

**II. RESULTS FROM PRIOR DOE SUPPORT**

The research described in this proposal will be the continuation of the ongoing research project which is currently being supported by the U.S. Department of Energy under the contract "Mathematical Models of Hysteresis" (contract No. DE-FG05-88ER13846). Thus, before discussing the proposed research in detail, it is worthwhile to describe and summarize the main results achieved in the course of the research work under the above contract.

The ongoing research has largely been focused on the development of mathematical models of hysteretic nonlinearities with "nonlocal memories". The distinct feature of these nonlinearities is that their current states depend on past histories of input variations. It turns out that memories of hysteretic nonlinearities are quite selective. Indeed, experiments show that only some past input extrema (not the entire input variations) leave their marks upon future states of hysteretic nonlinearities. Thus special mathematical tools are needed in order to describe nonlocal selective memories of hysteretic nonlinearities. The origin of such tools can be traced back to the landmark paper of Preisach.

Our research has been primarily concerned with Preisach-type models of hysteresis. All these models have a common generic feature; they are constructed as superpositions of simplest hysteretic nonlinearities-rectangular loops. During the past four years, our study has been by and large centered around the following topics:

- Further development of scalar and vector Preisach-type models of hysteresis.
- Experimental testing of Preisach-type models of hysteresis.
- Development of new models for viscosity (aftereffect) in hysteretic systems.
- Development of mathematical models for superconducting hysteresis in the case of gradual resistive transitions.
- Software implementation of Preisach-type models of hysteresis.
- Development of new ideas which have emerged in the course of the research work.

Next, I shall briefly describe the main scientific results obtained in the areas outlined above.

**Further development of scalar Preisach-type models of hysteresis**

In our previous research, we have performed experimental testing of the nonlinear

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(input dependent) Preisach model as well as the restricted Preisach model. This testing was carried out for two typical particulate materials: Co-coated  $\gamma - Fe_2O_3$  and iron magnetic tape materials. It turned out that the accuracy of the nonlinear model was about the same as the accuracy of the restricted model. However, it was observed that the actual (experimentally measured) reversal curves are "sandwiched" between the predictions of these models. This observation prompted the idea to use a "superposition" ("average") model:

$$f(t) = \int \int [\mu(\alpha, \beta, u(t)) + \bar{\mu}(\alpha, \beta, M_1)] \hat{\gamma}_{\alpha\beta} u(t) d\alpha d\beta + \frac{f_{u(t)} + f_{u(t)}^- + f_{M_1}^+ + f_-}{4} \quad (1)$$

This model was tested for the above mentioned two materials and it turned out to be remarkably accurate for all used (and densely distributed) reversal values of inputs (magnetic fields). This experimentally observed fact warranted further experimental testing of the "superposition" model as well as its thorough theoretical study. The extensive experimental testing of the average Preisach model has been carried out. This experimental testing has been performed for the following magnetic tape materials: Ampex-641, Ampex-797, Ampex-D1, TDK-VHS, Maxell-Beta, and Memorex-3480. Experimental results have revealed the consistent and remarkable accuracy of the average model. At the same time, a thorough theoretical study of the average model has been undertaken. This study has revealed that functions  $\mu(\alpha, \beta, u)$  and  $\bar{\mu}(\alpha, \beta, M_1)$  can be individually determined by matching first- and second-order reversal curves exactly as in the case of input dependent and restricted models. On the other hand, it has been shown that the average model further relaxes the congruency property of the restricted Preisach model and the property of equal vertical chords of the input-dependent model. Namely, the average model describes hysteresis nonlinearities whose minor loops have equal vertical chords only if they are formed after the same largest input maximum  $M_1$  was achieved. In other words, the property of equal vertical chords becomes history dependent. It is clear that the average model relaxes the congruency property of the classical Preisach model much deeper than the input-dependent or restrictive models. At the same time, the average model requires the same experimental data for its identification as the two afore-mentioned models.

We have been also concerned with numerical implementation of the feedback Preisach model. The main reason for this has been the fact that magnetostrictive and piezoelectric materials are currently widely used in fine positioning applications (such as scanning tunneling and atomic force microscopy). To reduce detrimental effects caused by hysteresis of these materials on the accuracy of positioning, various feedback controllers can be used. This has posed an interesting problem of investigation of Preisach models with feedback. It is important to emphasize that here the feedback appears as a result of applications of the classical Preisach model in control systems. The feedback Preisach model also naturally arises in magnetic measurements, as a result of the demagnetizing field phenomena. This is because hysteretic materials usually "feel" not only applied fields but the superpositions of applied fields and demagnetizing fields. Since demagnetizing fields are proportional to the resulting magnetization (output), the feedback connection is thus introduced.

The feedback Preisach model has been also suggested as a model of hysteresis and has been theoretically studied by many authors. However, the difficult problem of numerical implementation of this model has not been satisfactorily addressed. In the course of our research work, we have developed a new technique for the numerical implementation of the classical feedback Preisach model. This numerical technique is applicable for moderately small feedback factors  $k$  (i.e., when  $|k| < 0.3$ ).

This numerical technique has been implemented and tested. Computer simulation have been performed using two sets of first-order reversal curves measured for Co-coated  $\gamma - Fe_2O_3$  magnetic tape and Terfenol samples. High-order reversal curves computed by using this technique were compared to similar ones computed by an iterative technique which converges to the correct output values of the feedback model. These simulations have demonstrated that this new technique is of low error margin for moderately small feedback factors. It also has the advantage of being non-iterative and, as a result, it is efficient and straight-forward.

#### **Further development of vector Preisach-type models of hysteresis**

Here, our research has been focussed on the development of new generalized vector Preisach models which circumvent deficiencies of the classical vector Preisach model. Similar to the classical vector Preisach model, the developed generalized vector Preisach models

are constructed as superpositions of scalar Preisach models continuously distributed along all possible directions. However, these scalar Preisach models are driven not by input projections (as in the case of the classical vector model), but by "generalized" input projections. These "generalized" input projections are mappings of the vector input realized by some function  $g$ . In other words, these models can be mathematically defined as follows:

$$\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \bar{e}_\phi \left( \int_T \int \nu(\alpha, \beta) \hat{\gamma}_{\alpha\beta} |\bar{u}(t)| g(\theta(t) - \phi) d\alpha d\beta \right) d\phi, \quad (2)$$

where:  $\bar{f}(t)$  is the model output at time  $t$ ,  $\theta(t)$  is the popular angle which defines the orientation of the input  $\bar{u}(t)$  at time  $t$ ,  $\bar{e}_\phi$  is a unit vector along the direction specified by a polar angle  $\phi$ , and  $\hat{\gamma}_{\alpha\beta}$  are elementary hysteresis operators represented by rectangular loops with  $\alpha$  and  $\beta$  being "up" and "down" switching values, respectively, and  $T$  is the triangle  $\{-\alpha_0 \leq \beta \leq \alpha \leq \alpha_0\}$  which contains the support of  $\nu(\alpha, \beta)$ .

In the above models, functions  $\nu(\alpha, \beta)$  and  $g(\phi)$  are not specified in advance but rather should be determined by fitting these models to some experimental data. This is an identification problem.

The presence of additional unspecified function  $g$  opens the opportunity to match experimental data obtained from both "scalar" and "rotational" experiments. This is the most important additional feature of the generalized vector Preisach models as compared to the classical vector Preisach model. At the same time, it makes the identification of the generalized vector Preisach models much more difficult. For this reason, the identification problem has been the focus of our research effort and a special mathematical machinery for the solution of the identification problem for the generalized vector Preisach models of hysteresis has been developed.

In applications, vector hysteresis usually appears as a part of a system. As a result, vector Preisach-type models appear in equations which describe systems with hysteresis. To study the equations with vector Preisach nonlinearities, it is important to first understand the properties of these nonlinearities as input-output operators. This is a formidable mathematical problem because vector Preisach models are very complicated mathematical objects. During past years, some progress in this direction has been achieved in our joint

work with Russian mathematician M.A. Krasnosel'skii and his colleagues. Namely, some results have been established concerning the continuity of the vector Preisach operators and some estimates for the modulus of continuity of the output have been derived.

It is well-known that in the case of isotropic hysteresis a uniformly rotating magnetic field results in uniformly rotating magnetization. This magnetization lags behind the magnetic field due to hysteretic losses. Depending on the physical nature of hysteresis, these losses may decrease to zero or even continuously increase as the magnitude of the uniformly rotating magnetic field is increased. It is the generally held opinion that the former case is realized for magnetic hysteresis, while there is some credible evidence that the latter case is true for superconducting hysteresis. Successful Preisach models of vector hysteresis should be able to accommodate this broad spectrum of behavior of rotational hysteretic losses. We have demonstrated that this is true for the generalized isotropic vector Preisach models of hysteresis. Namely, we have shown that, with appropriate choice of function  $g$ , the generalized vector models can replicate various asymptotic behavior of rotational hysteretic losses.

The experimental testing of generalized isotropic vector Preisach models of hysteresis has been performed for an isotropic magnetic tape material Ampex-641 ( $\gamma - Fe_2O_3$ ). This has been accomplished by using a vibrating sample magnetometer equipped with orthogonal pairs of pickup coils. The performed testing has demonstrated that the generalized vector hysteresis models have superior ability to mimic the correlation between mutually orthogonal components of output and input than the classical vector Preisach model. It is important to note that the correlation between orthogonal components of input and output has long been regarded as an important "testing" property for any vector hysteresis model in the area of magnetics.

The testing described above has been performed in house by using a vibrating sample magnetometer (VSM) provided by the NSA researchers who have expressed some interest in our work. Initially, this VSM was only equipped with one pair of pickup coils to measure scalar hysteresis. However, we have managed, with the help of the engineers from LDJ Company (which is the VSM manufacturer) to upgrade its capabilities to include orthogonal pickup coils for vector hysteresis measurements as well. By using the upgraded

version of VSM, intensive experimental testing has been done to assess the accuracy of various scalar and vector Preisach models of hysteresis.

### Development of Preisach-type models for aftereffect

It is well-known that the physical origin of hysteresis is due to the multiplicity of metastable states. At equilibrium, large deviations of random thermal perturbations may cause a hysteretic system to move gradually from higher to lower energy metastable states. This phenomenon is generally referred to in the literature as "aftereffect," "viscosity," or "creep". Traditionally, the modeling of hysteresis and aftereffect has been pursued along two quite distinct lines. In phenomenological modeling of hysteresis, the Preisach approach has been prominent, while the aftereffect has been studied by using thermal activation type models. It is desirable to develop the uniform approach to the modeling of both hysteresis and aftereffect. Previously, it was suggested to use the classical Preisach model driven by stochastic inputs as a model for aftereffect. However, the stochastic inputs were modelled by discrete time i.i.d. (independent identically distributed) random processes. Then, we further extended this approach by modeling the stochastic inputs by continuous time diffusion processes. From the mathematical point of view, it made the problem much more complicated. In the course of our research, we have shown that these difficulties can be largely overcome by using the mathematical machinery of the "exit problem" for diffusion processes. By using this machinery, we have developed an analytical technique for the calculation of time evolutions of the expected value of the output of the classical Preisach model driven by a diffusion process.

The classical Preisach model has some well known limitations. For this reason, we have extended the aforementioned approach by employing the more accurate input dependent (nonlinear) Preisach model. To achieve this extension, we have computed the joint probability density function (PDF) of the time continuous diffusion process  $x_t$  and discrete random variable  $\hat{\gamma}_{\alpha\beta}x_t$ . By using this PDF, the whole problem once again has been reduced to the exit problem for stochastic processes. By employing the mathematical machinery of the "exit problem," the solution of our problem was given in terms of series of iterated convolutions, which were further reduced to geometric series by using the Laplace transform.



An effort has been made to extend the aforementioned approach to vector hysteresis models. In particular, the Stoner-wohlfarth vector hysteresis model has been employed and the stochastic input has been modelled as a discrete time i.i.d. random process. An expression for the expected value of the magnetic moment of Stoner-wohlfarth particles as a function of time has been derived. It has been found that this expected value for each particle decays exponentially to a history independent value. An expression for the expected value of the average magnetization as given by the Stoner-Wohlfarth model has also been derived. The performed analysis has been based upon the Preisach-type formalism previously developed for the Stoner-Wohlfarth model.

#### Analysis of penetration of electromagnetic fields into superconductors with gradual resistive transition

It is well known that models for superconducting hysteresis are based on the analytical study of penetration of electromagnetic fields into hard superconductors. In the critical state (Bean) model, this study is performed under the assumption of ideal (sharp) resistive transition. However, actual resistive transitions are gradual and it is customary to describe them by the following power law:

$$E = \left( \frac{J}{k} \right)^n, \quad (n > 1), \quad (3)$$

where  $E$  is an electric field,  $J$  is a current density and  $k$  is some parameter which coordinate the dimensions of both sides in expression (2).

The exponent "n" is a measure of the sharpness of the resistive transition and it may vary in the range 4-1000. Initially, the power law was regarded only as a reasonable empirical description for the resistive transition. Recently, there has been a considerable research effort to justify this law theoretically. As a result, models based on Josephson-junction coupling, sausaging and spatial distribution of critical current have been proposed. In our work, the power law has been used as a constitutive equation for hard superconductors. It is easy to show (by using Maxwell's equations), that this constitutive relation leads to the following nonlinear partial differential equation for the current density:

$$\frac{\partial^2 J^n}{\partial z^2} = \mu_0 k^n \frac{\partial J}{\partial t}. \quad (4)$$

We first considered the solution of this equation for the following boundary and initial conditions:

$$J(0, t) = ct^p, \quad (t \geq 0, p > 0), \quad (5)$$

$$J(z, 0) = 0, \quad (z > 0). \quad (6)$$

It is worthwhile to note here that the choice of the above boundary conditions has been dictated by considerations of finding simple self-similar analytical solutions. It may seem at first that these boundary conditions are of very specific nature. However, it can be remarked that these boundary conditions do describe a wide class of monotonically increasing functions as  $p$  varies from 0 to  $\infty$ . The close examination of self-similar solutions led to the following observation: in spite of the wide range of variation of boundary conditions, the profile of electric current density  $J(z, t)$  remains approximately the same. For typical values of  $n$  (usually  $n \geq 7$ ), this profile is very close to a rectangular one. This suggested that the actual profile of electric current density should be close to a rectangular one for any monotonically increasing boundary condition  $J_0(t) = J(0, t)$ . In this way, we have arrived at the following generalization of a critical state model.

*Current density  $J(z, t)$  has a rectangular profile with the height equal to the instantaneous value  $J_0(t)$  of electric current density on the boundary of superconductor. Magnetic field  $H(z, t)$  has a linear profile with a slope determined by instantaneous value of  $J_0(t)$ .*

To better appreciate the above generalization, it is useful to recall that in the critical state model the current has a rectangular profile of constant (in time) height, while the magnetic fields has a linear profile with constant (in time) slope. This generalization of the critical state model has also resulted in very simple analytical solutions.

### **Vector generalization of the critical state model for superconducting hysteresis**

Most of the literature on the critical state model is concerned with scalar superconducting hysteresis. This is because the study of vector superconducting hysteresis requires the investigation of penetration of electromagnetic fields into superconductors for the case when these fields are not linearly polarized. This is a very difficult analytical problem which requires the solution of coupled nonlinear partial differential equations. This problem has been solved only for circular polarization in the case of ideal (sharp) resistive

transition.

In our study, we have first considered the circular polarization of electromagnetic field in the case of gradual resistive transition described by the "power law." By exploiting the rotational symmetry of the problem, the exact analytical solution of nonlinear coupled PDE's has been found. In the limiting case of sharp transition the obtained solution is reduced to the solution previously asserted by Bean [1].

Then, we have considered vectorial polarizations of electromagnetic fields which can be treated as perturbations of circular polarizations. By using the perturbation technique, we have derived for perturbations a set of coupled linear partial differential equations of parabolic type with variable in time and space coefficients. These coupled PDE's for perturbations inherited some symmetry properties from the unperturbed problem corresponding to the circular polarization of the electromagnetic field. These symmetry properties facilitated the analytical solution of the above PDE's and resulted in the interesting effect of "gradual breeding of higher order harmonics." In the limiting case of  $n \rightarrow \infty$ , we have obtained the vector generalization of the critical state model for superconducting hysteresis.

#### Computation of static magnetic fields in hysteretic media

Several attempts were made in the past to utilize the Stoner-Wohlfarth model in the computation of static magnetic fields in hysteretic media by using the integral equation technique. Then, it was shown that the vector Preisach models had several advantages over the Stoner-Wohlfarth model. Among those advantages are the ability to describe non-symmetrical minor loops, the existence of well-defined identification procedures, and the relative numerical simplicity. This prompted the idea of using the vector Preisach model in computations of magnetostatic fields in media with hysteresis. This idea was realized and the solution of the 2-D magnetostatic problem in hysteretic media was attempted. Recently, more accurate generalized vector Preisach-type models have been developed. For this reason, we have extended the integral equation approach to solve 3-D magnetostatic problems in hysteretic media by utilizing those vector Preisach models. The very nature of the hysteresis phenomenon suggests that the magnetization at any instant of time is dependent on the past history of magnetic fields variations. For this reason, a time step-

ping approach has been used to trace the evolution of the magnetic field at each point of media. At each time step, the problem has been formulated in terms of integral equations and magnetization has been computed iteratively until convergence is achieved.

#### **Development of new ideas which have emerged in the course of our research**

Methods for the solution of 3-D eddy current problems have received considerable attention in past years. This is due to their importance in the design of various devices. It has been gradually realized that different approaches have to be developed for 3-D eddy current problems in the case of bulk (voluminous) conductors and in the case of thin conducting shells.

For the former case, we have developed a new iterative method for the solution of 3-D eddy current problems by using  $\vec{A} - V$ -formulation. This technique has two main advantages. First, the method has a global convergence, which means that it converges for any choice of initial guess. This makes the technique quite robust. Second, numerical implementation of this method requires successive solutions of the Laplace and Poisson equations and, consequently, it can be carried out with minimal new software developments. This technique also has some limitations. First, it is applicable for nonmagnetic conductors. Second, this iteration method converges fast only if the skin depth is comparable with the geometric dimensions of the conductor. Nevertheless, this method may find many practical applications. It is important to note that the mathematical development of the technique brings some new and interesting insights into the nature of eddy current problems. The value of these insights may well extend far beyond the utility of the iterative technique itself. For example, some sharp (i.e., easily computable) estimates (inequalities) for eddy current losses have been derived. These estimates represent a useful by-product of the mathematical machinery developed for the substantiation of the iterative technique. These estimates allow one to obtain some useful information concerning eddy current losses without resorting to laborious eddy current computations.

We have also developed a new technique for the calculation of eddy currents in conductors with small skin depths. This technique is based on impedance boundary conditions. These boundary conditions have been represented in terms of magnetic field. This has led to a magnetic scalar potential formulation of 3-D eddy current problems with small skin

depths. The scalar potential formulation has been then reduced to a weak Galerkin form. The finite element discretization of this form has resulted in two (volume and surface) "stiffness" matrices.

For the case of thin conducting shells, it was suggested in the past to use a scalar stream function for the description of eddy currents in the shells. We have advanced another approach to the calculation of 3-D eddy currents in thin conducting shells. This approach is based on the introduction of special impedance type boundary conditions on shell surfaces. These boundary conditions can be represented only in terms of magnetic field. This leads to a new magnetic scalar potential formulation for 3-D eddy currents in conducting shells. It has been shown that this scalar potential formulation can be reduced to weak Galerkin forms. The finite element discretization of these weak forms results in volume and surface "stiffness" matrices.

Together with Professor S. Belbas from the University of Alabama, we have conducted research on numerical solution of quasi-variational inequalities arising in stochastic game theory. We have studied the finite-difference approximation for the quasi-variational inequalities for a stochastic game involving discrete actions of the players and continuous and discrete payoff. We have proved convergence of iterative schemes for the solution of the discretized quasi-variational inequalities, with estimates of the rate of convergence (via contraction mappings) in two particular cases. Further, we have proved stability of the finite-difference schemes, and convergence of the solution of the discrete problems to the solution of the continuous problem as the discretization mesh goes to zero. We have provided a direct interpretation of the discrete problems in terms of finite-state, continuous-time Markov processes.

The detailed discussion of all the results described above can be found in our published papers. The list of these papers is given below.

#### **Published Papers**

1. A.A. Adly and I.D. Mayergoyz, "Experimental Testing of the Average Preisach Model of Hysteresis," IEEE Transactions on Magnetics, vol. 28, No. 5, 2268, 1992.
2. I.D. Mayergoyz and a.A. Adly, "Numerical Implementation of the Feedback

- Preisach Model," IEEE Transactions on Magnetics, vol. 28, No. 5, 2605, 1992.
3. G. Friedman and I.D. Mayergoyz, "Stoner-Wohlfarth Hysteresis Model with Stochastic Input as a Model of Viscosity in Magnetic Materials," IEEE Transactions on Magnetics, vol. 28, No. 5, 2262, 1992.
  4. A.A. Adly and I.D. Mayergoyz, "A New Vector Preisach-type Model of Hysteresis," Journal of Applied Physics, vol. 73, No. 10, 5824, 1993.
  5. I.D. Mayergoyz and A.A. Adly, "A New Isotropic Vector Preisach Type Model of Hysteresis and its Identification," IEEE Transactions on Magnetics, vol. 29, No. 6, 2377, 1993.
  6. A.A. Adly, I.D. Mayergoyz, R.D. Gomez, and E.R. Burke, "Computation of Magnetic Fields in Hysteretic Media," IEEE Transactions on Magnetics, vol. 29, No. 6, 2380, 1993.
  7. I.D. Mayergoyz and C.E. Korman, "The Preisach Model with Stochastic Input as a Model for Aftereffect," Journal of Applied Physics, vol. 75, No. 10, 5478, 1994.
  8. I.D. Mayergoyz, "2D Vector Preisach Models and Rotational Hysteretic Losses," Journal of Applied Physics, vol. 75, No. 10, 5686, 1994.
  9. I.D. Mayergoyz, "Penetration of Circularly Polarized Electromagnetic Fields into Superconductors with Gradual Resistive Transitions," Journal of Applied Physics, vol. 75, No. 10, 6963, 1994.
  10. M.A. Krosnosel'skii, I.D. Mayergoyz, A.V. Pokrovskii, and D.I. Rachinskii, "Variable States of Continuous Relay Systems," Russian Acad. Sci. Dokl. Math, vol. 47, No. 3, 513, 1994.
  11. I.D. Mayergoyz and G. Bedrosian, "Iterative Solution of 3-D Eddy Current Problems," IEEE Transactions on Magnetics, vol. 29, No. 6, 2335, 1993.
  12. I.D. Mayergoyz and G. Bedrosian, "On Finite Element Implementation of Impedance Boundary Conditions," Journal of Applied Physics, vol. 75, No. 10, 6027, 1994.
  13. Can E. Korman and I.D. Mayergoyz, "The Input Dependent Preisach Model with Stochastic Input as a Model for Aftereffect," IEEE Transactions on Magnetics,

vol. 30, No. 6, 4368, 1994.

14. I.D. Mayergoz, "On Penetration of Electromagnetic Fields into Superconductors with Gradual Resistive Transition," *Journal of Applied Physics*, vol. 76, No. 10, 7130, 1994.
15. I.D. Mayergoz, "On Vector Generalization of the Critical State Model for Superconducting Hysteresis," *Journal of Applied Physics*, vol. 76, No. 10, 6956, 1994.
16. S.A. Belbas and I.D. Mayergoz, "Numerical Solution of quasi-Variational Inequalities Arising in Stochastic Game theory," *Applied Mathematics and Optimization*, vol. 31, 19, 1995.
17. I.D. Mayergoz and G. Bedrosian, "On Calculation of 3-D Eddy Currents in Conducting and Magnetic Sheets," *IEEE Transactions on Magnetics*, vol. 31, No. 3, 1319, 1995.
18. M.A. Krasnoselskii, A.V. Pokrovskii, D.I. Rachinskii, and I.D. Mayergoz, "Operators of Hysteresis Nonlinearities of Vector Relay System Type," *Automatic Control and Remote Sensing*, No. 7, 49, 1994 (in Russian).
19. M. Krosnoselskii, I. Mayergoz and D. Rachinskii, "On Canonical States of Continual Systems of Relays," (*ZAMM*), *Z. angew. Math. Mech.* vol. 75, No. 7, 515, 1995.
20. C.E. Korman and I.D. Mayergoz, "Switching as an Exit Problem," to appear in *IEEE Transactions on Magnetics*, vol. 31, No. 6, 1995.

Copies of the above papers are attached in Appendix II.

During the past four years, our work has achieved the following recognition.

1. According to the Scientific Citation Index, there are 56 citations of my publications in 1992, 86 citations in 1993, and 93 citations in 1994.
2. I have been elected a distinguished lecturer of the Magnetic Society of IEEE for 1994 to give presentations on the topic "Mathematical Models of Hysteresis."
3. I received the Distinguished Scholar-Teacher Award for the 1995-1996 academic year. This is one of the highest honors the College Park campus can bestow on a member of the faculty.

However, the most important recognition of our research work is the acceptance and further scientific development of the Preisach type hysteresis models outside (and beyond) the area of magnetics. It has been always emphasized in our work that it is desirable to separate the Preisach model from its magnetic connotation and treat it as a general mathematical model. This emancipation of the Preisach model from its magnetic origin has resulted in the development of a new mathematical tool which can now be used for the description of hysteresis of any physical nature. This phenomenological and mathematical approach to the Preisach model has also been embraced by mathematicians. References [2] - [8] are selective examples of publications in which the aforementioned approach to the Preisach model has been adopted.