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INTRODUCTION

Anomalous transport in toroidal plasmas arises from the presence of more complicated magnetic and electric fields than are required by the macroscopic equilibrium of the plasma. If one knew these fields, then the transport coefficients, calculated by classical methods, would be correct except for the direct transport of energy by fields (which is usually considered negligible). The limitation of this procedure is the lack of knowledge of the field perturbations, but there are a number of features that are generally accepted whether the perturbations are due to microinstabilities or due to macroscopic effects such as tearing modes. In particular, the frequencies and the wave numbers of perturbations satisfy $\omega \ll \omega_c$, $k_\perp \rho_i \ll 1$, and $k_\parallel \rho_i \ll 1$ which imply that the magnetic moments of both the ions and the electrons are conserved and that the particle trajectories obey drift equations. The drift equations imply that if magnetic surfaces exist, particles can cross a surface only if there is a spatial variation in the field strength or the electric potential in the surface. If surfaces fail to exist, then particles can also cross the toroidal flux surfaces by moving along the field lines.

An important example of the type of problem which can be studied using these ideas is sawtooth activity of a tokamak. Until a few years ago, Kadomtsev mechanism /1,2/ was thought to explain the data on sawtooth. However, sawteeth on large tokamaks, such as TFTR and JET, have behaved in a manner which is inconsistent with this model /3,4/. By observing the relaxation times during a sawtooth crash of different types of particles, one should be able to determine the fundamental mechanisms that cause the sawtooth. This mechanism could be any of the three types: an ideal instability, which means a distortion of the pressure surfaces, but no

breaking of the surfaces; electrostatic, which means that the pressure surfaces remain fixed, but the electrical potential varies in the surfaces; tearing, which means that the pressure surfaces break and interconnect. One can numerically study the relaxation of various groups of particles that can be followed experimentally; runaway electrons, thermal electrons, passing beam-injected ions, trapped beam-injected ions, thermal ions and neutron producing ions; to find clear signatures for the various mechanisms.

Measurements of transport of various groups of particles can also help determine the nature of field perturbations that are responsible for the transport. If the transport coefficients for the particles and electron heat are comparable, then the ions must be affected comparably to electrons by the perturbations. Cross terms, such as the transport of ion energy by convection, give information on which particles in a roughly Maxwellian distribution of energies are primarily causing the transport. (The use of $5/2$ for this number is purely conventional, but one would expect the number to be positive and of order of magnitude of unity). The differences in the transport coefficients between high and low energy particles, such as beam and bulk ions, or runaway and bulk electrons, help determine whether magnetic or electric potential perturbations are responsible for transport. Enhanced transport of passing particles, such as beam ions, relative to the trapped particles is indicative of the loss of surfaces. The presence or absence of anomalous current profile spreading also addresses the issue of surface existence. If all plasma transport coefficients are approximately equal, then one explanation would be intermittent stochasticity of field lines. In this model, each magnetic surface in the plasma is broken during the course of confinement time, but at any particular time only a fraction of surfaces are

broken. Theoretical calculations, using Monte Carlo as well as analytic calculations, could clarify the level and type of perturbations that are required to obtain observed features of plasma transport.

TRANSPORT MECHANISMS

Since both the microscopic and MHD instabilities associated with toroidal plasmas satisfy $k_{\parallel} \rho_i \ll 1$, these instabilities conserve the magnetic moment μ of both the ions and the electrons. If the constant pressure surfaces are labelled by the toroidal flux ψ contained in the surface, there are only three ways in which μ conserving particles can cross such surfaces:

- (1) spatial variation of magnetic field strength in the surface, $B(\psi, \theta, \phi, t)$.
- (2) spatial variation of electric potential in the surface, $\Phi(\psi, \theta, \phi, t)$.
- (3) spatial variation in the poloidal flux outside the surface, $\chi(\psi, \theta, \phi, t)$.

The third mechanism is equivalent to the loss of magnetic surfaces.

FIELD STRENGTH VARIATION:

This factor is responsible for neoclassical and ripple transport in tokamaks and enhanced transport in stellarators /5/. Drift velocities for high energy particles are higher than for bulk particles. On surfaces which are not rational, the effect on the passing particles is small; trapped particles are affected considerably and they dominate the transport.

On rational surfaces, both trapped and passing particles are affected. In MHD theory, a pressure gradient cannot exist unless $\oint dl/B$ is the same on every field line of a rational surface.

The θ and ϕ dependence of B leads to Pfirsch-Schluter currents. If the shape of a pressure surface is perturbed, the resulting P-S currents may

reinforce the perturbation. Such effects are responsible for pressure-driven ideal MHD instabilities /6/.

The change in B due to a distortion δr in the surface is small,
 $\delta B/B \approx \delta r/R = 10^{-2}$.

ELECTRIC POTENTIAL VARIATION:

Electric potential variation on pressure surface causes the EXB drift. At a given point on the surface, the drift velocity is same for all particles, but passing particles generally average out the EXB drift.

It is not possible to determine locally whether a given electric field:
 (1) is associated with the distortion of the pressure surface. Surface distortions should be considered as modifying $B(\psi, \theta, \phi, t)$ in transport calculations.

(2) gives a drift of particles out of the pressure surface.

On non-rational surfaces, electric field in a surface is associated with a parallel electric field $B \cdot \nabla \Phi$. For periodicity implies

$$\Phi = \sum_{nm} \Phi_{nm}(\psi) \exp[i(n\phi - m\theta)] \quad (1)$$

and

$$B \cdot \nabla \Phi = B \cdot \nabla \left[\left(\frac{\partial \Phi}{\partial \theta} \right) + q \left(\frac{\partial \Phi}{\partial \phi} \right) \right]. \quad (2)$$

The importance to transport of variation in the potential in a surface is measured by $\delta \Phi/T$. It is generally thought that $\delta \Phi/T \approx 10^{-2}$ for microinstabilities in the central part of plasma /7/.

LOSS OF MAGNETIC SURFACES:

In magnetic coordinates, the poloidal flux is a function of the toroidal

flux ψ only, $\chi(\psi)$. Imperfect surfaces can be represented by adding θ and ϕ dependence to $\chi/8/$. Trapped particles are not affected by imperfect surfaces; only passing particles are. High energy passing particles, particularly electrons, are most affected. The characteristic level of transport associated with loss of surfaces is $D \propto 1/m^2$ with m the poloidal mode number of the perturbation.

For the mode numbers associated with MHD instabilities the transport is large, but could be much smaller if the loss of surfaces is intermittent. Intermittent loss of surfaces could appear as comparable transport rate for heat, runaway electrons, particles, impurities and momentum. It would also affect the transport of poloidal flux, but weakly affect the overall resistance of plasma.

EXPERIMENTAL MEASUREMENTS

Experimentally, the important measurements are the transport of:

- (1) high energy versus the bulk particles
- (2) passing versus trapped particles
- (3) magnetic flux versus heat
- (4) heat versus particles.

Cross terms in the transport equations also provide important information.

PARTICLE DRIFT EQUATIONS IN NON-CANONICAL CO-ORDINATES:

When the magnetic moment μ is conserved, the particle trajectories can be most efficiently followed using the drift Hamiltonian

$$H(\theta, P_\theta, \phi, P_\phi) = \frac{1}{2} v_{\parallel}^2 + \mu B + q\Phi \quad (3)$$

The canonical co-ordinates are a poloidal angle θ and toroidal angle ϕ and

their conjugate momenta

$$P_\theta = Iv_{\parallel}/B + C_p \psi \quad (4)$$

$$P_\phi = gv_{\parallel}/B - C_p \chi_{\text{total}} \quad (5)$$

where

$$\chi_{\text{total}} = \chi + \chi_{\text{axis}}(t). \quad (6)$$

The functions $g(\chi)$ and $I(\chi)$ are the poloidal current outside and the toroidal current inside a toroidal flux surface. Note that these equations are in dimensionless form. See Table 1.

Now we define the vector $X_{\text{canonical}}$ of canonical co-ordinates

$$X_{\text{canonical}} = (\theta, \phi, P_\theta, P_\phi, t) \quad (7)$$

and another vector $X_{\text{practical}}$ of non-canonical, practical co-ordinates

$$X_{\text{practical}} = (\theta, \phi, v_{\parallel}, \chi, t). \quad (8)$$

We make a transformation from the canonical co-ordinates to the practical co-ordinates

$$X_{\text{canonical}} \rightarrow X_{\text{practical}} \quad (9)$$

and the particle drift equations of motion then become /11/

$$\frac{dv_{\parallel}}{dt} = \frac{1}{\left(\frac{\partial P_\theta, P_\phi}{\partial v_{\parallel}, \chi} \right)} \left[\frac{1}{2\pi} \frac{\partial H, P_\theta}{\partial \chi, \theta} - \frac{1}{2\pi} \frac{\partial H, P_\theta}{\partial \chi, \phi} - \frac{\partial P_\theta, P_\phi}{\partial t, \chi} \right] \quad (10)$$

$$\frac{d\chi}{dt} = - \frac{1}{\left(\frac{\partial P_\theta, P_\phi}{\partial v_{\parallel}, \chi} \right)} \left[\frac{1}{2\pi} \frac{\partial H, P_\phi}{\partial \chi, \theta} - \frac{1}{2\pi} \frac{\partial H, P_\phi}{\partial v_{\parallel}, \phi} - \frac{\partial P_\theta, P_\phi}{\partial t, v_{\parallel}} \right] \quad (11)$$

$$\frac{d\theta}{dt} = \frac{1}{2\pi} \frac{1}{\left(\frac{\partial P_\theta, P_\phi}{\partial v_{\parallel}, \chi} \right)} \left[\frac{\partial H, P_\phi}{\partial v_{\parallel}, \chi} \right] \quad (12)$$

$$\frac{d\phi}{dt} = - \frac{1}{2\pi} \frac{1}{\left(\frac{\partial P_\theta, P_\phi}{\partial v_{\parallel}, \chi} \right)} \left[\frac{\partial H, P_\theta}{\partial v_{\parallel}, \chi} \right] \quad (13)$$

and

$$\frac{dH}{dt} = \frac{1}{\left(\frac{\partial P_{\theta, P\phi}}{\partial v_{\parallel, X}} \right)} \left[\frac{\partial H}{\partial t} \frac{\partial P_{\theta, P\phi}}{\partial v_{\parallel, X}} - \frac{\partial H}{\partial v_{\parallel}} \frac{\partial P_{\theta, P\phi}}{\partial t, X} + \frac{\partial H}{\partial X} \frac{\partial P_{\theta, P\phi}}{\partial t, v_{\parallel}} \right] \quad (14)$$

where

$$\frac{\partial u, v}{\partial x, y} = \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x}. \quad (15)$$

This is the set of equations that we use in our Monte-Carlo codes. The functions for determining drift trajectories are:

$B(x, \theta, \phi, t)$ for ideal instabilities,

$\Phi(x, \theta, \phi, t)$ for electrostatic instabilities,

and

$\psi(x, \theta, \phi, t)$ for resistive instabilities.

The effect of loop voltage is included using the equation

$$\frac{dx_{axis}(t)}{dt} = cV_{loop} \quad (16)$$

which gives

$$x_{axis}(t) = \text{constant} \cdot t. \quad (17)$$

Therefore,

$$x_{total} = x + x_{axis}(t). \quad (18)$$

There are obvious advantages to using the practical non-canonical co-ordinates. Also, one can test the correctness of results by comparing the change in energy in two independent ways /11/. For example, when $V_{loop} \neq 0$, $\Delta E = q\Delta\phi V_{loop}$, and ΔE is also obtained from the equation dH/dt .

MONTÉ CARLO METHOD:

Monte Carlo runs can ascertain the level of transport that one expects from various models of microinstabilities. The usual models of transport due

to microinstabilities do not take into account the complexities of particle drift orbits. Monte Carlo method can remedy this problem. Monte Carlo codes can also give values for cross terms in the transport coefficients, such as transport of energy by convection.

In our Monte Carlo method, a large group of particles is followed using at each time step the equations of motion (eqs. (10) to (14)) followed by a stochastic perturbation that represents the effects of collisions over that time-step. Due to the presence of collisional effects in the calculation, the group of particles tends to relax to a Maxwellian distribution and diffuse across the magnetic field. In this way, the Monte Carlo code is used to determine transport coefficients in fields of arbitrary complexity.

We use the Monte Carlo method developed by Boozer and Petravic for evaluating transport coefficients in asymmetric, toroidal geometries /9,10/. Unlike analytical theory, this method does not require an analytically tractable, simple expression for drift orbits of the particles. In this approach, exact numerical evaluation of drift orbits is made. Effects of Coulomb scattering is included by the appropriate random evolution of constants of motion. The magnetic field enters the calculations only as the magnitude of B . Coupled to this is the advantage that we employ a combination of magnetic and practical, non-canonical co-ordinates. This simplifies the study of transport. The method allows comparison with analytical results as well as the evaluation of cross terms in the transport equations. Three basic elements of a Monte Carlo method are: particle trajectory calculations, collision operator, and diffusion coefficient evaluation.

COLLISION OPERATOR:

In our Monte Carlo method, the Lorentz collision operator is represented by the random sequence /9/

$$\lambda_n = \lambda_0(1 - \nu_d \tau) \pm [(1 - \lambda_0^2) \nu_d \tau]^{1/2}, \quad (19)$$

where $\lambda = v_{\parallel}/v$, ν_d is the deflection collision frequency. To obtain good representation of Lorentz operator, we take $\nu_d \tau \ll 1$ where τ is the time step of integration. Criterion for determination of step-size is discussed later. This restriction assumes that a particle suffer several scatterings before its pitch changes significantly.

A Monte Carlo equivalent energy scattering operator is given by /a/

$$E_n = E_0 - (2\nu_E \tau) [E_0 - (3/2 + (E/\nu_E)(d\nu_E/dE))T] \pm 2[TE_0(\nu_E \tau)]^{1/2} \quad (20)$$

where, for ion scattering on ion

$$\nu_E = 3(\pi/2)^{1/2} \nu_B [\Psi(x)/x]. \quad (21)$$

ν_B is Braginskii collision frequency, and $\Psi(x)$ is error function.

DIFFUSION COEFFICIENT:

If $f(E, \psi)$ is normalized and highly localized in ψ , then one can show that /9/

$$\langle D \rangle = (1/2) (d/dt) (\langle \psi^2 \rangle - \langle \psi \rangle^2). \quad (22)$$

This is to be interpreted as: Let j th particle be located at position ψ_0 at time zero. Then $\psi_j(t_0)$ is its radial position at time t_0 . This position is determined by following the drift motion of the particle with appropriate pitch angle scattering after each time step. If one defines

$$D_j = (1/2t_0) (\psi_j - \psi_0)^2 \quad (23)$$

then the estimate of D from J particles is

$$D_* = (1/J) \sum D_j \quad (24)$$

Index j varies from 1 to J . The statistical distribution of D_j is chi-square distribution with one degree of freedom. Practical choice for t_0 is one collision time or the time to cover the surface. Each Monte Carlo run is made with

sufficiently large number of particles with fixed ψ_0 and E , but with different pitch $v_{\parallel}/v$, θ_0 and ϕ_0 . Thus, several of D_j 's can be determined by following these particles and using equation (23).

COMPUTATIONAL DETAILS:

Our Monte Carlo codes use a fourth order Runge-Kutta integrator. The step-size is determined by requiring that $|\Delta E_n/E_n| \leq 10^{-9}$ when energy is conserved, and $|(E_n - E'_n)/E_n| < 10^{-9}$ when energy is not conserved. Here E_n is the energy at n^{th} time step obtained from the expression for Hamiltonian (equation (3)) and E'_n is the energy at n^{th} step obtained from the solution of equations of motion, equation (14). In the presence of pitch angle scattering, a new value for magnetic moment μ is obtained using equations (3) and the expression $\lambda = v_{\parallel}/v$.

The codes that we have developed have numerous switches. For example, we can choose to have random phases for magnetic perturbations or may choose to supply them as input, and so on.

FIELD LINE TRAJECTORIES /14/:

To study the trajectories of magnetic field lines, we use the Hamilton's equations of motion for the magnetic field lines with Hamiltonian

$$\psi(\chi, \theta, \phi) = \psi_0(\chi) + \sum a_{mn}(\chi) \cos[2\pi(m\theta - n\phi + \delta_{mn})]. \quad (26)$$

Here $\psi_0(\chi)$ is the unperturbed Hamiltonian for the field line, and the second

term represents the magnetic perturbations which may lead to breaking of flux surfaces. Note that the unperturbed toroidal flux depends only on poloidal flux χ . Hamilton's equations of motion for field line trajectories are

$$\frac{d\chi}{d\theta} = - \frac{\partial \psi}{\partial \theta} \quad (27)$$

$$\frac{d\phi}{d\theta} = \frac{\partial \psi}{\partial \chi} \quad (28)$$

$$\frac{d\psi}{d\theta} = \frac{\partial \psi}{\partial \theta} \quad (29)$$

Later on we will discuss the resistive break up of flux surfaces. To study this phenomena, we solve the above equations numerically using fourth order Runge Kutta integrator. For the case of unperturbed flux surfaces, we use the conservation of the field line Hamiltonian to determine the step-size. We require that $|\Delta\psi_n/\psi_n| \leq 10^{-9}$ where ψ_n is the Hamiltonian at nth step. When magnetic perturbations are present, the criterion for determining the step-size is $|(\psi'_n - \psi''_n)/\psi'_n| \leq 10^{-9}$, where ψ'_n is the nth step field line Hamiltonian obtained from equation (20), and ψ''_n is the nth step Hamiltonian obtained from solution of equation (29). The poicare surfaces of section are made by intersection of field line with the $\phi = 0$ plane. The points of intersection are determined by interpolation.

The code that we have developed to study the break-up of flux surfaces is called FIELD4.

Now we turn to the application of our method to two toroidal devices: Reversed Field Pinch ZT-40 and tokamak TOKAPOLE II.

APPLICATION TO ZT-40/11,12,14/:

In order to apply above ideas regarding anomalous transport in toroidal device to the ZT-40, we use the Bessel function model to represent the equilibrium fields and currents in RFP ZT-40.

$$B = \text{constant } J. \quad (30)$$

This leads to the following expressions:

$$\psi(\chi) = a_1\chi + a_2\chi^2 + a_3\chi^3 + a_4\chi^4 \quad (31)$$

$$I(\psi) = \frac{k\psi}{2} \quad (32)$$

$$g(\chi) = \frac{1}{\epsilon_a} - \frac{k\chi}{2} \quad (33)$$

and

$$B = 1 - \epsilon_a \chi^{\frac{1}{2}} \cos(2\pi\theta). \quad (34)$$

Above equations are in dimensionless form. The profiles of $B_\phi(r)$, $\chi(r)$, $\psi(r)$, $v(r)$, $q(r)$ are shown in Fig. 1. Table 2 shows the ZT-40 parameters used in our simulations.

TESTING THE CODE:

The code that we have developed to study anomalous transport in ZT-40 is called RFPCH3. We have tested this code extensively for convergence, particle trajectories, pitch space relaxation etc. In Fig. 2 (a) - (h), we show the electron trajectories for trapped and untrapped particles with and without collisions and/or loop voltage. In Fig. 3, we show the temporal evolution of distribution function in pitch space.

BREAK-UP OF FLUX SURFACES:

We have studied the break-up of flux surfaces in ZT-40 due to $m=1$ mode. This is done by using the code FIELD4. The results of this study are shown in Fig. 4(a) - (d). We see that when $n=7,8,9,10$ and 11 , the core of ZT-40 roughly half way out in the minor radius is stochastic. When $n=7,9,\dots,-7$, entire volume is stochastic.

In Fig. 5, we show a typical collisional electron trajectory when the entire volume is stochastic with loop voltage of 40 volts. For this figure, $m=1$, $n=7,8,9,10,11,12,14,16,20,-20,-16,-14,-12,-10,-8$, and -7 . The amplitude of each of this mode is $a_n = .01$. See equation (20).

ANOMALOUS TRANSPORT IN ZT-40:

We have used our code RFPCH3 to study the particle diffusion in ZT-40 due to $m=1$ resistive perturbations. Results of this study are shown in Fig. 6(a) - (d). For these results, the collision frequency $\nu_d = 10^6 \text{ s}^{-1}$ and $V_{\text{loop}} = 40$ volts. Initial conditions are chosen randomly in these figures, a denotes the magnitude of the magnetic perturbations. We simulate 64 particles and calculate particle diffusion co-efficient in χ space. From these figures, we see that Rosenbluth-Rechester scaling of $D \propto a^2$ is verified when we have full stochasticity.

APPLICATION TO TOKAPOLE II /13/:

To apply our method to the tokamak TOKAPOLE II, we have written the code TKMKCH4. The TOKAPOLE II parameters used are shown in Table 3. In our code, we use the following expressions:

$$\psi(\chi) = \chi + \text{constant} \cdot \chi^2 \quad (35)$$

$$I(\psi) = 0 \quad (36)$$

$$g(\chi) = \frac{1}{\epsilon_a} \quad (37)$$

$$B = 1 - \epsilon_a \psi^{\frac{1}{2}} \cos(2\pi\theta). \quad (38)$$

The constant in equation (35) is evaluated by specifying q_{edge} .

TESTING THE CODE:

We have tested the code just as in case of ZT-40 by obtaining the particle trajectories for trapped and passing particles with/without collisions and/or loop voltage. One such typical particle trajectory is shown in Fig. 7. We have ascertained the correctness of the trajectories obtained from our code.

NEOCLASSICAL TRANSPORT IN TOKAPOLE II:

To further test our code TKMKCH4, we have used it to obtain the neoclassical particle diffusion coefficients in TOKAPOLE II as a function of collision frequency ν_d . The results of this study are shown in Fig. 8. From Fig. 8 we see that our method and the code give correct results for neoclassical transport. The transition from bananas regime to plateau regime, and from plateau regime to Pfirsch-Schultz regime occurs at correct values of ν_d , as can be seen from Fig. 8.

CONTRIBUTED PAPERS AND INVITED SEMINARS:

As a result of research done in this project, we have presented the following papers at professional meetings:

- (1) Application of Monte Carlo Method for Evaluation of Transport Coefficients to RFP
Maria Lam, Alkesh Punjabi and Allen Boozer
Bull. Am. Phys. Soc., 33, 2000 (1988).
- (2) Equations of Motion in Non-canonical Co-ordinates for Particle Trajectories in Toroidal geometries
Alkesh Punjabi, Maria Lam and Allen Boozer
Pwc. of Sherwood Meeting 1989, paper 2C24.
- (3) Monte Carlo Study of Transport in TOKAPOLE II
Alkesh Punjabi and Allen Boozer
Bull. Am. Phys. Soc., 34, 2168 (1989).
- (4) Monte Carlo Study of Anomalous Electron Transport in RFP
Kathy Burke, Maria Lam, Alkesh Punjabi and Allen Boozer.
Bull. Am. Phys. Soc., 34, 2148 (1989).

Two invited seminars were given on research done in this project. These are:

- (1) RFP Transport Coefficients Using Monte Carlo Calculation
Alkesh Punjabi
Given at Los Alamos National Laboratory
August 19, 1988.
- (2) Anomalous Transport in Toroidal Plasmas
Alkesh Punjabi
Given at Alabama A & M University
August 19, 1989.

PUBLICATION:

We plan to start writing a paper in January 1990 which will be based on the research work done up until now in this project.

TRAINING OF MINORITY GRADUATE STUDENT:

One female minority graduate student is supported by this project as a graduate research assistant. Her name is Ms. Kathy Burke. She has received training in scientific research using supercomputers. The research done by her under this project will form her thesis for the degree of Master of Science in Applied Mathematics from Hampton University. The expected date of her graduation is February 1990.

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FIGURE CAPTIONS

Fig. 1. Profiles for (a) poloidal flux χ , (b) toroidal flux ψ , (c) rotational transform ι , and (d) safety factor q as functions of r based on Bessel function model with RFP ZT-40 parameters. Note that all quantities are in dimensionless form.

Fig. 2. Particle trajectories in RFP ZT-40 computed from our code. We show the projection of the trajectory in x - y plane. We also show $\theta(t)$, $\phi(t)$, $\chi(t)$, $v_{\parallel}(t)$. All quantities are dimensionless. Time is expressed in units of cyclotron time.

- (a) trapped electron with $\lambda_0 = 0.4$, no loop voltage, no collisions.
- (b) passing electron with $\lambda_0 = 0.9$, no loop voltage, no collisions.
- (c) trapped electron in presence of loop voltage of $V_0 = 40$ volts and $\lambda_0 = 0.4$, no collisions.
- (d) passing electron in presence of loop voltage of $V_0 = 40$ volts and $\lambda_0 = 0.9$, no collisions.
- (e) trapped electron with $\lambda_0 = 0.4$. Pitch angle scattering is on. Loop voltage is off.
- (f) passing electron with $\lambda_0 = 0.9$. Pitch angle scattering is on. Loop voltage is off.
- (g) initially trapped electron with $\lambda_0 = 0.4$ when both, the pitch angle scattering and loop voltage, are on.

(h) passing electron with $\lambda_0 = 0.9$ when both the pitch angle scattering and loop voltage are on.

Fig. 3. Here we show the time evolution of distribution function in pitch space. We start with 1024 electrons at $r = .5$, $\theta = 0$, $\phi = 0$ and $\lambda = .1$ at $t = 0$. The relaxation of this delta function distribution is observed when time t is equal to
(a) $\tau_d/4$, (b) $\tau_d/2$, (c) $3\tau_d/4$, and (d) τ_d .
 $f(\lambda)$ is normalized to unity.

Fig. 4. Poincare surfaces of section for magnetic field lines. Here we show the points of intersection of field line with $\phi = 0$ plane. We use (χ, θ) as polar co-ordinates. These are computed by the code FIELD4. We start the integration at $\theta_0 = 0$, $\phi_0 = 0$ and various values of r .
(a) $m = 1$, $n = 7$, $r_0 = .1, .3, .4$ and $.5$
(b) $m = 1$, $n = 7$ and 8 , $r = .15, .2, .25, .3, .35, .4$, and $.45$
(c) $m = 1$, $n = 7, 8, 9, 10$, and 11 , $r_0 = .1, .2, \dots, .8$
(d) $m = 1$, $n = 7, 8, \dots, 12, 14, 16, 20, -20, -14, -10, -8, -7$,
 $r_0 = .1, .2, \dots, .8, .875, 1$.

Fig. 5 A typical collisional electron trajectory in stochastic fields when all flux surfaces are broken. Here a_n 's are all equals to $.01$.
 $r_0 = .5$, $\theta_0 = 0$, $\phi_0 = 0$, $\lambda_0 = .5$, $\nu_d = 10^6 \text{ s}^{-1}$, $V_{\text{loop}} = 40$.
We show χ , θ , $v_{\parallel}$ and h vs t . Also projection of trajectory on (x, y) plane is shown. Here $x = r \cos \theta$, and $y = r \sin \theta$.

Fig. 6 Particle diffusion coefficient as a function of amplitude of the perturbation. Each point is from the simulation of 64 particles which start out with random initial conditions. For this figure

$\nu_d = 10^6 \text{ s}^{-1}$, $V_{\text{loop}} = 40 \text{ volts}$. D is in χ space.

(a) $m = 1$, $n = 7$. a denotes the magnitude of a_7 .

(b) $m = 1$, $n = 7$ and 8 , $a_7 = a_8 = a$.

(c) $m = 1$, $n = 7, 8, 9, 10, 11$, $a_7 = a_8 = \dots = a_{11} = a$.

(d) $m = 1$, $n = 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, -20, -16, -14, -12, -10, -9, -8, -7$, $a_7 = \dots = a_{-7} = a$.

Note that the random phases δ_n are same as for Figure 4.

Fig. 7 A typical particle trajectory in TOKAPOLE II. Collisions are on. Loop voltage is zero. Collision frequency is determined from TOKAPOLE II parameters shown in Table 3. Initial conditions are $r_0 = .5$, $\theta_0 = 0$, $\phi_0 = 0$, $\lambda_0 = .1$, $\nu_d = 1.e6$.

Fig. 8 Neoclassical particle diffusion coefficients D as a function of collision frequency ν_d in TOKAPOLE II. Each point was obtained by simulating 64 particles with random initial conditions except $r_0 = .5$. Theoretically $\nu_1 = \epsilon^{3/2} v_T / R_q = 1.37 \times 10^5$ and $\nu_2 = v_T / RT = .5 \times 10^6$.

TABLE 1

Scales used in dimensionless equations

<u>Physical quantity</u>	<u>Symbol</u>	<u>Characteristic scale</u>
Length	L_0	a
time	t_0	$2\pi/\omega_c$
velocity	v_0	$a\omega_c/2\pi$
energy	H_0	mv_0^2
electric potential	Φ_0	$H_0/ q $
magnetic moment	μ_0	H_0/B_0
current	I_0	$cB_0a/2$
magnetic flux	ψ_0	πB_0a^2
angular momentum	P_0	$ma v_0$
angle	θ_0	2π

TABLE 2

ZT-40 parameters

minor radius	a	20 cms
major radius	R_o	114 cms
reversal parameter	k	2.6
electron temperature	T_e	150 eV
plasma density	n	10^{13} cm^{-3}
Loop voltage	V_{loop}	40 volts
magnetic field	B_o	2.4 KGauss

TABLE 3

TOKAPOLE II parameters

minor radius	a	9 cms
major radius	R_o	50 cms
electron temperature	T_e	100 eV (high q_a)
plasma density	n	$.5 \times 10^{13} \text{ cm}^{-3}$ (high q_a)
magnetic field	B_o	5 KGauss

(a)

(b)

(c)

(d)

FIG. 1

FIG. 2a

FIG. 2c

FIG. 2d

FIG. 2e

FIG. 2f

FIG. 2g

FIG. 2h

$$t = \tau_d/4$$

Run6, Run3

(a)

$$t = \tau_d/2$$

Run6, Run3

(b)

$$t = 3\tau_d/4$$

Run6, Run3

(c)

$$t = \tau_d$$

Run6, Run3

(d)

FIG. 3

Poincare Surfaces of Section:

$m = 1$
 $n = 7$ and 8

(b)

Poincare Surfaces of Section:

$m = 1$
 $n = 7$

(a)

FIG. 4

Poincare Surfaces of Section:

$$m = 1$$

$$n = 7, 8, 9, 10, 11$$

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(c)

Poincare Surfaces of Section:

$$m = 1$$

$$n = 7, \dots, 12, 14, 16, 20, 20, 14, 10, 8, 7$$

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(d)

FIG. 4

FIG. 6

FIG. 6

(e)

FIG. 6

FIG. 7

Neoclassical Particle Diffusion in Tokopole II

FIG. 8