*Information Content of Low-Dose Radiographs,
Part 1*

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*Edited by Chris Percy, Group CIC-1
Technical Illustration by AnnMarie Dyson, Group CIC-1
Photocomposition by Randi Bagley, Group CIC-1*

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ABSTRACT

Two measures (not independent) of radiographic image quality are contrast and spatial frequency response. At normal radiographic exposure levels, noise is not a significant factor in evaluating these parameters. However, when the photon fluence on the detector is less than 100 photons/pixel, the statistical noise in the image significantly degrades the image contrast. This paper presents a method of characterizing the image quality that utilizes both contrast and noise by calculating the net information content of the image (in bits) using the image entropy.

The information content (IC) is calculated from

$$IC = H(y) - H(y|x),$$

where $H(y)$ is the total entropy and $H(y|x)$ is the conditional entropy of the image intensity given the object thickness, x . An algorithm for calculating IC is developed and results are presented for simulated radiographs as various radiographic parameters are varied. This Mathematica code is listed in the Appendix. A copper step wedge is radiographed and the actual IC is measured for all the steps which are taken two at a time. This experimental data is compared to simulated results with good agreement.

A companion report will show how varying such radiographic parameters as x-ray energy and total dose affect the IC of the image.

INTRODUCTION

The concept of using the statistical properties of an image to judge the image quality has a long history. Most discussions¹ along these lines have used Shannon's theorem to calculate the information capacity of an imaging medium in bits/sqmm.

Shannon's information content assumes that the signal is averaged over time so that time variations in the signal can be ignored and only spatial variations are considered. If we reverse this thinking, averaging over the spatial dimensions and considering information as a function of time (dose), we can determine the information content (IC) of a radiographic image when the modulation transfer function (MTF) can be ignored (or is assumed to equal 1.0). The image quality is determined primarily by the time (statistical) fluctuations in the signal. This also means we are throwing away all of the spatial information and visual cues in the image. Two images with identical parameters and values of IC will not necessarily have the same visual appearance. The IC, when measured in bits, tells us the number of bits required to span the contrast range of the image when noise is taken into account.

If we consider an image as the output signal values of an information channel, where the signal is optical density or storage phosphor (SP) intensity, and ignore the location of this signal in the image plane, we can consider the output signal as a function of photon intensity. The input signal is a histogram of the object thickness. The output signal sequence is now a histogram of the number of signal values as a function of photon intensity or, if we use Beer's law, the thickness itself. When the object has a constant, uniform thickness, the input signal is a delta function and the output signal, the intensity histogram, is the point-spread function (PSF) along this thickness coordinate. In principle, we could apply Shannon's theorem and calculate the information capacity in bits of this new channel.

Instead, we will simply calculate the information content (not capacity) contained in a radiographic image when the object consists of a step wedge with two thicknesses. The information input to the channel is identically one bit. When system noise is added to the signal, the net information content received on the other end (the image) will be less than one bit. The case in which there is a large thickness difference between the two steps is relatively uninteresting; the steps are easily differentiated and the information transmitted is very close to one bit. However, when the thickness change approaches zero, the visibility of the step goes to zero as the noise increases and the information content also goes to zero.

This measure of information content of an image is a statistic equal to the difference between the entropy of the total image and the entropy of the noise within the image. The entropy of the noise is the conditional entropy of the image given the object thickness. This statistic has units of bits when the entropy is calculated to the base 2 and is the number of bits needed to represent the gray levels actually transmitted to the image from the object and shall be represented by IC. The formalism is presented in Appendix B of Ref. 2.

Since the locations of the pixels in the image plane are ignored, it does not include information encoded in the spatial distribution of the photon fluence in the x-y plane. This information is adequately described by the MTF in the image plane. The statistical information discussed in this report addresses the variations in photon fluence caused by the photon energy dependence of the linear attenuation coefficient of the material. While the concept of statistical image information applies to visible photography as well as radiography, the modeling described in this report assumes a radiographic object.

The usefulness of this measure of information content depends on the dose regime the radiograph was taken. If conventional high-dose radiographs are examined, the limiting factor in extracting information from the image is the spatial resolution and contrast of the image. Image noise is relatively unimportant. On the other hand, in the low-dose regime, the statistical noise in the image limits the observable contrast and hence the information content. We will assume that we are operating in the regime where the MTF is equal to 1.0. If we have image features that contain high-frequency components (where the $MTF < 1.0$), this obviously is a factor in determining its visibility. In fact, the IC can be considered a function of spatial frequency but, in this report, we will only consider the zero or low-frequency response.

It is instructive to look at limiting cases to get an intuitive feel for the statistical information contained in a radiograph. At one extreme is a perfectly uniform slab of any material. The radiograph shows a uniform average optical density (OD)—one average grey level. The total entropy and the conditional entropy are equal and the information content is zero. Since the input IC is zero (single, uniform thickness), the IC of the output signal must also be zero.

On the other hand, assume that the object has an arbitrary thickness or density variation along the z axis in the x-y plane but the attenuation coefficient of the material is zero; there is no interaction between the x-rays and the object. Again the image (assuming the photons do interact with the detector) consists of a single, uniform OD and the information content is zero.

Now let's postulate an object with two thicknesses with each thickness covering half the object area. We can also get real to the extent that we assume that the x-ray photons interact with the object according to Beer's law, the standard exponential absorption law. However, at first let's ignore statistical fluctuations in the x-ray fluence. The image consists of two regions with different ODs without any statistical noise. The information content is $IC = \log_2(2) = 1.0$. In the ideal situation without noise, the information content is always 1.0 no matter how small the OD difference between the two regions becomes. This is analogous to an imaging system that has perfect resolution in the imaging plane, perpendicular to the object thickness direction. If the PSF is a delta function, you can, in principle, detect any arbitrarily small feature.

In fact, the analogy can be carried further. If the signal is digitized, the analog-to-digital converter (ADC) bin width is equivalent to a sampling interval and a Nyquist frequency can be determined for the coordinate perpendicular to the image plane. The PSF in this direction is determined by the statistical noise in the x-ray fluence and is displayed as the intensity histogram through a uniform object.

If statistical noise is now added to the x-ray fluence, the information content approaches zero as the OD difference approaches zero. This real-world situation is what we are interested in. We want to find out how the information content varies with

- material absorption coefficient
- x-ray energy coefficient
- x-ray intensity (photons/pixel) coefficient
- relative object thickness changes
- relative areas in the x-y plane of the two thicknesses.

THEORY

The model in Ref. 2 assumes a source of input data represented by x and a set of output data represented by y . The two sets of data are connected by a data transmission channel that, in the real world, introduces some noise into the data set y . We could consider the functional relationship

$$y(i) = F[x(j)] + \text{noise},$$

where i and j are indexes that range over the number of events in the two sets of data.

We can define various statistics, such as the mean and variance, of the two data sets x and y . Another such statistic is the probability of observing a particular value of x or y , $p_x(i)$ and $p_y(j)$. If a value $y(i)$ is observed, the information we gain is defined as $\log_2[1/p_y(i)]$ and similarly for a value of x . The average value of these information values over the complete data set is the entropy

$$H(y) = -\sum [p_y(i) \log_2 p_y(i)],$$

where the summation is over the index i .

This is the total entropy of the output data set without any apriori knowledge about the input set x .

If the channel were noiseless, the information content of the output data set would be equal to that in the input data set and

$$IC = H(y) = H(x).$$

However, we assume the channel is not noiseless and the information lost in the noise is given by the conditional entropy

$$H(y|x) = -\text{Sum}[px(i)*py|x(j)*\text{Log}[2, py|x(j)]]$$

where,

$H(y|x)$ — conditional entropy of y given x

$px(i)$ — probability of observing x

$py|x(j)$ — probability of observing y given x

The summation is over i and j . The net information transferred by this noisy channel is simply

$$IC = H(y) - H(y|x).$$

The noise in the transmission channel is simply the quantum noise of the transmitted x-ray fluence plus the system noise in the detector and electronics. In the modeling code where the incident fluence is a parameter, the detective quantum efficiency (DQE) and quantum efficiency (QE) are assumed equal, and the only source of noise is in the photon fluence. All photon fluences in this report must be corrected for DQE and QE of the actual detector to get real fluences. Of course, when using experimental images, the measured IC takes into account all sources of noise.

The Appendix contains the Mathematica code used to simulate radiographic images and calculate IC.

EXPERIMENT

This section describes how the IC of the radiographic image of a copper step wedge was measured and compares those results with results from the simulation code. The copper step wedge was radiographed with a pulsed 150-pkV x-ray machine (Golden Inspector) using the SP detectors at two different doses. The IC of the complete image was not calculated. Sub-images, each consisting of two values of copper thicknesses, were constructed and the IC of each sub-image calculated.

MEASUREMENT AND SIMULATION OF COPPER STEP WEDGE

Two radiographs of a copper step wedge were taken and the information content between any two steps was calculated using the code in the Appendix. The step wedge consisted of seven steps, each

0.127 mm thick. The wedge was radiographed with the Golden machine at 4.0 meters with 10 and 50 pulses. The detector was a storage phosphor. Figures 1 and 2 show the radiographs.

After transferring the digital image to a Sparc10 workstation, a region of interest (ROI) 35 pixels on a side was defined for each step thickness. Each pixel was 352 x 352 microns. The histogram of the background intensity (zero thickness) appeared to be gaussian with a mean of 384 for the 10-pulse radiograph. A histogram of each thickness step was produced and named Cu0 to Cu7 where Cu0 is the background and Cu7, is the thickest step. These files were then analyzed with the code in the Appendix.

The effective attenuation coefficient of the x-rays through the copper was calculated from the experimental results and the IC's recalculated using the simulation code. This simulation used the unattenuated photon fluence as a free parameter.

The information content was calculated for all combinations of two steps. $IC(i,j)$ is a function of the step indexes i and j , and $i = j = 0$ is the background signal for zero thickness. If $i = j$, $IC(i,i) = 0.0$ for all i .

Figure 1. 150-pkV radiograph of copper step wedge. Background dose equals 150 microrad (10 pulses).

Figure 2. 150-pkV radiograph of copper step wedge. Background dose equals 750 microrad (50 pulses).

RESULTS

Figure 3 illustrates the image intensity histogram under the thickest step of the stepwedge for the 10-pulse radiograph. While noisy, the shape is reasonably gaussian with a mean intensity of 296. The other histograms are similar to Fig. 3, with the mean intensity of the background (zero copper) at about 384.

Figure 4 shows the information content when the IC between any two steps of the copper wedge is calculated. Obviously, when the two steps have the same thickness, the information is identically zero

(the diagonal). Again, these results are for an exposure of 10 pulses. A background intensity was not subtracted from any of the data.

Figure 3. Intensity histogram under 0.889 mm of copper.

Figure 4. Information content of the copper step wedge with the steps taken in pairs, Golden Inspector radiograph—10 pulses.

The IC of step 7 (thickest) compared to all the other steps was calculated using the simulation code and the results compared to the experimental data. The following parameters were used in the simulation:

$x_{\max} = 0.0889$ cm (copper),

$x_{\min} = 0.0127$ cm (copper),

$u = 9.92/\text{cm}$ (corresponds to an effective energy of 71.0 keV),

$px(1) = px(2) = 0.5$ (probability of a photon hitting each of the two thicknesses),

$i_o \approx 24.0$ and 7.0 photons/pixel,
 .1mm thick pure carbon fluor,
 pixel size = 0.352 mm^2 ,
 ROI = 35×35 pixels, and
 $\text{Noi} = 2 \times \text{ROI} = 2450$ pixels.

The value for i_o was assumed to be a virtual isotopic source that emits a monoenergetic photon at 71.0 keV with a rad output of 35.5 millirad/milliCurie/hour at one meter and an exposure time of one hour. If the total interaction crosssection is used, the fluor will interact with 0.33% of the incident photons producing an incident photon fluence of 24 photons/pixel.

Figure 5 shows the IC as a function of copper thickness compared to the thickest copper, 0.889 mm. The calculated value of IC for $i_o = 24$ is represented by the upper line, the IC for $i_o = 7$ by the lower line, and the experimental values by the dots. The value of 7.0 for the incident fluence is simply the value that produces the best visual fit between the experiment and the simulation.

The IC calculated with a value of $i_o = 7$ photons/pixel matches the experimental data quite well. Either the calculated value of the fluor efficiency is high by a factor of three to four or there is another source of random noise in the observed radiographic image. More than likely the answer is a mixture of both. In fact, we know that the fluor and the electronics contribute to the noise in the image. Another way of looking at the results is that the DQE of the fluor is 29%.

Figure 5. Information content of the 0.889-mm thickness compared to all the other steps.

CONCLUSIONS

Modeling

Keep in mind that in the simulation, two simplifying assumptions are made about the radiographic process: the radiation spectra is monoenergetic and scattering within the object is ignored. There is another constraint that is implicit in the calculations; all the results assume a

constant input fluence of photons on the object. Hence, the model is strictly accurate only for an imaginary monoenergetic isotopic source of constant Curie strength using 'good' geometry where scattering can be ignored.

The constant Curie strength and monoenergetic spectra assumptions are not a problem if the results are interpreted as a Green's function in the energy coordinate and are scaled properly to reflect variations in photon fluence. However, incorporating scattering into the model would require a fundamental change in the arithmetic.

Experimental Results

The experimental results obtained with the copper step wedge using the Golden x-ray machine illustrate the overwhelming influence photon statistics have on the image quality when we are operating in the low-dose regime. Figures 1 and 2 show that the thickest four steps cannot be distinguished with 10 pulses but are easily detected with 50. Except for the total exposure, the radiographic conditions were identical for the two radiographs.

Figure 5 compares the calculated values of IC for two values of incident photon fluence with the experimental values. The fluence of 24 photons/pixel represents the dose interacting with the fluor if the fluor is 0.1 mm thick and pure carbon and the source is 35.5 milliCuries of a fictitious isotope emitting a single 71.0-keV photon per disintegration. The experimental IC obviously corresponds to a much lower (noisier) fluence, 7 photons/pixel. The difference is presumably caused by the noise in the fluor and system noise and can also be interpreted as a DQE of 29%.

The statistical quality of the image can be improved by increasing the dose on the detector (time averaging) or increasing the pixel size (spatial averaging). Of course, this last method improves the statistics at the expense of spatial resolution. The IC of an image depends on the value of i_0 (incident photons/pixel) and not on the number of pixels used in the calculation. As the number of pixels increases, the precision of the results increases but not the value of IC. This implies that an analog image has an effective pixel size that is fixed by the particular film and screen used. If the image is digitized with a larger pixel than the effective pixel, the IC will improve because the data is averaged over uncorrelated values. If a smaller pixel than the effective pixel is used, you would not expect the IC to change significantly because adjacent pixel values are now highly correlated.

There might be another way of improving the statistical quality of the image. As mentioned in the introduction, the intensity histogram when the object thickness is constant can be considered a PSF for the z coordinate of the image. In a formal sense, an MTF can be calculated and classical image processing techniques might be used to improve the image contrast. In order to do this, we must still average over a number of pixels and this might be equivalent to plain old spatial averaging. Maybe generating two images, one using spatial enhancing techniques and the other enhancing the image information might be the answer.

Image processing techniques might be applied to the histogram of any arbitrary area of the image. The intent is to sharpen the peaks and reduce the influence of the random noise on the image contrast. Mathematically what we want to do is transform one histogram (the noisy one) to another, sharper one. Algorithms for this type of transformation are common.³ If we pick intensities from the original image, they will have a probability distribution defined by the original, noisy histogram. Going across to the cumulative probability curve for the new histogram and then down to a new intensity value 'should' reduce the noise in the image.

Without going into the details, the process described above is equivalent to devising an intensity look-up table that transforms the image intensity values into a new distribution that smooths out the random noise in the image. Unfortunately, this look-up table has discontinuities in it that must be fixed. This probably can be done and the resulting 'artifacts' taken out with a contraction operator. At this point, this last paragraph is pure speculation.

REFERENCES

1. C. N. Nelson, "Applied Optics," 11:87 (1972)
2. Edward L. O'Neill, "Introduction to Statistical Optics," Addison-Wesley Publishing Company, Inc. 1963.
3. William H. Press, et al., "Numerical Recipes in C," Cambridge University Press, 1988.

APPENDIX

ENTROPY OF SIMULATED IMAGE

This calculation of image entropy and information content imposes a Poisson distribution on the transmitted photon intensity rather than a normal distribution on the input intensity. The first task will be to turn the spelling checker off and load the discrete statistical package.

```
Off[General::spell1];
<<Statistics`ContinuousDistributions`
```

The parameters used in the calculation are:

Nos == Number of steps
io == input intensity, photons/pixel
pixel == pixel size, cm
Noi == Number of pixels sampled in each step
Nob == Number of bins in the histogram
Nor == Number of random numbers in each table
mu == attenuation coefficient, 1/cm
px[[i]] == probability of hitting each step, $i, i \leq \text{Nos}$
thick[[i]] == thickness in cm of step i.
avrg[[i]] == Average number of photons transmitted thru step i
intensity[[i,Nor]] == Table of Poisson distributed intensities thru step i.
xmin == minimum object thickness, cm
xmax == maximum object thickness, cm
dx == Maximum thickness change, $dx = \text{xmax} - \text{xmin}$

This initialization assumes the object to be a step wedge with circular steps with the diameters entered by the operator. The diameters can be in pixels or centimeters. The probabilities of hitting each step are calculated as the ratio of the area of that step to the total area. The second cell calculates the IC of the whole image: a step wedge with Nos steps.

```
Nos = 2;  
io = 20;  
pixel = 0.0176;  
Nob = 1000;  
Nor = 1000;  
mu = 0.549;  
xmin = 0.300;  
xmax = 0.400;  
dx = (xmax - xmin)/(Nos-1);  
px = Table[0.0,{Nos}];  
area = Table[0.0,{Nos}];  
thick = Table[0.0,{Nos}];  
avrg = Table[0.0,{Nos}];  
diam = Table[0.0,{Nos}];  
Noi = Table[0.0,{Nos}];  
tarea = 0.0;
```

```

Do[(
    diam[[i]] = Input["diam[[i]] = "];
    area[[i]] = N[ diam[[i]]*diam[[i]]*Pi/4.0];
    tarea = tarea + area[[i]];
    Noi[[i]] =Floor[ area[[i]]/(pixel*pixel)]
),
{i,Nos}
];
TNoi = Apply[Plus, Noi];
Do[(
    px[[i]] = area[[i]]/tarea;
),
{i,Nos}
];
px
Noi

```

This cell fills a table 'Nor' long with Poisson distributed random numbers. There is a table for each thickness in the step wedge. The thicknesses and average percent transmission are also calculated and stored.

After the tables are filled, the maximum, minimum, and bin width of the histograms are also calculated and stored.

As long as Nos, mu, and the thicknesses do not change, this cell does not have to be recalculated. This will save some time.

```

SeedRandom[Floor[SessionTime[]]];
intensity = Table[0.0,{Nos},{Nor}];
Do[(
    thick[[i]] = xmin+dx*(i-1);
    avrg[[i]] = io*Exp[-mu*thick[[i]]];
    stdev = Sqrt[avrg[[i]]];
    ND = NormalDistribution[avrg[[i]], stdev];
    intensity[[i]] = Table[Round[Random[ND]], {Nor}];
),
{i,Nos}
];
imin = Min[intensity];
imax = Max[intensity];
deltai = N[(imax-imin)/(Nob - 1)];

```

Now calculate the total entropy, conditional entropy, and the IC five times, each with a different set of random numbers.

The first section calculates the total entropy using Noi pixels in each step. The total number of pixels is Noi*Nos.

This next section calculates the conditional entropy given the thickness of the object. This is essentially the entropy of the noise within the image. The same lists of Poisson distributed numbers will be used in this section after initializing the random number generator. Noi pixels are sampled at each thickness. The probability density function for each thickness is plotted separately. Again, the total number of pixels used in the calculation is Noi*Nos. Nol equals the number of loops through the IC calculation.

```
SeedRandom[Floor[SessionTime[]]];
Nol = 5;
Hix = Table[0.0,{Nol}];
Info = Table[0.0,{Nol}];

Do[(
  h = Table[0.0,{Nol}];
  histo = Table[0, {Nob}];
  Do[(
    i = Random[Integer, {1,Nor}];
    s = 0;
    r = Random[];
    cp = 0.0;
    Do[(
      cp = cp + px[[ij]];
      If[(r<cp)&&(s==0)), s = jj , s = s];
    ),
    {jj, Nos}
  ];
  dum = intensity[[s,i]];
  index = (dum - imin)/deltai;
  k = Floor[index] + 1;
  If[k > Nob, k = Nob, k = k];
  If[k < 1, k = 1, k = k];
  histo[[k]] = histo[[k]] + 1;
),
{ij, TNol}
];
probi = histo/(TNol);
h[[1]] = 0.0;
Do[(
  If[probi[[i]] != 0, h[[1]] = h[[1]] + N[probi[[i]]*
  Log[2, probi[[i]]], h[[1]] = h[[1]]];
),
{i,Nob}
];
h[[1]] = -h[[1]];

SeedRandom[Floor[SessionTime[]]];
Hix[[1]] = 0.0;
Do[(
  histo = Table[ 0, {Nob}];
  Do[(
```

```

        i = Random[Integer, {1,Nor}];
        dum = intensity[[s,i]];
        k = Floor[(dum - imin)/deltai] + 1;
        If[k > Nob, k = Nob, k = k];
        If[k < 1, k = 1, k = k];
        histo[[k]] = histo[[k]] + 1;
    },
    {j, Noi[[s]]}
];

probi = histo/Noi[[s]];
Do[
    If[probi[[i]] != 0, Hix[[i]] = Hix[[i]] +
    N[px[[s]]*probi[[i]]*
    Log[2, probi[[i]]], Hix[[i]] = Hix[[i]]];
    ],
    {i, Nob}
];

),
{s, Nos}
];
Hix[[i]] = -Hix[[i]];
Info[[i]] = h[[i]] - Hix[[i]];
Print[h[[i]], " ", Hix[[i]], " ", Info[[i]]]
),
{l,Nol}
];
Print["mean = ",Mean[Info],", variance = ",Variance[Info]]

4.10706    4.09473    0.0123275
4.12977    4.09929    0.0304845
4.10964    4.07451    0.0351224
4.10925    4.07846    0.0307892
4.09686    4.10265    -0.00578704
mean = 0.0205873, variance = 0.000294283

```