Modelling crystal-field interaction for f-elements in LaCl$_3$

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Abstract

The results of crystal field calculations in the framework of exchange charge model (ECM) are reported for trivalent lanthanide and actinide ions doped into LaCl$_3$. Whereas the scalar strength of the model crystal field parameters are consistent with that previously determined by fitting the experimental data, the sign of the second-order parameter is found to be negative, in contrast to previous reports. The contribution from long-range electrostatic interactions exceeds that from the nearest neighboring ligands and leads to the negative sign of the second-order crystal field parameter. Other interaction mechanisms including overlap, covalence, and charge exchange are less important to the second order parameter, but dominate the fourth- and sixth-order parameters. This work provides a consistent interpretation of the previously controversial experimental results for both lanthanide and actinide ions in this classical host.

Keywords: crystal field, rare-earth ions, actinide ions, spectroscopy, lanthanum trichloride
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I. Introduction

Lanthanum trichloride is a classical "textbook" host for f-element experimental and theoretical spectroscopy. Studies of this compound doped with lanthanide (4f) ions have provided a basis for the use of crystal-field modelling to describe the Stark splitting of 4f-electronic energy levels [1]. Most of the crystal field calculations were done using this host as a model to determine the contribution of different interactions into observed splitting. Moreover, the set of LaCl₃ parameters are often used as a starting point for fitting experimental data in other hosts.

In earlier crystal-field modelling, the most difficult part was to evaluate the long range electrostatic interaction represented by the second-order crystal-field parameter $B_{20}$. The first calculation of crystal field parameters for LaCl₃ was performed by Hutchings and Ray in 1963, and was based on a simple electrostatic model [2]. However, due to the poor convergence and a rudimentary point charge field evaluation, the obtained value for $B_{20}$ was not reliable. As shown in Ref.3, the point dipole and point quadrupole contributions to this parameter were strongly overestimated. Since this early work, numerous attempts have been made [4-6] to modify the electrostatic model [2] by including additional interactions, in order to get satisfactory agreement with experimental crystal-field parameters. In a review by Newman [4], up to 10 different interactions were considered. Since most of these later calculations were done in the framework of superposition model [4], which utilises only the nearest neighbour environment of a 4f ion, contributions from long range electrostatic interaction were assumed to be the same as that from Ref. 2. In order to represent better the effects of long range electrostatic interactions, Malkin [3] developed an exchange charge model (ECM), which also includes other quantum chemical
corrections such as electronic orbital overlap, covalence and charge exchange. No attempt to review the previous crystal field model for LaCl$_3$ on the basis of this ECM [3] has been published to date.

For lanthanide ions in LaCl$_3$, a long time discrepancy between model crystal field and experiments was first realised in EPR studies of the ground state splitting of the S-state ions (Gd$^{3+}$ and Eu$^{2+}$) in LaCl$_3$[7-9]. The EPR assignment for the lowest of the four crystal field states in the J=7/2 free-ion ground state was |M|=±1/2, whereas the model crystal field predicted a |M|=±7/2 ground state. This prediction is the direct result of a positive second order crystal field parameter $B_{20}$. Whereas numerous efforts were made to introduce additional interaction mechanisms to balance the difference between the existing model crystal field and the EPR results, the problem has not been resolved. Recently, more sophisticated spectroscopic experiments, such as Zeeman and spectral hole-burning experiments on the excited states of Am$^{3+}$ in LaCl$_3$, have been performed [10-12]. These experiments showed that the existing model crystal field [13] is inconsistent with experimental observation for the J=1 states ($^5$D$_1$), which splits into a non-Kramers doublet (μ=1) and a singlet (μ=0) states. The model predicts that the doublet has lower energy level, whereas it was observed at 9 cm$^{-1}$ above the singlet in the Zeeman experiment. This result was further confirmed by the hyperfine energy-level structure obtained in spectral hole-burning experiments. The same experiments on Am$^{3+}$ doped into CaWO$_4$ [11] gave the reverse order for the two sublevels of the J=1 state, which is in agreement with the model crystal field established for lanthanides in CaWO$_4$. In order to resolve the discrepancy for Am$^{3+}$:LaCl$_3$, it is necessary to assume that the sign of the second-order parameter is negative. However, the theoretical basis of a negative $B_{20}$ for this system is unclear.
Based on ECM, we have recently performed a series of theoretical calculations on the crystal field parameters for trivalent lanthanide (4f) and actinide (5) ions in LaCl₃ [14]. The analysis provided in this paper focuses on the controversial sign and value for the second-order crystal field parameter $B_{20}$. It is shown that the calculation of conditionally convergent sums in the electrostatic contribution results in a negative $B_{20}$ for the La site in LaCl₃. This contribution is dominant for both lanthanide and actinide ions in LaCl₃. Contributions from other mechanisms are unlikely to change the sign.

II. Model calculation with ECM

In ECM, the effective Hamiltonian for electrons localised at an f-element ion is written as

$$H = H^{(q)} + H^{(s)}$$

(1)

where

$$H^{(q)} = \sum_{p,k} B^{(q)}_{pk} O_{pk}$$

(2)

is the energy of an electron in the field of point charges of the lattice, $p$ is the rank (or order) of crystal field parameters $B^{(q)}_{pk}$, and $k$ denotes components (from -p to p), $O_{pk}$ are Stevens spherical polynomials [3]. Representing the exchange charge interaction, $H^{(s)}$ is the energy associated with the overlap of the electron wavefunctions of the f-element ion with the wave functions of the ligand electrons,
\[ H^{(s)} = \sum_{p,k} B_{pk}^{(s)} O_{pk}, \quad (3) \]

where

\[ B_{pk}^{(s)} = B_{pk}^{(s)} (S, S\sigma, S\pi, G, G\sigma, G\pi), \]

and \( S, S\sigma, S\pi\) are the overlap integrals of 4f or 5f ion wavefunctions with 3s, 3p ligand wavefunctions \( \ln \ln m \), and \( n, l, m \) are quantum numbers.

The “exchange charge” contribution includes effects of covalence overlap and exchange, and depends on three dimensionless adjustable parameters \( G_s, G\sigma, \) and \( G\pi \) [3] that “scale” the overlap integrals. These three adjustable parameters depend on the radial wavefunctions used in the calculations of the overlap integrals and are constants for isostructural crystals for a given pair of ions [3].

Calculations of electrostatic contribution for the fourth- and sixth-order parameters can be performed simply by summing over a few neighbouring coordination shells of the f ion. However, special attention should be paid to the calculation of the conditionally convergent sums in the second-order parameters. In this case, the Ewald method is required in order to obtain correct results [3].

Our calculation was performed by using crystallographic data for a single crystal of LaCl\(_3\). The crystal lattice of LaCl\(_3\) is hexagonal (the space group is \( C_{6h}^2 - P6_3/m \)) with a bimolecular unit cell [15]. For convenience of interpretation, we write the electrostatic part of the second-order parameters as

\[ B_{2k}^{(q)} = (1 - \sigma_2)(r^2) \sum_i L_{2k} (l), \quad (4) \]
where $0 < \sigma_2 < 1$ is a Sternheimer constant introduced to allow for linear shielding of the external electrostatic field acting on the valence electrons by the filled electronic shells of the f ion, and $\langle r^2 \rangle$ is the radial integral for the f ion and is calculated together with $\langle r^4 \rangle$ and $\langle r^6 \rangle$ using Hartree-Fock numerical functions with relativistic corrections for the 4f and 5f ions, and $l$ denotes different sublattices. The lattice sum $L_{2k}(l)$ depends on the positions and charge of crystal lattice ions only. The values of $L_{2k}(l)$ are the same for all f-element ions in a given crystal environment. The calculated values of $L_{2k}(l)$ for LaCl$_3$ are listed in Table 1. The x and z axes of

<table>
<thead>
<tr>
<th>Ion position</th>
<th>$L_{20}(l)$</th>
<th>$L_{22}(l)$</th>
<th>$L_{2-2}(l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>La(1) 2/3; 1/3; 1/4</td>
<td>-4985</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>La(2) 1/3; 2/3; 3/4</td>
<td>-489</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cl(1A) u; v; 1/4</td>
<td>-469</td>
<td>-3826</td>
<td>559</td>
</tr>
<tr>
<td>Cl(1B) -u; -v; 3/4</td>
<td>1945</td>
<td>2276</td>
<td>2158</td>
</tr>
<tr>
<td>Cl(2A) -v; u-v; 1/4</td>
<td>-469</td>
<td>2396</td>
<td>3033</td>
</tr>
<tr>
<td>Cl(2B) v; v-u; 3/4</td>
<td>1945</td>
<td>732</td>
<td>-3049</td>
</tr>
<tr>
<td>Cl(3A) v-u; u; 1/4</td>
<td>-469</td>
<td>1428</td>
<td>-3592</td>
</tr>
<tr>
<td>Cl(3B) u-v; -u; 3/4</td>
<td>1945</td>
<td>-3006</td>
<td>891</td>
</tr>
<tr>
<td>Total</td>
<td>-1046</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Lattice sums for second-order parameters (cm$^{-1}$/Å$^2$)

$^a$ u=0.287$a_0$, and v=0.382$a_0$, where $a_0$ is the lattice constant [15].
the reference coordinate are parallel to the crystallographic $a$ and $c$ axes, respectively. Since all $L_{21}(l)$ and $L_{21}(l)$ are zeros, they are not listed.

It can be seen from Table 1, that the negative sign for the total value of $L_{20}$ is largely due to the negative contribution from the lanthanum sublattices. Although, 3 of the chlorine sublattices also give negative contributions to $L_{20}$. To understand the source of the positive sign for $B_{20}$ in previous work, we calculated the contributions from several coordination shells by direct summation over the lattice. Results are listed in Table 2.

**Table 2. Contributions to $L_{20}$ from different coordination shells LaCl$_3$ (cm$^{-1}$/ Å$^2$)**

<table>
<thead>
<tr>
<th>coordination shell number</th>
<th>ion</th>
<th>number of ions in shell</th>
<th>$L_{20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cl</td>
<td>9</td>
<td>1300</td>
</tr>
<tr>
<td>2</td>
<td>La</td>
<td>2</td>
<td>-4155</td>
</tr>
<tr>
<td>3</td>
<td>Cl</td>
<td>3</td>
<td>-973</td>
</tr>
<tr>
<td>4</td>
<td>La</td>
<td>6</td>
<td>1782</td>
</tr>
</tbody>
</table>

In calculations of the overlap integrals, it was found that, because of the more diffuse nature of the chlorine 3p orbital, the dominant contribution is from the $S_{\pi}$ integrals, which are significantly larger than that for oxygen or fluorine ions. Therefore, in calculation of $B_{pk}^{(6)}$, we used a simplified ECM model, in which $G_5=G_0=G$. Thus we have only two varied parameters: $G$ and $G_{\pi}$.
For comparison with previous work, we chose two systems, Nd$^{3+}$ and Am$^{3+}$ doped, respectively, into LaCl$_3$. The calculated crystal field parameters, $B_{pk} = B_{pk}^{(q)} + B_{pk}^{(s)}$, are listed in Table 3 in comparison with the previously determined values [13, 16]. All the previously determined crystal field parameters are from the least squares fitting of experimental data to the simplified model of D$_{3h}$ effective point symmetry. It is evident that the most dramatic change is the sign of $B_{20}$, and the leading contribution is from the electrostatic interaction. The calculated values for the fourth- and sixth-order parameters do not differ much from those established by fitting of the experimental spectra to a point charge model of D$_{3h}$ symmetry. For the sixth-order parameters, one can compare the calculated value of $[(B_{66})^2 + (B_{6-6})^2]^{1/2}$ for C$_{3h}$ model, which is 382 cm$^{-1}$ for Nd$^{3+}$ and 834 cm$^{-1}$ for Am$^{3+}$, with the fit value of $B_{66}$ for D$_{3h}$ model. It is also anticipated that, because of the more extended nature of the 5f electrons, the fourth- and sixth-order parameters differ significantly. 

### Table 3. Crystal field parameters for Nd$^{3+}$ and Am$^{3+}$ in LaCl$_3$ crystal (cm$^{-1}$)

<table>
<thead>
<tr>
<th>$B_{pk}^{(q)}$</th>
<th>$B_{pk}^{(s)}$</th>
<th>$B_{pk}$ (ECM, C$_{3h}$)</th>
<th>$B_{pk}$ (fit to D$_{3h}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pk</td>
<td>Nd$^{3+}$</td>
<td>Am$^{3+}$</td>
<td>Nd$^{3+}$</td>
</tr>
<tr>
<td>20</td>
<td>-117</td>
<td>-122</td>
<td>39</td>
</tr>
<tr>
<td>40</td>
<td>-21</td>
<td>-55</td>
<td>-57</td>
</tr>
<tr>
<td>60</td>
<td>-3</td>
<td>-10</td>
<td>-41</td>
</tr>
<tr>
<td>66</td>
<td>20</td>
<td>68</td>
<td>279</td>
</tr>
<tr>
<td>6-6</td>
<td>-13</td>
<td>-44</td>
<td>-226</td>
</tr>
</tbody>
</table>
order crystal field parameters for the actinide ion Am\(^{3+}\) are about two times of that for the lanthanide ion Nd\(^{3+}\).

Using the crystal field parameters resulted from our ECM calculations together with the free-ion interaction parameters provided by previous work [13], we have also performed calculations on the Stark splitting of the electronic states of Pr\(^{3+}\) and Nd\(^{3+}\) ions in LaCl\(_3\)[14]. A reasonable agreement was achieved between our calculation and the previous fitting of the experimental data with a point charge model based on an effective D\(_{3h}\) site symmetry.

III. Discussion

It is clear that the second order crystal field parameter characterises the long range electrostatic coupling as well as the nearest neighbouring interactions. From our calculations, the large contributions from the La shells leads to a negative \(B^{(q)}_{20}\), whereas a smaller positive value for \(B^{(s)}_{20}\) was obtained. The total value of \(B_{20} (= B^{(q)}_{20} + B^{(s)}_{20})\) is negative for lanthanide and actinide ions in LaCl\(_3\). We have shown that, for 4f and 5f ions in LaCl\(_3\), it is not appropriate with any crystal field model to include only the nearest neighbour ions for second-order parameter calculation, since such an approximation results in a wrong sign for \(B_{20}\). It is necessary to point out that the change in crystal lattice structure due to the f-element ion substitution does not change the sign of \(B_{20}\), whereas it could have pronounced effect on the fourth- and sixth-order parameters.

We have shown that the sign of \(B_{20}\) does not significantly affect the overall fitting of crystal field spectra for trivalent f ions in LaCl\(_3\). However, for some special states, such as the
J=1 states of the ions that have even number of f electrons, and the J=3/2 states of the ions that have odd number of f electrons, the ordering of the crystal field levels is reversed. This is because that the Stark splitting of these states is dominated by the second order crystal field parameter. For Am$^{3+}$ in LaCl$_3$, the ordering of the J=1 states observed in spectral hole burning and Zeeman experiments supports this calculation [10-12]. There is no alternative explanation nor additional mechanisms for the observed characteristics of this J=1 state. For other states, the effects induced by changing the sign of $B_{20}$ can be balanced by variation of the fourth- and sixth-order parameters.

Our conclusion about the sign of $B_{20}$ is supported also by an early experiment [17], in which it was shown that the experimental value of $B_{20}$ for Np$^{3+}$ in the isostructural LaBr$_3$ is negative. This negative $B_{20}$ was explained by introducing a correction[6], which closely resembles the shielding of crystal field by the filled external s and p orbitals of the f-element ion. This correction is considered in our calculation by the Sternheimer constant $\sigma_2$. Since the point charge electrostatic contribution from the crystal lattice already gives a negative sign for $B_{20}$, it is not necessary to introduce additional corrections of uncertain significance.

Another consequence of the negative $B_{20}$ for lanthanide and actinide ions in LaCl$_3$ is that the mechanisms previously evaluated for the ground state splitting of the S-state ions including Gd$^{3+}$, Eu$^{2+}$, and Cm$^{3+}$ in LaCl$_3$ become questionable. Since a negative $B_{20}$ results in a $\mu=1/2$ ground state, many of the higher order mechanisms previously introduced for interpreting the EPR experiments could be over estimated [7]. More detailed analysis is needed for this special case.

We did not evaluate contribution from higher electrostatic multipoles (point dipoles, quadrupoles and so on), because this procedure is far from being exact for two main reasons: 1)
the values of ion polarisability in crystals can be determined with a limited precision; 2) there is no convergence for multipole series (dipole and quadrupole contributions have the same order).

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References


14. V. V. Zhorin and G. K. Liu, to be published.

