.2, which was obtained under more general conditions. The importance of Fig. 4.6 or Table 4.6 is that FCR and EC are major "possible" influences on the TOC, and some effort should be made to determine these values as well as possible to obtain the most accurate prediction of future TOC values.

Table 4.5. Quantiles of the input variables corresponding to various probabilities

| Probability | SC | EC | FCR | RF | $\mathbf{L}_{\mathbf{s}} \mathbf{F}$ | $\mathbf{P}_{\mathbf{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.05 | 223.55 | 0.018 | 0.153 | 0.405 | 0.101 | 1.014 |
| 0.25 | 233.26 | 0.027 | 0.165 | 0.425 | 0.128 | 1.030 |
| 0.50 | 240.00 | 0.034 | 0.175 | 0.450 | 0.159 | 1.050 |
| 0.75 | 246.74 | 0.042 | 0.185 | 0.475 | 0.201 | 1.070 |
| 0.95 | 256.45 | 0.052 | 0.197 | 0.495 | 0.256 | 1.086 |

Table 4.6. TOC values as a function of changing input values

| Probability | SC | EC | FCR | $\mathbf{R F}$ | $\mathbf{L}_{\mathbf{s}} \mathbf{F}$ | $\mathbf{P}_{\mathbf{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.05 | 921.34 | 848.00 | 1003.35 | 922.53 | 912.46 | 923.02 |
| 0.25 | 931.79 | 899.93 | 966.16 | 929.88 | 924.63 | 930.08 |
| 0.50 | 939.06 | 939.06 | 939.06 | 939.06 | 939.06 | 939.06 |
| 0.75 | 946.33 | 981.22 | 914.90 | 948.25 | 957.86 | 948.22 |
| 0.95 | 956.79 | 1037.48 | 889.14 | 955.60 | 982.96 | 955.66 |



Fig. 4.6. Sensitivity of TOC to changes in inputs.

## 5. STATISTICAL DISTRIBUTIONS

The usefulness of probability distributions in simulating the $A$ and $B$ values is obvious. What is not obvious is how to select the appropriate distribution to use. This is not an easy problem and is discussed in several articles (see Draper 1995; Helton 1993; Beckman and McKay 1987; Downing, Gardner, and Hoffman 1985; and Kleijnen 1996). Numerous statistical papers are devoted to the subject of density estimation, and this section does not attempt to relate even a small fraction of the literature on the subject. We will introduce a few concepts, and the interested reader can find real depth on the subject in the literature cited. We shall restrict ourselves to continuous random variables, that is, to random variables that can take on any value in an interval. For example, the FCR may take on any value between 0 and 1 . It cannot be less than zero nor greater than unity, and many values in this interval have negligible probability of occurring. Usually we limit the number of significant digits that are reported, but this is for convenience rather than because of any limit on the true value of the random variable.

The distributions introduced in Sect. 4 (normal, uniform, triangle, and gamma) each have specific shapes, as shown in Figs. 4.1-4.4. These shapes and the limitations on the range of values that the associated random variable may take on are important in attempting to select a distribution. The typical process is to plot the data (observed values of the random variable) in a manner that will approximate the distribution. The plotting tool usually used is the histogram. A histogram is a visual way of describing the data. It is obtained by partitioning the range of the data into several intervals of equal length, counting the number of observations that lie in each interval, and plotting the counts as bar lengths. Figure 5.1 is a histogram for the A factors collected from several utilities across the United States for the year 1991.

The relative heights of the bars represent the relative density of observations in the intervals and in this way represent the probability density of the variable. Because the histogram has been used extensively and is familiar to most people, it is useful for conveying distributional information, but histograms suffer from several drawbacks. The primary drawback is the arbitrary choice of the number of intervals and their relative position. Changing these can alter the histogram markedly, and there is no known "best" choice. There are rules that some people use - for example, Sturges's number of bins rule [see Scott (1992)], which suggests using

$$
k=1+\log _{2} n,
$$

where $n$ is the number of observations in the sample and $k$ is the number of bins. For the data of Fig. 5.1, Doane (1976) suggested another rule if the data are skewed, rather than normal. Newer techniques using adaptive histograms focus on bin width rather than the number of bins (Scott 1992).

Putting these problems aside, the histogram does give a visual representation of the distribution of the data and allows one to see the symmetry of the data or whether it is skewed. Figure 5.1 reveals that the A factors are somewhat symmetric, but there are some rather large values extending the tail of the histogram to the right. This indicates a positive skewness. The large values are from Kauai and HELCO, both Hawaiian utility companies and perhaps not representative of the majority of utility companies. Omitting them would tend to make the histogram more symmetric and normal looking. We note also that the first interval of the histogram is from -2 to 0 . There are no negative values in the data, but there are two zero values. The plot of the histogram is largely a function of the computer code written to display it, and some things are beyond the control of the user.


Fig. 5.1. A factors for several U.S. utilities, 1991.

A second plot is the quantile plot. The term quantile is closely associated with the term percentile, which is a more common term. For example, in scholastic testing we speak of students who scored in the 99th percentile. This means that $99 \%$ of all scores fell below this student's score and only $1 \%$ of the scores fell above it. Similarly, we define the .99 quantile of a set of data to be a number on the scale of the data that divides the data into two groups, so that 0.99 of the observations fall below and 0.01 fall above. We will call this value $Q(.99)$. The only difference between percentile and quantile is that percentile refers to a percentage of a set of data and quantile refers to a fraction of the set of data. An obvious problem occurs when we want a quantile of a given value, say the .25 quantile, from a set of 10 data values. Each data value splits off $10 \%$ of the whole set, so we can find the .2 quantile and the .3 quantile, but not the .25 quantile. Another complication is that if we put the split point exactly at an observation, we would not know whether to include the observation in the lower part or the upper part.

The following definition helps overcome the above problems. Starting with the raw set of data $y_{i}$ $i=1,2, \ldots, n$, we order the data from smallest to largest, $y_{(i j} i=1,2, \ldots, n$. Letting $p$ represent any fraction between 0 and 1 , we begin by defining the quantile $Q(p)$ corresponding to the fraction $p$ as follows: Take $Q(p)$ to be $y_{(i)}$ whenever $p$ is one of the fractions $p_{i}=(i-.5) / n$, for $1,2, \ldots, n$.

Thus, the quantiles $Q\left(p_{i}\right)$ of the data are just the ordered data values themselves, $y_{(j)}$. Figure 5.2 is a quantile plot of the A factor data. It is a plot of the $Q\left(p_{i}\right)$ against $p_{i}$ for the A factor data. The horizontal scale shows the fractions $p_{i}$ and goes from 0 to 1 . The vertical scale is the scale of the original data. Note that if we ignore the horizontal scale, this is identical to a plot of $y_{(i)}$ vs $i$. The plot in Fig. 5.2 shows a series of connected line segments. Each line segment is an interpolation between $Q\left(p_{i}\right)$ and $Q\left(p_{i+1}\right)$. These line segments allow one to calculate $Q(p)$ for any fraction $p$.


Fig. 5.2. Quantile plot of the $A$ factor data for several U.S. utilities, 1991.

If $p$ is a fraction $f$ of the way from $p_{i}$ to $p_{i+1}$ then $Q(p)$ is defined to be

$$
Q(p)=(1-f) Q\left(p_{i}\right)+f Q\left(p_{i+1}\right)
$$

where

$$
f=\frac{p-p_{i}}{p_{i+1}-p_{i}} \quad p_{i} \leq p \leq p_{i+1} .
$$

Note that the above formula is for interpolation. It is tricky to extrapolate values, and thus for $p<.5 / n$ or $\mathrm{p}>1-.5 / n$ we choose to be cautious and use $y_{(1)}$ and $y_{(n)}$, respectively. There are some special quantiles that should be discussed. The median, $Q(.5)$ is the middle value that divides the data into two groups of equal size. If the sample size, $n$, is odd, the median is $y_{(n+1 / 2)}$. If $n$ is even, then
there are two values of $y_{(0)}$ equally close to the middle, and our interpolation rule tells us to average them, giving $\left(y_{(n / 2)}+y_{(n / 2+1)}\right) / 2$.

Two other quantiles are the lower and upper quartiles, $Q(.25)$ and $Q(.75)$. They split off $25 \%$ and $75 \%$ of the data, respectively. The distance between the first and third quartile, $Q(.75)-Q(.25)$, is called the interquartile range and can be used to judge the spread of the bulk of the data. For the data displayed in Fig. 5.2, we find that $Q(.25)=2.31, Q(.5)=3.12$, and $Q(.75)=4.16$; and the interquartile range is $Q(.75)-Q(.25)=1.85$. The quantile plot yields other pieces of information as well. The highest local density of points occurs when there are many measurements with exactly the same value. This is revealed in the quantile plot by segments with zero slope. The gentle slope also indicates that values are relatively close together. The large values from the Hawaiian utilities appear together after the sharply rising segment, indicating their separation from the bulk of the data.

The quantile plot can be used to assess how well a theoretical distribution approximates the data in hand. One calculates the quantiles of the original data and the corresponding quantiles of the theoretical distribution and plots these against one another. These plots are called theoretical quantile-quantile plots, often shortened to the expression theoretical Q-Q plots. The term theoretical is used to indicate that one distribution is an assumed theoretical distribution. This same plot can be used to compare two sets of observed data. In this setting, both quantiles are empirical, and the plot of the two sets of quantiles is called a $Q-Q$ plot.

The theoretical quantiles are calculated in the following way. Let $F(y)$ denote the theoretical cumulative distribution function. The $p$ quantile of $F$, where $0<p<1$, is a number that we shall call $Q_{T}(p)$, (the subscript $T$ stands for theoretical) which satisfies

$$
F\left[Q_{T}(p)\right]=p
$$

or

$$
Q_{T}(p)=F^{-1}(p)
$$

Thus, $Q_{T}(p)$ is the value for which the fraction $p$ of the distribution lies to the left of it. These values are calculated for $p_{i}=(i-.5) / n$, where $i=1,2, \ldots, n$. The empirical quantiles are obtained, as before, from the ordered observations, where $Q\left(p_{i}\right)=y_{(i)}$. The values of $Q\left(p_{i}\right)$ and $Q_{T}\left(p_{i}\right)$ are plotted on the theoretical Q-Q plot. Figure 5.3 is a theoretical Q-Q plot of the A factor data for 1991, where the theoretical distribution is a standard normal (i.e., has mean zero and unit variance). The theoretical Q-Q plot should follow a straight line if the theoretical distribution is a good approximation to the distribution of the data. A straight line is plotted in Fig. 5.3, which indicates that the data is poorly fit by this model, especially in the right tail.

The large values from the Hawaiian utilities have a strong effect on the distribution shown in Fig. 5.3. Since these are quite different from the rest of the mainland data, they are dropped, and the theoretical Q-Q plot is redrawn in Fig. 5.4. The straight line is a much better fit to the data in this figure, but the values to the right again separate from the line. This indicates right skewness and is indicative of log normal data. Log normal data are data whose logarithms (to the base $e$ ) are normally distributed.

Figure 5.5 is a theoretical Q-Q plot of the log-transformed data. The Hawaiian utility data were included in the data set, and all values were increased by adding 1 to their values because the $\log$ of 0 is minus infinity. The line here fits better for the majority of the data except at both extreme tails. Data values at the extremes tend to be more variable than those in the center of the distribution, and some variation off the line is expected. Large discrepancies from the line at the ends indicate long or short tails. Thus, the two zero values and the Hawaiian data both indicate longer tails than might be


Fig. 5.3. Theoretical quantile-quantile plot of the A factor data for several U.S. utilities, 1991.
expected for the log normal distribution. Since the remainder of the data follows the straight line reasonably well, the log normal distribution may be an acceptable distribution for the majority of the data.


Fig. 5.4. Theoretical quantile-quantile plot for the A factor data, 1991, omitting the extreme values.


Fig. 5.5. Theoretical quantile-quantile plot for the $\log$ of the $\mathbf{A}$ factor data, 1991.

## 6. UTILITY APPLICATIONS

### 6.1 INTRODUCTION

Any methodology needs to be tested on field data, and we were fortunate to find two utilities willing to share their data with us for test cases. These cases represent transformers from the residential, commercial, and industrial sectors. Section 6.2, below, gives the designs (that is, the price, no-load loss, and load loss) for each transformer, the base values of A and B that were given to the transformer manufacturers, and the corresponding parameters used in the calculation of A and B along with their assumed distribution. The results of the Monte Carlo simulation analysis are given in Sect. 6.3. For each of the transformer selection cases, the transformer with the lowest TOC is selected in the majority of the simulations. Collateral information, such as pseudo-efficiency (loss index) and cost of losses, indicates that the selection makes sense economically. In Sect. 6.4 we look at the results when we substitute uniform distributions for all of the parameters, which indicates a lack of knowledge about the parameter. The general lesson to be learned from substituting the uniform distribution is that the choice is not as clear as it was when we assumed other distributions but results in the same selection; this outcome makes sense because we are admitting a lack of knowledge about the parameters which reflects itself in wider variation in the values that the parameters take on and subsequently in the values of A and B themselves.

### 6.2 DESIGNS AND PARAMETER DISTRIBUTIONS

The utilities provided data on four residential transformer sizes/types: 15-kVA pole mount, 25kVA pole mount, $25-\mathrm{kVA}$ pad mount, and $50-\mathrm{kVA}$ pad mount. The values of A and B given to the transformer manufacturers for residential transformers were $\mathrm{A}=\$ 2.00 / \mathrm{W}$ and $\mathrm{B}=\$ 0.25 / \mathrm{W}$. Data were provided on two commercial transformer sizes: $300-\mathrm{kVA}$ pad mount and $500-\mathrm{kVA}$ pad mount. The values of A and B given to the transformer manufacturers for commercial transformers were $\mathrm{A}=$ $\$ 2.00 / \mathrm{W}$ and $\mathrm{B}=\$ 0.60 / \mathrm{W}$. There was one industrial transformer, a $1000-\mathrm{kVA}$ pad mount transformer with the value of $A=\$ 2.00 / \mathrm{W}$ and $\mathrm{B}=\$ 1.00 / \mathrm{W}$. Tables 6.1-6.3 present the transformer designs for each of these cases.

The designs for residential, commercial, and industrial transformers show a nice variation in price, no-load, and load loss values. Usually, the more expensive transformers have lower no-load and load loss values, as one would expect. The question to be answered is, given these choices, which transformer should we select and why? Sect. 6.3 discusses the results of our simulations, answers the selection problem, and gives some reasons for the selection.

In order to run a simulation analysis we must have distributions for the various parameters that go into determining the A and B values. The utility companies helped us with the choice of the central values, but the choice of the probability distribution was ours. We tried to use common sense and less-informative distributions (like the triangle and uniform) as choices to avoid assuming more than we should about the distributions. This is an area on which utilities need to spend more time and effort, since it is a very important part of this analysis. We investigate the influence of distribution choice in Sect. 6.4, where we substitute uniform distributions for all parameters. Table 6.4 gives the parameters, their distribution, and associated distribution values for the residential, commercial, and industrial conditions.

Table 6.1. Residential transformer bids, $\mathbf{A}=\mathbf{\$ 2 . 0 0}$ and $B=\$ 0.25$

| Transformer size | Design | Price | No-load loss (NL) | Load loss (LL) |
| :---: | :---: | :---: | :---: | :---: |
| $15-\mathrm{kVA}$ pole mount | 1 | 284 | 50 | 300 |
|  | 2 | 300 | 50 | 320 |
|  | 3 | 310 | 53 | 220 |
|  | 4 | 320 | 80 | 270 |
|  | 5 | 340 | 40 | 320 |
| 25-kVA pole mount | 1 | 340 | 70 | 440 |
|  | 2 | 375 | 75 | 495 |
|  | 3 | 380 | 120 | 420 |
|  | 4 | 390 | 55 | 320 |
|  | 5 | 400 | 55 | 430 |
|  | 6 | 380 | 70 | 300 |
|  | 7 | 390 | 55 | 350 |
| 25-kVA pad mount | 1 | 730 | 70 | 500 |
|  | 2 | 740 | 75 | 470 |
|  | 3 | 750 | 60 | 470 |
|  | 4 | 770 | 55 | 390 |
|  | 5 | 810 | 60 | 410 |
|  | 6 | 825 | 100 | 370 |
|  | 7 | 850 | 55 | 310 |
| 50-kVA pad mount | 1 | 830 | 160 | 700 |
|  | 2 | 875 | 115 | 820 |
|  | 3 | 885 | 140 | 640 |
|  | 4 | 890 | 100 | 850 |
|  | 5 | 910 | 95 | 580 |
|  | 6 | 930 | 135 | 610 |
|  | 7 | 1010 | 80 | 610 |

Table 6.2. Commercial transformer bids, $\mathbf{A}=\$ 2.00$ and $B=\$ 0.60$

| Transformer size | Design | Price | No-load loss (NL) | Load loss (LL) |
| :---: | :---: | :---: | :---: | :---: |
| 300-kVA pad mount | 1 | 4900 | 500 | 3100 |
|  | 2 | 5250 | 535 | 2500 |
|  | 3 | 5400 | 540 | 2750 |
| 500-kVA pad mount | 4 | 5800 | 620 | 2000 |
|  | 1 | 5530 | 1220 | 5150 |
|  | 2 | 5700 | 880 | 4900 |
|  | 3 | 5800 | 1100 | 5630 |
|  | 4 | 5810 | 1080 | 5500 |
|  | 5 | 5820 | 1090 | 5630 |
|  | 6 | 6080 | 750 | 5750 |
|  | 7 | 6275 | 895 | 6075 |
|  | 8 | 6400 | 850 | 3900 |
|  | 10 | 6800 | 615 | 5850 |

Table 6.3. Industrial transformer bids, $\mathbf{A}=\$ 2.00$ and $B=\$ 1.00$

| Transformer size | Design | Price | No-load loss (NL) | Load loss (LL) |
| :---: | :---: | :---: | :---: | :---: |
| 1000-kVA pad mount | 1 | 8500 | 1820 | 8820 |
|  | 2 | 8635 | 1950 | 7650 |
|  | 3 | 8700 | 2150 | 7750 |
|  | 4 | 9000 | 1550 | 7500 |
|  | 5 | 9300 | 1600 | 7650 |
|  | 6 | 9600 | 1020 | 9500 |
|  | 7 | 10350 | 1170 | 6900 |
|  | 8 | 10500 | 1450 | 6300 |

Table 6.4. Distributions for economic variables used in computing $A$ and $B$

| Economic parameter | Residential |  | Commercial |  | Industrial |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Probability distribution | Distribution parameters | Probability distribution | Distribution parameters | Probability distribution | Distribution parameters |
| SC | Normal | $\begin{aligned} & \text { mean }=31.522 \\ & \text { std dev }=1.0 \end{aligned}$ | Normal | $\begin{aligned} & \text { mean }=31.522 \\ & \text { std dev }=1.0 \end{aligned}$ | Normal | $\begin{aligned} & \text { mean }=31.522 \\ & \text { std dev }=1.0 \end{aligned}$ |
| EC | Normal | $\begin{aligned} & \text { mean }=0.0259 \\ & \text { std dev }=0.0075 \end{aligned}$ | Normal | $\begin{aligned} & \text { mean }=0.0259 \\ & \text { std dev }=0.0075 \end{aligned}$ | Normal | $\begin{aligned} & \text { mean }=0.0259 \\ & \text { std dev }=0.0075 \end{aligned}$ |
| PL | Uniform | $\begin{aligned} & \min =0.9 \\ & \max =1.1 \end{aligned}$ | Uniform | $\begin{aligned} & \min =0.9 \\ & \max =1.1 \end{aligned}$ | Uniform | $\begin{aligned} & \min =0.9 \\ & \max =1.1 \end{aligned}$ |
| FCR | Triangle | $\begin{aligned} & \min =0.10 \\ & \max =0.14 \\ & \operatorname{mode}=0.1241 \end{aligned}$ | Triangle | $\begin{aligned} & \min =0.10 \\ & \max =0.14 \\ & \operatorname{mode}=0.1241 \end{aligned}$ | Triangle | $\begin{aligned} & \min =0.10 \\ & \max =0.14 \\ & \operatorname{mode}=0.1241 \end{aligned}$ |
| RF | Triangle | $\begin{aligned} & \min =0.45 \\ & \max =0.47 \\ & \operatorname{mode}=0.46 \end{aligned}$ | Triangle | $\begin{aligned} & \min =0.775 \\ & \max =0.795 \\ & \operatorname{mode}=0.785 \end{aligned}$ | Triangle | $\begin{aligned} & \min =0.875 \\ & \max =0.895 \\ & \operatorname{mode}=0.885 \end{aligned}$ |
| LsF | Triangle | $\begin{aligned} & \min =0.054 \\ & \max =0.084 \\ & \operatorname{mode}=0.074 \end{aligned}$ | Uniform | $\begin{aligned} & \min =0.21 \\ & \max =0.216 \end{aligned}$ | Uniform | $\begin{aligned} & \min =0.43 \\ & \max =0.434 \end{aligned}$ |

### 6.3 RESULTS OF MONTE CARLO SIMULATION ANALYSIS

This section discusses the analysis results for each transformer purchase and summarizes the findings. The summary statistics are defined in Sect. 4.3.

### 6.3.1 15-kVA Pole-Mount Residential Transformer

For the $15-\mathrm{kVA}$ pole-mount residential transformer, there are five designs to choose from, shown in Table 6.1. A visual summary of the output of our analysis is shown in Fig. 6.1. Fig. 6.1(a) gives the TOC using $\mathrm{A}=\$ 2.00$ and $\mathrm{B}=\$ 0.25$. This shows that Design I has the lowest TOC $(\$ 459.00)$, while Design 4 has the highest ( $\$ 547.50$ ). The values of $A$ and $B$ in these runs vary around the
central values of $\$ 2.00$ and $\$ 0.25$, respectively; for this analysis the average value of $A$ is $\$ 2.14$ and $B$ is $\$ 0.25$. Thus, the average TOC values will be somewhat higher than those based on the fixed $A$ and $B$ values; however, the relative average TOC values between designs is the same.

Fig. 6.1(b) indicates the pseudo-efficiency of the transformers (defined as the square root of the product of the no-load loss and load loss values). In general, it can be stated that the more efficient the design the more expensive the transformer. For these, Design 3 is the most efficient, and Design 4 is the least efficient. This is an interesting result because Design 4 is the second most expensive design. Note that the designs have been ordered by their cost from lowest to highest for all purchases, so that Design 1 is the lowest priced transformer, Design 2 is the next lowest priced transformer, and so on. Depending on the values of NL and LL and the values of A and B, the lowest TOC will differ.

Fig. 6.1(c) indicates the number of times a particular design would be selected based on its having the lowest TOC. This is independent of its first cost, being based simply on the TOC value. In this case study, Design 1 is chosen every time. This design is the lowest-first-price unit and also has the lowest TOC. In the simulation analysis it consistently had the lowest computed TOC value.

Fig. 6.1(d) indicates the average amount of money a given design would cost above the one with the lowest TOC over the 500 simulations. Since Design 1 had the lowest TOC value in every case, its average cost is zero. Design 3 is a close competitor, costing on average $\$ 12.30$ more than the lowest TOC design (which in this case is always Design 1). Design 4 is an obviously poor choice, since it cost $\$ 92.55$ more than the lowest TOC design on average (which means that in some cases it could even be more costly). Thus, for this transformer purchase, Design 1 is a clear choice.

### 6.3.2 25-kVA Pole-Mount Residential Transformer

Table 6.1 lists the seven designs for the $25-\mathrm{kVA}$ pole-mount residential transformer case study. The results for the study are shown in Fig. 6.2. Design 4 has the lowest TOC value [Fig. 6.2(a)], but is the fourth most expensive transformer. The reason for this can be seen by looking at the efficiency bar plot [Fig. 6.2(b)], where Design 4 is the most efficient design. In terms of the percentage of times a design is selected (i.e., has the lowest TOC value), we see that Design 4 is selected $82 \%$ of the time, and Design 1 is selected $18 \%$ of the time [Fig. 6.2(c)]. No other designs were selected. Recall that Design 1, by definition, is the lowest priced unit; however, because of its NL and LL values, it is not as good a purchase as Design 4. The average cost gives us additional information. On average, Design 4 costs $\$ 1.33$ more than the alternative lowest TOC unit (in this case, Design 1, since it is the only design that was chosen over Design 4), while Design 1 costs $\$ 13.86$ [Fig. 6.1(d)]. This says that Design 4 is close to the lowest TOC unit overall and that Design 1 can be substantially more costly than the lowest TOC unit. Again, the choice of transformer is Design 4. It is interesting to note that for an FCR of $12.41 \%$, the return on the incremental cost of unit 4 is $14.9 \%$ per year. This return increases to $18.1 \%$ if uncertainty costs are included in the savings.

### 6.3.3 25-kVA Pad-Mount Residential Transformer

The seven designs for the $25-\mathrm{kVA}$ pad-mount residential transformer case study are given in Table 6.1. Fig. 6.3 shows the results of this case study. The lowest TOC design is Design 4, which again is in the middle with regard to initial cost, but due to its efficiency [see Fig. 6.3(b)] has the lowest TOC. Design 7 is the most efficient design, but it is also the most expensive design (in fact, $\$ 80.00$ more than Design 4). The high purchase price of Design 7 keeps it out of the competition even though it is a much more efficient unit than any of the other designs. Design 4 is the second most efficient unit. This and its middle price make it an attractive choice. Design 4 is chosen $93.4 \%$
of the time, while Design 1 (the lowest priced unit) is selected $6.6 \%$ of the time. On average, Design 4 costs $\$ 0.34$ above the lowest TOC unit (not unexpected, since it has the lowest TOC value $93.4 \%$ of the time), but the average cost of $\$ 20.58$ for Design 1 indicates that it is fairly far from the lowest TOC unit. Design 3 has the second lowest average cost and is a better competitor on the basis of cost than Design 1 even though it never had the lowest TOC.

### 6.3.4 50-kVA Pad-Mount Residential Transformer

The seven designs for the $50-\mathrm{kVA}$ pad-mount residential transformer case study are listed in Table 6.1. Figure 6.4 shows the results of this case study. Design 5 has the lowest TOC because of its high efficiency (or low NL and LL values; see Table 6.1). Design 7 has the best efficiency, but it is also the most expensive unit, and the initial cost makes this design unattractive. The bar plot for percentage selected in Fig. 6.4(c) clearly indicates that Design 5 is the design of choice, being selected $97.4 \%$ of the time. Design 1 is chosen $2.6 \%$ of the time; although its initial cost is low, it is the least efficient unit. In terms of average cost, Design 5 is clearly the best choice ( $\$ 0.47$ ), with its nearest competitor, Design 4 , having an average cost of $\$ 59.21$. Clearly, in this study the best design economically is Design 5 , even though its first cost is the fifth highest.

### 6.3.5 300-kVA Pad-Mount Commercial Transformer

Table 6.2 contains the data for the four designs in the $300-\mathrm{kVA}$ pad-mount commercial transformer case study. Figure 6.5 shows the results of the analysis for this case study. Design 1 has the lowest cost and lowest TOC even though it is not the most efficient unit. Design 4 (the most expensive) is the most efficient unit. Design 1 is selected most often - $80.6 \%$ of the time - while Design 2 is selected $19.4 \%$ of the time. The average cost is revealing in this case, showing $\$ 8.60$ for Design 1 and $\$ 66.49$ for Design 2. The other two designs have substantially higher TOC values and are not competitive.

### 6.3.6 500-kVA Pad-Mount Commercial Transformer

The 10 designs for the $500-\mathrm{kVA}$ pad-mount commercial transformer case study are listed in Table 6.2. Figure 6.6 shows the results of the analysis. Design 2 has the lowest TOC and is also the fourth most efficient design out of the 10 . Design 9 is the most efficient design, with Design 10 the second most efficient design. The initial price of these units makes them somewhat unattractive. The design chosen most often is Design 2, selected $92.4 \%$ of the time, but Design 9 (the second most expensive design) is selected $7.6 \%$ of the time. The average cost for Design 2 is $\$ 7.55$, while that for Design 9 is $\$ 235.12$. The other designs have significantly higher average costs. In this case, the efficiency of the second most expensive design (Design 9) is nearly enough to make it an attractive choice, but Design 2 (the second lowest priced unit) is clearly the unit of choice.

### 6.3.7 1000-kVA Pad-Mount Industrial Transformer

Table 6.3 contains the data corresponding to the eight transformers in the $1000-\mathrm{kVA}$ pad-mount purchase competition. Figure 6.7 shows the results of the analysis. The TOC values are fairly close for these large and expensive transformers. Design 7 has the lowest TOC $(\$ 19,590)$ but is closely followed by Design $4(\$ 19,600)$ and Design $8(\$ 19,700)$. These three designs are the more expensive transformers with good efficiency numbers. The most efficient design is Design 7, followed by Design 8 and then Design 6. Design 4 is the fourth most efficient of the eight designs. The closeness
of these units is shown in the percentage selected bar plot. For this case study, Design 7 is selected most often ( $51.4 \%$ ) with Design 4 being a close second (selected $44.6 \%$ of the time). Design 8 (the most expensive and the second most efficient unit) was selected $3 \%$ of the time, and Design 2 (the second least expensive and the third least efficient unit) was selected $1 \%$ of the time. The average cost shows Design 7 is the winner, with a value of $\$ 114.94$; but Design 4 is a competitor, with an average cost of $\$ 168.79$. Design 8 is also competitive, with an average cost of $\$ 232.81$; but the other units have substantially higher average costs.

### 6.4 SENSITIVITY TO THE CHOICE OF DISTRIBUTION

The analysis given in Sect. 6.3 is very much dependent on the selection of the distributions and their associated parameter values. To see what effect the choice of distribution has on the results, we ran the analysis again with all the distributions set to the uniform distribution. As mentioned earlier, the uniform distribution gives equal probability to the value lying in any interval of the same width between the lower and upper limits of the distribution. In this sense there is no information given as to where the most probable value lies in the interval.

The results of the two analyses are tabulated using the percentage of time selected and the average cost in Table 6.5. An examination of the table reveals that the percentage selected for the design most often selected decreases going from the original analysis to the analysis using all uniform distributions. This is expected, in that there is more variability and less certainty with regard to the economic variables, and this translates into more uncertainty in the choice. Is this significant? For most of the studies it makes little difference. For example, in the $15-\mathrm{kVA}$ study the percentage selected went from $100 \%$ in the original analysis to $99.6 \%$ in the uniform distribution analysis. In this study the lack of information with regard to the distribution was not important. The other studies are more intermediate, but in an absolute sense the same transformer design is still chosen most of the time. The most variable study is that of the 1000-kVA industrial transformer, in which Design 7 is chosen $51.4 \%$ of the time and Design 4 is chosen $44.6 \%$ of the time. In the analysis using all uniform probability distributions, these percentages change to $49.2 \%$ and $38.2 \%$, respectively. Design 7 is still selected more than Design 4 - in fact, by an even greater percentage difference but in the latter analysis other designs are now being selected, especially Design 2. Having greater uncertainty leads to more variability and more difficult decisions.

The average cost analysis reflects this also, in that the average cost increases from the original analysis to the all uniform analysis consistently. The same ranking of the designs would be made in all cases on the basis of average cost, so that there is no difference in choice of design on the basis of average cost, but simply an increase due to the increased variability in the inputs.

Thus, the change of distributions has an effect, but this effect would not change the selection of the most economical transformer design. The effect is that the decision is not as unquestionable as when more is known about the input distributions. This is why it is important for utilities to be concerned with these variables and to track them over time to get a good understanding of the distribution of values that they may take on. This information will give better and more informed decision making.

### 6.5 CONCLUSION

In all of the cases studies presented, the decision as to which transformer design to choose was rather well determined. Only in the $1000-\mathrm{kVA}$ case study was there any vagueness in the choice (Design 7 vs Design 4), but the average cost in these cases gives a quantitative dollar value that can
help in making a decision. The methodology given here appears to work rather well, but the major shortcoming of this method or any other that depends on simulation is not knowing the distribution of the terms that will be sampled from. This information is utility-specific and is not generally available, but this method can be given to utilities for their private use. The results here give another view rather than simply computing TOC values and selecting on the basis of the lowest TOC or some band of equivalence. It allows a further investigation into possible changes in the future and how those might impact the economic results of a given choice. The results obtained seem logical and help to combine the competitive relationship between price and efficiency. A $3 \%$ BoE would allow selection of the lowest-cost unit in almost all the above cases. Yet such a selection would always have a higher cost of uncertainty and generally fail to recognize the value of the savings in reducing the cost of losses.

Table 6.5. Comparison of results changing probability distributions

| Study | Variables | Analysis | Design |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 15 kVA | \% selected | Orig. analysis | 100 | 0 | 0 | 0 | 0 |  |  |  |  |  |
|  |  | All uniform | 99.6 | 0 | 0.4 | 0 | 0 |  |  |  |  |  |
|  | Av. cost | Orig. analysis | 0 | 21.03 | 12.30 | 92.55 | 39.66 |  |  |  |  |  |
|  |  | All uniform | 0 | 21.23 | 11.77 | 94.94 | 38.97 |  |  |  |  |  |
| 25 kVA | \% selected | Orig. analysis | 18 | 0 | 0 | 82 | 0 | 0 | 0 |  |  |  |
|  |  | All uniform | 31.6 | 0 | 0 | 68.4 | 0 | 0 | 0 |  |  |  |
|  | Av. cost | Orig. analysis | 13.86 | 73.50 | 155.86 | 1.33 | 39.20 | 18.38 | 8.93 |  |  |  |
|  |  | All uniform | 17.67 | 77.73 | 163.40 | 3.93 | 41.88 | 22.11 | 11.56 |  |  |  |
| 25 kVA padmount | \% selected | Orig. analysis | 6.6 | 0 | 0 | 93.4 | 0 | 0 | 0 |  |  |  |
|  |  | All uniform | 24.6 | 0 | 0 | 75.4 | 0 | 0 | 0 |  |  |  |
|  | Av. cost | Orig. analysis | 20.58 | 33.65 | 11.48 | 0.34 | 56.17 | 146.76 | 59.93 |  |  |  |
|  |  | All uniform | 22.22 | 35.44 | 13.14 | 2.25 | 58.05 | 149.11 | 62.13 |  |  |  |
| 50 kVA | \% selected | Orig. analysis | 2.6 | 0 | 0 | 0 | 97.4 | 0 | 0 |  |  |  |
|  |  | All uniform | 12.6 | 0 | 0 | 0 | 87.4 | 0 | 0 |  |  |  |
|  | Av. cost | Orig. analysis | 89.89 | 68.77 | 86.94 | 59.21 | 0.47 | 113.68 | 75.91 |  |  |  |
|  |  | All uniform | 95.06 | 72.05 | 90.95 | 61.84 | 2.07 | 117.35 | 76.85 |  |  |  |
| 300 kVA | Av. cost | Orig. analysis | 80.6 | 19.4 | 0 | 0 |  |  |  |  |  |  |
|  |  | All uniform | 72.6 | 27.4 | 0 | 0 |  |  |  |  |  |  |
|  | Av. cost | Orig. analysis | 8.60 | 66.49 | 380.52 | 493.71 |  |  |  |  |  |  |
|  |  | All uniform | 21.89 | 77.40 | 392.63 | 503.24 |  |  |  |  |  |  |
| 500 kVA | \% selected | Orig. analysis | 0 | 92.4 | 0 | 0 | 0 | 0 | 0 | 0 | 7.6 | 0 |
|  |  | All uniform | 0 | 79.8 | 0 | 0 | 0 | 0 | 0 | 0 | 20.2 | 0 |
|  | Av. cost | Orig. analysis | 721.28 | 7.55 | 1024.79 | 912.72 | 1023.26 | 624.44 | 549.03 | 1454.07 | 235.12 | 1114.68 |
|  |  | All uniform | 752.87 | 26.21 | 1056.99 | 943.11 | 1055.15 | 646.76 | 567.24 | 1483.90 | 243.74 | 1133.67 |
| $\begin{aligned} & 1000 \\ & \mathrm{kVA} \end{aligned}$ | \% selected | Orig. analysis | 0 | 1 | 0 | 44.6 | 0 | 0 | 51.4 | 3 |  |  |
|  |  | All uniform | 0 | 8.6 | 0 | 38.2 | 0 | 0 | 49.2 | 4 |  |  |
|  | Av. cost | Orig. analysis | 1580.90 | 788.65 | 1371.66 | 168.79 | 726.22 | 1714.16 | 114.94 | 232.81 |  |  |
|  |  | All uniform | 1785.80 | 979.25 | 1577.60 | 329.71 | 893.43 | 1878.94 | 238.68 | 363.48 |  |  |



## ©


(d)
Fig. 6.1. Results of simulation analysis for 15-kVA pole-mount transformer.



Fig. 6.2. Results of simulation analysis for 25-kVA pole-mount transformer.


Fig. 6.3. Results of simulation analysis for 25-kVA pad-mount transformer.

(a)

(b)

(c)

(d)

Fig. 6.4. Results of simulation analysis for $\mathbf{5 0 - k V A}$ pad-mount transformer.


Fig. 6.5. Results of simulation analysis for $\mathbf{3 0 0}-\mathrm{kVA}$ pad-mount transformer.

(a)

(b)

(c)

(d)

Fig. 6.6. Results of simulation analysis for $500-\mathrm{kVA}$ pad-mount transformer.

(a)

(c)

(c)

(d)

Fig. 6.7. Results of simulation analysis for 1000-kVA pad-mount transformer.

## 7. SUMMARY

### 7.1 OVERVIEW

In this report the question of uncertainty in transformer TOC evaluations has been examined. Following a brief introduction in Chapter 1, the classic deterministic method of evaluating transformers (total owning cost, or TOC) and the uncertainty associated with purchasing decisions based upon this method were discussed in Chapter 2.

A major source of uncertainty in many utility evaluation decisions is the impact of deregulation and restructuring on the classic regulated utility financial and expansion planning models. Indeed, in the restructured utility market, classic planning models may not even be applicable. In order to provide the reader with some feel for the present status of utility restructuring and the possible impacts, a summary of the status of the restructured utility market was presented in Chapter 3.

The uncertainty model is developed in Chapter 4, and a summary of the methods that can be used to provide the statistical distributions needed in the nondeterministic model is presented in Chapter 5. Examples of applications of the method are provided in Chapter 6.

### 7.2 IMPACT OF NONDETERMINISTIC APPROACH

As indicated in Chapter 1 and illustrated in Chapter 2, the development of the original TOC method involved a sensitivity study of the key input variables. While this type of analysis provides insight into the relative variation of the TOC, it does not establish any range of uncertainty in TOC values. Rather, an analysis of this type establishes that TOC varies $x \%$ with a $y \%$ variation in a specific input variable and does not address the probability of the input variable assuming a particular value within the range of variation. Thus, the original TOC calculations are deterministic; i.e., a specific set of input variables always produces a specific output, and there is no effective and statistically valid way to assign spread or band of equivalence (BoE) on TOC.

The methodology developed by this research effort and presented in Chapter 4 of this report allows for uncertainty in the input variables by requiring as input a statistical distribution for each input variable. In response to deterministic values of $A$ and $B$, the vendors supply the price; and the associated evaluated losses are a set of fixed values. A relatively large Monte Carlo simulation is run to sample the input variables and estimate multiple values of the cost of no-load losses (A) and load loses (B). TOC is then calculated for each of the simulated values of $A$ and $B$. With the distributions on $A$ and $B$ and this set of prices and losses, the purchaser can evaluate the relative impact of uncertainty on a purchasing decision, since the resultant statistical distribution of TOC is established. A discussion of statistical distributions is provided in Chapter 5, and an example using utility input is provided in Chapter 6.

An alternate method mentioned at the end of Chapter 2 would alter the bidding process and request that vendors supply transformer prices as a functions of excitation (no-load) losses and fullload losses. This input, which is deterministic, could then be used in the Monte Carlo simulation to establish a distribution of TOC that includes a more accurate indication of the impact of transformer price on TOC. Since price represents a significant portion (nominally about 50\%) of the TOC, this method would provide a new nondeterministic means of evaluating TOC. Because of the need for specific price variation with respect to losses, this alternative method has not been presented in this report.

To determine the effectiveness of the proposed methodology, we asked utilities to provide sets of transformer bid responses (price and losses) together with the associated A and B factors provided
the vendors. The utilities were also asked to supply the inputs used to calculate these $A$ and $B$ factors and a best estimate of the statistical distribution associated with each input. Since utilities were unable to supply any estimate of the distributions on the input variables, the authors provided estimates of the distribution for use in assessing the methodology. The details are presented in Chapter 6. Even with limited data, uniform distributions appear to establish of the validity of the Monte Carlo simulation developed in this report. The method provides a consistent set of results that can be used to enhance the quality of the purchasing decision by including uncertainty.

The values utilities have traditionally used for A and B are changing, and relatively extreme variations are sometimes seen from one year to the next in the same utility. The inability of the utilities to supply even a simple uniform distribution on any of the input variables could imply either a lack of understanding of the proposed methodology or a lack of understanding of what constitutes a statistical distribution. Alternatively, utilities in general seem to be uncertain about the specific form of the inputs used in the classic TOC analysis. To some degree this reflects uncertainty in utility planning and financing, perhaps caused by restructuring. A bit of truth can be found in each of the above observations. If the methods developed in this project are to be applied consistently, the manner in which the input parameters are estimated must be expanded to provide reasonable estimates of the statistical distributions of the variables.

## 8. REFERENCES

Barnes, P. R., et al. 1995. The Feasibility of Replacing or Upgrading Utility Distribution Transformers during Routine Maintenance, ORNL-6804/R1, Oak Ridge National Laboratory, Oak Ridge, Tenn., April.

Beckman, R. J., and M. D. McKay 1987. "Monte Carlo Estimation Under Different Distributions Using the Same Simulation," Technometrics 29(2), 153-60.
Chambers, J. M., et al. 1983. Graphical Methods for Data Analysis, Duxbury Press, Boston.
Commerce Clearing House, Inc. 1993. 1993 Depreciation Guide, Report 29, Vol. 80, Chicago, Ill.
Doane, D. P. 1976. "Aesthetic Frequency Classification," American Statistics 30, 181-83.
Downing, D. J., R. H. Gardner, and F. O. Hoffman 1985. "An Examination of Response-Surface Methodologies for Uncertainty Analysis in Assessment Models," Technometrics 27(1), 151-63.

Draper, D. 1995. "Assessment and Propagation of Model Uncertainty," Journal of the Royal Statistical Society, Ser. B, 57, 45-97.

EIA (Energy Information Administration) 1994. Electric Power Annual 1993. DOE/EIA-0348(93), U.S. Department of Energy, Washington, D.C., December.

EIA (Energy Information Administration) 1996a. Input data for the Annual Energy Outlook 1997, staff transmittal of electronic files, U.S. Department of Energy, Washington, D.C., October.
EIA (Energy Information Administration) 1996b. Annual Energy Outlook 1997, DOE/EIA-0383(97), U.S. Department of Energy, Washington, D.C., November.

Helton, J. C. 1993. "Uncertainty and Sensitivity Analysis Techniques for Use in Performance Assessment for Radioactive Waste Disposal," Reliability Engineering and System Safety 42, 327-67.

Hill, L. J. 1995. A Primer on Incentive Regulation for Electric Utilities, ORNL/CON-422, Oak Ridge National Laboratory, Oak Ridge, Tenn., October.

IEEE (Institute of Electrical and Electronics Engineers) 1997. Guide for Distribution Transformer Loss Evaluation, C57.12.33, Draft 4, IEEE, New York, June 30.

Jaffe, A. B., and F. A. Felder 1996. "Should Electricity Markets Have a Capacity Requirement: If So, How Should It Be Priced?" Pp. 25-1-25-7 in Proceedings: 1996 EPRI Conference on Innovative Approaches to Electricity Pricing-Managing the Transition to Market-Based Pricing. TR106232, Electric Power Research Institute, Palo Alto, Calif., March.
Kleijnen, J. P. C. 1996. "Sensitivity Analysis and Related Analyses: A Review of Some Statistical Techniques," Journal of Statistical Computation and Simulation 57(1-4), 77-110.

Nickel, D. L., and H. R. Braunstein 1981. "Distribution Transformer Loss Evaluation: II — Load Characteristics and System Cost Parameters," IEEE Transactions on Power Apparatus and Systems PAS-100(2), February.

Pleat, G. R. 1996. "Pricing and Profit Strategies for a Stand-Alone Electric Distribution Company," Public Utilities Fortnightly 135(2), 20-24, January 15.

Scott, D. W. 1992. Multivariate Density Estimation: Theory, Practice, and Visualization, John Wiley and Sons, New York.

Thomas, S. 1996. "Electric Reform in Great Britain: An Imperfect Model," Public Utilities Fortnightly 134(12), 20-25, June 15.
U.S. Advisory Commission on Intergovernmental Relations 1992. Significant Features of Fiscal Federalism, Vol. 1, Budget Processes and Tax Systems, PB-92-198860, Washington, D.C., February.

## Appendix: Derivation of Total Owning Cost

The usual expression for total owning cost expressed as an equivalent first cost $\left(\mathrm{TOC}_{\mathrm{EFC}}\right)$ is of the form

$$
\mathrm{TOC}_{\mathrm{EFC}}=\text { bid price }+ \text { cost of core losses }+ \text { cost of load losses },
$$

where

$$
\begin{aligned}
& \text { cost of core } \text { losses }_{\mathrm{EFC}}=\mathrm{A}(\$ / \mathrm{W}) \times \text { core loss }(\mathrm{W}) \times \text { loss multiplier } \\
& \text { cost of load losses } \mathrm{EFC}=\mathrm{B}(\$ / \mathrm{W}) \times \text { load loss }(\mathrm{W}) \times \text { loss multiplier , }
\end{aligned}
$$

and

$$
\begin{aligned}
& A=\text { equivalent first cost of no-load losses }(\$ / W) \\
& B=\text { equivalent first cost of load losses }(\$ / W) .
\end{aligned}
$$

The A and B factors are given by the expressions

$$
A=(S C+E C \times H P Y) /(F C R \times 1000)
$$

and

$$
\mathrm{B}=(\mathrm{SC} \times \mathrm{RF}+\mathrm{EC} \times \mathrm{LsF} \times \mathrm{HPY}) \times \mathrm{PL}^{2} /(\mathrm{FCR} \times 1000)
$$

where

$$
\mathrm{SC}=\text { avoided cost of system capacity }=\mathrm{GC}+\mathrm{TD} .
$$

The basic financial, cost, and load parameters are defined as follows:
SC = avoided cost of system capacity ( $\$ / \mathrm{kW}$ ) — The levelized avoided (incremental) cost of generation, transmission, and distribution capacity necessary to supply the next kilowatt of load to the transformer coincident with peak load.
$\mathrm{GC}=$ avoided cost of generating capacity $(\$ / k W)$
$\mathrm{TD}=$ avoided cost of transmission and distribution capacity ( $\$ / \mathrm{kW}$ )
$\mathrm{EC}=$ avoided cost of energy ( $\$ / \mathrm{kW}$ ) — The levelized avoided (incremental) cost for the next kilowatt produced by the utility's generating system.

HPY = energized hours per year - Usually 8760, but lower in special cases (for example, seasonal loads).

FCR $=$ fixed charge rate (\%) - The cost of carrying a capital investment, made up of the weighted cost of capital, depreciation, taxes, and insurance. Expressed in decimal form.

RF = peak responsibility factor (unit-less) - A measure of the load diversity on the transformer. It is never greater than 1 and is expressed in decimal form.
$\mathrm{L}_{\mathrm{s}} \mathrm{F}=$ transformer loss factor (unit-less) - The ratio of average load losses to peak load losses. It is never greater than 1 and is expressed in decimal form.

PL = equivalent annual peak load (unit-less) - The transformer's levelized annual peak load. It is generally assumed that the load grows from an initial peak load with an estimated growth rate to some maximum level where the transformer is changed out to a lower load site. By its very nature, there is great uncertainty in this parameter. However, levelizing tends to reduce the impact of this uncertainty.

The loss multiplier (unit-less) is a measure of transmission and distribution system losses between the generating unit and the transformer being evaluated. It is generally about 50 to $75 \%$ of total system losses ( 5 to $7 \%$ ).

The transformer cost and performance parameters are as follows:
P = bid price (\$) - The price for which a manufacturer will supply the transformer delivered to a specified point.
$\mathrm{NL}=$ no-load or core losses (watts) - The excitation losses at rated voltage when the transformer is not supplying a load. These losses are continuous and are not loaddependent.
$\mathrm{LL}=$ load losses (watts) - Losses that are a result of $\mathrm{I}_{2} \mathrm{R}$ losses and eddy current losses in the transformer windings. They are dependent on the square of the per unit load, and specifications should state the allowed temperature rise. Load at less than full load should be corrected to account for the effects of lower temperature.

## INTERNAL DISTRIBUTION

| 1-10. | P. R. Barnes |
| :--- | :--- |
| 11. | J. E. Christian |
| 12. | S. M. Cohn |
| 13. | G. E. Courville |
| 14. | T. R. Curlee |
| 15-24. | S. Das |
| 25. | D. J. Downing |
| 26. | S. W. Hadley |
| 27. | R. James |
| 28-32. | B. W. McConnell |
| 33. | C. I. Moser |

34. N. Myers
35. 
36. 
37. 
38. 

39-46.
47.
48.
49.
50.
R. D. Perlack
D. T. Rizy
R. B. Shelton
J. P. Stovall
J. W. Van Dyke
R. B. Wolfe
J. VanCoevering

Central Research Library
Laboratory Records - RC

## EXTERNAL DISTRIBUTION

51. Mike Agudo, Rural Utility Service, 1400 Independence Ave., SW, Stop 1569, Washington, DC 20250-1569
52. Paul Allen, Nashville Electric Service, 1214 Church Street, Nashville, TN 37203
53. Glenn Anderson, Duke Power Corp., 422 South Church Street, Box 33189, Charlotte, NC 28242
54. Joe Baker, Jr., ABB Power T\&D Company, Inc., P.O. Box 920, South Boston, VA 24592
55. Donald Ballard, 1603 Fourth Street, NE, Conover, NC 28613
56. William T. Black, CDA Cooper Development Association Inc., 260 Madison Ave., New York, NY 10016
57. Rusty Blackwood, Marietta Power, 205 Lawrence Street, Box 609, Marietta, GA 30060-0609
58. Jim Bohlk, Rural Security Service, 1400 Independence Ave., SW, Stop 1569, Washington, DC 20250-1569
59. John Borst, ABB Power T\&D Company, Inc., 500 Westinghouse Drive, Jefferson City, MO 65101
60. Robert Brewer, DOE Office of Energy Management, EE-14, 1000 Independence Ave., SW, Washington DC 20585
61. Thomas Brown, A\&N Electric Cooperative, P.O. Box 1128, Parksley, VA 23421
62. Dr. Edwin F. Brush, BBF \& Associates, 68 Gun Club Lane, Weston, MA 02193
63. Glenn Cannon, Waverly Light and Power, 1002 Adams Parkway, P.O. Box 329, Waverly, IA 50677
64. Jacob Chaco, Ames Municipal Electric Systems, 502 Carroll, Ames, IA 50010
65. Chris Cockrell, U.S. Department of Energy, 7129 Park Road, Kansas City, MO 64129

66-67. Kurt Conger, American Public Power Associates, 2301 M Street, NW, Washington, DC 20037-1484
68. Lake Coulson, National Electrical Manufacturers Association, 1300 North 17 th Street, Suite 1847, Rosslyn, VA 22209
69. Greg Coulter, General Electric Company, P.O. Box 2188, Hickory, NC 28603-2188
70. Rae Cronmiller, National Rural Electric Cooperative Association, 4301 Wilson Blvd., Arlington, VA 22203-1860
71. Russ Dantzler, Mid-Carolina Electric Cooperative, Inc., I-20 and Road 204/254 Longs Pond Road, P.O. Drawer 669, Lexington, SC 29072
72. Dave D'Avanzo, Midstate Electric Cooperative, P.O. Box 127, La Pine, OR 97739
73. A. Berl Davis, Jr., Palmetto Electric Cooperative, Inc., 111 Mathews Drive, P.O. Box 23619, Hilton Head, SC 29925
74. Jim Dedman, National Rural Electric Cooperative Association, 4301 Wilson Blvd., Arlington, VA 22203-1860
75. Andrew Deleski, Consortium for Energy Efficiency, One State Street, Suite 1400, Boston, MA 02109-3507
76. Thomas Dismantis, Engineering Standards, Bldg. D-G, Niagara Mohawk Power Corp., 300 Erie Boulevard West, Syracuse, NY 13202
77. Don Duckett, 2600 Lake Lucine Dr., Suite 400, MAC MT 3B, Maitland, FL 32751-7234
78. Kathi Epping, U.S. Department of Energy, Office of Energy Management, EE-141, 1000 Independence Ave., SW, Washington, DC 20585-0121
79. Wayne Gallimore, Northern Indiana Public Service Co., 801 East 86th Ave., Merrillville, IN 46410
80. John A. Gauthier, National Electrical Manufacturers Association, 1300 North 17 th Street, Suite 1847, Rosslyn, VA 22209
81. Howard Geller, American Council for an Energy-Efficient Economy, 1001 Connecticut Ave., NW, Washington, DC 20036
82. Martin Gorden, NRECA, 1800 Massachusetts Ave., NW, Washington, DC 20036
83. Kenneth L. Hall, Edison Electric Institute, 701 Pennsylvania Ave., NW, Washington, DC 20004-2696
84. Tony Harfield, ERMCO, 2225 Industrial Road, Dyersburg, TN 38024
85. Joe Hegwood, Peace River Electric Cooperative, P.O. Box 1310, Wauchula, FL 33873
86. Gerald R. Hodge, Vice President and General Manager, Howard Industries, Inc., P.O. Box 1588, Laurel, MS 39441
87. Allan Hofftnan, DOE Office of Utility Technologies, EE-10, Forrestal Building, Room 6C-036, 1000 Independence Ave., SW, Washington, DC 20585-0121
88. Tim Holdway, Federal Pacific Transformer Company, Old Airport Road, P.O. Box 8200, Bristol, VA 24203-8200
89. Philip J. Hopkinson, Square D Company, 1809 Airport Rd, P.O. Box 500, Monroe, NC 28110
90. Tam Huynh, City of Riverside, 3900 Main Street, 4th Floor, Riverside, CA 92522
91. Mike Hyland, American Public Power Assoc., 2301 M Street, NW, Washington, DC 20037-1484
92. Mike Iman, MGM, 5701 Swithway Street. Commerce, CA 90040
93. David Kanly, Moorhead Public Service, P.O. Box 779, Moorhead, MN 56560
94. J. Kappenman, Minnesota Power, 30 W. Superior Street, Duluth, MN 55802
95. Alexander D. Kline, Southern Transformer Company, 3015 Martin Street, P.O. Box 90460 , East Point, GA 30344-0460
96. Joseph L. Koepfinger, Director, Systems Studies and Research, Duquesne Light Company, One Oxford Center, 301 Grant Street (19-5), Pittsburgh, PA 15279
97. Rick Larsen, The Northeast Utilities Service Company, 107 Seldon Street, Berlin, CT 06037
98. Kenneth R. Linsley, ABB Power T\&D Company, Inc., 1021 Main Campus Drive, Raleigh, NC 27606-5021
99. Terry Logee, DOE Office of Energy Management, EE-431, 1000 Independence Ave., SW, Washington, DC 20585-0121
100. Mark Loveless, Oklahoma Gas \& Electric, P.O. Box 321, 101 N. Robinson, Oklahoma City, OK 73101-0321
101. Jeff Martin, Tampa Electric Company, P.O. Box 11, Plaza 2, Tampa, FL 33601
102. Fran Mayko, United Illuminating Company, 157 Church Street, New Haven, CT 06506
103. Dennis McEntire, City of Newnan, Newnan Water \& Light Commission, P.O. Box 578, Newnan, GA 30263
104. Jim McMahon, Standards Group Leader, Lawrence Berkeley Laboratory, One Cyclotron Road, 90-4000, Berkeley, CA 94720
105. Emil Milker, Phelps Dodge Magnet Wire Company, 2131 South Coliseum Blvd., Fort Wayne, IN 46803
106. Norvin Mohesky, Pauwels Transformers, Inc., One Pauwels Drive, P.O. Box 189, Washington, MO 63090
107. Gene Morehart, Acme Electric Corp., 4815 West 5th Street, Lumberton, NC 28358
108. David M. Nemtzow, President, The Alliance to Save Energy, 1725 K Street, NW, Suite 509, Washington, DC 20006
109. Harry Ng, Electric Power and Research Institute, 3412 Hillview Ave., P.O. Box 10412, Palo Alto, CA 94303
110. John O’Brien, Lane Electric Cooperative, 767 Bailey Hill Road, P.O. Box 21410, Eugene, OR 97402-0407
111. Andy Onestie, Shawano Electric Utilities, 122 N. Sawyer Street, P.O. Box 436, Shawano, WI 54166-0436
112. Steve Ott, Tri-County Electric Cooperative, Inc., P.O. Box 217, St. Mathews, SC 29135
113. Phil Overholt, DOE Office of Energy Management, EE-141, 1000 Independence Ave., SW, Washington, DC 20585-0121
114. Wesley F. Patterson, Jr., Technical Director, ABB Power T\&D Company Inc., Highway 58 West, P.O. Box 920, South Boston, VA 24592-0920
115. Oskars Petersons, NIST, Bldg 220, Room B-168, Gaithersburg, MD 20899
116. Rick Poltevecque, Central Florida Electric Cooperative, P.O. Box 9, Chiefland, FL 32626
117. Mike Poucher, Ocala Electric Utility, P.O. Box 1270, Ocala, FL 34478
118. James E. Powers, General Electric Company, P.O. Box 2188, Hickory, NC 28603-2188
119. Chris Pruess, Allied Signal, 6 Eastmons Road, Parsipany, NJ 07054
120. Rip Psyck, Manager, ARMCO Advanced Materials Co., 101 Three Degree Road, P.O. Box 832, Butler, PA 16003-0832
121. Richard Remijan, Commonwealth Edison Company, 1319 South First Ave., Maywood, IL 60153-2496
122. David Rollins, Cooper Industry, 1900 E. North Street, Waukesha, WI 53188-3899
123. Walt Ros, General Electric Company, P.O. Box 2188, Hickory, NC 28603-2188
124. Steven Rosenstock, Edison Electric Institute, 701 Pennsylvania Ave., NW, Washington, DC 20004-2696
125. Dean Sherrick, Edmond Electric, 315 SW 33rd Street, Edmond, OK 73013-3802
126. Ed Smith, Central Moloney, Inc., 2400 West Sixth Ave., Pine Bluff, AR 71601
127. Edward Smith, H-J Enterprises, Inc., 3010 High Ridge Blvd., High Ridge, MO 63049
128. Ron Stahara, Central Moloney, Transformer Division, 2400 West 6th Ave., Pine Bluff, AR 71601
129. Ernie Starnes, TU Electric, Distribution System Analysis, 115 West 7th Street, Fort Worth, TX 76101-0970
130. Margaret Suozzo, ACEEE, 6000 Shepherd Mountain Cove, No. 503, Austin, TX 78730
131. F. M. Tesche, Consulting Scientist, 6714 Norway Road, Dallas, TX 75230
132. Allen Traut, Kuhlman Electric Corp., 101 Kuhlman Blvd., Versailles, KY 40383
133. Diana Tringali, American Physical Plant Association, 1446 Duke Street, Alexandria, VA 22314
134. Charles Underhill, Vermont Public Power Supply Authority, Route 100, Stowe Road, Waterbury Center, VT 05677
135. D. R. Volzka, Senior Project Engineer, Wisconsin Electric Power Company, 333 West Everett Street, Milwaukee, Wl 53201
136. Ingrid Watson, U.S. Department of Energy, Office of Energy Management, EE-141, 1000 Independence Ave., SW, Washington, DC 20585-0121
137. Ray Wegley, Adams Electric Cooperative, 1338 Biglerville Road, Gettysburg, PA 17325
138. James Weid, Wisconsin Electric Power Co., P.O. Box 2046, Milwaukee, WI 53202
139. John Weimer, Allegheny Power Systems, 800 Cabin Hill Drive, Greensburg, PA 15601
140. Bill Wemhoff, American Public Power Assoc., 2301 M Street, NW, Washington, DC
141. 20037-1484
142. Larry K. Wendahl, Cooper Power Systems, 1900 E. North Street, Waukesha, WI 53188-3899
143. Steve West, Portland General Electric Co., 121 SW Salmon Street, Portland, OR 97204
144. Alan L. Wilks, ERMCO, P.O. Box 1228, Dyersburg, TN 38025-1228
145. James Will, Speciality Steel Industry of North America, Attn: Skip Hartquirst, 3050 K Street, NW, Washington, DC 20007
146. Matt Williams, National Electrical Manufacturers Association, 1300 N. 17th Street, Suite 1847, Rosslyn, VA 22209
147. Ken Winder, Moon Lake Electric Assoc., 188 West 2nd North, Roosevelt, UT 84066
148. Stephen Wood, COM/Electric, 2421 Cranberry Highway, Wareham, MA 02571
149. H. W. Zaininger, ZECO, 1590 Oakland Road, Suite B21 1, San Jose, CA 95131

150-200. NCI Information Systems, Inc., 8260 Greensboro Drive, Suite 325, McLean, VA 22102
201. Office of Assistant Manager for Energy, Research and Development, DOE-ORO, P.O. Box 2001, Oak Ridge, TN 37831
202-3. OSTI, U.S. Department of Energy, P.O. Box 62, Oak Ridge, TN 37831


[^0]:    * The approach in this study does not attempt to model time-dependent sequences. Uncertainty is handled by providing statistical distributions for the input parameters.
    ** The relationship $\mathrm{LF}^{2}<\mathrm{L}_{\mathrm{s}} \mathrm{F}<\mathrm{LF}$ can be established.

