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Analytical Theory of Intensity Fluctuations in SASE

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Abstract

Recent Advances in SASE experiments [1] stimulate interest in quantitative comparison of measurements with theory. Extending our previous analysis [2] of the SASE intensity in guided modes, we provide an analytical description of the intensity fluctuations by calculating intensity correlation functions in the frequency domain. Comparison of our results with experiment yields new insight into the SASE process.

1. Introduction.

Recently, there have been a number of experimental demonstrations of SASE [1]. All of them measured the SASE radiation pulse energy as a function of pulse charge, showing a deviation from the linear dependence. The theory on the average power, bandwidth, and transverse mode expansion for SASE has been developed years ago [2,3], and can be used to compare with the new results. However, the measurements also show that when the pulse charge increases, the pulse to pulse radiation energy fluctuation also increases. About this fluctuation, there have been analyses based on numerical simulations [4], showing that the relative fluctuation decreases as a function of the ratio of pulse length l over the coherence length l_c . This stimulated our interest to develop an analytical formula for the fluctuation. Previous works on start-up noise [3], and specially our 3D calculation [2] made this task straight forward, because the expansion of radiation in terms of the guided modes in this work can be used to derive a formula directly comparable with the experiments.

One particular result of our analysis is that in the 1D limit our 3D fluctuation formula is significantly simplified to:

$$\frac{\sigma_w}{W} = \frac{1}{\sqrt{l/l_c}}, \quad (1)$$

where σ_w is the rms fluctuation of the output SASE energy W per pulse, l is the length of a flat-top pulse and l_c is a correlation length characterizing SASE coherence. We find

$$l_c = N_w \lambda_s \left(\frac{2\pi L_G}{3 L_w} \right)^{1/2}, \quad (2)$$

where N_w is the number of undulator periods, λ_s is the radiation wavelength, L_w the undulator length, and L_G is the power e-folding length.

In a recent BNL ATF SASE experiment: the radiation wavelength is 1μ ; the MIT micro undulator is about 0.54m long, with 60 periods of length 0.88cm; the peak current is about

320 ampere, with normalized emittance of the electron beam in this peak current portion of the bunch about 0.7 mm-mrad; the electron beam energy is 34 Mev. Thus the gain length is calculated to be about 0.11 m , i.e., the wiggler has 4.9 gain lengths. The slippage is $60 \times 1 \mu m = 60 \mu m$, and the coherence length is $60 \mu / \sqrt{\frac{3}{2\pi} \times 4.9} = 40 \mu$, from eq.(2). The pulse length is measured to be about 4ps, i.e. , about $1300 \mu m$, hence the fluctuation $\sigma_w / \langle W \rangle$ is calculated from eq.(1) to be $\sqrt{40/1300} \cong 17\%$. This is consistent with the measured fluctuation of about 15%, considering that the pulse shape is actually not a step function and the calculated beam size is not really large enough to be at the 1D limit.

We remark here that if the current is increased and the pulse length remains the same, the gain length and coherence length should be reduced, so the fluctuation should be less. However, this is incorrect if the bunch shape is not a step function. For a nonuniform bunch profile, the effective bunch length is determined by the peak current portion of the electron pulse, because the gain is much higher at this part of the bunch, and most of the radiation energy is contributed by this part. Therefore, as the current increases, it may happen that the effective bunch length decreases, and the relative fluctuation increases, depending on the specific details of the electron bunch profile. Hence applying eq.(1) yields insight into the electron distribution. When the electron distribution possesses a narrow, high current peak, the fluctuation formula can be used to estimate the peak width.

In the following, we shall first give a brief review of the guided mode expansion. Then, we shall derive the intensity fluctuation formula for pulse energy, followed by a simple formula for a correlation function in the single shot frequency spectrum.

2. A Brief Review of the Start-up Noise in Guided Modes

In our 3D start-up calculation [2], we considered an electron distribution without energy spread and with zero angular spread, i.e., we assumed a beam with zero emittance. We showed that in the exponential growth regime the electric field of the output radiation can be written

as a sum of contributions from each individual electron. The spontaneous radiation from each of the electrons is amplified by the FEL interaction to contribute to the sum. The contribution of each electron is represented by a green's function \dot{g} with the source point located at the electron j (see eq.(5.8) of [2]):

$$E(\tau, \zeta, \vec{x}) = \frac{b\kappa}{n_0} \sum_j e^{-i\zeta_j} \dot{g}(\tau, \zeta - \zeta_j, \vec{x}, \vec{x}_j), \quad (3)$$

where $\tau = k_w z$ with k_w the wiggler wavenumber, and z is the longitudinal position in the wiggler; $\zeta = (k_s + k_w)z - \omega_s t$ is the ponderomotive phase of the system with $k_s = \omega_s/c$ the radiation wavenumber and t the time; $\vec{x} = \sqrt{2k_s k_w} \vec{r}$ is the scaled transverse position (see eq.(2.20) of [2]); n_0 is the electron density; $b = 2k_w k_s (k_s + k_w) \approx 2k_w k_s^2$ and $\kappa = \frac{n_0 \mu_0 e^2 c A_w}{2\pi k_w \gamma_0}$ with μ_0 the vacuum permeability, γ_0 the beam energy, and A_w the vector potential of the wiggler (see eq.(5.5-5.6) of [2]).

The green's function is found by solving a third-order partial differential equation, which describes the FEL interaction. The green's function is longitudinally Fourier transformed in the frequency domain, and then transversely expanded in terms of the guided modes (eq.(5.11) of [2]):

$$\dot{g}(\tau, \zeta, \vec{x}, \vec{x}') = \int \frac{dq_{\parallel}}{2\pi} e^{iq_{\parallel}\zeta} \sum_n \dot{G}_n(q_{\parallel}, \tau) \psi_n(q_{\parallel}, \vec{x}) \psi_n(q_{\parallel}, \vec{x}'), \quad (4)$$

where \dot{G}_n is the Fourier transform of the green's function of the guided mode n , $q_{\parallel} = \frac{\Delta\omega}{\omega}$ is the relative frequency detuning, ψ_n is the guided mode function [5], which can be found by solving an eigenvalue problem similar to the Schroedinger equation but with complex eigenvalues. We remark that the index n is used here to abbreviate a set of discrete indices, for example, it could be actually a set like $\{m, l\}$ with m the azimuthal node number and l the radial node number. We use $n=0$ to denote the fundamental mode with the highest growth rate.

For a beam with a transverse step function profile, the guided mode problem is solved [2]. For this case, an important parameter is the scaled beam size \tilde{a} . This is defined by eq.(6.13)

of reference [2] as $\tilde{a} = 2\rho\sqrt{2k_w k_s}R_0$, where R_0 is the radius of the step profile; and ρ the Pierce parameter [6], the growth rate per wiggler period in 1-D theory, defined by:

$$(2\rho)^3 = \frac{n_0 Z_0 e^2 K^2 [JJ]^2}{2m\gamma^3 k_w^2 c},$$

with n_0 the particle density, $Z_0 = 377\Omega$ the vacuum impedance, K the wiggler parameter, and γ the electron energy, $[JJ]$ the Bessel factor, which is equal to 1 if the wiggler is helical, as we assumed in [2]. Once the value of \tilde{a} is given, we can calculate $\dot{G}_n(q_{\parallel}, \tau)$ and the mode function $\psi_n(q_{\parallel}, \vec{x})$, as described in detail by [2]. The physical meaning of the scaled beam size is that \tilde{a}^2 is about equal to the ratio of the Rayleigh range of an optical gaussian beam with the same beam waist as the electron beam size, over the 1D e-folding length of an electron beam with the same current density. If the beam size is sufficiently large so that $\tilde{a} \gg 1$, the diffraction effect is negligible, and the power e-folding length approaches the 1D limit $L_G = \lambda_w / (4\pi\sqrt{3}\rho)$. We call this the large beam size limit.

For the sake of simplicity, we shall not give the specific expression of these mode functions (see eq.(6.27) of [2] and the related discussions), but only point out here that the mode functions are not orthogonal to each other, even though they are normalized according to

$$\int d^2x \psi_n^2(q_{\parallel}, \vec{x}) = 1. \quad (5)$$

Notice that in the integrand, the mode function does not take the absolute value. In the large beam size limit, these functions become nearly orthogonal to each other.

The Fourier transform of the green's function \dot{G}_n is given by (This expression is not limited to step function profile):

$$\dot{G}_n(q_{\parallel}, \tau) = \frac{e^{-i\Omega_n(q_{\parallel})\tau}}{-i\Omega_n(q_{\parallel}) [1 - F_n(q_{\parallel})]}.$$

Again, we shall not give the specific expressions for $\Omega_n(q_{\parallel})$ and $F_n(q_{\parallel})$ (see eq.(6.24-26) and the Appendix C and D of [2]), but only point out here that in the large beam size limit $\tilde{a} \gg 1$,

$\Omega_0(q_{\parallel} = 0)$ approaches $2\rho e^{i2\pi/3}$, so the growth rate $\text{Im}(\Omega_0(q_{\parallel} = 0))$ approaches $\sqrt{3}\rho$, and the coefficient $1 - F_0(q_{\parallel} = 0)$ approaches 3.

Another important point is that $\Omega_n(q_{\parallel})$, $F_n(q_{\parallel})$ and $\psi_n(q_{\parallel}, \vec{x})$ are smooth functions of the detuning q_{\parallel} near resonance $q_{\parallel} = 0$. However, the green's function \dot{G}_n changes exponentially with wiggler distance τ due to the exponential factor, and hence is a sharply peaked function of the detuning q_{\parallel} , of a width

$$\sigma_{q_{\parallel}} \approx \left(\frac{3\sqrt{3}\rho}{\tau} \right)^{1/2} = \left(\frac{3L_w}{2L_G} \right)^{1/2} \frac{1}{2\pi N_w}, \quad (6)$$

in the large beam size limit (see eq.6.49 of [2]). This will be used for the calculation of many integrals over the detuning q_{\parallel} , where slow varying functions such as $\psi_n(q_{\parallel}, \vec{x})$ can be taken as constant, and moved outside the integrals.

Now we return to the guided mode expansion eq.(4). If the difference of the growth rate of different modes is sufficiently large, then at the end of the wiggler, the fundamental mode with the highest growth rate will dominate over other modes, and the sum will be dominated by the term with $n=0$. In this paper we shall assume this domination of the fundamental mode, and use the equations 3 and 4 to derive the intensity fluctuation from pulse to pulse.

3. The Intensity Fluctuation

We assume the electron beam pulse shape is a step function with pulse length l , and the slippage distance is much smaller than l , then the radiation pulse energy is

$$W = \frac{1}{Z_0 c} \int_0^l dz \int d^2r |E|^2 = \frac{1}{Z_0 bc} \int d^2x \int_0^s d\zeta |E(\zeta, \vec{x})|^2, \quad (7)$$

where we have changed the variables z and \vec{r} into the scaled variables ζ and \vec{x} . The scaled pulse length is $s \cong k_s l$. We define the Fourier transform of $E(\zeta, \vec{x})$ by:

$$E(\zeta, \vec{x}) = \int \frac{dq_{\parallel}}{\sqrt{2\pi}} \tilde{E}(q_{\parallel}, \vec{x}) e^{iq_{\parallel}\zeta}. \quad (8)$$

Then we can rewrite the pulse energy in the frequency domain:

$$W = \int dq_{\parallel} W(q_{\parallel}), \quad (9)$$

with the energy spectrum

$$W(q_{\parallel}) = \frac{1}{Z_0 b c} \int d^2 x |\tilde{E}(q_{\parallel}, \vec{x})|^2. \quad (10)$$

Now it is easy to show that the pulse energy fluctuation is

$$\frac{\sigma_w^2}{\langle W \rangle^2} = \frac{\langle W^2 \rangle - \langle W \rangle^2}{\langle W \rangle^2} = \frac{\int dq_{\parallel} \int dq'_{\parallel} C(q_{\parallel}, q'_{\parallel})}{\left[\int dq_{\parallel} \langle W(q_{\parallel}) \rangle \right]^2}, \quad (11)$$

where the angled brackets represent ensemble average over the electron random distributions, and $C(q_{\parallel}, q'_{\parallel})$ is the correlation function

$$C(q_{\parallel}, q'_{\parallel}) \equiv \langle W(q_{\parallel}) W(q'_{\parallel}) \rangle - \langle W(q_{\parallel}) \rangle \langle W(q'_{\parallel}) \rangle. \quad (12)$$

We need to calculate the averages $\langle W(q_{\parallel}) \rangle$ and $\langle W(q_{\parallel}) W(q'_{\parallel}) \rangle$. For this, we first calculate the Fourier transform $\tilde{E}(q_{\parallel}, \vec{x})$ of the electric field using its definition eq.(8) and the equations eqs.(3, 4). Substituting into eq.(10), yields:

$$W(q_{\parallel}) = \frac{1}{2\pi} \frac{b\kappa^2}{Z_0 n_0^2 c} |\dot{G}_0(q_{\parallel})|^2 N_1(q_{\parallel}) \sum_{i,j} e^{-i(\zeta_i - \zeta_j)(1+q_{\parallel})} \psi_0(q_{\parallel}, \vec{x}_i) \psi_0^*(q_{\parallel}, \vec{x}_j), \quad (13)$$

where

$$N_1(q_{\parallel}) \equiv \int d^2 x |\psi_0(q_{\parallel}, \vec{x})|^2 \quad (14)$$

is similar to a normalization factor, but not quite, because the modes are not orthogonal, and we have replaced the sum over the modes by a single fundamental mode labeled $n=0$. The ensemble average of eq.(13) over the random distribution of ζ_i eliminates all the terms except those with $i=j$, thus we replace the sum over particle index i and j by \sum_i , which in turn can be replaced by an integral over the electron density distribution $\frac{n_0}{b} \int_0^s d\zeta \int d^2 x u(\vec{x})$. As an

example, if we assume the profile function u is a step function in longitudinal and transverse directions, then the transverse distribution function is $u = 1$ inside the beam ($x \leq a$) and $u = 0$ outside the beam. In the more general cases we obtain:

$$\langle W(q_{\parallel}) \rangle = \frac{s}{2\pi} \frac{\kappa^2}{Z_0 n_0 c} |\dot{G}_0(q_{\parallel})|^2 N_1(q_{\parallel}) N_2(q_{\parallel}), \quad (15)$$

where

$$N_2(q_{\parallel}) \equiv \int d^2x |\psi_0(q_{\parallel}, \vec{x})|^2 u(\vec{x}). \quad (16)$$

Notice that $N_1(q_{\parallel}) \neq N_2(q_{\parallel})$, because the integration in $N_2(q_{\parallel})$ is limited to within the electron beam due to the factor $u(\vec{x})$. The product $N_1(q_{\parallel})N_2(q_{\parallel})$ is defined as "overlap integral" N_{00} , with an analytical expression given by the eq.(C6) of [2].

The calculation of $C(q_{\parallel}, q'_{\parallel})$ proceeds in a similar way and leads to

$$C(q_{\parallel}, q'_{\parallel}) = \frac{1}{4\pi^2} \left(\frac{b\kappa^2}{Z_0 n_0^2 c} \right)^2 |\dot{G}_0(q_{\parallel})|^2 |\dot{G}_0(q'_{\parallel})|^2 N_1(q_{\parallel}) N_1(q'_{\parallel}) (|\Sigma_1|^2 - \Sigma_2), \quad (17)$$

where

$$\begin{aligned} \Sigma_1 &= \sum_i e^{-i\zeta_i(q_{\parallel} - q'_{\parallel})} \psi_0(q_{\parallel}, \vec{x}_i) \psi_0^*(q'_{\parallel}, \vec{x}_i), \\ \Sigma_2 &= \sum_i |\psi_0(q_{\parallel}, \vec{x}_i)|^2 |\psi_0(q'_{\parallel}, \vec{x}_i)|^2. \end{aligned} \quad (18)$$

A rough estimate shows that $|\Sigma_1|^2/\Sigma_2$ is of the order of the number of electrons in the beam of length l , so we can ignore Σ_2 . Next, we replace the sum over particles in Σ_1 by an integral as we did previously for $W(q_{\parallel})$:

$$\Sigma_1 = \frac{n_0}{b} \int_0^s d\zeta e^{-i\zeta(q_{\parallel} - q'_{\parallel})} \int d^2x u(\vec{x}) \psi_0(q_{\parallel}, \vec{x}) \psi_0^*(q'_{\parallel}, \vec{x}).$$

To simplify this, we need an approximation: if there is a significant exponential growth, the factors $|\dot{G}_0(q_{\parallel})|^2 |\dot{G}_0(q'_{\parallel})|^2$ in eq.(17) are sharply peaked around $q_{\parallel} = 0$ and $q'_{\parallel} = 0$ with a width of $\sigma_{q_{\parallel}}$ (see eq.6), which is smaller than $1/N_w$ even near saturation. Hence, we can take

both q_{\parallel} and q'_{\parallel} to be small and within this peak. The function $\psi_0(q_{\parallel}, \vec{x})$ changes very little within the peak width, therefore, we can replace q'_{\parallel} by q_{\parallel} in the argument of ψ_0^* and obtain

$$\Sigma_1 = \frac{n_0}{b} s \operatorname{sinc} \left(\frac{s}{2} (q_{\parallel} - q'_{\parallel}) \right) N_2(q_{\parallel}),$$

where $\operatorname{sinc}(x) = \sin(x)/x$.

Substituting this into eq.(17), we get

$$C(q_{\parallel}, q'_{\parallel}) = \left(\frac{s}{2\pi} \frac{\kappa^2}{Z_0 n_0 c} \right)^2 |\dot{G}_0(q_{\parallel})|^2 |\dot{G}_0(q'_{\parallel})|^2 (N_1(q_{\parallel}) N_2(q_{\parallel}))^2 \operatorname{sinc}^2 \left(\frac{s}{2} (q_{\parallel} - q'_{\parallel}) \right) \quad (19)$$

Now we assume the pulse length l is long enough so that the width of the sinc function is much narrower than the width $\sigma_{q_{\parallel}}$ of $|\dot{G}_0(q_{\parallel})|$, i.e., $1/s \ll \sigma_{q_{\parallel}}$. In other words, we assume the bunch length is much longer than one coherence length. Then we can calculate the pulse energy fluctuation

$$\begin{aligned} \langle W^2 \rangle - \langle W \rangle^2 &= \int dq_{\parallel} \int dq'_{\parallel} C(q_{\parallel}, q'_{\parallel}) \\ &\cong \frac{s\kappa^4}{Z_0^2 n_0^2 c^2} \int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^4 (N_1(q_{\parallel}) N_2(q_{\parallel}))^2. \end{aligned} \quad (20)$$

where the integration over q'_{\parallel} of eq.(19) was carried out by moving the slowly varying factor $|\dot{G}_0(q'_{\parallel})|^2$ outside the integral, and integrating only over the sinc function. Substituting eq.(20) and eq.(15) into the eq.(11), we obtain the relative pulse energy fluctuation when one single mode dominates:

$$\frac{\sigma_w^2}{\langle W \rangle^2} = \frac{1}{s} \frac{\int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^4 (N_1(q_{\parallel}) N_2(q_{\parallel}))^2}{\left(\int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^2 N_1(q_{\parallel}) N_2(q_{\parallel}) \right)^2}. \quad (21)$$

As we mentioned before, the width of the integrand is determined by the $\dot{G}_0(q_{\parallel})$ factor to be $\sigma_{q_{\parallel}}$, the "overlap integral" $N_1(q_{\parallel}) N_2(q_{\parallel})$ is a slow varying function of q_{\parallel} , and changes very little within this width. So we can shift it outside the integrals. The result is a significant cancelation of the transverse factors (factors which depend on \vec{x}) between the denominator and the numerator, where the only remaining 3D effect comes in through the green's function

$\dot{G}_0(q_{\parallel})$:

$$\frac{\sigma_w^2}{\langle W \rangle^2} = \frac{1}{s} \frac{\int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^4}{\left(\int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^2 \right)^2}. \quad (22)$$

The integrals in this expression can be calculated using the saddle point method because the green's function $\dot{G}_0(q_{\parallel})$ has a sharp peak around zero detuning, $q_{\parallel} = 0$. The exponent $\Omega_0(q_{\parallel})$ in $\dot{G}_0(q_{\parallel})$ can be expanded around zero to 2nd order in q_{\parallel} , and used in the saddle point calculation. For example, in the large beam size limit, as explained in eq.(6.41) of [2], we write $\Omega_0(q_{\parallel})$ as

$$\Omega_0(q_{\parallel}) = 2\rho\lambda_0,$$

with λ_0 expanded to second order of $\Delta \equiv 2\rho q_{\parallel}$:

$$\lambda_0 = e^{2\pi i/3} + \frac{1}{3}\Delta + \frac{1}{9}e^{-2\pi i/3}\Delta^2. \quad (23)$$

Then using the saddle point approximation, we find

$$\begin{aligned} \int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^2 &= \frac{1}{8\pi\rho^2} \frac{1}{9} e^{2\rho\tau\sqrt{3}} \sqrt{2\pi}\sigma_{q_{\parallel}}, \\ \int \frac{dq_{\parallel}}{2\pi} |\dot{G}_0(q_{\parallel})|^4 &= \frac{1}{32\pi\rho^4} \frac{1}{9^2} e^{4\rho\tau\sqrt{3}} \sqrt{\pi}\sigma_{q_{\parallel}}. \end{aligned}$$

Thus, eq.(22) is simplified to

$$\frac{\sigma_w}{\langle W \rangle} = \frac{\sqrt{\pi}}{\sigma_{q_{\parallel}} s}, \quad (24)$$

where the bandwidth $\sigma_{q_{\parallel}}$ is given in eq.(6) and $s = k_s l$. So using the 1D gain length $L_G = \lambda_w / (4\pi\sqrt{3}\rho)$, we finally get the eqs.(1,2) given in the Introduction.

The coherence length is equal to the slippage distance $N_w\lambda_s$ when the wiggler is about 2 gain lengths. When it is smaller than 2 gain lengths, there is essentially no gain, so the derivation here is not valid, but the result clearly approaches that calculated by the spontaneous radiation theory. In this case eq.(1) is still valid, with the coherence length equal the slippage distance $N_w\lambda_s$. When the wiggler is longer than 2 gain lengths, the coherence

length is reduced. Near saturation, when it is 20 gain lengths long, the eq.(2) shows that the coherence length is equal to about one third of the slippage distance.

Finally, using the correlation function eq.(19) and the single shot spectrum eq.(15), we find the relative spectral correlation

$$\begin{aligned}
 C_r(q_{\parallel}, q'_{\parallel}) &\equiv \frac{\langle W(q_{\parallel})W(q'_{\parallel}) \rangle - \langle W(q_{\parallel}) \rangle \langle W(q'_{\parallel}) \rangle}{\langle W(q_{\parallel}) \rangle \langle W(q'_{\parallel}) \rangle} \\
 &= \text{sinc}^2\left(\frac{s}{2}(q_{\parallel} - q'_{\parallel})\right) = \text{sinc}^2\left(\frac{\Delta\omega}{\omega_s} \frac{\pi l}{\lambda_s}\right),
 \end{aligned}
 \tag{25}$$

where $\Delta\omega$ is the frequency difference between the two detunings q_{\parallel} and q'_{\parallel} .

The last expression can be directly used in comparison with the single shot spectral correlation experiment in SASE. It is remarkable that this formula does not depend on current or gain length, and hence the relative correlation is only determined by the bunch shape.

As a final remark, we note here that after this work was completed and we had submitted the abstract describing the fluctuation formulae, eq.(1,2), we received an article authored by Saldin, E.A. Schneidmiller, M.V. Yurkov [7]. The paper carried out a 1D analysis, and for 1D limit derived many of the results presented here, in particular, an intensity fluctuation formula identical to our eq.(24), and a spectral correlation formula identical to our eq.(25).

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