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ON THE DEFINITION OF NEUTRON LIFETIMES
IN MULTIPLYING AND NON-MULTIPLYING SYSTEMS

(invited)

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ABSTRACT

Historically, the term neutron lifetime has been used in the literature to describe a wide variety of different time intervals associated with a neutron's trek through a given system. This duplication of usage of the term neutron lifetime has undoubtedly resulted in some confusion concerning its physical meaning. In hopes of reducing some of this confusion, we suggest in this work that the various time intervals characterizing the life of a neutron be divided into three general categories: 1) neutron lifespans, 2) reaction rate lifetimes, and 3) neutron generation times. In this report, we define these three different time intervals and give deterministic and Monte Carlo transport expressions that can be used to calculate them.

I. INTRODUCTION

It is unknown who first defined a neutron lifetime in a quantitative sense. However, estimates of the neutron lifetime in a thermal system and in a fast system are mentioned in the Frisch-Peierls Memorandum [1] written in early 1940. By the time the Los Alamos National Laboratory was formed in March 1943, the basic concept that the neutron lifetime is inversely proportional to the absorption rate must have been well understood, judging from the mundane manner in which Serber discussed it in the Los Alamos Primer [2]. Serber defined the neutron lifetime as the ‘mean time between fissions’ based on two assumptions: 1) the system was so large that neutrons would not be lost by leakage and 2) all neutrons absorbed in the system would only be absorbed in fission reactions (i.e., no parasitic absorption). Hence, Serber’s lifetime definition corresponded to

\[ \tau = \frac{1}{v \Sigma_f}, \]  

where \( v \) is the average neutron velocity and \( \Sigma_f \) is the average macroscopic fission cross section.

In October 1943, Feynman [3] extended Serber’s definition to an ideal \( 1/v \) absorber, including both parasitic absorption and fission absorption. Feynman defined the absorption lifetime in the same manner as Serber, but he used the macroscopic absorption cross section rather than the fission cross section.

\[ \tau = \frac{1}{v \Sigma_a}. \]  

As stated by Feynman, the reciprocal of the absorption lifetime was, by definition, the ‘probability per second of being absorbed.’

About a year later, de Hoffmann et al. [4] developed a more general expression for the definition of a fission lifetime. In their work, de Hoffmann et al. defined \( P_p \) as the probability that a prompt neutron, emitted at time \( t' \), will cause a fission at any later time. The chance of producing a
fission at time \( t \) was found by weighting \( P_p \) by a function \( R(t - t')dt \) such that \( P_p R(t - t')dt \) is the probability that a neutron born promptly at time \( t' \) will produce a fission in the interval \( dt \) at time \( t \). By writing down an expression governing the variation of the time-dependent fission rate and expanding this expression in a Taylor series, de Hoffmann et al. showed that the 'average time from one fission to the next fission due to prompt neutrons' is given by

\[
t_f = \int (t - t') R(t - t') \, dt ,
\]

(3)

where the integral of \( R(t - t')dt \) is appropriately normalized to 1.0.

In March 1946, Kupferberg [5] wrote another Los Alamos report in which the neutron lifetime for a non-multiplying reflector was defined such that the loss rate due to absorption and leakage were both included. In his report, Kupferberg stated (without any formal derivation) that the mean neutron-lifetime in the reflector, \( \tau_{ref} \), is

\[
\frac{1}{\tau_{ref}} = \frac{1}{\tau_a} + \frac{1}{\tau_l} ,
\]

(4)

where \( 1/\tau_a \) is the probability per unit time of an absorption and \( 1/\tau_l \) is the probability per unit time of a leakage. In accordance with Kupferberg's definition, \( 1/\tau_{ref} \) is the probability per unit time of either an absorption or a leakage.

In 1948, Hurwitz [6] presented his derivation of the point kinetic equations in which he defined a quantity \( \tau \) that he referred to as the 'prompt neutron generation time.'

\[
\tau = \frac{\int P(r, r', T) \, dT \, dV}{\int P_r(r, r', T) \, dT \, dV} ,
\]

(5)

where \( P(r, r', T) \) is the probability that a neutron liberated in a fission at point \( r' \) and time \( t = 0 \) will give rise to a fission at point \( r \) and time \( t \) per unit time and volume where \( T \) is the time from birth-to-fission and \( V \) is the system volume. Hurwitz described \( \tau \) as the 'average time it takes a prompt neutron to produce a fission.'

In 1955, Ussachoff [7] derived the point-kinetic equations from the Boltzmann transport equation. From that derivation, he obtained an expression for the neutron generation time, \( \Lambda \), written in terms of the angular fluxes and angular adjoint fluxes.

\[
\Lambda = \frac{\int \frac{\Psi \Phi}{\nu} d\Omega dV dE}{\int \Psi \chi_f \nu \Sigma_f \Phi^* d\Omega dE' dV dE} ,
\]

(6)

where \( \Psi \) is the adjoint angular flux and \( \Phi \) is the forward angular flux. Ussachoff described this quantity as "the mean lifetime of the neutron in the reactor."

During the next few years, many articles and textbooks were published in which each author presented one or more expressions to calculate a neutron lifetime (usually based on diffusion theory) while simultaneously adding their own nuance to the verbal definition. Most of these expressions assumed the neutron life-cycle model whereby an average neutron in the system was followed over a somewhat nebulous neutron generation time that was related to the average time from birth-to-death of an average neutron. To be consistent with the life-cycle model, the effective multiplication factor, \( k_{eff} \), was formally defined as the ratio of the neutron population in successive generations.

In 1958, Weinberg and Wigner [8] published their book on reactor theory in which neutron lifetimes were discussed extensively. They quickly pointed out that it is very difficult to rigorously define a neutron lifetime based solely on the concepts of the life-cycle model since a generation time is not well defined. Instead, they suggested a neutron-balance model in which \( k_{eff} \) is redefined to be the ratio of the neutron production rate divided by the neutron loss rate, and the neutron lifetime is defined by the equation

\[
\frac{N}{\tau} = L + A ,
\]

(7)

where \( L \) is the neutron leakage rate and \( A \) is the neutron absorption rate. When \( k_{eff} \) and \( \tau \) are defined in this manner, the life-cycle model and the neutron-balance model yield consistent equations that describe the time rate of change of the total neutron population. Furthermore, when defined in this manner, the physical interpretation of \( 1/\tau \) is consistent with the interpretation given by Feynman and Kupferberg, namely—\( 1/\tau \) is the probability per unit time that a neutron will be either absorbed or leak from the system.

As we continued to review all of the various publications [written in English or translated into English] dealing with the definition of neutron lifetimes, these authors noticed that, in many instances, the terms neutron lifetime and neutron generation time were used to describe a wide variety of time intervals associated with a neutron's trek through a given system even though these time intervals were clearly not the same. This duplication of usage of the these terms has undoubtedly led to some confusion concerning the quantity that is actually being calculated. In hopes of reducing some of this confusion, we suggest in this work that the various time intervals associated with the life of a neutron be divided into three general categories: 1) neutron lifespans, 2) reaction rate lifetimes, and 3) neutron generation times. In this report, we define these three different time intervals and give deterministic and Monte Carlo transport expressions that can be used to calculate them.

II. LIFETIME DEFINITIONS

Neutron Lifespan

In any system, there are only two ways in which a neutron can be lost—absorption or leakage. If we were able to perform an experiment where we could measure the length of time required for an individual neutron to be
absorbed after being born at time \( t=0 \) and we repeated this experiment for a large number of neutrons, we could calculate the average time to absorption, \( t_a \), of those neutrons destined to be absorbed. That is,

\[
t_a = \frac{1}{N_a} \sum_{j} t_j \tag{8}
\]

where \( t_j \) is the time interval from birth-to-absorption for each neutron that is absorbed and \( N_a \) is the total number of neutrons that are absorbed.

By performing a similar experiment for those neutrons destined to leak from the system, we could also calculate the average time to leakage, \( t_l \).

\[
t_l = \frac{1}{N_l} \sum_{k} t_k \tag{9}
\]

where \( t_k \) is the time interval from birth-to-leakage for each neutron that leaks and \( N_l \) is the total number of neutrons that leak.

Both \( t_a \) and \( t_l \) represent the average length of time required for a neutron to be terminated by either an absorption event or a leakage event following its birth. We refer to these two average birth-to-event time intervals as neutron lifespans.

Following the same logic, we could just as easily have subdivided the absorption lifespan into various components representing, for example, a fission lifespan, a radiative capture lifespan, an \((n,\alpha)\) lifespan, etc., and any other absorption process that might be applicable. These additional lifespans would all have the same functional relationships as Eqs. (8) and (9) and each of these lifespans would represent the average time from birth-to-event for that particular process.

From a computational standpoint, neutron lifespans can be easily calculated in Monte Carlo codes such as MCNP [9] and KENO [10]. In MCNP, three different lifespans are estimated—the fission lifespan, the absorption lifespan, and the escape (or leakage) lifespan [11]-[12]. In KENO, only the fission lifespan is calculated which, because of prior interpretations, is labeled as the neutron generation time rather than the fission lifespan [13].

Knowing the absorption and leakage lifespans, the average time that a neutron will survive in a system before being removed via an absorption event or a leakage event can be estimated by weighting the two lifespans by their respective probability of occurrence. That is,

\[
\tau_r = P_a t_a + P_l t_l \tag{10}
\]

where \( P_a \) is the fraction of neutrons that are absorbed and \( P_l \) is the fraction of neutrons that leak. We refer to \( \tau_r \) as the removal lifetime since it represents the mean time between the removal of a neutron from that system by either an absorption or a leakage. And, by definition, if \( \tau_r \) is the mean time between removal events, then \( 1/\tau_r \) is the probability per unit time of a neutron removal by absorption or leakage.

Although Eq. (10) is well suited for Monte Carlo type calculations, it is not particularly useful for deterministic solutions; deterministic transport codes do not track individual neutrons from birth-to-death and, as such, the absorption and leakage lifespans are not known. Nevertheless, we can easily derive a deterministic expression that defines a removal lifetime using the neutron-balance model proposed by Weinberg and Wigner. This derivation is discussed in the following section.

### Reaction Rate Lifetimes

The time-dependent behavior of the total adjoint-weighted neutron population, \( N^+ \), in a multiplying system can be written as

\[
\frac{dN^+}{dt} = P^+ - (L^+ + A^+) + S^+ \tag{11}
\]

where \( P^+ \) is the total adjoint-weighted neutron production rate due to fission, \( L^+ \) is the total adjoint-weighted neutron leakage rate, \( A^+ \) is the total adjoint-weighted neutron absorption rate, and \( S^+ \) is the adjoint-weighted intrinsic/external source rate. Equation (11) is derived by multiplying the Boltzmann transport equation by the adjoint flux obtained from a \( k \)-eigenvalue adjoint solution and then integrating over all angles, energy, and space. In this particular form, delayed neutrons have been neglected.

If we neglect \( S^+ \) and we assume that the system is increasing or decreasing on a stable period,

\[
N^+ = N_0^+ e^{\alpha t} \tag{12}
\]

then Eq. (11) reduces to

\[
\alpha N^+ = P^+ - (L^+ + A^+) \tag{13}
\]

Dividing both sides of Eq. (13) by the adjoint-weighted neutron loss rate, we obtain the very familiar equation

\[
\alpha = \frac{K-1}{\tau^+} \tag{14}
\]

where \( K \) is the prompt multiplication factor defined by

\[
K = \frac{P^+}{L^+ + A^+} \tag{15}
\]

and \( \tau^+ \) is the adjoint-weighted neutron removal lifetime defined by

\[
\tau^+ = \frac{N^+}{L^+ + A^+} \tag{16}
\]
When written in terms of the adjoint and forward angular fluxes, Eq. (16) becomes
\[
\tau^+ = \frac{\int \Psi \Phi d\Omega dr dE}{\int \Psi \Omega \cdot \nabla \Phi d\Omega dr dE + \int \Psi \Sigma_a \Phi d\Omega dr dE}.
\] (17)

The prompt multiplication factor, \( K \), in Eqs. (14) and (15) corresponds to the \( k \)-eigenvalue solution when based on \( \bar{\psi}_p \) rather than \( \bar{\psi}_t \). Because \( \bar{\psi}_p = (1 - \beta) \bar{\psi}_t \), we can write the prompt multiplication factor in terms of the effective multiplication factor, \( k_{\text{eff}} \), as
\[
k_{\text{eff}} = \frac{K}{1 - \beta}.
\] (18)

(Note, to be totally consistent with our own nomenclature, we should write \( k_{\text{eff}} \) as \( k^+ \). However, the \( k \)-eigenvalue has always been written as \( k_{\text{eff}} \) with the understanding that it is equivalent to an adjoint-weighted quantity.)

Substituting Eqs. (15) and (18) back into Eq. (16), we obtain the more familiar form of the adjoint-weighted removal lifetime written in terms of \( k_{\text{eff}} \) and the adjoint-weighted neutron production rate.
\[
\tau_r^+ = \frac{k_{\text{eff}} \int \Psi \Phi d\Omega dr dE}{\int \Psi \bar{\psi}_t \Sigma_a \Phi d\Omega dr dE + \int \Psi \bar{\psi}_t \Sigma_a \Phi d\Omega dr dE}.
\] (19)

Upon rearrangement of Eq. (16), we note consistency with the neutron-balance model. That is,
\[
\frac{N^+}{\tau_r^+} = L^+ + A^+.
\] (20)

Similarly, we can express the adjoint-weighted leakage rate and the adjoint-weighted absorption rate as
\[
\frac{N^+}{\tau_l^+} = L^+,
\] (21)
and
\[
\frac{N^+}{\tau_a^+} = A^+.
\] (22)

where \( \tau_r^+ \) is the adjoint-weighted neutron leakage lifetime and \( \tau_a^+ \) is the adjoint-weighted neutron absorption lifetime. It follows, therefore, that
\[
\frac{1}{\tau_r^+} = \frac{1}{\tau_l^+} + \frac{1}{\tau_a^+},
\] (23)
which is consistent (at least in form) with Kupferberg's work, but has a slightly different meaning. Since the removal lifetime in Eq. (23) is adjoint-weighted, then \( 1/\tau_r^+ \) is the probability per unit time that a neutron of average importance will be removed from the system by either absorption or leakage.

So far in this section, we have defined a series of adjoint-weighted neutron lifetimes. These lifetimes were derived from the adjoint-weighted transport equation which was obtained by multiplying the transport equation by the adjoint flux and then integrating over all angles, energy, and space. When written in this form, the adjoint-weighted transport equation represents a balance of neutron importance. If, however, we integrate the transport equation over all angles, energy, and space without first multiplying through by the adjoint flux, we would obtain an equation that represents a balance of neutrons rather than importance.

Following a similar methodology used above, we can derive a series of expressions representing the unweighted neutron removal lifetime as well as the unweighted leakage and absorption lifetimes. This yields
\[
\tau_r = \frac{\int \Phi d\Omega dr dE}{\int \Omega \cdot \nabla \Phi d\Omega dr dE + \int \Sigma_a \Phi d\Omega dr dE},
\] (24)
and
\[
\tau_l = \frac{\int \bar{\psi}_t \Omega \cdot \nabla \bar{\psi}_t d\Omega dr dE}{\int \Omega \cdot \nabla \bar{\psi}_t d\Omega dr dE},
\] (25)
and
\[
\tau_a = \frac{\int \bar{\psi}_t \Sigma_a \Phi d\Omega dr dE}{\int \Sigma_a \Phi d\Omega dr dE}.
\] (26)

Just as before, we note that the reciprocal of the removal lifetime is the harmonic sum of the absorption and leakage lifetimes.
\[
\frac{1}{\tau_r} = \frac{1}{\tau_l} + \frac{1}{\tau_a}.
\] (27)

The meaning of \( 1/\tau_r \) is very similar to Kupferberg's interpretation; it is the probability per unit time of a neutron being removed from the system by either absorption or leakage, regardless of its importance.

The difference between the unweighted neutron removal lifetime and the adjoint-weighted neutron removal lifetime can be quite significant depending on the type of system. To illustrate this, consider a bare uranium sphere and a uranium sphere surrounded by a graphite reflector. Using the deterministic code, ONEDANT [14], Eq. (24) was evaluated over a range of \( k_{\text{eff}} \) and then compared to the
removal lifetime ascertained from an α-eigenvalue solution. The results are shown in Figs. 1 and 2. (Note, when $keff < 1.0$, the α-eigenvalue would not converge. Hence, there are no values for the adjoint-weighted removal lifetime below $keff=1.0$).

When these two hypothetical systems were analyzed using the Monte Carlo code MCNP4B, the removal lifetime, as estimated by Eq. (10) agreed to within 10% of the unweighted removal lifetime calculated using ONEDANT. The differences between the two sets of results were caused by differences in the neutron velocities associated with each energy group. In ONEDANT, the velocities are specified on the cross section input file whereas, in MCNP4B, the velocities are evaluated at the mid-point energy of each energy group. For the fast groups, the differences in velocities were relatively small; but, for the thermal groups, the group velocities differed by as small as 50%.

Notwithstanding the differences in the velocities, we surmise that the removal lifetime, as estimated by Eq. (10), must be identical to the unweighted removal lifetime as defined by Eq. (24). This identity seems logical since the neutron lifespan estimated by Monte Carlo techniques do not involve adjoint-weighting of the individual neutron birth-to-event time intervals.

The reaction-rate lifetimes defined in this section can also be related to the lifespans defined in the previous section. First, we take the ratio of Eq. (26) to Eq. (24), which results in,

$$\tau_a = \frac{\tau_r}{P_a},$$  \hspace{1cm} (28)

where $P_a$ is the fraction of neutrons that are absorbed. Next, we divide Eq. (10) through by $P_a$, which results in the following expression for $\tau_a$ in terms of the absorption and leakage lifespans. That is,

$$\frac{\tau_r}{P_a} = \tau_a + \frac{P_l}{P_a} \tau_l = \tau_a .$$  \hspace{1cm} (29)

Hence, from this expression we conclude that the absorption lifetime is always greater than or equal to the absorption lifespan. Similar expressions can be derived the leakage lifetime, fission lifetime, etc.

**Neutron Generation Time**

The reaction-rate lifetimes discussed in the previous section are associated with the termination of a neutron in the system. The neutron generation time, on the other hand, is associated with the mean time between fission neutrons. From the definition of the adjoint-weighted removal lifetime [see Eq. (17)] and the definition of the adjoint-weighted effective multiplication factor [see Eqs. (15) and (18)], the adjoint-weighted neutron production rate can be expressed as

$$k_{eff} \frac{N^+}{\tau^+} = \int \Phi d\Omega d\theta d\phi.$$

The left-hand side of this equation can be rewritten as

$$k_{eff} \frac{N^+}{\tau^+} = \frac{N^+}{\Lambda^+},$$  \hspace{1cm} (31)

where $\Lambda^+$ is defined as the adjoint-weighted neutron generation time and is equal to $\tau^+ / k_{eff}$. This generation time represents the mean time per unit neutron importance between the production of fission neutrons; it does not include any contributions from $(n,n)$ reactions, $(\gamma,n)$ reactions, etc., or intrinsic/external neutron sources. From its definition, we also note that the adjoint-weighted neutron generation time is only equal to the adjoint-weighted removal lifetime at delayed critical.

The adjoint-weighted neutron generation time, of course, has special significance in conjunction with reactor point kinetics; the prompt neutron time constant that appears in those equations is the adjoint-weighted neutron generation time.
III. CONCLUSIONS

The phrase "neutron lifetime" has been used too often in the past to describe a wide variety of time intervals associated with a neutron's trek through a given system. In an effort to clarify the meaning of some of the more important time intervals associated with the life of a neutron, we suggest that these various time intervals be divided into three categories: 1) neutron lifespans, 2) reaction-rate lifetimes, and 3) neutron generation times.

A neutron lifespan is the average time interval from birth-to-event (i.e., absorption, leakage, etc.). Monte Carlo codes are most adept at estimating this type of time interval.

A reaction-rate lifetime is the mean time per neutron (or unit of importance) between events of a particular type. It has the general form of

\[ \tau_x = \frac{\int \Phi \, d\Omega \, dr \, dE}{\int \sum_x \Phi \, d\Omega \, dr \, dE} \]  

(32)

as an unweighted quantity, and the general form of

\[ \tau_x^+ = \frac{\int \Psi \Phi \, d\Omega \, dr \, dE}{\int \Psi \sum_x \Phi \, d\Omega \, dr \, dE} \]  

(33)

as an adjoint-weighted quantity. Reaction-rate lifetimes are most generally associated with a neutron termination event and can easily be calculated using deterministic solutions.

A neutron generation time is a special case of a reaction-rate lifetime; it is the mean time per neutron (or unit of importance) between fission production events.

\[ \Lambda^+ = \frac{\int \Psi \Phi \, dr \, dE \, d\Omega}{\int \Psi \sum \Phi \, d\Omega \, dr \, dE \, d\Omega} \]  

(34)

The inverse of a reaction-rate lifetime is interpreted as the probability per unit time that a neutron (or a unit of importance if adjoint-weighted) will be involved in a reaction of a particular type.

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