LIMITED WEIGHTS NEURAL NETWORKS: VERY TIGHT
ENTROPY BASED BOUNDS

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Limited Weights Neural Networks: Very Tight Entropy Based Bounds

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Topics: 

- neural networks
- limited (integer) weights
- entropy bounds (number of bits)
- complexity of neural networks
- classification problems

This paper is a perfect fit for the following workshop/tutorial mentioned on the home page of SOCO'97 at "http://www.compusmart.ab.ca/icsc/soco97.htm#Workshop/Tutorials"

Workshop/Tutorial: 

Information Theory and Artificial Neural Networks
1. Statement of the problem

Being given a set of $m$ examples (i.e., data-set) from $\mathbb{R}^n$ belonging to $k$ different classes, the problem is to compute the required number-of-bits (i.e., entropy) for correctly classifying the data-set. Very tight upper and lower bounds for a dichotomy (i.e., $k = 2$) will be presented, but they are valid for the general case.

2. Results achieved

The paper presents an upper bound (tighter than the ones previously known [4, 8]) of:

$$\#\text{bits} < mn \left[ \log \left( \frac{D}{d} \right) + 0.5111 \right] / 2$$

if $|\text{weights}| < \sqrt{2 \left[ \log \left( \frac{D}{d} \right) + 0.5111 \right]}$.

A tight lower bound will also be detailed starting from the bound presented in [10]. In this case $|\text{weights}| < p$ (i.e., integer weights in the range $[-p, +p]$). We improve on the bound detailed there, and show that:

$$\#\text{bits} > \frac{mn}{2} \left[ \log \left( \frac{D}{d} \right) - 1.5359 + \frac{\log n}{n} \right]$$

which clearly gives us:

$$\frac{mn}{2} \left[ \log \left( \frac{D}{d} \right) - 1.5359 \right] < \#\text{bits} < \frac{mn}{2} \left[ \log \left( \frac{D}{d} \right) + 0.5111 \right].$$

3. Significance

- The bounds are proven in a constructive way.
- Although they do not lead to complexity reductions, they should be judged in the context of lowering certain constants for very difficult (i.e., NP-complete or NP-hard) problems [9].
- An interesting aspect is that a constructive algorithm based on the upper bound has already been designed and used to generate both classical Boolean circuits and threshold gate circuits, or a mixture of them [2, 3, 6].
- Work is in progress for designing a constructive algorithm based on the lower bound.

4. Comparison with previous work

A recent result has shown that [4] $\#\text{bits} < mn \left[ \log \left( \frac{D}{d} \right) + 2.0471 \right]$ with weights bounded as $|\text{weights}| < \sqrt{2 \left[ \log \left( \frac{D}{d} \right) + 2.0471 \right]}$. This upper bound has been very recently [8] improved to:

$$\#\text{bits} < mn \left[ \log \left( \frac{D}{d} \right) + 1.8396 \right] / 2$$

with weights bounded as $|\text{weights}| < \sqrt{2 \left[ \log \left( \frac{D}{d} \right) + 1.8396 \right]}$.

A lower bound (but not an absolute one) has been recently [10] detailed for the case when the weights are integers in the range $[-p, +p]$:

$$\#\text{bits} > mn \left[ \log \left( 2pD \right) \right] / 2$$

This bound is consistent with the upper bounds presented in [4,8] as in this case it was proven [10] that $d = 1/2p$, which gives:

$$\frac{mn}{2} \left[ \log \left( \frac{D}{d} \right) \right] < \#\text{bits} < \frac{mn}{2} \left[ \log \left( \frac{D}{d} \right) + 1.8396 \right].$$
Appendix: selected references


