TITLE: SELM HUGURAL PROCEDURES FOR
22 CETI STARS AND AN APPLICATION TO G117-B15A

AUTHOR(S): P.A. BRADLEY

SUBMITTED TO: Conference Proceedings for
"A HALF-CENTURY OF STELLAR PULSATION
INTERPRETATIONS: A TRIBUTE TO
ARTHUR N. COX"
June 16-20, 1997

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
Seismological Procedures for ZZ Ceti Stars and An Application to G 117–B15A

P. A. Bradley

XTA, MS B220, Los Alamos National Laboratory,
Los Alamos, NM 87545

Abstract. In this paper, we outline the procedure for seismological analysis of the ZZ Ceti stars, which are pulsating white dwarfs with hydrogen atmospheres. We use G 117–B15A as the example for this process and derive constraints on the mass and internal structure. The hydrogen layer mass is either about $10^{-4}M_\odot$ or $10^{-7}M_\odot$, depending on whether the $e=1$ mode near 215 s is $k=2$ or $k=1$, respectively. In both cases, the best fitting mass is $0.60M_\odot$, in agreement with spectroscopic log $g$ values.

1. Introduction

The goal of asteroseismology is to derive the internal structure of pulsating stars by comparing the observed pulsation properties with those predicted by theoretical models and pulsation theory. ZZ Ceti stars are pulsating hydrogen atmosphere (DA) white dwarfs, and they hold the key to understanding the internal structure of the predominant spectral class of white dwarf. Until recently, there has been relatively little progress on ZZ Ceti star seismology, because the pulsation spectra are so simple in the hotter variables that there is no good way to choose from the many possible models that fit the observational data. On the other hand, cooler ZZ Ceti stars have more complicated pulsation spectra with numerous harmonics ($nf_1$), frequency sum ($f_1 + f_2$), and frequency difference ($f_1 - f_2$) peaks due to the strongly nonlinear pulse shapes; many of these stars also have pulsation spectra that vary dramatically with time, further complicating mode identification. The problem of mode identification is the crucial obstacle for seismology of all the ZZ Ceti stars.

Here, we study G 117–B15A, which has three observed pulsation modes at 215, 270, and 304 s. This star has an $\ell = 1$ identification for the 215 s mode from multiwavelength photometry (Robinson et al. 1995; Fontaine et al. 1996) G 117–B15A provides a useful test on what asteroseismology can do to constrain the interior structure of a “typical” hot ZZ Ceti star, which only has a few observed modes. If we can derive meaningful constraints on G 117–B15A, then we have a hope for seismology of
other hotter ZZ Ceti stars; if not, then we must find out what is needed to make seismology possible.

2. Modeling Strategy

Bradley (1997) describes the seismology of G 117–B15A and R 548 in considerable detail. We will not repeat this discussion, but rather examine by way of example the methodology of choosing starting points for seismology and refining the model fits.

The first constraints for seismology are the observed effective temperature and the stellar mass as estimated from the surface gravity. Bergeron et al. (1995) is the best reference here, both for completeness and uniformity of analysis. Unfortunately, Koester & Vauclair (1997) show that the real error bars on the temperature and gravity are distressingly large because of the behavior of the hydrogen line profiles near 12,000 K.

Next, we need at least an initial constraint on the $\ell$ and $k$ identification of the observed modes. Robinson et al. (1995) and Fontaine et al. (1996) provide an additional constraint for G 117–B15A by showing that the 215 s mode must be $\ell = 1$. A 215 s $\ell = 1$ period is short enough that it can only be the $k = 1$ or $k = 2$ mode; the hydrogen layer mass is then about $10^{-6}M_\odot$ or $10^{-4}M_\odot$ respectively. G 117–B15A has two additional modes at 270 and 304 s, which Clemens (1994) suggests fit a common mode pattern displayed by all the hotter white dwarfs and Bradley (1996) show that these three modes can be satisfactorily fit as three consecutive overtone $\ell = 1$ modes, although Bradley (1993) discusses model fits involving $\ell = 2$ modes.

To go further, we have to examine the period trends of the modes corresponding to the 270 and 304 s periods when we change various structural parameters one at a time, keeping the stellar mass fixed. Bradley (1996, 1997) presents the results of a parameter search like this. When we apply this to the case of G 117-B15A, the 270 s mode period changes relatively little with changes in the helium layer mass or the structure of the C/O core. By contrast, the 304 s mode is strongly dependent on the extent of the C/O core. The independent behavior of these two modes to changes in the helium layer mass and the C/O core structure means that we can obtain a good initial seismological fit by tuning the fit each mode almost independent of the others. In outline form, the procedure is:

1. Pick a stellar mass and effective temperature consistent with observations.
2. Use a "standard" post-Asymptotic Giant Branch star helium layer mass of $10^{-2}M_\odot$ and a "nominal" C/O profile (see below).
3. Adjust the hydrogen layer mass to obtain an $\ell = 1$, $k = 1$ or 2 mode near 215 s.
4. Adjust the temperature and hydrogen layer mass slightly to bring the 270 s period into agreement.

5. Adjust the C/O profile to make the 304 s mode agree with observations.

6. Refine the fit further and/or perform sensitivity studies by changing the hydrogen layer mass, helium layer mass, and core structure.

To decide which model fits the observed periods the best, we simply average the absolute value of the difference between the observed and theoretical periods, and denote this average by $\Delta$ in the period comparison tables.

3. A Model Fitting Example for G 117–B15A

For G 117–B15A, we use Bergeron et al.’s effective temperature of 11,600 K and $\log g = 7.97$, which implies a mass of $0.59M_\odot$. Because we already have an extensive model grid for $0.60M_\odot$, we use this mass and attempt to match the three observed periods, assuming that the 215 s mode is $k = 1$.

First, we look at plots (or tables) of $\ell = 1$ periods as a function of hydrogen layer mass, looking for where the 215 s period is well matched for $k = 1$ at temperatures near 11,600 K. This occurs when the hydrogen layer mass is near $10^{-7}M_\odot$ (see Table 1). At this point, we also consider the 270 s mode, and the correct period spacing of 55.8 s occurs only when the hydrogen layer mass is $2 \times 10^{-7}M_\odot$. The period spacing is more relevant, because the relatively coarse temperature grid used in the initial sweep of evolutionary models does not guarantee the $k = 1$ mode will be at 215.2 s.

Table 1. 0.60$M_\odot$ models versus G 117–B15A: Vary H Layer mass

<table>
<thead>
<tr>
<th>$M_H/M_\odot$</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$P$ (s)</th>
<th>$P$ (s)</th>
<th>$P$ (s)</th>
<th>$\Delta$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 $\times 10^{-7}M_\odot$</td>
<td>11,620</td>
<td>215.2</td>
<td>271.0</td>
<td>304.4</td>
<td>—</td>
</tr>
<tr>
<td>2.0 $\times 10^{-7}M_\odot$</td>
<td>11,560</td>
<td>213.5</td>
<td>276.3</td>
<td>293.8</td>
<td>5.9</td>
</tr>
<tr>
<td>2.0 $\times 10^{-7}M_\odot$</td>
<td>11,590</td>
<td>213.2</td>
<td>268.0</td>
<td>286.2</td>
<td>7.7</td>
</tr>
<tr>
<td>3.0 $\times 10^{-7}M_\odot$</td>
<td>11,590</td>
<td>213.1</td>
<td>259.6</td>
<td>284.5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

At this point, the first two modes fit, but the $k = 3$ periods are too short for the "nominal" C/O core structure used here. The "nominal" C/O core structure has 20% carbon out to 0.75$M_\odot$, whereupon we impose a linear ramp to pure carbon by 0.90$M_\odot$. This profile captures the essence of the more complicated C/O profiles of Mazzitelli & D’Antona (1986). When we change the core structure, we vary only two parameters. First,
we change the amount of carbon in the core, but leave the linear ramp boundaries intact. Alternatively, we can fix the core carbon abundance at 20% and move the inner boundary point of the linear ramp from the default of 0.75$M_\ast$. Other possibilities exist, but these two parameters capture most of the possible effects on the theoretical periods for a minimum number of additional free parameters.

With this said about the core structure of the models, we fix the hydrogen layer mass at $2 \times 10^{-7}M_\ast$ and vary the core structure (see Table 2). A model with the inner transition point moved out to 0.85$M_\ast$ provides a good fit to all the periods and the model temperature is within 100 K of the spectroscopic temperature. Alternatively, a 50:50 C/O core out to 0.75$M_\ast$ at 11,460 K fits the three periods to an average of better than 1 s. At this point, both models are equally plausible representations for the structure of G 117-B15A, and they say that the core is at least 50% oxygen, consistent with the oxygen mass fraction in the evolutionary models of Mazzitelli & D'Antona (1986a) and Salaris et al. (1997). The predicted C/O transition region may be a bit further in than the above evolutionary calculations suggest, but the core/envelope construction of our models (Wood 1990) does not allow us to put C/O transition regions past about 0.90$M_\ast$.

Table 2. 0.60$M_\odot$ models versus G 117-B15A (215 s is $k = 1$).

<table>
<thead>
<tr>
<th>Fraction C/O</th>
<th>$M_{c/o}/M_\ast$</th>
<th>$T_{\text{eff}}$</th>
<th>(1,1)</th>
<th>(1,2)</th>
<th>(1,3)</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(K)</td>
<td>(s)</td>
<td>(s)</td>
<td>(s)</td>
<td>(s)</td>
</tr>
<tr>
<td>G 117-B15A</td>
<td>11,620</td>
<td>215.2</td>
<td>271.0</td>
<td>304.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20:80</td>
<td>0.85</td>
<td>11,510</td>
<td>214.7</td>
<td>269.2</td>
<td>301.7</td>
<td>1.7</td>
</tr>
<tr>
<td>20:80</td>
<td>0.85</td>
<td>11,430</td>
<td>215.6</td>
<td>269.9</td>
<td>302.9</td>
<td>1.0</td>
</tr>
<tr>
<td>20:80</td>
<td>0.80</td>
<td>11,560</td>
<td>213.8</td>
<td>268.5</td>
<td>293.1</td>
<td>4.7</td>
</tr>
<tr>
<td>20:80</td>
<td>0.75</td>
<td>11,590</td>
<td>213.2</td>
<td>268.0</td>
<td>286.2</td>
<td>7.7</td>
</tr>
<tr>
<td>50:50</td>
<td>0.75</td>
<td>11,590</td>
<td>213.2</td>
<td>268.0</td>
<td>286.2</td>
<td>7.7</td>
</tr>
<tr>
<td>80:20</td>
<td>0.75</td>
<td>11,460</td>
<td>217.3</td>
<td>270.9</td>
<td>304.3</td>
<td>0.8</td>
</tr>
<tr>
<td>80:20</td>
<td>0.75</td>
<td>11,990</td>
<td>213.4</td>
<td>268.5</td>
<td>306.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Having found a set of “best fitting” models at 0.60$M_\odot$ when the 215 s mode is $k = 1$, we would now assume the 215 s mode is $k = 2$ and use the same procedure to determine the structure of the best fitting model(s). Here, the best fitting hydrogen layer mass is about $8 \times 10^{-5}M_\ast$ the helium layer mass is about $10^{-2}M_\ast$, and again there are two equally possible C/O core profiles. One C/O core possibility has the inner transition point at 0.83$M_\ast$ and a 20:80 C/O core, while the other possible core profile has a
50:50 C/O core out to 0.75$M_\odot$. Both profiles fit all three periods to within 2.5 s or better.

Finally, to bracket the observed uncertainties in $T_{\text{eff}}$ and $\log g$, we examine models at the extreme spectroscopic masses, which are 0.55 and 0.65$M_\odot$. Again, we consider the possibility of the 215 s being $k = 1$ or $k = 2$. We derive hydrogen layer masses of $1 \times 10^{-6}M_\odot$ or $1.5 \times 10^{-4}M_\odot$ for 0.55$M_\odot$ models, and values of $5 \times 10^{-8}M_\odot$ or $3 \times 10^{-5}M_\odot$ for 0.65$M_\odot$ models. In all cases, the core is oxygen-rich and the core composition profiles are similar to the 0.60$M_\odot$ profiles.

4. Summary and Conclusions

We briefly illustrate the procedure for seismology of the hotter ZZ Ceti stars by using G 117-B15A as an example. We can fit all three periods quite well, but we cannot distinguish between a $k = 1$ or $k = 2$ 215 s mode based on period fits alone; rather, we will need Rotational fine structure splittings or finding additional periods to do this. We refer the reader to Bradley (1997) for a complete description of the search through parameter space for models that fit the periods of G 117-B15A and R 548, which has similar periods.

5. REFERENCES


Report Number (14) \( \text{LA-UR--97-4012} \)
\( \text{CONF-9706195} \)

Publ. Date (11) \( 1997 \)

Sponsor Code (18) \( \text{NASA, XF} \)

JC Category (19) \( \text{UC-000, DOE/ER} \)