SAN098-0543C

SAND--98-0543C
CONF-980610--

APPLICATION OF SENSITIVITY COEFFICIENTS FOR HEAT CONDUCTION PROBLEMS[1]

Kevin J. Dowding, Ben F. Blackwell, and Robert J. Cochran

Thermal Sciences Department 9113, M/S 0835
Sandia National Laboratories
Albuquerque, New Mexico

ABSTRACT

In parameter estimation considerable insight is provided by examining sensitivity coefficients. This paper focuses on the use of sensitivity coefficients in connection with estimating thermal properties in the heat conduction equation. A general methodology for computing sensitivity coefficients can be an important design tool. The use of such a tool is demonstrated in this paper.

A control volume, finite element program is used, and briefly described, to implement numerical sensitivity coefficient calculations. In this approach general problems can be studied. Several example problems are presented to demonstrate the insight gained from sensitivity coefficients. The problems are selected from experimental studies to characterize the thermal properties of carbon-carbon composite. Sensitivity coefficients show that in an experiment that is not well designed, additional materials in the experimental configuration can have a larger impact on the temperature than the material of interest. Two-dimensional configurations demonstrate that there can be isolated areas of insensitivity and the difficulty of estimating multiple parameters.

NOMENCLATURE

a	carbon-carbon body dimension, m
a_1	heating length, m
b	carbon-carbon body dimension, m
c	specific heat, $J/kg-°C$
k	thermal conductivity, $W/m-°C$
k_x, k_y	orthotropic thermal conductivity, $W/m-°C$
L	thickness of slab, m
q	heat flux, W/m^2
q_0	boundary heat flux, W/m^2
T	temperature, $°C$
T_o	initial temperature, $°C$
T_β	scaled temperature sensitivity coefficient for parameter β, $°C$
x,y	spatial coordinates

Greek:

α	thermal diffusivity, m^2/s
β	parameter
δ	silicon carbide thickness
ρ	density, kg/m^3

Subscripts

al	aluminum
cc	carbon-carbon
eff	effective
kap	kapton
mica	mica
si	silicon grease
sic	silicon carbide

INTRODUCTION

Sensitivity analysis can be an important and useful design tool. By sensitivity analysis it is meant that the partial derivative of the state variable (temperature for thermal problems) with respect to

1. This work was performed at Sandia National Laboratories and supported by the U. S. Department of Energy under Contract DE-AC/04-94AL85000.

DISCLAIMER

model parameters (thermal conductivity, volumetric heat capacity, convection coefficient, emissivity, etc.) is observed. This partial derivative is called a sensitivity coefficient. Valuable insight is gleaned from sensitivity coefficients. Use and computation of sensitivity coefficients is discussed in this paper. In particular it is demonstrated how sensitivity coefficients can provide design information concerning the effect of parameters on the model response.

Sensitivity coefficients have many applications. They are used in parameter estimation, optimization, optimal experimental design, and uncertainty or error analysis. In this paper, however, the focus is on the engineering insight that they provide for parameter estimation and optimal experimental design. An optimal criteria is not rigorously studied, but general conditions that result in an experiment being closer to optimal are suggested by observing the sensitivity coefficients. In this context they serve as a design tool. General criteria for desirable characteristics of the sensitivity coefficients are provided along with discussion.

The importance of observing sensitivity coefficients has been widely advocated in parameter estimation by Beck. See Beck and Arnold (1977), Beck (1970), Beck (1996). Other researchers have also used sensitivity coefficients to derive insight for parameter estimation, see Marchall and Milos (1997) and Vozar and Sramkova (1997). Their use for designing an optimal experiment to estimate parameters is discussed in Beck and Arnold (1977, Chapter 8); Taktak (1992), Taktak et al. (1993); and Emery and Fadale (1990, 1996, and 1997). Moffat (1985 and 1982) has suggested sensitivity coefficients be computed in analysis codes to quantify the experimental uncertainty associated with complex analyses. Computing experimental uncertainty and propagating error through an analysis code using perturbation methods is straightforward when sensitivity coefficients are available.

Despite the insight that can be gained from sensitivity coefficients, they are used infrequently as a design tool. This is especially true in connection with experimental studies and analyses. Although parametric investigations may be done - i.e., if parameter b changes $x\%$ the results vary $y\%$ - rarely are sensitivity coefficients computed. Typically, issues of what information is available from experimental measurements and how it is impacted by other experimental parameters are not addressed until *after* conducting an experiment. At this time it is probably too late. Later in this paper it is demonstrated that seemingly insignificant effects in an experiment can be important. In many instances intuition or experience may suggest an inappropriate option. Sensitivity coefficients help to understand the parametric dependence of an experiment and shape our experience and intuition for future cases.

The focus of this paper is not specific results of a sensitivity analysis or experimental design. The intent is to demonstrate the use of sensitivity coefficients as a design/analysis tool. To assist, several thermal models used in experimental investigations to estimate thermal properties are studied, and sensitivity coefficients are shown. Insight derived from sensitivity coefficients is described.

INSIGHT FROM SENSITIVITY COEFFICIENTS

The use of sensitivity coefficients to provide insight requires some experience and background. As noted previously, the sensitivity coefficient is the partial derivative of temperature with respect to a parameter, which is $\partial T/\partial\beta$ for a parameter β. Because a comparison of magnitudes for different coefficients is often of interest, a scaled (sometimes called "modified") sensitivity coefficient is used:

$$T_\beta \equiv \beta\frac{\partial T}{\partial\beta}. \tag{1}$$

The nomenclature should not be confused with the convention that the subscript signifies the partial derivative only. Some papers represent the sensitivity coefficient for parameter β with X_β. In this paper it is represented as T_β because it has units of temperature, which is a main reason that scaled sensitivity coefficients are used. Because Eq. (1) has units of temperature for all parameters, magnitudes for various parameters can be directly compared. If the sensitivity coefficients are not scaled, they would have different units and possibly different magnitudes. For example, the sensitivity to thermal conductivity and volumetric heat capacity would have units of $[(m^\circ C^2)/W]$ and $[(m^3{}^\circ C^2)/J]$, respectively, and could not be directly compared. By scaling the sensitivity coefficients their magnitudes can be compared with a representative temperature change for the case. For example, the temperature change from the initial temperature, or temperature rise, is a typical representative temperature for transient cases. This gives a reference for the magnitude of the scaled sensitivity coefficient.

It depends upon one's perspective whether sensitivity is desired to be large or small. Small sensitivity coefficients, or general insensitivity, are beneficial in instances where the parameters are not well quantified, such as a material with thermal properties that are not well characterized. Then the parameter is not influential on the thermal response. Experiments to estimate parameters, however, require that the measured response be sensitive to the parameters. In this case, the scaled sensitivity coefficients are desired to be large in magnitude (compared to the representative temperature) and linearly independent (having different shapes). The desire for these characteristics can be physically motivated. Since the temperature is measured and used to infer information about other parameters in the model, it is essential that the temperature be sensitive to the parameters. The more sensitive the temperature (or large the sensitivity coefficient) is, the more valuable the temperature measurements are. By similar reasoning, to estimate multiple parameters requires the sensitivity, or the effect on temperature of each parameter, to be different or independent of one another for each parameter. If two parameters have similar effects on temperature, their individual influence is difficult to distinguish.

The basis for the general characteristic that the sensitivity coefficients be large (that is, the scaled sensitivity coefficients be large compared to the temperature rise) and linearly independent can be motivated from an optimality criteria (Beck and Arnold, 1977, pp. 432). With standard statistical assumptions it can be shown that the larger the sensitivity coefficients are, the more optimal is the experiment, assuming the sensitivity coefficients are (linearly) independent for multiple parameter. Another beneficial characteristic is that sensitivity coefficients have sign change(s), with respect to time or location. The sign changes result in the sensitivity coefficients for multiple parameters having a more pronounced independence, displaying different effects on the temperature.

SENSITIVITY COEFFICIENTS FOR COMPLEX PROBLEMS

Solving complex problems with irregular geometry or multiple materials requires a numerical method. To obtain sensitivity coefficients with a numerical method there are typically two options. The first uses a finite difference approximation of sensitivity coefficients. By performing two numerical solutions, one with the base parameter and the second with the parameter perturbed, the sensitivity coefficient is approximated. The second option is to derive sensitivity equations from the describing equations and numerically solve them. The latter option is used in this paper.

The details of the methods and algorithm to compute the sensitivity coefficients are described in Blackwell, et al., 1998. An unstructured grid solver based on a control volume finite element formulation for spatial discretization and implicit time discretization is used. The code architecture has been designed such that multiple equations can be solved. With this design, computing sensitivity coefficients requires writing additional element routines for the desired sensitivity coefficients.

EXAMPLE CASES

In this section problems with practical applications are studied. The cases are based upon experimental investigations to estimate thermal properties of carbon-carbon composite (Dowding et al., 1995 and Dowding et al., 1996). The first example demonstrates how measurements are impacted by other materials in an experimental apparatus. Through observation of the sensitivity coefficients, the impact is assessed, and an alternate experimental configuration is suggested. Sensitivity coefficients for two experimental configurations are contrasted. The first example is based on one dimensional heat flow. For the second example the experiment configuration is modified for two dimensional heat flow to investigate the orthotropic thermal conductivity of the carbon-carbon. In the final example the arrangement of a thin film or coating on a substrate is studied. It is discussed in the context of a thin, high conductivity layer on an orthotropic substrate, which is the case for carbon-carbon with silicon-carbide coating that protects it from oxidation. The two dimensional examples focus on identifying sensitive regions and studying the sensitivity distributions for multiple parameters.

Property Estimation - One-Dimensional Experiments

During the course of an experimental investigation to estimate the thermal properties of carbon-carbon composite two experimental configurations were used. In the reported results (Dowding et al., 1995) only the final configuration is considered. Due to the difficulties encountered with an early configuration, a study and contrast of theses configurations is instructive to demonstrate experimental design and the application of sensitivity coefficients. There are two main distinctions between the configurations. The first configuration considered, experimental configuration 1, has the novel approach of measuring temperature within a kapton heater assembly. Additionally a high conductivity material is put in contact with the back surface of the composite test specimen. In the second configuration, experimental configuration 2, temperature is not measured in the heater assembly but at the surface of the carbon-carbon and a low conductivity material is put at the back surface of the composite test specimen.

Experimental Configuration 1. The model of an experiment to estimate the thermal properties of a carbon-carbon composite is shown schematically in Figure 1. The model represents one-half of the actual experimental apparatus. The configuration has nominally identical halves on both sides of an electric heater. By assuming symmetry and measuring the electrical power to the heater, the surface heat flux (q_0) is quantified. There are three main materials in the model: a kapton heater assembly, carbon-carbon composite, and aluminum block. A small amount of silicon grease (approximately 0.13 mm thick) is put between the layers to promote thermal contact. The goal of the experiment is to measure q_0 and T and estimate the thermal properties of the carbon-carbon. The aluminum block is intended to maintain a nearly constant, but measured, temperature boundary condition. The heater assembly is a kapton material with a resistance heater and resistance temperature detector (RTD) sensor assemblies. The heater is modeled with effective thermal properties that represent the combined effect of the heater and silicon grease as a single lumped material with thermal properties ($k_{kap,eff}$, $\rho c_{kap,eff}$). In future references to the kapton heater it is understood that the effects of the silicon grease (at this interface only) are lumped with the heater and described with effective thermal properties. An experiment for estimating the effective thermal properties of the kapton heater is discussed later.

The experiment is intended for estimating the thermal properties of the carbon-carbon composite without instrumenting it - i.e., attaching the thermocouples to the specimen. The heater assembly has a resistance heater as well as integral RTD sensors that are calibrated for measuring temperature. Thermocouples are also placed between the carbon-carbon and aluminum block. It is naturally advantageous to conduct an experiment without having to attach sensors to the carbon-carbon. Furthermore, since the carbon-carbon composite has an oxidation protective layer, instrumenting the specimen may change its thermal characteristics. The possible difficulty with such an arrangement is the importance of the other material's thermal properties in the model. That is, how sensitive are the measured temperatures to the (effective) thermal properties of the kapton heater and silicon grease between the composite and aluminum block? To quantify the effect of the material properties on the temperature response in the model, sensitivity coefficients are studied.

The simplified model for this experiment, which neglects the thermal effects of the kapton heater and aluminum block, is a single material with a prescribed (constant) surface heat flux on one surface and prescribed (constant) temperature on the other surface. This model has been studied to identify optimal experimental conditions to estimate thermal properties (Taktak et al., 1993). Similar

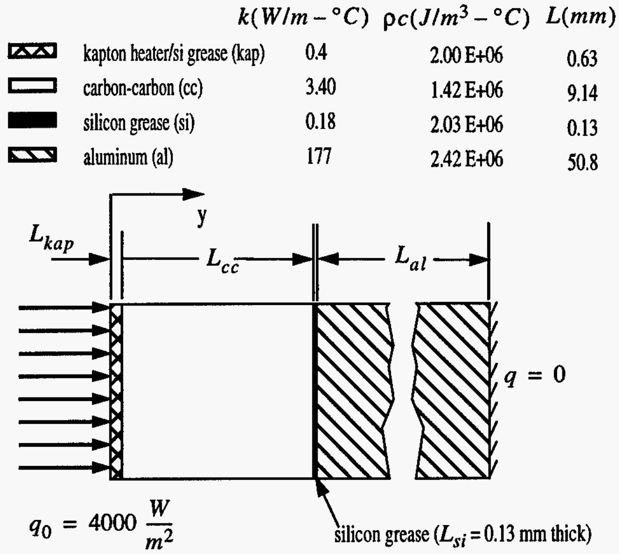

Figure 1. Experiment for estimating thermal properties of carbon-carbon composite (y = 0 at left face), experimental configuration 1

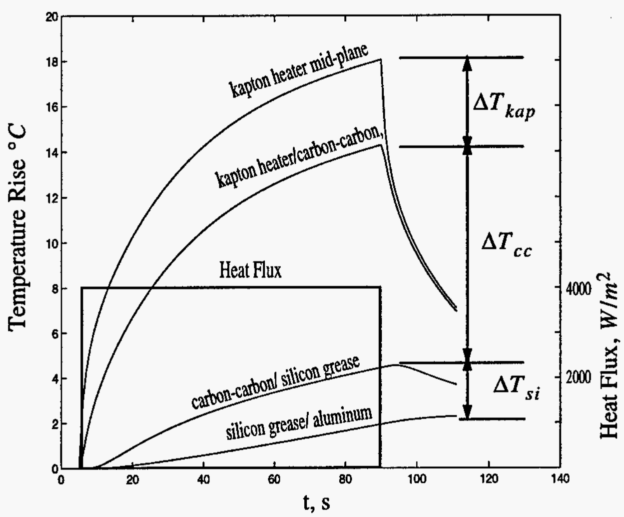

Figure 2. Surface heat flux and temperature at various locations for experimental configuration 1; used to characterize thermal properties of carbon-carbon

experimental conditions are used to generate temperature and sensitivity coefficients in this study.

The temperature at various locations in the experimental apparatus is shown in Figure 2 along with the surface heat flux. Locations selected included the mid-plane of the kapton heater ($y = L_{kap}/2$), the heater/carbon-carbon interface ($y = L_{kap}$), the carbon-carbon/silicon grease interface ($y = L_{cc} + L_{kap}$), and the silicon grease/aluminum interface ($y = L_{cc} + L_{kap} + L_{si}$). Observing the temperature difference across (one-half) the heater assembly shows that it is inappropriate to neglect the thermal effects of the heater. Although it is thin compared to the carbon-carbon, the low

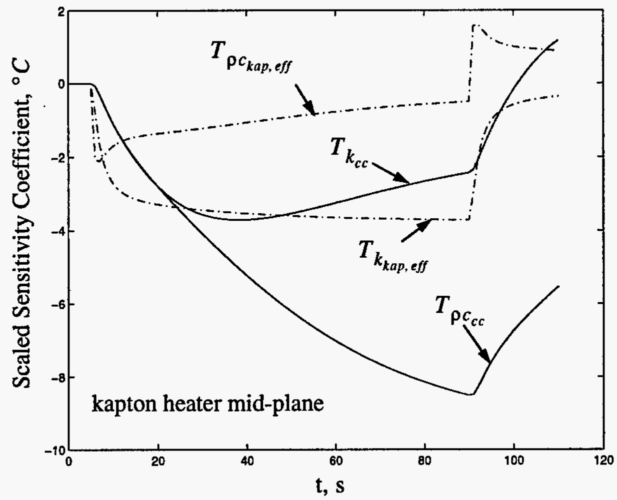

Figure 3. Sensitivity coefficients for experiment to characterize thermal properties of carbon-carbon at the location of RTD sensors within the kapton heater assembly ($y = L_{kap}/2$), experimental configuration 1

(effective) thermal conductivity results in its effect being thermally important. Similarly, there is a large temperature difference across the silicon grease. The carbon-carbon has a larger temperature differential, suggesting it is (thermally) important. The magnitudes of the temperature differentials indicate that the effective properties of the kapton heater and silicon grease may be important as well. However, measuring the temperature drop across these thin layers is difficult and not typically available. Hence, sensitivity coefficients are useful to quantify the effect of the kapton heater and silicon grease on temperature and compare with the carbon-carbon's effect.

The sensitivity coefficients at the mid-plane of the kapton heater assembly, where the RTD sensors are located, are shown in Figure 3. At early time ($t < 10$ s) the sensitivity coefficients for the thermal properties of the heater respond with a larger magnitude than those for the carbon-carbon; this point is discussed in more detail later. The effective thermal conductivity of the kapton heater is as important as the thermal conductivity of the carbon-carbon. Sensitivity to the effective thermal conductivity of the kapton heater is of the same order as the sensitivity to the conductivity of the carbon-carbon. Only for a short time is the sensitivity to the thermal conductivity of the carbon-carbon larger than the kapton heater's sensitivity. The sensitivity coefficient for the effective volumetric heat capacity of the kapton heater is not as important; it is an order of magnitude smaller than that of the carbon-carbon and decreases with time.

The small amount of silicon grease (0.13 mm thick) at the interface of the carbon-carbon/aluminum block may appear innocuous. However, considering the magnitude of the sensitivity coefficients at this location, shown in Figure 4, it is important. The sensitivity to the conductivity of the silicon grease is as large as the sensitivity to the conductivity of the carbon-carbon at this location. The magnitude is also comparable to sensitivity coefficients for conductivity at the mid-plane of the heater (Figure 3) as well. It may

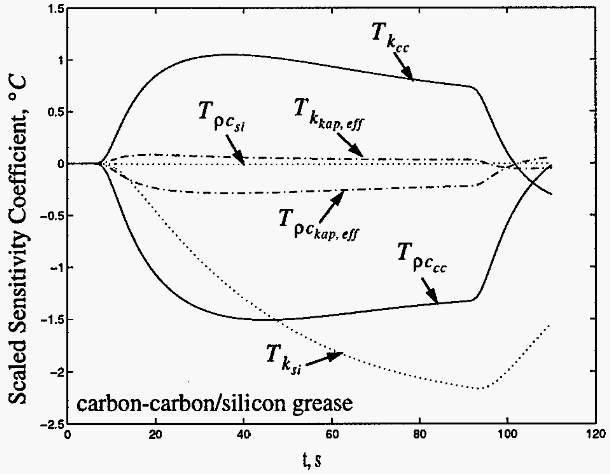

Figure 4. Sensitivity coefficients for experiment to characterize thermal properties of carbon-carbon at carbon-carbon/silicon grease interface ($y = L_{kap}+L_{cc}$), experimental configuration 1

be better in this situation to use the temperature measurement at this location as a boundary condition and not include the aluminum block in the model. However, the sensors need to be (thermally) attached to the carbon-carbon specimen. If sensors are placed between the layers and allowed to "float" in the silicon grease, the small uncertainty in the sensor location can significantly effect the temperature (see Figure 2) and influence the estimated properties of the carbon-carbon.

Notice in Figure 2 that temperature data are acquired after the heat flux stops. Continuing to acquire data after stopping the heat flux results in an improved experiment. This is because it causes the sensitivity coefficients (Figure 3 and 4) to dramatically change shape after the heating stops. These effects result in a more accurate estimation of multiple thermal properties (Beck and Arnold, 1977, pp. 459).

The outcomes shown in Figure 3 and 4 are not ideal for estimating thermal properties of the carbon-carbon test specimen. Because the effective thermal conductivity of the kapton heater and conductivity of silicon grease are important, as demonstrated by their sensitivity coefficients, the accuracy of the estimated properties of the carbon-carbon will depend upon accurately specifying properties for these materials. Since the sensitivity coefficients for the other materials are appreciable in size (compared to the temperature rise), it may be possible to simultaneously estimate properties for all materials. It is, however, better to estimate as few properties as needed from a single experiment

It was previously noted at early time that the sensitivity to the effective properties of the kapton heater is proportionally larger than the sensitivity to the carbon-carbon (See Figure 3). Figure 5 shows data from an experiment conducted for a shortened time duration. Sensitivity coefficients for the experiment are plotted in Figure 6. In this time region the temperature is more sensitive to the effective thermal properties of the kapton heater than to the thermal properties of the carbon-carbon. Consequently, by conducting an experiment that heats for a short time, the effective

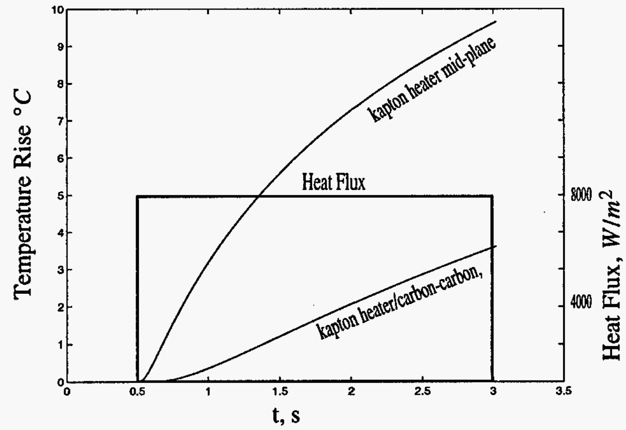

Figure 5. Surface heat flux and temperature for experiment to characterize thermal properties of kapton heater/silicon grease, experimental configuration 1

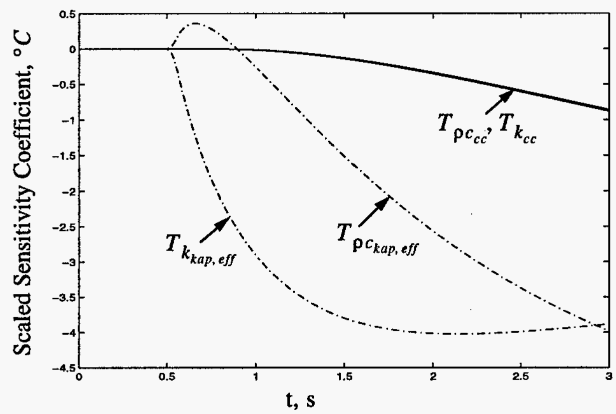

Figure 6. Sensitivity coefficients at location of RTD sensors within kapton heater assembly ($y = L_{kap}/2$) for experiment to characterize kapton heater/silicon grease, experimental configuration 1

properties of the kapton heater (including the effects of the silicon grease) can be estimated without detailed knowledge of the carbon-carbon's thermal properties. Conducting this type of experiment is important because the properties of the kapton heater and silicon grease are typically not accurately known and it is difficult to accurately model the thin grease layer. By conducting this type of experiment, the properties of the heater and silicon grease are estimated as a combined effect and represented with the estimated effective properties.

The experimental configuration in Figure 1 is a preliminary design to measure thermal properties of carbon-carbon composite. Before conducting experiments the sensitivity coefficients should be studied to design the experiment. However, to do so the thermal properties of all materials are required. Since the experiments are being conducted to estimate thermal properties, some properties must be approximated. Carbon-carbon composite is quite variable depending on the processing, and the thermal conductivity was an order of magnitude larger than previously studied materials. The

	$k(W/m-°C)$	$\rho c(J/m^3-°C)$	$L(mm)$
mica heater (mica)	0.14	2.03 E+06	0.44
carbon-carbon (cc)	3.40	1.42 E+06	9.14
insulation (ins)	0.09	0.42 E+06	50.8

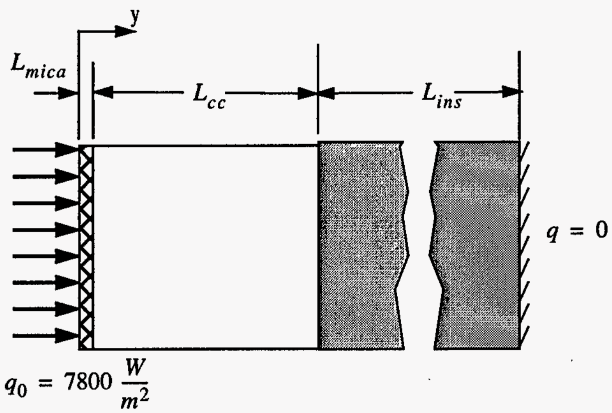

Figure 7. Experiment for measuring thermal properties of carbon-carbon composite, experimental configuration 2

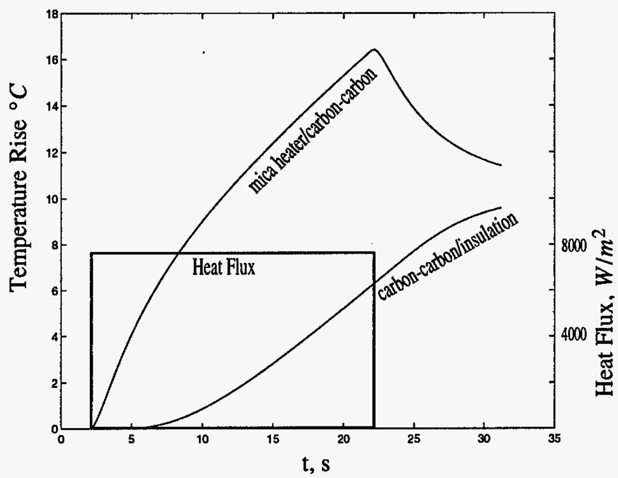

Figure 8. Surface heat flux and temperature on either side of carbon-carbon specimen for experimental configuration 2; used to characterize thermal properties of carbon-carbon,

result was an experiment not suited to accommodate a material with high thermal conductivity. An examination of the sensitivity coefficients indicated deficiencies in the experimental design. If sensitivity coefficients had not been studied it is possible that the impact of the other materials may not have been realized.

Experimental Configuration 2. To accommodate the carbon-carbon's higher thermal conductivity, and the desire to investigate temperatures above the capability of the kapton heater/RTD, the experimental configuration is redesigned. Two major changes are adopted. First, the specimens are instrumented with temperature sensors. Grooves are machined on the surface, and thermocouples are cemented into the grooves. This reduces the sensitivity of the temperature (at the measurement location) to the effective properties of the mica heater. Because higher temperature experiments

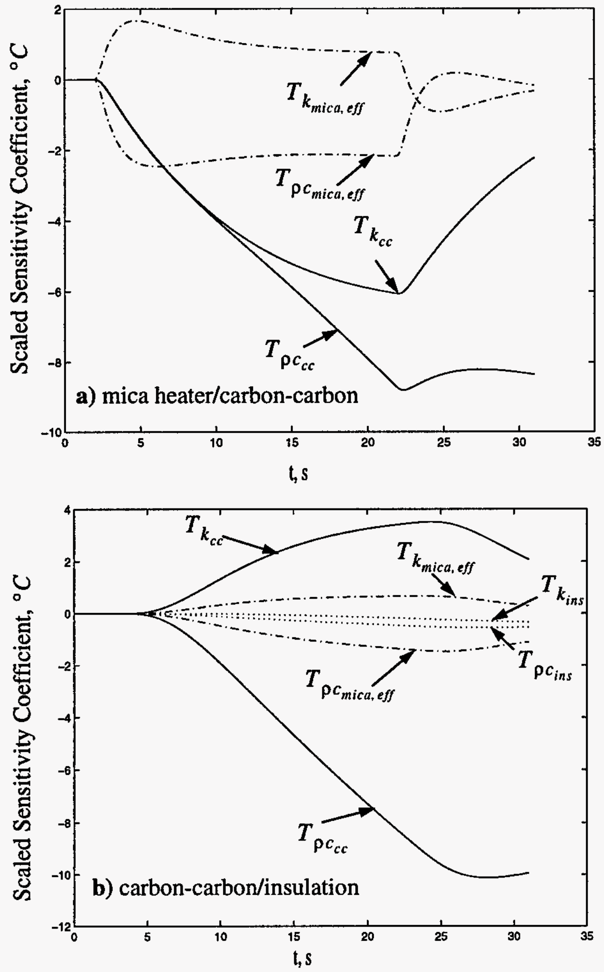

Figure 9. Sensitivity coefficients for experiment to estimate properties of carbon-carbon **a)** at heater/carbon-carbon interface ($y = L_{mica}$) **b)** at carbon-carbon/insulation interface ($y = L_{mica} + L_{cc}$), experimental configuration 2

are intended, silicon grease is not used as in the previous design, and the contact resistance between the heater and specimen is expected to be greater. As previously described, a short duration experiment is used to quantify the contact resistance by lumping its effect with the heater and describing it with effective properties. The second change is that the aluminum block was replaced with an insulating material. With the insulation, temperature gradients are small at the carbon-carbon/insulation interface, and the measurement is not as sensitive to the specified location or contact resistance between the layers. The redesigned experiment is shown schematically in Figure 7. A mica heater assembly is used, and a short duration experiment (not shown for this configuration) is analyzed to quantify the effective properties of the mica heater/contact resistance. In addition, these effective properties account for the cement used to attach the thermocouples to the surface. See Dowding et al. (1995) for a more detailed discussion of the experiment. The back surface of the carbon-carbon is bounded by the insulation.

The temperature on both sides of the carbon-carbon is shown in Figure 8. The optimal experiment for this simplified case, a single material with a prescribed heat flux on one surface and the other surface adiabatic, is much shorter in duration (Beck and Arnold, 1977, p. 454). The experimental duration is 20 s, compared to 80 s for the previous configuration. The maximum temperature rises are similar.

Sensitivity coefficients at one location for temperature measurement ($y = L_{mica}$) are displayed in Figure 9a. Although, the effective conductivity of the mica heater is one-third that of the kapton heater, in this configuration the temperature is not as sensitive to the effective thermal properties of the mica heater. Except for early time ($t < 7$ s) the sensitivity to the effective properties of the mica heater is small in comparison. (It is seen at early times that the sensitivity coefficients for the effective properties of the mica heater are correlated. The sensitivity coefficients have proportional magnitudes with opposite signs.) At later time the sensitivity to thermal conductivity is an order of magnitude larger for the carbon-carbon. Sensitivity to the volumetric heat capacity of the carbon-carbon is four times that of the mica heater/contact resistance.

The sensitivity coefficients at the opposite side of the carbon-carbon are shown in Figure 9b. At this location the sensitivity coefficients are larger then they are for experimental configuration 1, particularly for the volumetric heat capacity. More information is gained at this surface by using an insulating material, when applicable. (For testing low conductivity materials it may not be applicable.) The temperature at this surface is quite insensitive to the thermal properties of the insulating material.

Property Estimation - Two-Dimensional Experiment

Carbon-carbon composite is typically modeled with an orthotropic thermal conductivity. An approach to characterize orthotropic properties is an experiment with two dimensional heat flow (Dowding et al. 1996). The previous one dimensional experiment is made two dimensional by heating over a portion of the surface, the experiment is show schematically in Figure 10. The assembly is identical to the previous experiment; however, the heat flux extends over one-third of the surface to produce two dimensional heat flow. The input heat flux is known ($q_0 = 15{,}000\ W/m^2$), and it is therefore possible to simultaneously estimate thermal conductivity and volumetric heat capacity. Representative values for the thermal properties and dimensions are given in Figure 10. Thermal conductivity in the x-direction is nearly 20 times greater than the value in the y-direction for the carbon-carbon. Temperature and sensitivities to thermal properties ($k_{cc,x}$, $k_{cc,y}$, ρc_{cc}) are investigated after applying the heat flux for 35 s starting at a uniform initial temperature. There are two relevant dimensionless times of interest: time based on information in the y-direction ($(k_{cc,y}/\rho c)t/L_{cc}^2 = 1.00$) or based on the x-direction ($(k_{cc,x}/\rho c)t/a^2 = 0.25$). The duration of the experiment needs to be selected to provide adequate response in both directions. These dimensionless times suggest the duration of the experiment should provide adequate response (sensitivity) for both directions.

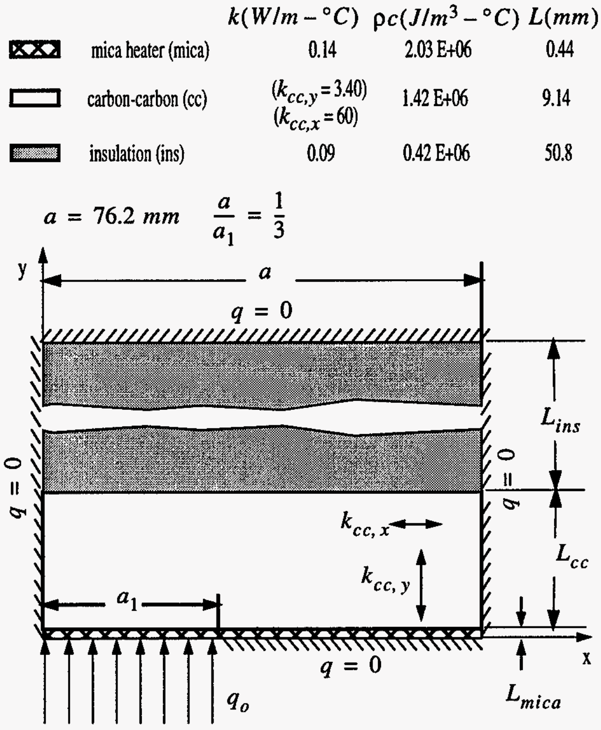

Figure 10. Two dimensional model of proposed experiment to estimate orthotropic thermal properties of carbon-carbon composite material

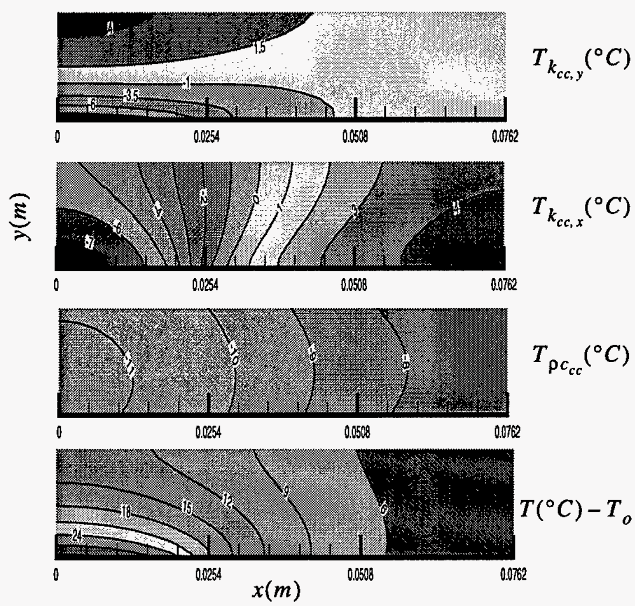

Figure 11. Temperature and sensitivity coefficients throughout spatial domain of the carbon-carbon body ($0 < x < a$, $L_{mica} < y < L_{mica} + L_{cc}$) at $t = 35\ s$

The bottom contour plot in Figure 11 shows the temperature distribution at 35 s for the previously specified properties and dimensions. Only the carbon-carbon body is shown in Figure 11. The orthotropic nature of the material results in the elongated isotherms. The maximum temperature rise is at the corner of the heated region. Sensitivities to the thermal properties ($k_{cc,x}$, $k_{cc,y}$, ρc_{cc}) are shown in the three contour plots above the temperature in Figure 11. In the sensitivity distributions it is noted that the magnitude of the sensitivity coefficients for thermal conductivities (top two contour plots) are largest (both positively and negatively signed) along the boundaries. It is a fortuitous result that the most sensitive locations are easily accessible along the boundaries. This is not an uncommon outcome for experiments with gradient boundary conditions. The sensitivity coefficient for $k_{cc,x}$ changes sign traversing in the x-direction, and $k_{cc,y}$ changes sign traversing in the y-direction. This means at intermediate locations, the sensitivities are zero. It is not as important for $k_{cc,y}$ because the sign change occurs inside the body. But for $k_{cc,x}$ it occurs on an outer surface where measurements may be taken. In addition to having the largest magnitudes at the boundaries (corners), the sensitivities for $k_{cc,y}$ and $k_{cc,x}$ have opposite signs at two corners; this is beneficial and further discussed later. The sensitivity for ρc_{cc} is negative and decreases in magnitude with distance from applied heat flux. From these sensitivity distributions it is concluded that temperature measurements at the corners of the body are the most sensitive.

Transient plots of the sensitivity coefficients at the corners of the carbon-carbon ($x = 0,a$) and ($y = L_{mica}, L_{mica}+L_{cc}$) are shown in Figure 12 as a function of time. To ease the discussion, the heater/carbon-carbon interface is identified along $y = L_{mica} \equiv y_{cc1}$ and carbon-carbon/insulation interface is identified along $y = L_{mica} + L_{cc} \equiv y_{cc2}$. Sign changes with location in the thermal conductivity sensitivity coefficients are clear in this figure; sensitivity to $k_{cc,x}$ is oppositely signed at $x = 0$ and $x = a$ and $k_{cc,y}$ is oppositely signed at $y = y_{cc1}$ and $y = y_{cc2}$. The combination of opposite signs for $k_{cc,y}$ and $k_{cc,x}$ improves the experimental design. Physically this is a result of the sign changes meaning the sensitivity coefficient having much different shapes.

The sensitivity to $k_{cc,y}$ is approaching a steady-state value, while the sensitivity to $k_{cc,x}$ has not. Dimensionless times computed previously show that a longer experiment is needed for the sensitivity to $k_{cc,x}$ to reach steady-state. Although a longer duration experiment would result in a larger sensitivity for $k_{cc,x}$, the sensitivity to ρc_{cc} would likewise increase. Sensitivity to ρc_{cc} increases linearly with time for large times in this model. The result of a longer experiment would be sensitivity to ρc_{cc} dominating the other sensitivity coefficients. To estimate several parameters simultaneously, it is better to not have one coefficient dominate the others. For estimating both thermal conductivities and volumetric heat capacity, the experiment needs to be of sufficient length to provide adequate information in both directions but not too long as to allow the sensitivity for volumetric heat capacity to dominate the sensitivity for the thermal conductivities.

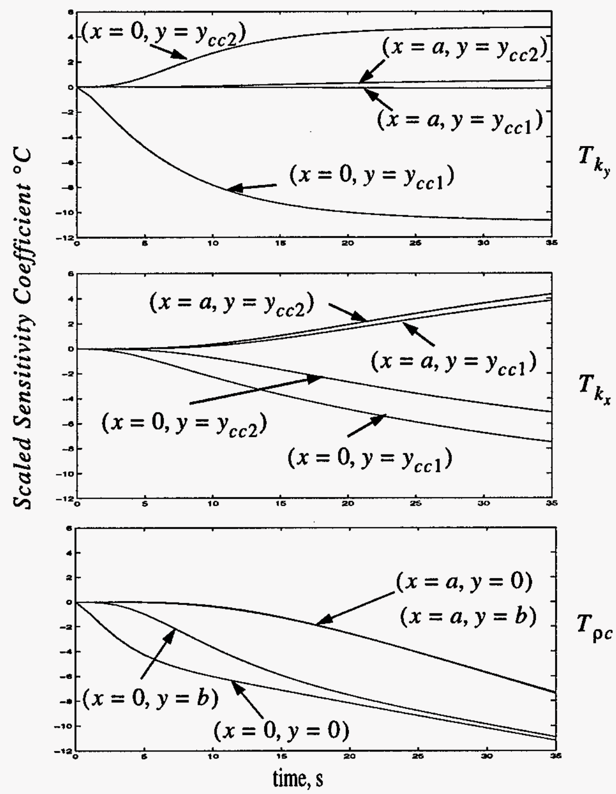

Figure 12. Sensitivity coefficients at the corners of the carbon-carbon body for a two dimensional experiment to estimate orthotropic properties of carbon-carbon

Property Estimation - Two-Dimensional: Thin film on a thick substrate

An important problem with many applications is the arrangement of a thin film or coating on a thick substrate. Examples include the use of films to protect composites from oxidation, chemical vapor deposition (CVD) of diamond film on a silicon substrate, and the

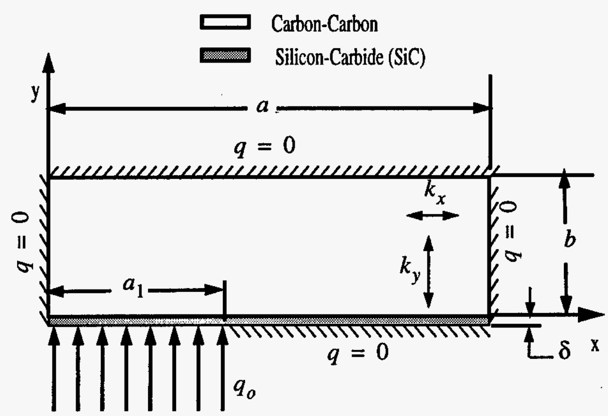

Figure 13. Model of proposed two dimensional experiment to estimate thermal properties of carbon-carbon composite material and silicon carbide coating

use of ceramic coatings to protect high temperature components, such as turbine blades. Carbon-carbon composite has a thin layer of silicon-carbide (SiC) coating on its surface that protects the underlying carbon-carbon from oxidation. A schematic of an experimental set-up for determining the properties of the carbon-carbon material with a SiC layer is shown in Figure 13. To develop insight to the problem, the silicon-carbide layer is neglected on all but the bottom surface. All surfaces are assumed adiabatic except over a portion of the bottom surface where the heat flux is applied. A mathematical description for the model is

$$\frac{k_x}{\rho c_{cc}}\frac{\partial^2 T}{\partial x^2}+\frac{k_y}{\rho c_{cc}}\frac{\partial^2 T}{\partial y^2}=\frac{\partial T}{\partial t} \quad 0<x<a\ ,\ 0<y<b \qquad (2)$$

$$\left[-\left(\frac{k_{sic}\delta}{\rho c_{sic}}\right)\frac{\partial^2 T}{\partial x^2}-\left(\frac{k_y}{\rho c_{sic}}\right)\frac{\partial T}{\partial y}+\delta\frac{\partial T}{\partial t}\right]\Bigg|_{y=0} = \begin{Bmatrix} \dfrac{q_o}{\rho c_{sic}} & (0\le x\le a_1) \\ 0 & (a_1<x\le a) \end{Bmatrix} \qquad (3)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=b}=0, \qquad \left.\frac{\partial T}{\partial x}\right|_{x=0,a}=0 \qquad (4)$$

$$T|_{t=0}=0 \qquad (5)$$

The effect of the SiC film is modeled in the boundary condition on $y = 0$, Eq. (3), which assumes negligible temperature variation across the thickness of the SiC. Notice that this boundary condition contains a second order derivative on x, which can be made a second order derivative on y by substituting from the differential equation. If an analytical solution is sought, this is a necessary substitution. For developing insight into the parameters, it is not used.

In an experiment where the heat input is unknown, which is typical of heating with a radiating source or laser, the heat input must be estimated. In such cases it is not possible to simultaneously estimate the thermal conductivity and volumetric heat capacity. Since q_o is unknown, only thermal diffusivities ($k/\rho c$) can be estimated. The thermal conductivity k_y appears as $k_y/\rho c_{cc}$ and $k_y/\rho c_{sic}$ in Eq. (2) and Eq. (3). Since it is not possible to simultaneously estimate both ratios with an unknown heat input, the volumetric heat capacities are assumed equal, $\rho c_{si} = \rho c_{cc} = \rho c$. This is not an unreasonable assumption for these materials. The temperature, which has already been scaled by the volumetric heat capacity in Eq. (2) - Eq. (3), is shown to depend on the following parameters:

$$T = T\left(\frac{k_{sic}}{\rho c}, \frac{k_y}{\rho c}, \delta, \frac{q_o}{\rho c}, \frac{k_x}{\rho c}\right) \qquad (6)$$

To determine thermal properties for an unknown heat input requires estimating $q_o/\rho c$ as a nuisance parameter. That is, $q_o/\rho c$ is estimated even though its magnitude is not directly of interest, although it is a necessary parameter to obtain the thermal diffusivities. In principle the sensitivities for all five groups are independent and all can be estimated. The thickness, δ, is usually available from microscopic optical measurements. To investigate the practicality of estimating the other parameters the sensitivity of the temperature to these parameters is investigated.

Measurements at early time and near the region where the heating is applied have information about the properties of the coating. Unlike the previous two-dimensional case, the magnitude of the heat flux is assumed unknown and must also be estimated. For the previous case without the SiC coating, the body was heated over one-third of the surface. For the SiC coating to be thermally important (have a large sensitivity coefficient), the heating should occur over a distance that is on the order of the coating thickness. If the heating occurs over a much larger area, the coating is not important in the model. This is demonstrated in the limit by considering that the entire surface is heated, resulting in one-dimensional heat flow. Since the coating is thin and has a relatively high thermal conductivity, the heat flow is dictated mainly by the lower conductivity substrate, and the coating is not significant. Conceptually, this experiment tries to exploit the two dimensional effects caused by the SiC coating to estimate its properties. Sensitivity coefficients are studied to understand this experiment (model).

The thermal properties and dimensions are $k_{sic} = 200\ W/m{-}°C$, $k_x = 60\ W/m{-}°C$, $k_y = 3.4\ W/m{-}°C$, $\rho c = 1.42E{+}06\ J/m^3{-}°C$, $a/b = 2.54/0.914$, $\delta/b = 0.5/9.14$, and $a_1/a = 0.5/25.4$. The conditions are representative of a small carbon-carbon specimen (2.54 cm x 0.914 cm x 1.2 cm) with a 0.5 mm thick layer of silicon-carbide. The surface is assumed to be uniformly heated over a region 0.5 mm x 1.2 cm (on the 2.54 cm x 1.2 cm face) with a constant heat flux of $q_o = 100{,}000\ W/m^2$. A laser can experimentally provide this type of heat flux. It is assumed that the volumetric heat capacity is known, and the thermal conductivities are to be estimated. Sensitivities with respect to thermal conductivities are, therefore, discussed. If volumetric heat capacity is unknown, then the thermal diffusivities can be estimated instead. In either case the scaled sensitivity coefficients for the diffusivity and conductivity are the same (for constant properties).

The temperature distribution is shown in Figure 14 at $t = 8$ s. The scaled sensitivity coefficient for the input heat flux, which is also to be estimated, is equal to the temperature rise for the case of all gradient boundary conditions. The temperature effects are localized near the region of heating. Due to the high thermal conductivity of SiC coating and orthotropic substrate, the isotherms are elliptical and highly elongated. Sensitivities to the three thermal conductivities are shown in the top three contour plots of Figure 14 at $t = 8$ s. Similar to the temperature, the sensitivity coefficients are largest near the region of heating for the three conductivities.

Sensitivity to the conductivity of the SiC coating diminishes rapidly with distance from the heated region. Within a distance of five times the coating thickness (2.5 mm), the sensitivity coefficient is reduced by one-half. This means that to obtain information about the coating with this experiment, temperature measurements are required in a small region near the applied heat flux. Furthermore,

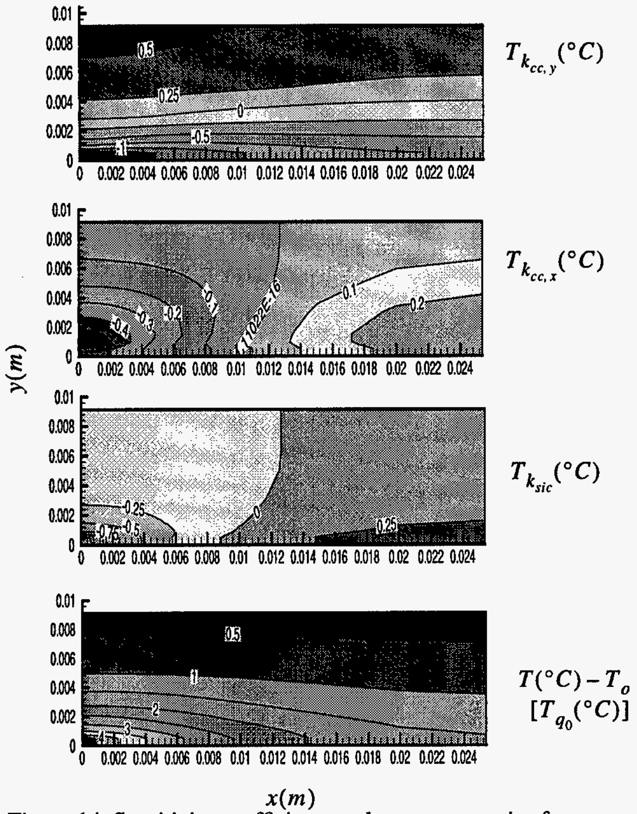

Figure 14. Sensitivity coefficients and temperature rise for a two dimensional experiment on an orthotropic body with a silicon-carbide coating, $t = 8\ s$

within this small region the measurements are quite sensitive to the specified location because temperature gradients are large there. The temperature will depend on accurately representing the distribution of the heat flux provide by the heat source in this region as well.

Sensitivity to the thermal conductivities of the carbon-carbon ($k_{cc,y}, k_{cc,x}$), shown in top two contour plots of Figure 14, are quite different from each other. Sensitivity to $k_{cc,y}$ is larger and has appreciable magnitude throughout the entire domain. It has a sign change across the body also. Sensitivity to $k_{cc,x}$ is less than one-half in magnitude as that for $k_{cc,y}$ at the maximum. Its magnitude is greatest in a localized area near the heated region, like the sensitivity to k_{sic}. Comparing contour sensitivity plots for $k_{cc,x}$ and k_{sic} shows similarity in their shapes suggesting correlation. Because of this correlation and the magnitude of the sensitivity for $k_{cc,x}$, it is unlikely that $k_{cc,x}$ can be estimated from this experiment measurements along $y=0$.

Since the most information is available at the corner where the heat flux is applied, transient sensitivity coefficients are plotted at this location. Figure 15a shows a plot of the sensitivity coefficients as a function of time at $x = 0, y = 0$. As noted in the contour plots, the shape of the curves for $k_{cc,x}$ and k_{sic} are proportional, and the magnitude of the sensitivity for k_{sic} is five times as large as that of $k_{cc,x}$. The SiC coating masks the effect of conductivity in the x-

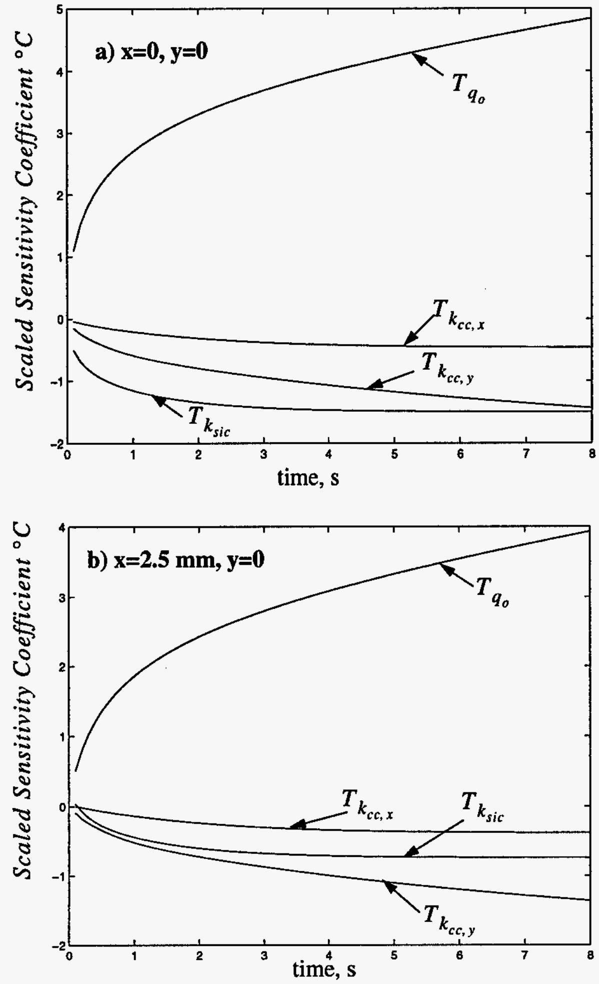

Figure 15. Sensitivity coefficients for a two dimensional experiment on a body with a silicon-carbide coating **a)** at location of heating ($x = 0, y = 0$) **b)** at location ($x = 2.5\ mm, y = 0$)

direction of the substrate. If the intent is to estimate $k_{cc,x}$, this is not a good result. However, if $k_{cc,x}$ is not of main interest, and possibly not well known, such insensitivity is beneficial. At $x=0, y=0$ the sensitivity to k_{sic} is larger than it is for $k_{cc,y}$ But sensitivity to $k_{cc,y}$ increases with time while the sensitivity to k_{sic} quickly reaches a steady value; this experiment is nearly equally dependent on $k_{cc,y}$ and k_{sic}. If time ranges greater than 8 s are considered, then $k_{cc,y}$ becomes the dominant thermal parameter. In contrast, consider a location that is five coating thickness away ($x=2.5\ mm, y=0$). The sensitivity in Figure 15b shows that $k_{cc,y}$ is larger in magnitude than k_{sic} for all time. Sensitivity for k_{sic} is one-half its value at x=0, while sensitivity to $k_{cc,y}$ is comparable in magnitude. This demonstrates the local nature of the information for the SiC coating. Beyond this x location the quality of information about the coating rapidly diminishes.

It is concluded from the sensitivity coefficients that the experiment is insensitive to $k_{cc,x}$, early time measurements near the applied heat flux are required for sensitivity to k_{sic} to be appreciable, for later time and greater distance from the applied flux $k_{cc,y}$ has the largest sensitivity. Due to practical issues of obtaining measurements near the applied flux, estimating k_{sic} from this experiment is will be difficult.

SUMMARY

Application of sensitivity coefficients have been discussed and presented. In the context of parameter estimation, it is desirable to have large sensitivity coefficients, compared to a representative temperature, and have them vary in shape for different parameters. For materials with thermal properties that are not well characterized or understood, it may be desirable to have their sensitivity coefficients be small. Much insight can be gained by observing the sensitivity coefficients.

The sensitivity for two experimental configurations to estimate the thermal properties of carbon-carbon composite were contrasted. Sensitivity coefficients demonstrated that an experimental design can depend significantly on additional materials in the experimental apparatus. Observation of sensitivity coefficients showed the importance of additional materials in the model on the thermal response. With this information an alternate configuration was used, which was not as sensitive to the thermal properties of the other materials. A second example showed the sensitivity coefficients for a two dimensional orthotropic body. Cases with, and without, a thin, high conductivity coating were shown. In both instances, the most sensitive locations, and ability to estimate the desired parameters, were determined by the sensitivity coefficients.

ACKNOWLEDGEMENTS

The experiments discussed were conducted at Michigan State University and supported by US Air Force contract number FY1456-91-N0058 and the Research Excellence Fund of the State of Michigan through the Composite Materials and Structures Center at Michigan State University.

The authors would like to acknowledge Professor J. V. Beck of Michigan State University for having introduced us (BFB and KJD) to the useful concept of sensitivity coefficients. In addition, the paper was strongly influenced by many discussions with Professor Beck concerning the experimental configurations presented.

REFERENCES

Beck, J. V., 1996, "Parameter Estimation Concepts and Modeling: Flash Diffusivity Application," Proceeding of the Second International Conference on Inverse Problems in Engineering: Theory and Practice, eds. D. Delaunay, K. Woodbury, and M. Raynaud, 9-14 June 1996, LeCroisic, France, ASME Engineering Foundation.

Beck, J. V., Cole, K, Haji-Sheikh, A., and Litkouhi, B., 1992, *Heat Conduction Using Green's Functions*, Hemisphere, New York.

Beck, J. V., and Arnold, K. J., 1977, *Parameter Estimation*, Wiley, New York.

Beck, J. V., 1970, "Sensitivity Coefficients Utilized in Nonlinear Estimation With Small Parameters in a Heat Transfer Problem," *Journal of Basic Engineering*, June 1970, pp. 215-222.

Dowding, K. J., Beck, J. V., and Blackwell, B. F., 1996, "Estimation of Directional-Dependent Thermal Properties in a Carbon-Carbon Composite," *International Journal of Heat and Mass Transfer*, Vol. 39, No. 15, pp. 3157-3164.

Blackwell, B. F., Cochran, R. J., and Dowding, K. J., 1998, "Development and Implementation of Sensitivity Coefficient Equations For Heat Conduction Problems," presented at 7th AIAA/ASME Joint Thermophysics and Heat Transfer Conference, June 15-18, 1998, Albuquerque, NM.

Dowding, K., Beck, J., Ulbrich, A., Blackwell, B., and Hayes, J., 1995, "Estimation of Thermal Properties and surface Heat flux in a Carbon-carbon Composite Material," *Journal of Thermophysics and Heat Transfer*, Vol. 9, No. 2, pp. 345-351.

Emery, A. F., and Fadale, T. D., 1990, "Stochastic Analysis of Uncertainties in Emissivity and Conductivity by Finite Element," paper 90-WA/HT-11, presented at Winter Annual Meeting Dallas, TX, November 1990.

Emery, A. F., and Fadale, T. D., 1996, "Design of Experiments Using Uncertainty Information," *ASME Journal of Heat Transfer*, Vol. 118, pp. 532-538.

Emery, A. F., and Fadale, T. D., 1997, "Designing Thermal Systems with Uncertainty Properties using Finite Element/Volume Methods," Proceeding of the 32nd National Heat Transfer Conference, Vol. 2, eds. G. S. Dulikravich and K. A. Woodbury, Baltimore, MD, HTD-Vol 340, pp. 75-81.

Marschall, J., and Milos, F., 1997, "Simultaneous Estimation of Sensor Locations During Thermal Property Parameter Estimation," Proceeding of the 32nd National Heat Transfer Conference, Vol. 10, eds. R. Clarksean et. al, Baltimore, MD, HTD-Vol 348, pp. 79-89.

Moffat, R. J., 1985, "Using Uncertainty Analysis in the Planning of an Experiment," *Journal of Fluids Engineering*, Vol. 107, pp. 173-182.

Moffat, R. J., 1982, "Contributions to the Theory of Single-Sample Uncertainty Analysis," *Journal of Fluids Engineering*, Vol. 104, pp. 250-260.

Taktak, R., 1992, "Design and Validation of Optimal Experiments for Estimating Thermal Properties of Composite Materials," Ph.D thesis, Michigan State University, East Lansing, Michigan.

Taktak, R., Beck, J.V., and Scott, E., 1993, "Optimal Experimental Design for Estimating Thermal Properties of Composite Materials," *International Journal of Heat Mass Transfer*, Vol. 36, No. 12, pp. 2977-2986.

Vozar, L., and Sramkova, T., 1997, "Two data Reduction Methods for Evaluation of Thermal Diffusivity from Step-Heating Measurements," *International Journal of Heat and Mass Transfer*, Vol 40, No. 7, pp. 1647-1655.

M98004238

Report Number (14) SAND--98-0543C
CONF-980610--

Publ. Date (11) 199802
Sponsor Code (18) DOE/DP, XF
UC Category (19) UC-706, DOE/ER

DOE