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Vortex structure in rotating Rayleigh-Bénard convection

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Abstract

We investigate the flow patterns in a cylindrical rotating Rayleigh-Bénard convection cell with radius-to-height ratio $\Gamma = 0.5$. The Rayleigh number R is 2×10^8 , the dimensionless rotation rate Ω varies from 10^4 to 5×10^4 , and the convective Rossby number Ro is between 2 and 0.4. Measurements of the velocity field in the volume adjacent to the top of the cell are acquired with a scanning particle image velocimetry (PIV) system. We present quantitative results for velocity and vorticity of the cyclonic and anticyclonic vortices characterizing the convection, as well as for the dependence of the vortex size on the rotation rate and variation of vorticity with depth.

1 Introduction

Turbulent thermal convection with rotation about a vertical axis is an important laboratory realization of phenomena in geophysical systems such as the earth's atmosphere and oceans and in solar and planetary atmospheres. Despite the early roots of these types of investigations in the pioneering theoretical and experimental studies of Chandrasekhar[1] and Nakagawa and Frenzen[2], there has been remarkably little quantitative work on the flow structures and in particular the velocity field in rotating convection. The primary tools of past work have been global heat transport measurements, local temperature probes, and qualitative flow visualization with dye, aluminum flakes, streak photography, and shadowgraph[2-7]. A semi-quantitative investigation of the vertical temperature structure of a transient thermal plume in the presence of rotation indicated a complex three-dimensional form for non-axially-concentric vortices[8]. Much of this omission of quantitative detail stems from the lack of good experimental tools for obtaining the velocity or

temperature field over a spatial volume combined with the difficulties of a rotating laboratory experiment. Advances in particle-image-velocimetry (PIV) allow acquisition of the velocity field in a plane within the fluid illuminated by a narrow light sheet. In the work presented here, we make use of the PIV technique to provide quantitative information about vortical flow structures in the vicinity of the upper cold boundary layer of the convection cell. Recent numerical investigations of turbulent, rotating convection[9] have suggested an interesting and complicated structure of vortices erupting from an unstable thermal boundary layer. Our measurements confirm those predictions, showing strong Ekman pumping along a ring with weak suction in the vortex core. This Ekman pumping may explain the enhancement of heat transport in turbulent rotating convection[6,7] and have important consequences for understanding the general problem of heat transport scaling developed over the past decade[10]. The measurements reported here demonstrate the richness of the experimental system that we have developed and open the way for many more quantitative studies of rotating turbulent thermal convection.

Rayleigh-Bénard convection occurs in a fluid layer of depth d bounded on the top and the bottom by rigid surfaces. Heating of the bottom surface introduces a temperature differential ΔT . If ΔT does not exceed a critical value ΔT_c , the fluid layer remains stable and heat is transported solely by thermal diffusion. If $\Delta T > \Delta T_c$, the fluid layer becomes unstable and heat is transported by a combination of diffusion and advection. In a reference frame rotating about a vertical axis, one also has to take into account the Coriolis and centrifugal forces. The dimensionless parameters that determine the state of convection are the Rayleigh number characterizing the potential energy in the fluid $R = \alpha g \Delta T d^3 / \nu \kappa$, the dimensionless rotation rate proportional to the strength of the Coriolis force $\Omega = \Omega_D d^2 / \nu$, and the Prandtl number representing the properties of the fluid $\sigma = \nu / \kappa$, where α is the coefficient of thermal expansion, g is the acceleration of gravity, ν is the kinematic viscosity of the fluid, κ is the fluid coefficient of thermal diffusivity and Ω_D is the angular rotation rate about the vertical axis. An extremely useful combination of these three parameters is the convective Rossby number $Ro = (\sqrt{R/\sigma}) / (2\Omega)$ which is a measure of the relative importance of buoyancy and rotation[9,7]. The condition $Ro \approx 1$ should separate regimes of weak rotation $Ro \gg 1$ and strong rotation $Ro \ll 1$. The aspect ratio of lateral to vertical dimensions for a cylindrical cell is $\Gamma = r_0/d$, where r_0 is the cell radius. Finally, centrifugal effects can be ignored provided that $\Omega^2 r_0/g \ll 1$, where r_0 is the cell radius. This condition is satisfied for measurements reported here with $\Omega^2 r_0/g < 0.1$.

A thorough review of the numerous investigations of rotating convection is presented by Boubnov and Golitsyn[11]. As noted above, however, many issues important for the quantitative understanding of the balance between buoyancy and rotation in convection remain unaddressed. In particular, there are no quantitative measurements of the flow in the vortices that form in a

rotating convection cell. Numerical investigations by Julien *et al.*[9] show that these vortices may possess a ring-like structure. as the influence of a thermal boundary layer would lead to reduction of pumping in the core region of the vortex (plume) and enhancement of pumping at the edge of the vortex, leading to reshaping of the vortex into a circular ridge. This mechanism may also be of relevance in formation of the quasi-stationary ring-like patterns in transient states (e.g. spin-up) of rotating convection, as our earlier study indicates[12].

To investigate the structure of the vortices in rotating convection, the present investigation employs the particle image velocimetry (PIV) technique. Traditional PIV produces information about the velocity field components lying in a planar section of the flow. Detailed overviews of the technique are given by Adrian[13] and Rockwell *et al.*[14]. To recover information about the three-dimensional structure of the flow, an extension of PIV known as scanning PIV can be implemented. A description of a state-of-the-art PIV scanning system is provided by Brucker[15].

With scanning PIV, we produce quantitative measurements of horizontal velocity and axial vorticity in cyclonic and anticyclonic vortices. We also investigate the decrease in the characteristic width of the vortex core with the growth in rotation rate Ω and confirm the existence of ringlike structures in downwelling cyclonic vortices emitted from the top boundary layer.

2 Experimental Setup and Data Acquisition System

The cylindrical Rayleigh-Bénard cell employed in the experiments (Fig. 1) has height $d = 12.7$ cm and radius $r_0 = 6.35$ cm. Its top surface is maintained at constant temperature, with constant heater power applied to the bottom surface. The cell and the image acquisition/scanning equipment are placed on a rotating table. Details of the experimental arrangement are similar to those described in our earlier work[12]. We use scanning digital PIV for data acquisition. A Sony digital camera (Fig. 1A) scans 30 frames per second with a resolution of 640 by 480 pixels. The light source used for visualization is a 300 W xenon arc lamp. The light of the lamp passes through a fluid-filled flexible light conduit connected to the optical system consisting of a spherical lens (Fig. 1F), a cylindrical lens (Fig. 1E), a collimating slit and a mirror (Fig. 1D). The optical system produces a narrow (3 mm) light sheet that illuminates horizontal sections within the convection cell. The system is mounted on a vertically moving table driven by a stepping motor. With a sawtooth profile of motion set for the motor, the light sheet can scan 4 cm of the depth of the cell every 2 s. An array of vertical slits is mounted in front of the cell (Fig. 1, B). As the light sheet traverses the depth of the cell, it illuminates only vertical positions determined by the slits. The width of the slits in the array is 1.5 mm.

Fig. 1. Rayleigh-Bénard cell with data acquisition system, side and top views

During the scanning pass of the optical system, the plane of each slit is fully illuminated for 0.075 s, the time necessary to acquire two frames for double-frame cross-correlation. Unlike the system described by Brucker[15], this setup does not use stereoscopy and thus only the horizontal velocity components are acquired in four horizontal planes within the 8 by 6 by 3 cm volume.

The particles used for flow seeding are neutrally-buoyant 250-micron polystyrene spheres. Analysis of the errors inherent in PIV interrogation due to limitations of the optical resolution of the system and due to turbulent particle drift is presented elsewhere[12]. For a characteristic dimensionless rotation rate $\Omega = 4 \times 10^4$ the cumulative error in PIV-reconstructed velocity was found to be on the order of 1%.

3 Observations

Fig. 2 shows instantaneous velocity and sectional streamline patterns near the top of the cell for different dimensionless rotation rates starting from zero. The size of the flow area shown in the pictures is 7.72 by 5.69 cm. For the non-rotating case (Fig. 2A), the characteristic features of the flow are the upwelling plumes, appearing as the mass sources in the streamline pattern, and the long thin zones of downwelling flow visible as limit lines for the sectional streamlines. Addition of rotation introduces considerable changes to the topol-

Fig. 2. Instantaneous sectional velocity and streamline patterns in the plane adjacent to the top of the cell. A - no rotation, B - $\Omega = 9.6 \times 10^3$, $Ro = 1.9$, C - $\Omega = 2.9 \times 10^4$, $Ro = 0.63$, D - $\Omega = 4.8 \times 10^4$, $Ro = 0.38$. The arrow at the lower left indicates the velocity scale.

ogy of the flow. Vortical structures corresponding to cyclonic (downwelling, clockwise orientation of rotation in the figures) and anticyclonic (upwelling, counterclockwise) thermal plumes emerge. In the streamline pattern, these structures are represented with spiraling streamlines, cyclonic vortices carrying mass away from the plane near the upper boundary of the cell and anticyclones injecting mass. For the lowest rotation rate with $\Omega = 9.6 \times 10^3$ and $Ro = 1.9$, Fig. 2B shows the limit-line patterns characteristic of zero rotation. But as the rotation rate increases and Ro decreases, they are no longer observed. This observation is consistent with the separation between regimes of weak and strong rotation by the condition $Ro \approx 1$. It is also interesting that, although the rotation rate and the flow patterns change dramatically over the sequence of images in Fig. 2, the characteristic velocity remains roughly the same - 0.25 to 0.75 cm/s.

An important feature of the vortices in rotating convection is the decay of vorticity in the plumes with increasing distance from the top/bottom bound-

Fig. 3. Vorticity as a function of distance away from the top surface showing decay in the magnitude of the vorticity away from the boundary layer.

ary. Fig. 3 presents plots for instantaneous vorticity in the cores of 12 vortices, cyclonic and anticyclonic, from measurements acquired at different rotation rates. Regardless of the rotation rate and vortex strength at the top, descent to a depth of 3 cm (approximately 1/4 of the height of the cell) produces a 50% decrease in vorticity relative to that near the top surface.

4 Analysis

In the previous section, we described some features of the velocity and vorticity fields associated with turbulent convection. The properties of these fields have traditionally been used to characterize flows with vortices. As Chong *et al.*[16] point out, however, the definition of a vortex in terms of velocity field topology is insufficiently robust, as the streamline patterns depend on the velocity of the reference frame of the observer. Vorticity has the advantage of being reference-frame independent, but not all concentrations of vorticity correspond to vortical structures in the velocity field. A definition of a vortex from the local flow topology point of view is presented by Chong *et al.*[16]. They suggest that vortex cores must be defined as regions of space with vorticity sufficiently strong to cause the rate-of-strain tensor to be dominated by the rotation tensor. In these regions, the rate-of-deformation tensor must have complex eigenvalues. This definition also is independent of the reference frame of the observer.

Fig. 4. Characteristic radius of vortical structures near the top surface of the cell for different dimensionless rotation rates.

Near vertical boundaries, rotation in convection vortices takes place primarily in the horizontal plane, so the above definition can be applied to produce a simple method of measurement of the characteristic size of the vortices in the plane adjacent to the top surface of the convection cell. First, at each grid point in the plane, components of the two-dimensional rate-of-deformation tensor are constructed from the derivatives of the horizontal velocity components u and v :

$$A = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}. \quad (1)$$

The eigenvalues of the matrix A are complex if the expression

$$f = (\text{Tr} A)^2 - 4 \det A \quad (2)$$

is negative. Areas of negative values of f thus denote the presence of vortical structures. A computer program calculates f , distinguishes simply-connected areas of negative f in the interior of the image plane, computes their areas and estimates each corresponding characteristic vortex size as the square root of the area divided by π . Fig. 4 shows the averaged characteristic radii retrieved by this program after analyzing ten instantaneous velocity maps for each dimensionless rotation rate shown. The error bars indicate the standard deviation. The characteristic radius decreases with increasing rotation rate.

Representations of the flow field in a cyclonic vortex ($\Omega = 10^4$) with velocity vectors, streamlines, contours of out-of-plane vorticity and local topology type are compared in Fig. 5. The shortcoming of the function f as defined by Eq. 2 when used for the purposes of visualization lies in its inability to differentiate between stable and unstable foci, so instead we visualize contours of the function

$$g = \begin{cases} -\text{sgn}(\text{Tr}A) f, & f < 0 \\ 0, & f \geq 0 \end{cases}, \quad (3)$$

which equals zero at the points where A has real eigenvalues. In the complex eigenvalue case, its sign corresponds to the sign of the real part of the eigenvalues, thus determining the stability of the fixed point. Contours of constant g are shown in Fig. 5D.

The correspondence between the swirl in the velocity field (A) and the concentration of out-of-plane vorticity (C) is what one would expect. The streamline pattern, however, exhibits some interesting peculiarities. In the simplest case of a swirling and downwelling flow, the sectional streamlines should spiral in towards some focal point serving as a sink of mass out of the plane. What Fig. 5B shows is a combination of a spiraling-in zone on the periphery of the zone and a spiraling-out area near the core winding together into a limit-cycle trajectory. This instantaneous streamline pattern was observed for several cyclonic vortices (in some cases it probably could not be resolved because of grid limitations). In anticyclonic vortices, the only streamline pattern we recorded was simple spiraling out.

Additional insight into the structure of the flow is provided by Fig. 5D showing the local topology in terms of contours of the function g defined by Eq. 3. One can see a zone of unstable-focus topology near the core surrounded by the stable-focus topology closer to the periphery of the vortex. Both zones are somewhat distorted by the grid effects – one sees several areas of stable-focus topology surrounding the unstable-focus core region rather than a continuous circle of stable-focus points representing the limit cycle. In three dimensions, this would suggest a vortex structure with upwelling at the core, as the sketch in Fig. 6 illustrates. Unstable foci in the horizontal plane suggest that the corresponding three-dimensional topology according to the classification of Chong *et al.*[16] is of the type unstable-focus-compression. This indicates vertical motion towards the plane and an unwinding spiral in the streamline pattern. Likewise, stable foci suggest stable-focus-stretching type of topology and motion away from the plane, corresponding to the area of the limit-cycle trajectory and the spiral winding in on it from the exterior in the streamline plane. The structure of the vortex in Fig. 6 is constructed on the basis of these considerations. It is similar to that predicted in numerical simulations

Fig. 5. Structure of a cyclonic vortex. A – velocity field, B – instantaneous streamlines, C – contours of out-of-plane vorticity (solid – negative, dashed – positive), D – local instantaneous topology (thick lines - unstable focus, thin lines - stable focus)

Fig. 6. Schematic of three-dimensional flow in a cyclonic vortex

of Julien *et al.*[9]

5 Conclusion

We have made, for the first time in studies of rotating Rayleigh-Bénard convection, experimental measurements of the horizontal velocity field and directly measured the decay of axial vorticity for individual vortices as a function of depth for cyclonic and anticyclonic vortices at dimensionless rotation rates Ω between 10^4 and 5×10^4 with corresponding convective Rossby numbers Ro between 2 and 0.4. The rate of decay does not show strong dependence on either the vortex strength at the top of the cell or the rotation rate, whereas the characteristic vortex size decreases with increase in dimensionless rotation rate. The overall flow structure supports the criterion $Ro \approx 1$ for separating regimes of weak and strong rotation-dominated convective phenomena.

Our investigation also revealed the flow structure in cyclonic vortices consistent with the predictions of numerical simulations carried out by Julien *et al.*[9], with thermal boundary layer effects suppressing Ekman pumping in the core of the vortex and thus leading to formation of a narrow zone of upwelling flow near the core and a ring-shaped downwelling region surrounding it. The structure of the boundary layer vortices may prove important in understanding recent heat transport results which show an enhancement by rotation of the amount of heat transported by convection[6,7]. Previous suggestions on the mechanism for this enhancement have been that Ekman pumping is more effective at extracting heat from the boundary layer than buoyancy alone. Our measurements provide some evidence for this picture by revealing the horizontal structure of the flow around a thermal vortex. A complete characterization of the thermal vortex including thermal structure and vertical velocity would elucidate this problem further and are currently in progress.

In summary, the system of turbulent rotating thermal convection is providing new insights into the structure of thermal boundary layers and the vortical structures that arise from instability of such boundary layers. In addition, the properties of rotating convection are such that they allow for different degrees of freedom in probing the general state of turbulent convection[7]. This contributes a new tool for testing scaling theories of turbulence applied to Rayleigh-Bénard convection[10].

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