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SUBMITTED TO: DYMAT 97 Proceedings to be published
in J. de Physique
Toledo Spain - Sept 22-26, 1997

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Impact Produced Stress Waves in Composites

By

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Abstract The Nonhomogenized Dynamic Method of Cells (NHDMOC) is used to study the propagation of stress waves through laminates. The accuracy of the theory is tested by applying it to a plate-impact experiment and checking its ability to resolve a propagating shock wave front. The theory is then compared to Lagrangian hydrodynamic calculations, where it is found that the NHDMOC consistently requires less fine spatial and temporal grids, and less artificial viscosity to control numerical noise. The theory is then used to treat the impact of an epoxy-graphite bilaminate. When the viscoelastic properties of the epoxy are accounted for, the theory agrees well with the experiment.

Résumé La méthode dynamique non-homogène de cellules (NHDMOC) est utilisée pour étudier la propagation d'onde de stress au travers laminé. La précision de la théorie est vérifiée par l'application à une expérience d'impact de plaque et de comparant son abilité à résoudre la propagation d'une onde de choc. La théorie est aussi appliquée aux calculs hydrodynamique Lagrangien, où on trouve que NHDMOC demande de façon consistante des réseaux spatiaux et temporels moins fin et aussi moins de viscosité artificiel à fin de contrôler le bruit numérique. La théorie est alors utilisé pour traiter l'impact de graphite-epoxy bilaminé. Quand on compare les propriétés viscoélastique de l'époxy, la théorie est en agrément avec les expériences.

1. INTRODUCTION

Because of their engineering applications, there is considerable interest in understanding the mechanical properties of composites. The method of cells provides a powerful, computationally efficient method for studying these systems. There are two different versions of the method of cells: the homogenized version, abbreviated here by MOC, and a nonhomogenized dynamical version (NHDMOC). Both are attributed to J. Aboudi [1,2].

The MOC theory is a micromechanical analysis based on the assumption that a single representative volume element (RVE) can be identified that repeats itself throughout the space of the entire composite. Obviously, such an approach is best suited for composites having microstructure that is nearly periodic. The MOC theory is based on conditions of stress and displacement continuity at all interfacial boundaries associated with the RVE, plus conditions of equilibrium. One obtains an effective stiffness matrix, in terms of the microstructural parameters, as a result of carrying out the MOC analysis. Since the homogenized version is frequently applied to complex three-dimensional problems, computational tractability forces approximations to be made that limits its utility to problems where deformational variations occur over a length scale that are long compared to the variations in the microstructure. The MOC theory can be used in conjunction with a hydrodynamical calculation. In this context, the MOC theory determines the effective stiffness matrix and the stress, at all points in the homogenized space, given the hydrodynamic strain rates at those points.

The NHDMOC theory is the object of our study in the present work. A brief review of the NHDMOC theory is given in the next section. One begins the NHDMOC formalism by expanding the particle displacement field in a Legendre series. In practice, the expansion must be truncated at some order in the Legendre polynomials. For example, Aboudi [2] found that the second-order theory, which uses the expansion truncated at second order, gives a good representation of the stress-wave profile for a dynamically loaded bilaminate.

Our goal in this paper is to provide an explanation of why the NHDMOC is a viable method for investigating wave propagation in shock loaded materials, especially laminates. The merits of the method, compared to direct applications of standard hydrodynamic calculations, will be stressed. In section 3, we address a fundamental question regarding the particle displacement expansion, namely, how fast does the expansion converge? We supply a partial answer to this question by carrying out a comparative study of the first-, second-, and third-order NHDMOC theories.

Next, in section 4, we will compare the third-order NHDMOC theory with the results generated from a standard Lagrangian hydrodynamic calculation. We emphasize that the physical content of the NHDMOC and hydrodynamic theories is essentially identical; in both the same constitutive relations and equations of motion are used. The difference in the two approaches lies in the numerical solution scheme. It is there that different approximations for the two theories lead to somewhat different numerical results.

Finally, having tested the NHDMOC theory, in section 5, the second-order theory is used to model a viscoelastic epoxy-graphite bilaminate. The results are compared to a plate-impact experiment done on that system.

2. NHDMOC THEORY

We begin this section with a review of the NHDMOC theory. This is followed by a brief discussion of the equations solved in the hydrodynamical theory. In past implementations of the NHDMOC theory [2,3], artificial viscosity has been omitted but here it is incorporated into the analysis because it has a non-negligible effect in reducing the numerical ringing in the hydrodynamical calculations.

For the purposes of testing these theories we will focus on using them to model plate-impact experiments. A schematic of the flyer-target setup for these experiments is shown in Fig. 1. We consider the case where the impact faces are parallel to each other and lateral dimensions of the system are large. Then, any particle motion in lateral directions can be ignored prior to the arrival of edge refraction waves. Consequently, the displacement and strain can be treated as uniaxial.

Figure 1. Experimental setup modeled in this work.

A suitable spatial grid must be chosen in the NHDMOC theory. The spatial region between two adjacent grid points will be referred to as a cell. Letting p denote the p^{th} cell, the continuous local spatial coordinate, measured from the origin of that cell, will be denoted by $\bar{x}_p$. The NHDMOC method uses a Legendre expansion in the particle displacement,

$$u^{(p)}(\bar{x}_p, t) = \sum_{l=0}^{\mathcal{N}} U_l^{(p)}(t) P_l(2\bar{x}_p/d_p). \quad (1)$$

Here, d_p is the width of cell p , and all the time dependence in the particle displacement is now contained in the *cell coefficients*, $U_l^{(p)}(t)$. $\mathcal{N}$ is the order of the expansion. The particle displacement and strain are completely known once the cell coefficients have been determined. To determine the cell coefficients we will invoke stress and displacement continuity between all cell boundaries, plus the equation of motion (and any relevant constitutive equations). The equation of motion (EOM) relates the stress gradients to the particle accelerations $\ddot{u}^{(p)}(\bar{x}_p, t)$:

$$\frac{\partial}{\partial \bar{x}_p} [\sigma^{(p)}(\bar{x}_p, t) + q^{(p)}(\bar{x}_p, t)] = \rho_p \ddot{u}^{(p)}(\bar{x}_p, t). \quad (2)$$

For cell p , ρ_p is the mass density, $\sigma^{(p)}(\bar{x}_p, t)$ is the stress, and the additional term $q^{(p)}(\bar{x}_p, t)$ represents the artificial viscosity, discussed above and used to reduce unphysical numerical ringing of the velocity and stress solutions.

The EOM, along with stress and displacement continuity between neighboring cells, *i.e.*,

$$u^{(p-1)}(+\frac{d_{p-1}}{2}, t) = u^{(p)}(-\frac{d_p}{2}, t) \quad (3)$$

$$\sigma^{(p-1)}\left(+\frac{d_{p-1}}{2}, t\right) = \sigma^{(p)}\left(-\frac{d_p}{2}, t\right), \quad (4)$$

is not sufficient to determine the cell coefficients. We need to supplement these equations and this is done by multiplying both sides of the EOM by $\bar{x}_p^m$ ($m \leq \mathcal{N}$), and integrating the result over the p^{th} cell. In the third-order theory, for example, we can generate four more independent equations by using the $m = 0, 1, 2$, and 3 equations. We refer to these equations as the moment integrals.

For purposes of studying the rate of convergence of the particle displacement expansion, and making comparisons with the hydrodynamic calculations, we will use a simple, linear elastic constitutive relation,

$$\sigma^{(p)}(\bar{x}_p, t) = E_p \epsilon^{(p)}(\bar{x}_p, t), \quad (5)$$

to relate the p^{th} cell stress to its strain, $\epsilon^{(p)}(\bar{x}_p, t)$, where,

$$\epsilon^{(p)}(\bar{x}_p, t) = \frac{\partial u^{(p)}(\bar{x}_p, t)}{\partial \bar{x}_p}. \quad (6)$$

Here, E_p is the elastic moduli for the material in cell p ; it is related to the longitudinal sound velocity by $c_L = \sqrt{E_p/\rho_p}$. In section 5, viscoelastic materials are considered and Eq. (5) will be duly modified.

It remains to substitute the particle displacement expansion into the stress-strain relation. The result of this manipulation is then substituted into the moment integrals. Similarly, the second time-derivative of the displacement equation can be substituted into the acceleration term of the moment integrals.

A standard form, in the literature, for the artificial viscosity is constructed from linear and quadratic strain rate dependent terms. Since the linear term is known to encompass much of the effects of the quadratic term, here we will simply use it as:

$$q^{(p)}(\bar{x}_p, t) = \eta_p d_p \sqrt{E_p \rho_p} \dot{\epsilon}(\bar{x}_p, t). \quad (7)$$

In a typical hydrodynamical calculation of shock-wave propagation, the "adjustable" parameter η_p has a value in the neighborhood of 0.1 - 0.5. The relative size of the parameter η_p needed to achieve satisfactory, stable numerical solutions will provide one of our indicators of the overall goodness of the numerical method.

We now outline the algorithm for solving these equations. For each (internal) cell there are four independent equations in the third-order theory. The entire set of equations can be mapped onto a matrix equation of the form,

$$\mathbf{A}\ddot{\mathbf{U}}(t) + \mathbf{B}\dot{\mathbf{U}}(t) = \mathbf{R}(t), \quad (8)$$

where the zeroth, first and second time derivatives of the cell coefficients have been separated into individual matrices. Matrices, $\mathbf{A}$ and $\mathbf{B}$ are $4N \times 4N$ parameter matrices constructed from the d_p and ρ_p . The column matrix $\dot{\mathbf{U}}(t)$ and $\ddot{\mathbf{U}}(t)$ are composed of first- and second-time derivatives of the cell coefficients, respectively. The matrix $\mathbf{R}(t)$ consists of all terms in these equations containing no time derivatives of cell coefficients. Representing the first and second time derivatives as finite differences,

$$\frac{\mathbf{A}}{(\Delta t)^2} [U(t + \Delta t) - 2U(t) + U(t - \Delta t)] + \frac{\mathbf{B}}{2\Delta t} [U(t + \Delta t) - U(t - \Delta t)] = \mathbf{R}(t), \quad (9)$$

solving for $U(t + \Delta t)$, and then stepping through in time in increments of Δt , the solution of the entire set of differential equations is obtained. This is the only finite-difference equation encountered in the NHDMO theory. Initial conditions associated with the impact interface follow from displacement and stress continuity applied to the cells boundaries near and at the impact interface.

3. CONVERGENCE OF THE DISPLACEMENT EXPANSION

Having reviewed the essential formalism, we now address the first question raised in section 1 on how fast the displacement expansion converges. We provide a partial answer to this by investigating the degree to which the various orders of the NHDMO can resolve rapid spatial variations in the stress. The stress profile at the shock wave front is a good example. We model the symmetric impact of an 0.20 cm homogeneous steel flyer striking a 0.20 cm homogeneous steel target. The mass density and moduli are 7.90 g/cm³ and 1.1258 Mbar, respectively. The impact velocity is 1 km/s.

In Fig. 2, a plot of the stress wave shock front is shown as a function of distance along the target. The impact occurred at 0.0 cm, and the elapsed time over which the wave has propagated into the target is 0.4 μ s. We limited the number of curves to only those cell widths that are near the point of resolving the wave front.

To resolve the shock front, the third-order theory requires significantly courser gridding than in second order, and moreover in the first-order theory. Computationally, this means that although a first-order theory requires time stepping through a 2N set of equations, in contrast to the 4N set of equations in third-order, the running time for the two calculations is not significantly different. For a slightly finer gridding, the second-order theory gives a respectable representation of the stress wave, while the first-order theory gives only qualitative agreement. The closeness of the second- and third-order theories indicates that the particle displacement expansion converges rapidly. The results in this section were generated without the use of artificial viscosity.

Figure 2. NHDMOCT calculation of the stress in the steel target.

4. COMPARISON WITH HYDRODYNAMIC CALCULATIONS

We now turn to a brief discussion of the finite-difference equations solved in the hydrodynamical theory. We make no attempt to discuss the complexities encountered in advanced finite-difference algorithms, nor do we introduce any complex constitutive relations into our discussion (plastic flow in metals, cracking in ceramics, etc.). In fact, the constitutive equation we use in the hydrodynamical calculations is completely equivalent to Eq. (5). Furthermore, the same expression for the artificial viscosity is used.

The EOM is most conveniently written in terms of the particle velocity $v^{(p)}(x_p, t)$, rather than the displacement. Expressed as a finite difference, the EOM has the form

$$v^{(p)}(x, t + \Delta t) = v^{(p)}(x, t) + \frac{\Delta t}{\rho_p d_p} [\sigma^{(p)}(x + \Delta x, t) - \sigma^{(p)}(x, t) + q^{(p)}(x + \Delta x, t) - q^{(p)}(x, t)] . \quad (10)$$

With the velocity updated by Eq. (10), the strain-rate follows from

$$\dot{\epsilon}^{(p)}(x, t) = \frac{1}{d_p} [v^{(p)}(x + \Delta x, t) - v^{(p)}(x, t)] . \quad (11)$$

Thus, in the hydrodynamic calculation, finite-difference solutions in both space and time are used. This in contrast to the single finite-difference solution used in the NHDMOCT theory.

In Fig. 3, the velocity profiles, obtained by the third-order NHDMOCT and hydrodynamic calculations, are shown. Here, a 0.10 cm homogeneous steel flyer struck a 0.30 cm homogeneous steel target, moving at a speed of 1 km/s. The stress is monitored at the midpoint in the target. The NHDMOCT results were generated using 40 equal-spaced cells. By going to 80 cells, it was determined that the 40 cell profile was converged. A time step of 0.001 μ s was sufficient in the NHDMOCT calculation, whereas 0.0001 μ s was needed in the hydrodynamic calculation. With zero artificial viscosity, the

NHDMOC theory remains giving stable solutions, although small numerical ringing is observable. By increasing η from zero to a value of 0.006 we could completely remove these oscillations. By comparing the two calculations, it is apparent that the hydrodynamical calculation requires finer spatial and time grids to achieve the same resolution of the wave profiles. Also, in the hydrodynamical calculation, larger values of artificial viscosity are needed to reduce the numerical ringing to a tolerable amount. (The figure was generated with $\eta = 0.1$.) At 400 cells, the hydrodynamic and NHDMOC calculations (the later used 40 cells and $\eta = 0.006$) essentially match. A 800 cell hydrodynamical calculation revealed little change over the 400 cell one, indicating that convergence had been reached by 400 cells. The hydrodynamical velocity profiles for $\eta = 0.01$ were also calculated. Even by increasing the cell number to 1600, the amplitude of the ringing was 10% of the velocity profile maxima. No substantial improvement was observed by going to still finer spatial and temporal grids. It is obvious that one must use artificial viscosity much more liberally to achieve satisfactory results in the hydrodynamical calculation. The fact that smaller artificial viscosity is required in the NHDMOC theory is very important when the theory is applied to complex heterogeneous structures. There multiple reflections and transmission at all true material boundaries will produce true physical high-frequency oscillations that should not be, *a priori*, damped out of the solutions.

Figure 3. NHDMOC and Hydrodynamic calculations of the particle velocity.

5. APPLICATION TO AN EPOXY-GRAPHITE BILAMINATE

We now apply the second-order NHDMOC theory to model the graphite-epoxy bilaminate shown in Fig. 4. The flyer consists of a 1.557 mm Z-cut quartz impactor backed by a 5 mm PMMA slab. The bilaminate has 19 bilayers: one layer of each bilayer is an epoxy-graphite mixture, and the second, much thinner layer, is pure epoxy. The graphite, in the mixture, is distributed uniformly throughout the layer. The bilaminate is backed by a 5 mm PMMA window. In the experiment, VISAR was used to measure the particle velocity record at the bilaminate-PMMA interface. The impact velocity of the flyer was 0.5003 km/s. The mass densities of the PMMA, Z-cut quartz, epoxy, and epoxy-graphite mixture are taken to be, 1.15, 2.65, 1.27, and 1.63 g/cm³, respectively.

To model this problem, we must modify Eq. (5) to include large strains and viscoelastic components. We use a stress-strain constitutive law that contains second-order strain terms. The viscoelastic materials will be treated by introducing a relaxation time τ_p into a *dynamic* elastic stress-strain equation, *via* the "standard viscoelastic-solid model":

$$\begin{aligned} \sigma^{(p)}(\bar{x}, t) = & \left(E_p \delta_{t,t'} + \left[\frac{M_p - E_p}{\tau_p} \right] \int_0^t dt' e^{(t'-t)/\tau_p} \right) \epsilon^{(p)}(\bar{x}, t') \\ & + \frac{1}{2} \left(E'_p \delta_{t,t'} + \left[\frac{M'_p - E'_p}{\tau_\alpha} \right] \int_0^t dt' e^{(t'-t)/\tau_p} \right) (\epsilon^{(p)}(\bar{x}, t'))^2. \end{aligned} \quad (12)$$

Each integral acts on the $\epsilon^{(\alpha,p)}(\bar{x}, t)$ to the right of it. The first- and second-order moduli are denoted by E_p and E'_p , and the corresponding relaxed moduli by M_p and M'_p . The first-order moduli for the

PMMA, Z-cut quartz, epoxy, and epoxy-graphite mixture are taken to be, 0.089, 1.072, 0.081, and 0.146 Mbar, respectively. Similarly, the second-order moduli are -0.72, -5.83, -0.32, and -0.88 Mbar. The relaxed moduli for the PMMA and epoxy have values of 75% and 90% that of their unrelaxed values and their relaxation times are 0.01 and 0.02 μ s, respectively.

Figure 4. Setup of the epoxy-graphite bilaminate experiment.

In Fig. 5, the experimental and theoretical particle velocity profiles are shown. The origin of the time axis is placed at the instant of impact determined by the theory. In both the experiment and the theory, particle velocities were recorded at the composite-window interface. Overall, the agreement between the NHDMO C theory and the experiment is quite satisfactory. Future work consists of carrying out this calculation to third-order, plus implementing artificial viscosity to reduce the unphysical numerical ringing observed, in Fig. 5, ahead of the shock front.

Figure 5. Particle velocity from experiment and the NHDMO C theory.

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