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Disorder in High- T_c Superconductors***

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VORTEX DYNAMICS AND CORRELATED DISORDER IN HIGH- T_c SUPERCONDUCTORS

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ABSTRACT

We develop a theory for the vortex motion in the presence of correlated disorder in the form of the twin boundaries and columnar defects. Mapping vortex trajectories onto boson world lines enables us to establish the duality of the vortex transport in the systems with correlated disorder and hopping conductivity of charged particles in 2D systems. A glassy-like dynamics of the vortex lines with zero linear resistivity and strongly nonlinear current-voltage behavior as $V \propto \exp[-const/J^\mu]$ in a Bose glass state is predicted.

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The vortex dynamics in the systems with correlated disorder such as twin boundaries and/or the artificial *columnar defects* resulting from the irradiation of the sample with energetic heavy ions of tin, lead or gold, has attracted a great deal of recent attention. Twin boundaries were found to give rise to a characteristic shoulder on the temperature resistive curve for magnetic field aligned with the c-axis and the corresponding sharp deep in the angular dependence of resistivity ¹. It was found in addition that even few twin boundaries can change significantly the regime of motion of the vortex lines ². The columnar defects appear especially important since they are expected to provide the maximum possible pinning of flux lines parallel to the heavy ions tracks. Indeed, columnar defects were observed to enhance dramatically flux pinning, to shift upward the irreversibility line, and to depress magnetic relaxation ³⁻⁶. On the theoretical side, the theory of vortex pinning by correlated disorder was proposed. The formation of the specific Bose glass state characterized by the infinite tilt modulus c_{44} , and zero linear resistivity $\rho_{lin} = \left(\frac{\partial V}{\partial J} \right)_{J=0} = 0$, where J is the applied current and V is the voltage induced by the vortex motion, was predicted ⁷. The reported results on magnetization measurements revealed nonlinear (in time logarithm) magnetic relaxation with creep barriers diverging with the decreasing persistent current ^{9,10}, and confirmed conclusively the theoretical predictions of ⁷. In this communication we present the further development of the theory of the vortex dynamics in the Bose glass state with the emphasis on the analogy of the vortex dynamics with the quantum mechanical transport in 2D systems ^{7,10}.

The thermally activated vortex transport in the Bose glass state where vortices are localized on the columnar defects has been formulated in terms of the nucleation creep model ⁷. This model was originally proposed to describe the motion of dislocation in the random field of point defects ¹¹, and was later adapted to the description of vortex creep in high temperature superconductors ¹². Thermally activated vortex motion is considered as the sequence of the thermally activated jumps of the vortex segments or vortex bundles between the metastable states generated by disorder. Every elementary jump is viewed

as the nucleation of the vortex loop, and the mean velocity of the vortex configuration is determined by the nucleation rate of the optimal loop or the critical nucleus:

$$v \propto \exp(-\delta F/T). \quad (1)$$

Here δF is the free energy for the formation of the critical nucleus and is determined by the balance between elasticity, pinning and driving Lorentz forces.

Before the description of the dynamics itself let us review briefly the pinning properties of the Bose glass state ^{7,10}. One can distinguish two main regions on the $B - T$ plain with respect to the pinning. Below some characteristic *interaction field* $B^*(T)$, which we will define in Appendix, vortices are pinned individually, whereas at $B > B^*(T)$ interactions between vortex lines become essential and one has to consider pinning of vortex bundles. Let us consider first low fields $B < B^*$ and low temperatures (the exact criteria will be given below), where each vortex is trapped near a single columnar pin. We model a columnar pin by a cylindrical cavity with radius r_0 . Solving the London and Ginzburg-Landau equations one can find the following interpolation formula for the vortex binding energy ^{7,10}:

$$U_0 = \frac{1}{2} \epsilon_0 \ln \left(1 + \left(\frac{r_0}{\sqrt{2}\xi} \right)^2 \right), \quad (2)$$

where $\epsilon_0 = (\Phi_0/4\pi\lambda)^2$, $\Phi_0 = \pi\hbar c/e$ is the flux quantum, and λ and ξ are the London penetration length and the coherence length in ab -plane (we consider magnetic field aligned with the c -axis). The critical current J_c corresponding the maximal pinning force the vortex experiences near the line defect is given with the good accuracy by ^{7,10}:

$$J_c \cong \frac{3\sqrt{3}}{4\sqrt{2}} J_{pb} \frac{\left(\frac{r_0}{\sqrt{2}\xi} \right)^2}{1 + \left(\frac{r_0}{\sqrt{2}\xi} \right)^2}, \quad (3)$$

where $J_{pb} = c\Phi_0/12\sqrt{3}\pi^2\lambda^2\xi$ is the pair-breaking current. Thermal fluctuations affect both static and dynamic behavior of vortex configuration. On the other hand thermal

fluctuations cause thermally activated jumps of vortex lines over pinning barriers at small $J \ll J_c$ currents leading to creep phenomenon. We will see below that both types of thermal fluctuations can be described in a natural unified way by means of the concepts from the elementary quantum mechanics. The thermal fluctuations of vortex lines near their equilibrium positions renormalize pinning potential reducing thus critical currents. The quantity characterizing pinning at finite temperatures is the binding *free* energy $U(T) = U_0 - TS$, where entropy term accounts for thermal excursions of vortex lines. It was shown¹³ that the classical partition function of the vortex system is equivalent to the imaginary time path integral representation of quantum mechanics of 2D Bose particles. In the thermodynamic limit $L_z \rightarrow \infty$, where L_z is the size of the system along z direction, the binding free energy is dominated by the lowest eigenvalue, $E_0 = -U(T)$ of the two dimensional Schrodinger equation

$$\left[-\frac{T^2}{2\epsilon_1} \nabla_{\perp}^2 + V_D(\mathbf{r}) \right] \psi(\mathbf{r}) = E_0 \psi(\mathbf{r}), \quad (4)$$

where $V_D(\mathbf{r})$ represents pinning potential and $\mathbf{r}$ denotes the position in the plane perpendicular to z axis. Note that temperature T plays the role of Planck constant $\hbar$ and vortex linear tension $\epsilon_1 = [\Phi_0^2 / (4\pi\lambda)^2 \gamma^2] \ln \kappa$ plays the role of particle mass ($\gamma^2 = M_z / M_{\perp}$, $\kappa = \lambda / \xi$, and ξ and λ are coherence length and London penetration depth in ab -plane).

Let us consider the pinning of a vortex line near a single columnar defect. The solution to Eq. (4) introduces important characteristic temperatures describing pinning by columnar defects^{13,7,10}. Approximating the potential of the columnar defect by the cylindrically symmetric rectangular potential with depth U_0 , one finds for the binding energy¹³

$$U(T) = U_0 f(T/T^*), \quad (5)$$

where the characteristic energy T^* is given by

$$T^* = b_0 \sqrt{\epsilon_1 U_0}, \quad (6)$$

where $b_0 = \max(r_0, \xi)$, $f(x) \approx 1 - (c_1/2)x^2$ at $x \rightarrow 0$, and $f(x) \approx (x^2/2)\exp[-2(x^2)]$ at $x \rightarrow \infty$ and c_1 is a constant related to the first zero of the Bessel function $J_0(x)$.

Note that in order to liberate vortex line from the columnar defect one has to pull out of the rod the segment of the vortex line by means of formation the double kink configuration. Then the vortex depinning can be completed by the adiabatic propagating of kinks along rod which costs no energy and the depinning energy is that needed for the formation of such a double kink configuration. The quantity T^* corresponds to formation of the minimal possible kink and, therefore represents the characteristic pinning energy. We are mostly interested in the temperature region where $\sqrt{2}\xi > r_0$. The characteristic temperature T_1 separating regimes of strongly- and weakly localized vortex line can be defined as the solution of the self consistency relation $T^*(T_1) = T_1$ following from Eq.(5) where $b_0 = \sqrt{2}\xi(T)$ is used¹⁰. Taking $r_0 = 35\text{\AA}$, $\xi_0 = 12\text{\AA}$, $\kappa = 10^2$, and Ginzburg number $Gi = 10^{-2}$ (see Appendix) for YBCO one gets $T_1 = 0.94T_c$.

The quantity $\kappa^{-1} = T/\sqrt{2\epsilon_1 U(T)}$ describes the decay of the wave function into the classically forbidden region. At $T \ll T_1$ the wave length $\kappa^{-1} \ll b_0$ and vortex line is localized within the rod, at $T \gg T_1$ thermal transverse excursions of vortex line exceed well the rod effective radius b_0 . Vortex line is bound weakly with the binding energy¹³

$$U(T) = \frac{1}{2}U_0 \left(\frac{T}{T^*} \right)^2 \exp \left[-2 \left(\frac{T}{T^*} \right)^2 \right], \quad (7)$$

and the transverse localization length

$$l_{\perp} \approx \kappa^{-1} \approx b_0 e^{(T/T^*)^2}. \quad (8)$$

As the localization length exceeds the mean distance d between rods, the vortex line cannot be pinned by a single pinning site any more, and is localized now by the fluctuations in the density of rods. This pinning regime can be described as the collective pinning of the single vortex line, and from the condition $l_{\perp}(T_{dl}) = d$ one finds the self-consistent equation for the crossover temperature $T_{dl} = T^*(T_{dl}) \sqrt{\ln \left[\frac{d}{\sqrt{2}\xi_0} \left(1 - \frac{T_{dl}}{T_c} \right)^{1/2} \right]}$.

At high fields $B > B^*$ the wandering of the vortex lines are confined by the interactions with another vortices rather than with the pinning potential (see ¹⁰ and Appendix B. This is the region of the vortex lattice collective pinning ^{7,10}.

Critical currents at $B < B^*$ can be found as the ionization field necessary to pull out the particle out of the 2D potential well analogously to the usual Frantz-Keldysh effect ¹⁴. At $T_1 < T < T_{dl}$ those are the wells associated with the single rod, whereas at $T_{dl} < T$ vortices are localized by the Gaussian fluctuations in the defects density. A straightforward calculation gives

$$J_c = J_0 \frac{3\sqrt{2}}{8} \left(\frac{r_0}{\sqrt{2}\xi} \right)^2 \left(\frac{T}{T^*} \right)^2 \exp \left[-3 \left(\frac{T}{T^*} \right)^2 \right], \quad T_1 < T < T_{dl}, \quad B < B^* \quad (9)$$

and

$$J_c \approx J_0 \left(\frac{r_0}{d} \right)^2 \frac{r_0}{\xi} \left(\frac{T^*}{T} \right)^4, \quad T > T_{dl}, \quad B < B^*. \quad (10)$$

One can immediately check that at $T = T_{dl}$ these expressions match each other up to the logarithmic factor.

Now we consider flux dynamics at subcritical (but not too small) currents $J_b < J < J_c$, and moderate magnetic fields $B < B_b < B^*$, where vortex dynamics is governed by the single vortex motion. The crossover current J_b and magnetic field B_b , where the crossover from the single-vortex creep to the creep of vortex bundles occurs, is defined by the condition ¹⁴:

$$L_z = a_0, \quad (11)$$

where $L_z(J)$ is the characteristic length of the fluctuating segment appearing in the single-vortex nucleation creep process. The creep of the single vortex escaping from the single rod occurs either via the thermally activated formation of the half loop saddle point configuration or via the throwing a vortex segment onto an adjacent rod favored by the applied current. The average vortex velocity can be found as the rate of formation of these saddle configurations. The energy of the saddle configuration δF can be found by means of the

standard variational procedure from the free energy functional for one flux line near the single rod:

$$F_1 = \int dz \left[\frac{1}{2} \epsilon_1 \left(\frac{dr}{dz} \right)^2 + V_1(r(z)) - f_L r(z) \right], \quad (12)$$

where $f_L = \frac{1}{c} \Phi_0 J$. An estimate of the saddle point energy can be obtained from the dimensional analysis. The energy of the half-loop configuration can be approximated as

$$\delta F \simeq \frac{\epsilon_1 u^2}{L} + UL - f_L u L, \quad (13)$$

where u and L are the transverse and the longitudinal sizes of the half-loop. Optimization with respect to the loop dimensions gives

$$u^* \simeq \sqrt{\frac{U}{\epsilon_1}} L^* \simeq \frac{cU}{\Phi_0 J} \quad (14)$$

and the saddle configuration energy

$$\delta F^* \simeq \frac{c \epsilon_1^{1/2} U^{3/2}}{\Phi_0 J}. \quad (15)$$

More quantitative description can be achieved by means of the quantum mechanical analogy. Within this analogy the vortex creep is viewed as the quantum mechanical decay of the metastable state analogous to the process of the atom ionization in the external electric field which is assumed to be small as compared to the ionization field. The process of the half-loop formation maps onto the process of the ionization of the isolated atom, and the process of the vortex hopping between the different rods corresponds to the particle tunneling in the external electric field from the one localized state to another. This hopping motion of the vortex lines is isomorphic, therefore, to the hopping conductivity in the 2D semiconducting systems.

Note that the free energy in Eq.(15) is identical to the action in the standard expression for the so-called *cold emission*, with the substitution $\epsilon_1 \rightarrow m$, $c/\Phi_0 J \rightarrow e\mathcal{E}$ ($\mathcal{E}$ is the electric field). A standard WKB calculation gives the half loop generation nucleation rate W as:

$$W \simeq \frac{A^2 \sqrt{\pi}}{2^{3/4}} \omega_0 \frac{U(T)}{\epsilon_1} \sqrt{\frac{TJ}{E_k(T)J_1}} \exp\left(-\frac{4\sqrt{2}}{3} \frac{E_k(T)}{T} \frac{J_1}{J}\right), \quad (16)$$

where $J_1 = cU(T)/\Phi_0 d$, $E_k(T) = d\sqrt{\epsilon_1 \bar{U}(T)}$ is the energy to form a kink connecting the segments of vortex line located on the adjacent rods separated by the distance d and ω_0 is the attempt frequency which can be estimated as $\omega_0 \simeq \sqrt{T^*/\eta b_0^2}$. This gives rise to the vortex velocity $v \simeq (d^2/r_0)\sqrt{2u_d \bar{W}}$, where $u_d \simeq \Phi_0 J/\eta c$, $\eta = \Phi_0 H_{c2}/\rho_n c^2$ is the vortex friction, and ρ_n is the resistivity in a normal state. As a result one finds for the current-voltage characteristic

$$V \propto J \exp\left(-\frac{2\sqrt{2}}{3} \frac{E_k(T)}{T} \frac{J_1}{J}\right). \quad (17)$$

This result holds for in the interval $J_1 < J < J_c$. In this interval $u^* < d$ and, accordingly, the free in Eq. (17) is $\delta F \approx 0.943 E_k J_1/J < E_k$. At $J < J_1$ the hopping distance $u^* > d$, the free energy for the half loop formation exceeds the energy of the double-kink configuration, and the hopping vortex transport via the the vortex jumps between the adjacent rods dominates the creep process.

If the binding energies are all identical for each rod, the hopping vortex transport exhibits the nearest neighbour hopping conductivity features with vortices following percolating trajectories in the plain perpendicular to z direction. This gives rise to a linear resistivity $\rho \propto \exp(-c_1 E_k/T)$, where the constant c_1 can be found from the 2D percolation theory¹⁵. This scenario holds provided the dispersion γ in the binding energies is small enough: $\gamma L_z < E_k$. The dispersion in binding energies arises both from the random environment near different pinning sites and because of the possible transitions of vortices between the randomly distributed pinning sites. The latter effect results in a different entropy contributions to the binding free energy and gives rise to a localization of the vortex lines near the pairs of the pinning sites separated by the smallest distance. In the quantum mechanical description this localization implies the dispersion in the bound state energy levels and is the exact analog of the *Lifshitz localization* of the quantum mechanical particle in the presence of the structure disorder.

The dispersion of flux binding energies becomes relevant for $J < J_1$ and $L_z > L_1 = E_k/\gamma$. The flux line can hop now only on the pinning site with the low energy level. The

elementary jump on the higher level is not possible since the total energy cost of such jump is proportional to the thickness of the sample L_z and becomes infinite in the thermodynamic limit. Hopping transport occurs now via the formation of the formation of the double *superkink* configurations connecting the vortex to the available state which optimizes the hopping probability. In the quantum mechanical analogy this process maps onto the variable range hopping (VRH) conductivity, and the description of the corresponding variable range flux hopping is achieved by the transcription of the 2D VRH formulas into the vortex language by $m \rightarrow \epsilon_1$, $\hbar \rightarrow T$, and $e\mathcal{E} \rightarrow \frac{1}{c}\Phi_0 J$ ^{7,10}. The energy of the saddle trajectory is give then by

$$\delta F_{sk}^* = \frac{E_k}{d} \left(\frac{c}{g(\mu)\Phi_0 J} \right)^{1/3}. \quad (18)$$

The corresponding current-voltage characteristic takes the form

$$V \propto \exp \left[-\frac{E_k}{T} \left(\frac{J_{VR}}{J} \right)^{1/3} \right]. \quad (19)$$

We have introduced the density of states $g(\mu)$ just above the chemical potential μ , which defines the total number of states $\mathcal{N}$ with energies in the interval $\delta\epsilon$ at distance less than or equal to R from the given site as $\mathcal{N} = g(\mu)R^2\Delta\epsilon$. From the condition $g(\mu)R^2\Delta\epsilon = 1$ the typical energy difference (per unit length) for a hop over a distance R is $\Delta\epsilon = 1/g(\mu)R^2$. The characteristic current J_{VR} is given by

$$J_{VR} = \frac{c}{\Phi_0 g(\mu) d^3}. \quad (20)$$

Note that in the limit $L_z \rightarrow \infty$ the percolation regime of vortex transport cannot occur. The half loop tunneling regime recovers at currents $J > J_2$, where

$$J_2 = \frac{J_1^{3/2}}{J_{VR}^{1/2}} = J_1 [U(T)g(\mu)]^{1/2} d. \quad (21)$$

The density of states at low temperatures $T < (d/\xi)^2 T^*$ is estimated as $g \simeq 1/Ud^2$. In this temperature range $J_1 \approx J_2 \approx J_{VR} = cU/\Phi_0 d$.

Now we turn to the vortex dynamics in the regime of the collective motion of the vortex bundles in the region $B^*(T) < B < B_M(T)$ ^{7,10}. We restrict ourselves to a brief discussion of collective creep where the transverse size of the vortex bundle $R_\perp$ exceeds the columnar defects spacing d . The creep rate as usual is determined by the saddle trajectory of the vortex configuration free energy

$$F = \int dz d\mathbf{r} \left[\frac{1}{2} C_{44} \left(\frac{\partial \mathbf{u}}{\partial z} \right)^2 + \frac{1}{2} C_{11} (\text{div} \mathbf{u})^2 + \frac{1}{2} C_{66} (\nabla_\perp)^2 + V_D(\mathbf{r} - \mathbf{u}(z, \mathbf{r})) - \frac{1}{c} J \mathbf{u}_T \right], \quad (22)$$

where $\mathbf{u}_T$ is the component of the displacement $\mathbf{u}$ parallel to the Lorentz force. Because of the specific form of the pinning potential $V_D(\mathbf{r})$ not containing an explicit dependence of the z -coordinate, the quantum mechanical analogy enables us to reduce the problem of *classical* collective creep through columnar disorder to the problem of *quantum* tunneling of a two dimensional pancake vortex lattice between metastable states generated by the quenched point disorder. The tilt modulus C_{44} plays a role of the fictitious particle "mass", and the tilt deformation represents a fictitious "kinetic energy." The creep rate is controlled again by the saddle point trajectory which can be found by a standard dimensional method^{7,10}. As a result one finds the following current dependencies^{7,10} for the creep activation energy:

$$U(J) \propto \text{const} \begin{cases} J^{-19/8}, & a_0 < R_\perp < \lambda \\ J^{-14/8}, & \lambda < R_\perp < \gamma\lambda \\ J^{-9/8}, & \gamma\lambda < R_\perp, \end{cases} \quad (23)$$

for the hopping distance $u^* < a_0$. When the hopping distance exceeds the vortex spacing, creep is governed by the CDW-like pinning¹⁴, and the corresponding pinning energy read:

$$U_{CDW}(J) \propto \text{const} \begin{cases} J^{-15/8} \ln^2 \left(\frac{J_b}{J} \right), & a_0 < R_\perp < \lambda \\ J^{-5/4} \ln^2 \left(\frac{J_b}{J} \right), & \lambda < R_\perp < \gamma\lambda \\ J^{-5/8} \ln^2 \left(\frac{J_b}{J} \right), & \gamma\lambda < R_\perp, \end{cases} \quad (24)$$

where J_b is some characteristic current describing pinning of vortex bundles^{7,10}. For more details concerning collective creep through the system of columnar defects see^{7,10}.

Now we consider vortex creep at very low temperatures. At small fields the vortex creep is controlled by the quantum vortex tunneling from the columnar defects. The tunneling rate is determined by the saddle trajectory of the Euclidean action generating vortex motion near the rod ¹⁴:

$$S_E = \int \frac{d\omega}{2\pi} \frac{dq}{2\pi} \left[\frac{1}{2} \left(|\omega|\eta + \epsilon_1 q^2 \right) |u(\mathbf{q}, \omega)|^2 + V_D(\mathbf{q}, u) \right]. \quad (25)$$

from the usual dimensional estimates one finds for the tunneling time

$$t_t^{-1} = \omega_t \simeq (\epsilon_1/\eta) q_t^2 \simeq (\epsilon_1/\eta) L^{-2}, \quad (26)$$

where L is the characteristic length of the fluctuating segment.

First, we estimate the creep rate $d \ln J / d \ln t \approx \hbar / S_E^*$ at currents close to the critical one $J \approx J_c$. Using Eqs.(14) and (2) one immediately finds $t_{tc} \simeq \eta r_0^2 / U_0$ and the creep rate is given by

$$\frac{\hbar}{S_{Ec}} \simeq \frac{\hbar \gamma}{\eta r_0^3}. \quad (27)$$

Note that the creep rate of quantum tunneling does not depend upon the strength of the pinning potential.

Now we turn to the creep at small currents $J \ll J_c$. We restrict ourselves to consideration of the half loop creep process and VRFH regimes. For the half loop process one immediately gets from Eq.(14):

$$t_t \simeq t_{tc} \left(\frac{J_c}{J} \right)^2 \quad (28)$$

and

$$\frac{S_{EHL}}{\hbar} = \frac{\hbar}{S_{Ec}} \left(\frac{J_c}{J} \right)^3. \quad (29)$$

Analogously one finds for the VRFH one finds:

$$\frac{S_{EVRH}}{\hbar} = \frac{\hbar}{S_{Ec}} \frac{d}{r_0} \left(\frac{g(\mu) d^3 U_0}{r_0} \right)^2 \left(\frac{J_1}{J} \right)^{7/3} \simeq \frac{\hbar}{S_{Ec}} \left(\frac{d}{r_0} \right)^3 \left(\frac{J_{VR}}{J} \right)^{7/3}, \quad (30)$$

where the estimate $g \simeq (U_0 d^2)^{-1}$ has been used.

A very interesting regime of tunneling may take place in very clean superconductors associated with the Hall motion of the vortex lines. In the absence of the pinning forces the general equation of the vortex motion has the form¹⁴:

$$\eta \mathbf{v} + \alpha \mathbf{v} \times \mathbf{z}_0 = \frac{\Phi_0}{c} \mathbf{J} \times \mathbf{z}_0, \quad (31)$$

where $\mathbf{z}_0$ is the unit vector along the vortex line and the second term at lhs of the Eq. (31) describes the Hall effect, α being the Hall constant. In the clean superconductors $\alpha = \pi \hbar n_s$. Note that the Hall term has the same structure with respect to velocity as the dissipative term, and the term generating the Hall motion analogous to the first dissipative term can be added to action in Eq. (25). If the dissipation is very small $\eta \ll \alpha$, the quantum tunneling is governed by the Hall effect¹⁵. The simple estimates for the quantum Hall tunneling can be easily obtained by dimensional analysis. In particular the half loop and VRHF regimes are described by Eqs. (27) and (30) respectively with the substitution $\eta \rightarrow \alpha$. The quantum Hall tunneling is discussed in details in¹⁵.

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APPENDIX. The characteristic *interaction field* $B^*(T)$ below which interactions are negligible is given by the following expressions⁹:

$$B^*(T) \approx B_\Phi, \quad T < T_0 \quad (A1)$$

$$B^*(T) \approx B_\Phi \left(\frac{r_0}{2\xi_0} \right)^2 \left(1 - \frac{T}{T_c} \right), \quad T_0 < T < T_1, \quad (A2)$$

$$B^*(T) \approx 4B_\Phi \frac{T^2}{T_c(T_c - T)}$$

$$\frac{Gi}{\ln \kappa} \exp\left[-2\left(\frac{4\xi_0}{r_0}\right) \frac{Gi}{\ln \kappa} \frac{T^2}{(T - T_c)^2}\right] \quad T_1 < T < T_{dp}, \quad (A3)$$

$$B^*(T) \approx 2_1 3 B_\Phi \left(\frac{r_0}{\xi_0}\right)^6 \left(\frac{r_0}{d}\right)^2 \left(\frac{\ln \kappa}{Gi}\right)^3 \frac{(T - T_c)^6}{T^6}, \quad T_1 < T < T_c \quad (A4)$$

where $B_\Phi = \Phi_0/d^2$ is the so called matching field at which vortex line spacing a_0 is equal the mean distance between defects, d , r_0 is the radius of the columnar defect, ξ_0 is the coherence length at zero temperature. The Ginzburg number $Gi = \frac{1}{2} \frac{M_z}{M_L} \left(\frac{T_c}{H_c^2 \xi_0^3}\right)^2$ is the reduced temperature $(T_c - T)/T_c$ at which fluctuations of the superconducting order parameter become comparable to the order parameter itself.

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