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Heat Transfer Conditions

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ANALYSIS OF A HTSC CURRENT LEAD UNDER NONLINEAR HEAT TRANSFER CONDITIONS

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INTRODUCTION

Vapor cooled cryogenic current leads have been used extensively to supply current from an electrical power source at ambient temperature to superconducting magnets. In the literature one finds two approaches to the design procedure concerning the major assumption that involves the heat convection mechanism between the lead and the coolant. The first assumes that the heat transfer is ideal, i.e., the heat transfer coefficient between the coolant and the lead is infinite, Ref. /1/. The second approach is relevant in situations when the heat transfer cannot be assumed as ideal, i.e., the heat transfer coefficient has a finite value, Ref. /2/, Ref. /3/. The assumption that the ideal heat transfer cooling model may be used is based on the engineering design practice. An unspecified 'rule of thumb' criterion states that the heat transfer is in most cases 'good enough' to assume the equality between adjacent local lead and coolant temperatures. According to Wilson, Ref. /1/, 'rough estimates' are usually sufficient to ensure that this criterion is satisfied. A rigorous proof of this assertion does not exist.

The analysis of the non-ideal heat convection model is far more complex. From the mathematical point of view to solve the non-ideal problem is even more difficult if the temperature dependencies of the current lead thermal conductivity and electrical resistivity have to be taken into account, as is the case of the high temperature superconductor.

A high temperature superconducting current lead model was analyzed by Sekulic et al., Ref. /2/, for both the superconducting and normal (resistive) modes of operation. The set of design parameters was defined and the optimal values of some of the relevant parameters were determined using a thermodynamic objective function. One of the important idealizations made in that previous study assumes that the heat transfer coefficient has a finite but constant value along the lead.

This study, however, presents preliminary results for even more elaborate model. It is assumed that the heat transfer coefficient has finite but variable values along the lead. The major objective of this effort is to prove the hypothesis that the simplifying assumption of a constant heat transfer coefficient does not cause sizable deviations in optimal design parameters.

NON-IDEAL VAPOR-COOLED CURRENT LEAD MODEL WITH VARIABLE HEAT TRANSFER COEFFICIENT

The mathematical model is represented by two coupled nonlinear differential equations.
The set of standard assumptions, Ref. /2/, has been adopted.

In the dimensionless form the model is as follows:

\[
\frac{d}{d\xi} \left[ a(\bar{T}) \frac{d\bar{T}}{d\xi} \right] + b(\bar{T})\Gamma = Bi^*(\bar{T})(\bar{T} - \Theta) \\
\frac{d}{d\xi} \Theta = St^*(\bar{T})(\bar{T} - \Theta) \\
\xi \in [0, 1], \quad \bar{T} \in [0, 1], \quad \Theta \in [0, \Theta_H] \\
B.C. \quad \bar{T} = 0 \land \Theta = 0 \land \xi = 0; \quad \bar{T} = 1 \land \xi = 1
\]

The functions \( a(\bar{T}) \) and \( b(\bar{T}) \) represent highly non-linear, dimensionless thermal conductivity and electrical resistivity of the lead material, respectively. Functions \( Bi^*(\bar{T}) \) and \( St^*(\bar{T}) \) represent modified Biot and Stanton number functions, respectively. Correspondingly, modified Biot and Stanton numbers vary along the lead. The variation of the modified Biot and Stanton numbers is the major feature of this model comparing it to the previous one, Ref. /2/.

Two characteristic functional relations for \( Bi^*(\bar{T}) \) and \( St^*(\bar{T}) \) have been considered: (i) the linear change of \( Bi^*(\bar{T}) \) (and correspondingly of \( St^*(\bar{T}) \)) with the change of the local temperature, and (ii) the exponential change. In both situations the local value of the corresponding parameter varies between \( 2Bi^* \) and \( (1/2)Bi^* \) (the same holds for the local value of \( St^*(\bar{T}) \)) between the low temperature and the high temperature end of a current lead (4.2/77 K). \( Bi^* \) and \( St^* \) are nominal, constant values of the respective parameters. Two parameter variation trends have been analyzed (i.e., the increase and the decrease of parameter values), but only the results for the second one will be discussed here.

RESULTS

In order to compare the results of the present, more general model to the more simplified approach, let us first assume that the variation of heat transfer coefficient does not influence substantially the conditions of optimal design determined by using the model of the constant heat transfer coefficient. Such an assumption is important not only for preliminary analysis but also for the final design purposes. To prove this assertion it is enough to prove that the change of the optimal design parameters is negligible despite of the local variation in heat transfer coefficient. In this study the minimum of entropy generation caused by heat transfer irreversibilities will be used as a criterion of optimality. Consequently, this study is not intended to consider the all of the possible temperature variations of a heat transfer coefficient along the lead and their influence on the heat transfer phenomenon. Rather, it provides a result of the sensitivity study of the influence of a particular change in relevant parameters (i.e., \( Bi^* \), \( St^* \), \( Bi^*/St^* \), and \( \Gamma \)) on the optimal level of entropy generation caused by heat transfer phenomena and Joule heating (the later being present only in normal, resistive mode of operation). It is important to note, however, that other
nonlinearities that characterize high-temperature superconducting material (the YBCO compound) are also taken into account (i.e., thermal conductivity and electrical resistivity are complex functions of temperature, Ref. /2/). In the normal mode of operation, the validity of the Wiedemann-Franz-Lorenz law is assumed.

The highly nonlinear model (Eqs.(1-4)) has been solved numerically. The same numerical procedure as used by Sekulic et al., Ref. /2/, has also been utilized in the present study. The additional nonlinearities introduced by the variation of \( \text{Bi}' \) and \( \text{St}' \) have been taken into account. The non-linear difference equations, obtained after the reformulation of the continuous problem into the discrete one, were solved using Newton's method, followed by a combination of Runge-Kutta and Euler's method. The initial approximations have been obtained from the Laplace transform solution of the linearized problem. The resulting temperature distribution data have been used to calculate the entropy generation caused by the heat transfer irreversibilities. The final results of the analysis are presented concisely in Figs. 1 and 2.

\[
\begin{align*}
\text{Bi}' &= 2\text{Bi}^* \exp(-aT), \quad \text{Bi}^* = 100, \quad a = 2\ln(2), \quad \Gamma = 0 \\
\text{St}' &= 2\text{St}^* \exp(-aT), \quad \text{St}^* = 100 \neq f(T), \quad \Gamma = 0
\end{align*}
\]

---

In Fig. 1 the numerical solutions for a superconducting mode of operation are presented in terms of entropy generation as a function of the flow parameter, \( \text{Bi}'/\text{St}' \), with \( \text{Bi}' \) and \( \text{St}' \).
being held constant (note, $Bi^*$ (or $St^*$) is a function of dimensionless temperature $\widetilde{T}$). Three characteristic situations are compared. The first one (dashed curve) corresponds to the constant heat transfer coefficient case. The other two correspond to the variable $Bi^*$ and $St^*$ along the lead (see the inset diagram): (i) linear change of $Bi^*(\widetilde{T})$ and $St^*(\widetilde{T})$ (circle symbols), and (ii) exponential (asterisk symbols).

The following important results may be noted. At small $Bi^*/St^*$ values (small local temperature differences) variation of the heat transfer coefficient does not have any significant influence on the overall entropy generation level, as expected. The optimal design corresponds to the minimum entropy generation and the influence of variable heat transfer coefficient is obviously negligible. At high $Bi^*/St^*$ values (large local temperature differences) the variable convective heat transfer coefficient influences significantly the entropy generation. It is obvious that the linear change in $Bi^*$ and $St^*$ has more prominent influence than the exponential one (between the same limiting values of parameters involved).

Fig. 2 demonstrates that the same trend may be noted for both, superconducting and normal (resistive) mode of operation. The lines represent the calculations for three different
dissipated power levels (i.e., the dissipated power parameter, $\Gamma$, takes values 0, 2, and 5) assuming constant heat transfer coefficients ($Bi'$ and $St'$ are constant). The symbols represent the calculations with the variable $Bi'$ and $St'$ parameters. The increase of the Joule heating effect contribution ($\Gamma$ increases) significantly changes (increases) the entropy generation level and shifts the optimal $Bi'/St'$ toward higher values, as expected. It is interesting to note that the influence of the variable heat transfer coefficient shows the same pattern in both, superconducting and normal mode of operation. The optimal design is not affected while at large $Bi'/St'$ values the influence of the variable heat transfer conditions should be taken into account. The influence of the variation in heat transfer coefficient is relatively more pronounced for the superconducting mode of operation.

CONCLUSION

The hypothesis that the variation in heat transfer coefficient along the current lead does not have a significant influence on the optimal design has been proved for a selected set of influential parameters. The minimum of entropy generation caused by heat transfer irreversibilities has been used as a criterion of optimality. The simplifying assumption of a constant heat transfer coefficient for a non-ideal, gas-cooled current lead made of a high temperature superconducting material is acceptable as long as the flow parameter stays relatively low or moderate (i.e., small $Bi'/St'$). For a large flow parameter (i.e., large $Bi'/St'$) the influence of the variation of heat transfer conditions has to be taken into consideration. The superconducting mode of operation is more sensitive to the influence of variable heat convection conditions.

NOMENCLATURE

- $A$ Cross-section area of a current lead [m$^2$]
- $a(\overline{\ell})$ Dimensionless thermal conductivity $[=k(\overline{\ell})/k_{ref}]$
- $b(\overline{\ell})$ Dimensionless electrical resistivity $[=\rho(\overline{\ell})/\rho_{ref}]$
- $Bi'(\overline{\ell})$ Modified Biot number $[=h(\overline{\ell})PL^2/k_{ref}A]$
- $c_p$ Specific heat of coolant (He) at constant pressure [J kg$^{-1}$K$^{-1}$]
- $h$ Heat transfer coefficient [W m$^{-2}$K$^{-1}$]
- $I$ Current [A]
- $k$ Thermal conductivity [W m$^{-1}$K$^{-1}$]
- $L$ HTSC current lead length [m]
- $m$ Coolant mass flow rate [kg s$^{-1}$]
- $P$ Current lead effective heat exchange perimeter [m]
- $S$ Entropy generation rate [W K$^{-1}$]
- $S^*$ Dimensionless entropy generation rate $[=S/(k_H A/L)]$
- $St'$ Modified Stanton number $[=hPL/(\dot{m}c_p)]$
- $t$ Coolant temperature [K]
- $\overline{\ell}$ Dimensionless current lead temperature $[=(T-T_C)/(T_H-T_C)]$
- $x$ Axial coordinate [m]
Greek symbols

$\Gamma$ Dissipated power parameter \[= (IL/A)^2 \rho_{\text{ref}}/k_{\text{ref}}(T_H-T_C)\]

$\Theta$ Dimensionless coolant temperature \[= (t-T_C)/(T_H-T_C)\]

$\xi$ Dimensionless axial coordinate \[= x/L\]

$\rho$ Electrical resistivity \[= \Omega m\]

Indices

C Low temperature end
H High temperature end
ref at reference temperature, $T_{\text{ref}}=T_H$

REFERENCES


ABSTRACT

The main objective of this paper is to consider the influence of the temperature dependence of the convective heat transfer coefficient on the optimal thermal design of the high temperature superconducting [a HTSC] part of a cryogenic current lead. In the previous study [D.P.Sekulic, Z. Uzelac and F. Edeskuty, Cryogenics, 32, 1992, pp.1154-1161] the analysis of a HTSC lead was performed with the assumption that the heat transfer coefficient between the coolant (boil-off liquid helium) and the lead is finite but constant along the lead. In the present effort the heat transfer coefficient is allowed to vary along the lead causing an additional nonlinearity in the model. Numerically obtained current lead temperature profiles and corresponding helium vapor temperature profiles were used to calculate entropy generation within the lead as a function of the relevant design parameters. It has been demonstrated that additional nonlinearities do not cause a significant change of optimal design parameters.
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