**ELECTROSEISMIC CHARACTERIZATION
OF LITHOLOGY AND FLUID TYPE IN
THE SHALLOW SUBSURFACE**

FINAL REPORT

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ABSTRACT

The U.S. Department of Energy funded the M.I.T. Earth Resources Laboratory to investigate electroseismic phenomena. Because electroseismic phenomena in fluid-saturated porous media provide geophysicists with a unique opportunity to detect a seismic-wave-generated flow of pore fluid with respect to the porous matrix. The term "electroseismic" describes phenomena in which a seismic wave induces an electrical field or causes radiation of an electromagnetic wave. Electrostatic phenomena take place in fluid-saturated porous rocks, because the pore fluid carries an excess electrical charge. When the charged pore fluid is forced to flow through the rock by pressure gradients within a seismic wave, a streaming electrical current is generated. This electrical current results in charge separation, which induces an electrical field. Measuring this seismic-wave-induced electrical field allows detection of the fluid flow generated by the wave in the porous medium. In turn, detecting the fluid flow allows characterization of fluid transport properties of the medium.

Research conducted at the Earth Resources Laboratory resulted in a significant advancement in the understanding of electroseismic phenomena. The major contribution of our research is in the following three areas.

1. **Theory.** Theoretical models of various electroseismic phenomena in fluid-saturated porous media were developed. Numerical algorithms were developed for modeling electroseismic measurements in surface (Paper 1 in this report) and VSP (Paper 2) geometries. A closed-form analytical expression was obtained for the logging geometry (Paper 8). An analysis of the seismic-wave-induced electrical field sensitivity to porosity, permeability, and pore fluid chemistry was performed (Paper 3; Paper 8). The major result is the theoretical models' prediction that porosity, permeability, and fluid chemistry can be characterized using electroseismic measurements.
2. **Laboratory Experiments.** A number of laboratory experiments were performed in surface (Paper 4), VSP (Paper 4), and logging (Paper 5) geometries. In addition, conversion of electrical energy into seismic energy was investigated (Paper 6). The major result is that all the electroseismic phenomena predicted by the theory were observed in the laboratory experiments.
3. **Field Measurements.** Field experiments were conducted in surface (Paper 7) and logging (Paper 8) geometries. Data processing algorithms were developed (Paper 7). All the phenomena predicted by the theoretical models were observed in the field. Moreover, it was demonstrated that the theory correctly predicted amplitudes and phases of the electrical fields measured in the experiments (Paper 8). The major contribution is that we demonstrated that porosity and permeability can be derived from field data (Paper 8).

Research conducted at the Earth Resources Laboratory under DOE grant #DE-FG02-93ER14322 resulted in understanding how to make electroseismic measurements, how to process and interpret field data, and how to deduce information about porosity and permeability of the formation from the field data. We suggest that electroseismic measurements can be developed into a VSP or a logging tool with applications in environmental geophysics and the oil industry.

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1. ELECTROSEISMIC WAVES FROM POINT SOURCES IN LAYERED MEDIA

1.1. SUMMARY

The macroscopic governing equations controlling the coupled electromagnetics and acoustics of porous media are numerically solved for the case of a layered poro-elastic medium. It is shown that these coupled equations decouple into two equation sets describing two uncoupled wavefield pictures. That is, the *PSVTM* picture where the compressional and vertical polarized mechanical waves drive currents in the *PSV* particle motion plane that couples to the electromagnetic wavefield components of the *TM* mode. And the *SHTE* picture where the horizontal polarized rotational mechanical waves drive currents in the *SH* particle motion plane that couples to the electromagnetic wavefield components of the *TE* mode. The global matrix method is employed in computing electroseismograms in layered poro-elastic media in the *PSVTM* picture.

The principal features of the converted electromagnetic signals are the following: (1) contacts all antennas at approximately the same time; (2) arrives at the antennas at half of the seismic travelttime at normal incidence reflected *P* waves; and (3) changes sign on opposite sides of the shot.

The seismic pulse is shown to induce electric fields that travel with the compressional wavespeed and magnetic fields that travel with the rotational wavefield. The frequency content of the converted electromagnetic field has the same frequency content of the driving incident seismic pulse, as long as the propagation distances are much less than the electromagnetic skin depth.

Snapshots in time and converted electromagnetic amplitudes versus seismic point source-antenna offset-are calculated for contrasts in mechanical and/or electrical medium property. Conversion happens there where the seismic wavefront passes a contrast in medium properties due to generated imbalances in current across the contrast. The *TM* component amplitude radiation pattern away from the interface shows similarities with an effective electric dipole radiation pattern, or its dual, an effective magnetic current loop radiation pattern centered right beneath the source at the contrast's depth. The *TM* mode amplitudes decay rapidly with traveled distance and suggest the importance of a Vertical Electrostatic Profiling geometry to enhance recording of the converted electromagnetic signal by positioning the antennas closer to the target (contrast) of interest.

1.2. INTRODUCTION

When seismic waves propagate through a fluid saturated sedimentary material, the motion of the pore fluid to the solid matrix causes relative flow. The driving force for the relative flow is a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain accelerations. The relative flow caused by grain accelerations can therefore be both due to compressional and shear waves.

A fluid electrolyte in contact with a solid surface chemically absorbs the anions from the electrolyte to the solid wall leaving behind a net excess of cations distributed near the wall. This region is known as an electric double layer (Bockris and Reddy, 1970). The diffuse distribution of mobile cations, with a higher concentration cations in the region close to the absorbed layer and more and

more diffuse towards the neutral electrolyte, is free to move when the fluid moves.

The seismic wave motion, which generates the relative flow, also induces a "streaming" electrical current due to the cation motion. This induced streaming current acts as a current source in Maxwell's equations. Therefore, when seismic waves travel through fluid-saturated sedimentary materials, current systems are set up in the material inducing nonradiating fields. But when the seismic waves hit a contrast in electrical and/or mechanical properties, the current systems on both sides form a complex dynamic current system across the interface generating electromagnetic waves.

In this paper, the governing macroscopic equations controlling such behavior will be laid out and these equations will be numerically solved for the case of a layered medium. A global matrix method (Chin *et al.*, 1984; Mal, 1988a,b) is employed in computing the synthetic seismograms in the horizontally stratified medium. The global matrix method solves simultaneously the macroscopic electromagnetic and acoustic-elastic wavefields which drive the current and fluid flow, resulting in a mechanical stress/displacement and electromagnetic wavefield component vector as a function of depth. The equations solved are the coupled poro-elastic (modified Biot's equations) and electromagnetic (Maxwell's) equations. Biot's equations are modified with an induced body force on Biot's equation of relative flow (where the electromagnetic field induces a mechanical force, an osmosis phenomenon). Amperes law in Maxwell's equations has an external current source (where the relative flow has induced an electrical streaming current, an electrokinetic phenomenon). The coupling is given by the transport equations, these flux/force relations relate the current and the relative flow to potential and pressure gradients. The EM field converted from seismic waves traversing boundaries is associated with a change in the following properties: changes in elastic properties (e.g., porosity, changes in fluid flow permeability, changes in fluid-chemistry that affect the amount of cations free to move in the double layer (e.g., bulk free-ion concentration, pH).

A thick, permeable sand layer example will be presented to show the effect of a mechanical contrast on the mechanical and electromagnetic wavefield components. A fresh water/brine contrast model is used to show the effect of an electrical contrast on the mechanical and electromagnetic wavefield components. Both models are also used in the calculation of converted magnetic and electric field amplitudes versus seismic source-antenna offset at different distances from the interface. Snapshots in time are calculated to follow the conversion evolution of mechanical waves into electromagnetic waves. The rapid decay in amplitude with traveled distance of the converted electromagnetic signal, driven by the seismic source with a seismic center frequency, suggests the Vertical Electrostatic Profiling geometry, where antennas are positioned close to the target of interest. The last two numerical examples are calculations in a VESP geometry.

1.3. MACROSCOPIC COUPLED ELECTROMAGNETIC AND PORO-ELASTIC FIELD EQUATIONS

The macroscopic, fully coupled, mechanical, electromagnetic equations and constitutive relations in volume averaged from describing the coupled field behavior in two phase porous medium are (Pride, 1994):

The electromagnetic field equations,

$$\nabla \times \underline{E} = i\omega \underline{B} \quad (1)$$

$$\nabla \times \underline{H} = -i\omega \underline{D} + \underline{J} \quad (2)$$

$$\underline{D} = \epsilon_0 \left[\frac{\phi}{\alpha_\infty} (\kappa_f - \kappa_s) + \kappa_s \right] \underline{E} \quad (3)$$

$$\underline{B} = \mu_0 \underline{H} \quad (4)$$

$$\underline{J} = \sigma(\omega) \underline{E} + L(\omega) \left[-\nabla P + \omega^2 \rho_f \underline{u}_s \right] \quad (5)$$

$$-i\omega \underline{w} = L(\omega) \underline{E} + \frac{k(\omega)}{\eta} \left[-\nabla P + \omega^2 \rho_f \underline{u}_s \right] \quad (6)$$

and the mechanical field equations,

$$\nabla \cdot \underline{\tau}_B = -\omega^2 [\rho_B \underline{u} + \rho_f \underline{w}] \quad (7)$$

$$\underline{\tau}_B = [K_G \nabla \cdot \underline{u}_s + C \nabla \cdot \underline{w}] \underline{I} + G_{fr} \left[\nabla \underline{u}_s + \nabla \underline{u}_s^T - \frac{2}{3} \nabla \cdot \underline{u}_s \underline{I} \right] \quad (8)$$

$$-P = C \nabla \cdot \underline{u}_s + M \nabla \cdot \underline{w}. \quad (9)$$

Equations (1)–(4) describe the electromagnetic wavefields and equations (7)–(9) the mechanical wavefields. All coupling is present in the transport equations (5)–(6).

The coefficients in the flux equations relating the equations of mass—here fluid flow and current flow when there are potential and pressure gradients—are the frequency dependent conductivity of the two phase medium, $\sigma(\omega)$, the dynamic permeability, $k(\omega)$ and the fluid viscosity, η . The first term in equation (5) is the conduction current contribution, and the second term is the streaming current contribution to the total current. The forces which drive the relative flow are a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain acceleration. The relative flow caused by grain acceleration can, therefore, be both due to compressional and shear waves. The fact that the cross terms have the same coupling coefficient $L(\omega)$ is a statement of Onsager reciprocity. The coefficient equality implicates the action of the electric field on the charge density to be reciprocal to the action of the fluid pressure on the charge, or the mobility of the ion is independent of the charge moving through the fluid or of the fluid moving around the charge.

The frequency dependent coefficients are obtained by Pride (1994), where they are explicitly obtained by a volume averaging procedure of Maxwell's equations coupled to Biot's equations. Two possible functions that smoothly connect the low and high frequency regimes of the transport coefficients are,

$$\frac{k(\omega)}{k_0} = \left[\left[1 - i \frac{\omega}{\omega_t} \frac{4}{m} \right]^{1/2} - i \frac{\omega}{\omega_t} \right]^{-1} \quad (10)$$

and,

$$\frac{L(\omega)}{L_0} = \left[1 - i \frac{\omega}{\omega_t} \frac{m}{4\alpha_\infty^2} \left(1 - 2\alpha_\infty \frac{\tilde{d}}{\Lambda} \right)^2 \left(1 - i^{3/2} \tilde{d} \sqrt{\frac{\omega \rho_f}{\eta}} \right)^2 \right]^{-1/2}. \quad (11)$$

The transition frequency ω_t separates the low frequency viscous flow behavior from the high frequency inertial flow and is defined as,

$$\omega_t = \frac{\phi}{\alpha_\infty k_0} \frac{\eta}{\rho_f} \quad (12)$$

The dimensionless number m is defined as,

$$m \equiv \frac{\phi \Lambda^2}{\alpha_\infty k_0} \quad (13)$$

This number consists only of geometry terms. The low frequency coupling coefficients L_0 is,

$$L_0 = -\frac{\phi}{\alpha_\infty} \frac{\epsilon_0 \kappa_f \zeta}{\eta} \left[1 - 2\alpha_\infty \frac{\tilde{d}}{\Lambda} \right]. \quad (14)$$

With κ_f , the relative fluid permittivity, ζ , the zeta potential, and α_∞ , the tortuosity (normally a number between 3 and 10). The “ $\sqrt{\frac{\omega \rho_f}{\eta}}$ ” factor in equation (11) determines the viscous skin depth and $\tilde{d} \leq \frac{3 \times 10^{-10}}{\sqrt{C}} \equiv$ Debye length (which is a measure of the electric double layer thickness) and with C the salinity of the pore fluid. The Λ parameter is the same as defined by Johnson *et al.* (1987). An important factor affecting the electrokinetic coefficient is the ζ -potential. The ζ -potential employed in the numerical calculations is taken from experimental studies. These ζ -potential values are based on saturated NaCl, KCl quartz samples at $T = 25^\circ\text{C}$ and $pH = 7$, determined by Gaudin and Fuerstenau (1955), Sidorova and Fridiriksberg (1975), and Hidalgo-Alvarez and Pardo (1985). Regression analysis applied on their combined data sets resulted in the following ζ -potential as function of electrolyte concentration.

$$\zeta(V) = 0.008 + 0.026 \log_{10}(C) \quad (15)$$

The coefficients in the deformation equations are,

$$K_G = H - \frac{4}{3}G = \frac{K_{fr} + \phi K_{fr} + (1 - \phi)K_s \Delta}{1 + \Delta} \quad (16)$$

$$C = \frac{K_f + K_s \Delta}{1 + \Delta} \quad (17)$$

$$M = \frac{1}{\phi} \frac{K_f}{1 + \Delta} \quad (18)$$

where the parameter Δ is defined as,

$$\Delta = \frac{K_f}{\phi K_s^2} [(1 - \phi)K_s - K_{fr}]. \quad (19)$$

The moduli K_{fr} and G_{fr} are the bulk and shear moduli of the framework of the grains, when fluid is absent. The frame moduli may either be considered experimentally determined, or may be obtained from approximate theoretical models for specific pore grain geometries. C and M are the incompressibilities used by Biot (1962) and Pride *et al.* (1992), they are complex and frequency dependent, allowing for losses in addition to those associated with relative flow.

1.3.1. Transform-Domain Electrostatic Wavefield Constituents in an Isotropic Poro-Elastic Layered Medium

In this section, the compressional (P_f, P_s), rotational (SV, SH) and electromagnetic fields (TM, TE modes) in layered isotropic fully saturated poro-elastic media are investigated. Only mechanical waves are generated by a localized explosive source situated at the interface between two adjacent layers (or subdomains) of the stratified configuration. The seismic wave motion which causes relative flow induces a "streaming" electrical current due to the excess cation motion, in the electric double layer. The induced streaming current acts as a current source in Amperes current law, equations (2) and (5). Therefore, when seismic waves travel through fluid saturated porous media, current systems are set up. When compressional waves travel through homogeneous porous material, the pressure gradients set up by the peaks and troughs cause the charge to separate (Figure 1). This induces within the seismic pulse a system of electric fields that travel with the compressional wave speed. By symmetry arguments within the seismic pulse (a ricker wavelet in the numerical modeling), there is no net current imbalance and, therefore, the seismic pulse cannot act as an electromagnetic source and radiate electromagnetic waves away from the pulse.

When rotational waves travel through homogeneous porous material, the grain accelerations which are equivoluminal set up current sheets, (Figure 2). This induces within the seismic pulse a magnetic field system that travels with the rotational mechanical wavespeed. By similar symmetry arguments for the induced current sheets within the seismic pulse, the seismic pulse does not radiate electromagnetic fields.

Compressional and vertical polarized rotational mechanical waves can only generate currents in the PSV particle motion plane in homogeneous isotropic poro-elastic media. This couples to the electromagnetic wavefield components of the TM mode. We will call this from now on the $PSVTM$ coupled electrostatic picture. The horizontal polarized rotational mechanical waves can only generate current in the SH particle motion plane in homogeneous isotropic poro-elastic media. This couples to the electromagnetic wavefield components of the TE mode. We will call this from now on the $SHTE$ coupled electrostatic picture.

But when seismic waves hit a contrast in electrical and/or mechanical properties, the current systems within the seismic pulse on both sides become unbalanced and form a complex dynamic current system across the interface generating electromagnetic waves. These EM fields converted from seismic waves traversing boundaries are associated with a change in the following properties: changes in elastic properties (e.g., porosity), changes in fluid flow permeability, and changes in fluid-chemistry that affect the amount of cations free to move in the double layer (e.g., bulk free-ion concentration, pH).

1.3.2. Transform-Domain Field Description

The following geometry description is used: the domain or layer occupied by the m th layer is denoted by D_m , $m = 1, \dots, ND$. Each layer is characterized by its (frequency dependent) elastic, electromagnetic properties, and hydraulic and electrical transport properties. The fundamental input properties, used in the numerical modeling, which determine all wavefield velocities, the electrical conductivity, and the electrokinetic coupling coefficients, are: porosity, dc permeability, bulk modulus of the solid and fluid phase, the frame's bulk and shear modulus, the densities of the solid and fluid phase, the viscosity of the fluid phase, the temperature of the bulk material, the tortuosity, the salinity of the fluid, and the relative permittivities of the solid and fluid phases.

First, the transform-domain field equations, describing the wave behavior in each layer, need to be determined. Next, those field components that are discontinuous across an interface between two media with different poro-elastic and/or electromagnetic properties are eliminated. The resulting differential equations are written in the form of a first-order ordinary matrix differential equation. Since the geometry has azimuthal symmetry, it is natural to use cylindrical coordinates (r, ϕ, z) with z being depth. First, the following definitions involving the horizontal components and horizontal derivatives are employed (e.g., Hudson, 1969; Kennett, 1983),

$$u_V = \frac{1}{r} \left[\frac{\partial}{\partial r} (r u_r) + \frac{\partial}{\partial \phi} u_\phi \right] \quad (20)$$

$$u_H = \frac{1}{r} \left[\frac{\partial}{\partial r} (r u_\phi) - \frac{\partial}{\partial \phi} u_r \right] \quad (21)$$

$$\tau_{Vz} = \frac{1}{r} \left[\frac{\partial}{\partial r} (r \tau_{rz}) + \frac{\partial}{\partial \phi} \tau_{\phi z} \right] \quad (22)$$

$$\tau_{Hz} = \frac{1}{r} \left[\frac{\partial}{\partial r} (r \tau_{\phi z}) - \frac{\partial}{\partial \phi} \tau_{zr} \right] \quad (23)$$

$$\langle E, H \rangle_V = \frac{1}{r} \left[\frac{\partial}{\partial r} (r \langle E, H \rangle_r) + \frac{\partial}{\partial \phi} \langle E, H \rangle_\phi \right] \quad (24)$$

$$\langle E, H \rangle_H = \frac{1}{r} \left[\frac{\partial}{\partial r} (r \langle E, H \rangle_\phi) - \frac{\partial}{\partial \phi} \langle E, H \rangle_r \right] \quad (25)$$

$$\nabla_1^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \quad (26)$$

to rewrite the governing macroscopic, coupled electromagnetic and poro-elastic field equations. It is advantageous to employ the invariance properties of the configuration. The equations are first Fourier transformed to the temporal frequency, where linear time invariance of the system is assumed (note $\frac{\partial}{\partial t} \rightarrow -i\omega$). To take advantage of the shift invariance of the configuration in the

horizontal plane, a Finite Fourier Hankel transform is performed over the horizontal coordinates r and ϕ thus providing a cylindrical wave decomposition.

This transform pair is defined as,

$$HF[\psi(r, \phi)] = \hat{\psi}(k, n) = \frac{1}{2\pi} \int_0^\infty dr r J_n(kr) \int_0^{2\pi} d\phi e^{-in\phi} \psi(r, \phi) \quad (27)$$

$$HF^{-1}[\hat{\psi}(k, n)] = \psi(r, \phi) = \int_0^\infty dk k \sum_{n=-N}^N J_n(kr) e^{in\phi} \hat{\psi}(k, n). \quad (28)$$

Note the property $HF[\nabla_1^2 \psi] = -k^2 \hat{\psi}$, (Watson, 1966). The magnitude N in the summation is determined by the azimuthal symmetry of the point source. For spherically symmetric sources there is no azimuthal dependence, then $N = 0$. If the source can be described using an arbitrarily directed force vector, then $N \leq 1$, while if it can be described using a second order moment tensor, then $N \leq 2$ (Kennett, 1983).

The first-order linear matrix differential equation satisfied by $B_J^{(m)}$ is,

$$\frac{\partial}{\partial z} B_I^{(m)} = A_{I,J}^{(m)} B_J^{(m)} + S_I^{(m)} \quad \text{for } z_{m-1} < z < z_m, \quad m = 2, \dots, ND - 1 \quad (29)$$

$$\frac{\partial}{\partial z} B_I^{(1)} = A_{I,J}^{(1)} B_J^{(1)} \quad \text{for } -\infty < z < z_1 \quad (30)$$

$$\frac{\partial}{\partial z} B_I^{(ND)} = A_{I,J}^{(1)} B_J^{(ND)} \quad \text{for } z_{ND-1} < z < \infty \quad (31)$$

where the field vector B_J , representative of the transform-domain displacement-stress-EM field components in the subdomain D_m , is denoted by $B_J^{(m)}$ and defined as,

$$B_J^{(m;PSVTM)} = [\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_1, \hat{\tau}_{zz}, \hat{T}_p, \hat{H}_2, \hat{E}_1]^T \quad (32)$$

$$B_J^{(m;SHTE)} = [\hat{u}_2, \hat{\tau}_2, \hat{H}_1, \hat{E}_2]^T. \quad (33)$$

These wavefield vector components are just the ones that are continuous across an interface. The field vector components are arrived at by requiring the continuity of the electromagnetic and poro-elastic Poynting vector in the direction perpendicular to the wavefront upon crossing an interface. This also guarantees a consistent macroscopic theory of energy transfer (Haartsen, 1995, chapter 3). The necessary boundary conditions are sufficient for uniqueness of the solution of the macroscopic field equations, as far as this uniqueness is based on energy considerations. Since there are four different up and down going traveling wavetypes in each poro-elastic layer (P_{fast} , P_{slow} , SV and TM) in the $PSVTM$ picture, they impose eight boundary conditions that require continuity of eight wavefield components at both sides of the interface. The $SHTE$ has two up and down traveling wavetypes and, therefore, requires four boundary conditions. The explicit boundary conditions that determine uniquely the displacement-stress-EM field components, (32) and (33), are discussed in Haartsen (1995, chapter 3).

For isotropic media, the first-order ordinary differential equations decompose into two sets that describe the *PSVTM* picture, the compressional P_{fast} and P_{slow} , vertical polarized shear and *TM* polarized *EM* wavefield behavior, and the *SHTe* picture, the horizontal polarized shear and *TE* polarized *EM* wavefield behavior. The first picture describes the changes in z of the field quantities

$$B_J^{(PSVTM)} = [\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_1, \hat{\tau}_{zz}, \hat{T}_p, \hat{H}_2, \hat{E}_1]^T.$$

The second picture describes the changes in z of the field quantities

$$B_J^{(SHTe)} = [\hat{u}_2, \hat{\tau}_2, \hat{H}_1, \hat{E}_2]^T.$$

The horizontal components are defined as follows,

$$\hat{u}_1 = \frac{\hat{u}_V}{ik} \quad \hat{\tau}_1 = \frac{\hat{\tau}_{Vz}}{ik} \quad \hat{H}_2 = \frac{\hat{H}_H}{ik} \quad \hat{E}_1 = \frac{\hat{E}_V}{ik} \quad (34)$$

$$\hat{u}_2 = \frac{\hat{u}_H}{ik} \quad \hat{\tau}_2 = \frac{\hat{\tau}_{Hz}}{ik} \quad \hat{H}_1 = \frac{\hat{H}_V}{ik} \quad \hat{E}_2 = \frac{\hat{E}_H}{ik}. \quad (35)$$

If a line source parallel with the stratification is considered instead, then a plane wave decomposition is employed. Working in the (x, y, z) Cartesian coordinates, assuming the line source to be parallel with the y axis, and taking a Fourier transform of the equations of motion with respect to x results in exactly the same equations as are obtained when the following identification of the horizontal components in the displacement-stress-*EM* vectors are made: $\hat{u}_1 = \hat{u}_x, \hat{u}_2 = \hat{u}_y, \hat{\tau}_1 = \hat{\tau}_{xz}, \hat{\tau}_2 = \hat{\tau}_{yz}, \hat{H}_H = \hat{H}_y, \hat{H}_V = \hat{H}_x, \hat{E}_H = \hat{E}_y, \hat{E}_V = \hat{E}_x$. Thus, using this mapping, it follows that the solution of $\frac{\partial}{\partial z} B_I = A_{IJ} B_J$ applies directly to the line source problem as well. Additionally, the cylindrical wave reflection/transmission coefficients to be obtained are necessarily identical to the plane wave coefficients.

Where $A_{IJ}^{(PSVTM)}$ is a 8 by 8 system matrix, and $S_I^{(PSVTM)}$ is a 8 by 1 notional source matrix, $A_{IJ}^{(SHTe)}$ is a 4 by 4 system matrix, and S_I^{SHTe} a 4 by 1 notional source matrix. The system matrix entries depend on frequency and frequency dependent poro-elastic, electric, and magnetic medium properties and on the horizontal slowness p (note $k = \omega p$). The first-order linear differential matrix equations describing the *PSVTM* case in the transform domain becomes now,

$$\frac{\partial}{\partial z} B_I^{(PSVTM)} = \begin{bmatrix} -i\omega p \underline{a}_{11} & \underline{a}_{12} & \underline{a}_{13} \\ -\omega^2 \underline{a}_{21} & -i\omega p \underline{a}_{11}^T & \underline{a}_{23} \\ -\omega^2 \underline{a}_{31} & -i\omega p \underline{a}_{32} & \underline{a}_{33} \end{bmatrix} B_J^{(PSVTM)} \quad (36)$$

with, $B_J^{(PSVTM)} = [\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_1, \hat{\tau}_{zz}, \hat{T}_p, \hat{H}_2, \hat{E}_1]^T$.

With the submatrices given by,

$$\underline{a}_{11} = \begin{bmatrix} 0 & 1 & 0 \\ \alpha & 0 & 0 \\ -\beta & 0 & 0 \end{bmatrix}, \quad \underline{a}_{12} = \begin{bmatrix} \frac{1}{G} & 0 & 0 \\ 0 & \frac{M}{\Delta} & -\frac{C}{\Delta} \\ 0 & -\frac{C}{\Delta} & \frac{H}{\Delta} - \frac{p^2}{\rho_E} \end{bmatrix}, \quad \underline{a}_{13} = \underline{0} \quad (37)$$

$$\underline{a}_{21} = \begin{bmatrix} B - 2p^2G(1 + \alpha) & 0 & 0 \\ 0 & \rho_B & \rho_f \\ 0 & \rho_f & \rho_E \left(1 + i\omega^3 \frac{\rho_E L^2}{\epsilon}\right) \end{bmatrix}, \quad \underline{a}_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\omega^2 p \frac{\rho_E L}{\epsilon} & 0 \end{bmatrix} \quad (38)$$

$$\underline{a}_{31} = \begin{bmatrix} L\rho_f & 0 & 0 \\ 0 & 0 & \frac{L\rho_E}{\epsilon} \end{bmatrix}, \quad \underline{a}_{32} = \begin{bmatrix} 0 & 0 & L \\ 0 & 0 & 0 \end{bmatrix}, \quad \underline{a}_{33} = \begin{bmatrix} 0 & i\omega\epsilon \\ -\omega\mu \left(i - \frac{p^2}{\mu\epsilon}\right) & 0 \end{bmatrix}. \quad (39)$$

The first order differential matrix equation describing the *SHTE* case in the transform domain becomes now,

$$\frac{\partial}{\partial z} B_I^{(SHTE)} = \begin{bmatrix} 0 & \frac{1}{G} & 0 & 0 \\ -\omega^2 (\beta - p^2 G) & 0 & 0 & 0 \\ \omega^2 L\rho_f & 0 & 0 & -i\omega\epsilon \left(1 - \frac{p^2}{\mu\epsilon}\right) \\ 0 & 0 & -i\omega\mu & 0 \end{bmatrix} B_J^{(SHTE)} \quad (40)$$

with, $B_J = [\hat{u}_2, \hat{\tau}_2, \hat{H}_1, \hat{E}_2]^T$. Where the 6 by 6 submatrix, *PSVTM* picture, and 2 by 2 submatrix, *SHTE* picture,

$$A_{IJ}^{(Biot;PSVTM)} = \begin{bmatrix} -i\omega p \underline{a}_{11} & \underline{a}_{12} \\ -\omega^2 \underline{a}_{21} & -i\omega p \underline{a}_{11}^T \end{bmatrix}, \quad A_{IJ}^{(Biot;SHTE)} = \begin{bmatrix} 0 & \frac{1}{G} \\ -\omega^2 (\beta - p^2 G) & 0 \end{bmatrix} \quad (41)$$

relate the mechanical stress-displacement wavefields with depth.

Where the 2 by 2 submatrix, *PSVTM* picture, and 2 by 2 submatrix, *SHTE* picture,

$$A_{IJ}^{(Maxwell;PSVTM)} = \underline{a}_{33}, \quad A_{IJ}^{(Maxwell;SHTE)} = \begin{bmatrix} 0 & -i\omega \left(1 - \frac{p^2}{\mu\epsilon}\right) \\ -i\omega\mu & 0 \end{bmatrix} \quad (42)$$

relate the electric and magnetic wavefield components with depth.

Where the 6 by 2 submatrix, *PSVTM* picture, and 2 by 2 submatrix, *SHTE* picture,

$$A_{IJ}^{(electrokinetic;PSVTM)} = \begin{bmatrix} -\omega^2 \underline{a}_{31} & -i\omega p \underline{a}_{32} \end{bmatrix}, \quad A_{IJ}^{(electrokinetic;SHTE)} = \begin{bmatrix} \omega^2 L\rho_f & 0 \\ 0 & 0 \end{bmatrix} \quad (43)$$

relate the electric and magnetic wavefield components, converted from the mechanical wavefields, with depth.

And where the 2 by 6 submatrix, *PSVTM* picture, and 2 by 2 submatrix, *SHTE* picture,

$$A_{IJ}^{(osmosis;PSVTM)} = \begin{bmatrix} \underline{a}_{13} \\ \underline{a}_{23} \end{bmatrix}, \quad A_{IJ}^{(osmosis;SHTE)} = \underline{0} \quad (44)$$

relate the mechanical wavefield components, converted from electric and magnetic wavefields, with depth.

The following coefficients have been used in the matrices above.

$$\alpha = 1 - \frac{2GM}{\Delta} \quad \beta = \frac{\rho_f}{\rho_E} - \frac{2GC}{\Delta} \quad (45)$$

$$\Delta = HM - C^2 \quad B = \rho_B - \frac{\rho_f^2}{\rho_E} \quad (46)$$

$$\epsilon = \epsilon_0 \kappa(\omega) + i \frac{\sigma(\omega)}{\omega}. \quad (47)$$

1.3.3. Solution Procedure to Obtain Wavefield Constituents

To obtain the wavefields in each layer, the continuity conditions at each boundary are used.

$$\lim_{z \downarrow z_m} B_J^{(m+1)} - \lim_{z \uparrow z_m} B_J^{(m)} = 0 \quad m \neq s \quad (48)$$

At the source level $z = z_s$, where the localized source is situated, B_J jumps by a finite amount. At the source level,

$$\lim_{z \downarrow z_s} B_J^{(s+1)} - \lim_{z \uparrow z_s} B_J^{(s)} = S_J \quad (49)$$

with S_J the source vector. Furthermore, $B_J^{(1)}$ and $B_J^{(ND)}$ must satisfy the radiation conditions,

$$B_J^{(1)} = 0 \quad z \rightarrow -\infty \quad (50)$$

$$B_J^{(ND)} = 0 \quad z \rightarrow \infty. \quad (51)$$

A linear transformation on each of the ND field matrices $B_J^{(m)}$ is carried out. Through it, a field-vector-formalism is obtained in which a decomposition of $B_J^{(m)}$ into up and downgoing fields is manifest. Let $W_N^{(m;PSVTM)}$ be the 8 by 1 matrix and $W_N^{(m;SHTE)}$ the 4 by 1 matrix that is related to $B_J^{(m)}$ by the linear transformation,

$$B_J^{(m)} = D_{J,N}^{(m)} W_N^{(m)} \quad (52)$$

where the composition matrix $D_{J,N}^{(m)}$ is subject to a convenient choice. From the theory of matrices $D_{J,N}^{(m)}$ is taken the eigencolumn matrix of the system matrix $A_{I,J}^{(m)}$

$$D_{J,N}^{(m;PSVTM)} = \begin{bmatrix} b_J^{(m;-P_f)}, b_J^{(m;-P_s)}, b_J^{(m;-SV)}, b_J^{(m;-TM)}, b_J^{(m;+P_f)}, b_J^{(m;+P_s)}, \\ b_J^{(m;+SV)}, b_J^{(m;+TM)} \end{bmatrix} \quad (53)$$

$$D_{J,N}^{(m;SHTE)} = \begin{bmatrix} b_J^{(m;-SH)}, b_J^{(m;-TE)}, b_J^{(m;+SH)}, b_J^{(m;+TE)} \end{bmatrix} \quad (54)$$

where $b_J^{(m;\pm r)}$, $r \in P_{fast}, P_{slow}, SV, SH, TM, TE$ denote the eigenvectors of the system matrices $A_{I,J}^{(m)}$. The eigenvectors are derived in Haartsen (1995, chapter 3).

For later convenience the elements of $W_N^{(m)}$ are $W_r^{(m;\pm)}$ with the convention

$$W_N^{(m;PSVTM)} = \begin{bmatrix} W_{P_f}^{(m;-)}, W_{P_s}^{(m;-)}, W_{SV}^{(m;-)}, W_{TM}^{(m;-)}, W_{P_f}^{(m;+)}, W_{P_s}^{(m;+)}, \\ W_{SV}^{(m;+)}, W_{TM}^{(m;+)} \end{bmatrix}^T \quad (55)$$

$$W_N^{(m;SHTE)} = \begin{bmatrix} W_{SH}^{(m;-)}, W_{TE}^{(m;-)}, W_{SH}^{(m;+)}, W_{TE}^{(m;+)} \end{bmatrix}^T. \quad (56)$$

Substituting equation (52) into equation (29) and premultiplication by $D_{M,I}^{(m)-1}$ yields the desired matrix differential equation for $W_N^{(m)}$

$$\frac{\partial}{\partial z} W_M^{(m)} = \Lambda_{M,N}^{(m)} W_N^{(m)} \quad (57)$$

where $\Lambda_{M,N}^{(m)}$ is the diagonal matrix of the eigenvalues $q^{(m;\pm r)}$ of system matrix $A_{I,J}^{(m)}$, and the diagonal matrix $\Lambda_{M,N}$ is defined as,

$$\Lambda_{M,N}^{(m;PSVTM)} = \text{diag} \left[q^{(m;-P_f)}, q^{(m;-P_s)}, q^{(m;-SV)}, q^{(m;-TM)}, q^{(m;+P_f)}, q^{(m;+P_s)}, \right. \\ \left. q^{(m;+SV)}, q^{(m;+TM)} \right] \quad (58)$$

$$\Lambda_{M,N}^{(SHTE)} = \text{diag} \left[q^{(m;-SH)}, q^{(m;-TE)}, q^{(m;+SH)}, q^{(m;+TE)} \right]. \quad (59)$$

For each subdomain D_m the system of first-order linear differential equations is now mutually uncoupled. The field matrix $W_N^{(m)}(z)$ of the subdomain D_m is coupled to the field matrix $W_N^{(m+1)}(z)$ of the subdomain D_{m+1} through the boundary conditions, equation (48) at the interface $z = z_m$ between D_m and D_{m+1} .

$$\lim_{z \downarrow z_m} D_{J,N}^{(m+1)} W_N^{(m+1)} - \lim_{z \uparrow z_m} D_{J,N}^{(m)} W_N^{(m)} = 0 \quad m \neq s \quad (60)$$

while at the source level $z = z_s$,

$$\lim_{z \downarrow z_s} D_{J,N}^{(s+1)} W_N^{(s+1)} - \lim_{z \uparrow z_s} D_{J,N}^{(s)} W_N^{(s)} = S_J. \quad (61)$$

Since there are $ND - 1$ interfaces, equations (60) and (61) represent $8 \times (ND - 1)$, the *PSVTM* picture, or $4 \times (ND - 1)$, the *SHTE* picture, linear algebraic equations of which the eight or four that follow from equation (61) are inhomogeneous. Next, for each subdomain D_m , the field matrix $W_N^{(m)}(z)$ is written in terms of the linear independent functions $\Gamma^{(m;\pm r)}(z)$ that express the decomposition into up and downgoing fields in D_m .

$$\Gamma_{M,K}^{(m;PSVTM)} = \text{diag} \left[\Gamma^{(m;-P_f)}, \Gamma^{(m;-P_s)}, \Gamma^{(m;-SV)}, \Gamma^{(m;-TM)}, \Gamma^{(m;+P_f)}, \Gamma^{(m;+P_s)}, \right. \\ \left. \Gamma^{(m;+SV)}, \Gamma^{(m;+TM)} \right] \quad (62)$$

$$\Gamma_{M,K}^{(m;SHTE)} = \text{diag} \left[\Gamma^{(m;-SH)}, \Gamma^{(m;-TE)}, \Gamma^{(m;+SH)}, \Gamma^{(m;+TE)} \right] \quad (63)$$

where $\Gamma_{M,K}^{(m)}(z)$ are solutions of differential equations (29).

$$\frac{\partial}{\partial z} \Gamma_{M,K}^{(m)} = \Lambda_{M,N}^{(m)} \Gamma_{N,K}^{(m)}. \quad (64)$$

The diagonal elements $\Gamma^{(m;\pm r)}(z)$ of $\Gamma_{M,K}^{(m)}(z)$ are given by,

$$\Gamma^{(m;-r)}(z) = \exp[i\omega q^{(m;-r)}(z - z_m)] \quad \text{for } z_{m-1} < z < z_m \quad (65)$$

$$\Gamma^{(m;+r)}(z) = \exp[-i\omega q^{(m;+r)}(z - z_{m-1})] \quad \text{for } z_{m-1} < z < z_m. \quad (66)$$

The source matrix S_I contains a concentrated source and has the form,

$$S_I = \hat{\phi}(\omega) X_I \delta(z - z_s) \quad (67)$$

where $\hat{\phi}(\omega)$ is the Fourier transform of the source signature $\phi(t)$ and X_I is the 8 by 1 or 4 by 1 vector that depends on the nature of the source. Combining equations (57), (60)–(62) and (67) the following expression for $W_N^{(m)}(z)$ is arrived at,

$$W_N^{(m)}(z) = \hat{\phi}(\omega) \Gamma_{N,M}^{(m)}(z) \tilde{W}_M^{(m)}. \quad (68)$$

The coefficient $\tilde{W}_M^{(m)}$ expresses the action of the source as well as the influence of the poro-elastic and electromagnetic properties of each layer. They follow from the boundary conditions at each of the ND-1 interfaces, equations (60) and (61). The transform-domain electromagnetic field matrix $B_J^{(m)}$ of the subdomain D_m is related to $\tilde{W}_M^{(m)}$ via the linear transformation,

$$B_J^{(m)}(k, \omega, z) = \hat{\phi}(\omega) D_{J,N} \Gamma_{N,M}^{(m)}(z) \tilde{W}_M^{(m)}. \quad (69)$$

The righthand side of equation (69) can be recognized as the superposition of eighth (the *PSVTM* picture), four (the *SHTE* picture) terms, each corresponding to a transform-domain field component. In the *PSVTM* picture, four are upward and four are downward traveling field constituents in subdomain D_m , while in the *SHTE* picture two upward and two downward field constituents are propagating in the same subdomain. In accordance with equations (55) and (56) $\tilde{W}_N^{(m)}$ is written as,

$$\tilde{W}_N^{(m;PSVTM)} = \left[\tilde{W}_{P_f}^{(m;-)}, \tilde{W}_{P_s}^{(m;-)}, \tilde{W}_{SV}^{(m;-)}, \tilde{W}_{TM}^{(m;-)}, \tilde{W}_{P_f}^{(m;+)}, \tilde{W}_{P_s}^{(m;+)}, \right. \\ \left. \tilde{W}_{SV}^{(m;+)}, \tilde{W}_{TM}^{(m;+)} \right]^T \quad (70)$$

$$\tilde{W}_N^{(m;SHTE)} = \left[\tilde{W}_{SH}^{(m;-)}, \tilde{W}_{TE}^{(m;-)}, \tilde{W}_{SH}^{(m;+)}, \tilde{W}_{TE}^{(m;+)} \right]^T. \quad (71)$$

The imposed conditions, equations (50) and (51) on $B_J(z)$ as $|z| \rightarrow \infty$ result in,

$$\tilde{W}_r^{(1;-)} \equiv 0 \quad \tilde{W}_r^{(ND;+)} \equiv 0 \quad (72)$$

This leaves a total of $8 \times (ND-1)$ unknown coefficients $\tilde{W}_r^{(m;\pm)}$ (the *PSVTM* picture) or $4 \times (ND-1)$ unknown coefficients in the *SHTE* picture, being just equal to the number of linear algebraic equations (60) and (61).

1.4. THE GLOBAL MATRIX METHOD

The global matrix method is used to solve simultaneously the unknown $\tilde{W}_r^{(m;\pm)}$ coefficients in all layers. The formulation was developed by Chin *et al.* (1984). The global matrix formulation has several important features: (1) multiple sources can be treated, the produced wavefields are simply superposed; (2) the up and down going wavefield coefficients are simultaneously solved for in all layers, therefore the wavefield is known to all receivers at all places in the medium for each solution pass; and (3) time stability problems do not occur because only decaying exponents are needed. In contrast, the Thomson-Haskell propagator matrix approach (Schmidt and Tango, 1986) does not have these advantageous features.

1.4.1. Numerical Solutions of the PSVTM and SHTE Problems

In the *PSVTM* picture the eigencolumn matrix, for each subdomain m , of the system matrix $A_{I,J}^{(m;PSVTM)}$ is defined as

$$D_{J,N}^{(m;PSVTM)} = [\underline{d}^{(m;-P_f,-P_s)}, \underline{d}^{(m;-SV,-TM)}, \underline{d}^{(m;+P_f,+P_s)}, \underline{d}^{(m;+SV,+TM)}] \quad (73)$$

$$= \begin{bmatrix} M_{P,Q}^{(m,PSVTM;-)} & M_{P,Q}^{(m,PSVTM;+)} \\ N_{P,Q}^{(m,PSVTM;-)} & N_{P,Q}^{(m,PSVTM;+)} \end{bmatrix} \quad (74)$$

with $M_{P,Q}^{(m,PSVTM;\pm)}$ and $N_{P,Q}^{(m,PSVTM;\pm)}$ 4×4 matrices.

In the *SHTE* picture the eigencolumn matrix for each subdomain m , of the system matrix $A_{I,J}^{(m;SHTE)}$ is defined as

$$D_{J,N}^{(m;SHTE)} = [\underline{d}^{(m;-SH,-TE)}, \underline{d}^{(m;+SH,+TE)}] = \begin{bmatrix} M_{P,Q}^{(m,SHTE;-)} & M_{P,Q}^{(m,SHTE;+)} \\ N_{P,Q}^{(m,SHTE;-)} & N_{P,Q}^{(m,SHTE;+)} \end{bmatrix} \quad (75)$$

with $M_{P,Q}^{(m,SHTE;\pm)}$ and $N_{P,Q}^{(m,SHTE;\pm)}$ 2×2 matrices.

The submatrices $\underline{d}$ contain the eigenvectors $b_j^{(m;\pm r)}$, displacement/powerflow normalized, with $\pm r \in P_f, P_s, SV, TM$ the up and downgoing wavefield types. The propagator matrix is defined as, see equations (65) and (66),

$$\Gamma_{M,N}^{(m)} = \exp[\Lambda_{M,N}^{(m)}(z_m - z_{m-1})] = \begin{bmatrix} E_{P,Q}^{(m)} & 0 \\ 0 & E_{P,Q}^{(m)-1} \end{bmatrix} \quad (76)$$

with the $E_{P,Q}^{(m)}$ matrices defined to be,

$$E_{P,Q}^{(m;PSVTM)} = \text{diag} \left(\exp[-i\omega q^{(m;P_f)} \Delta z], \exp[-i\omega q^{(m;P_s)} \Delta z], \exp[-i\omega q^{(m;SV)} \Delta z], \exp[-i\omega q^{(m;TM)} \Delta z] \right) \quad (77)$$

$$E_{P,Q}^{(m;SHTE)} = \text{diag} \left(\exp[-i\omega q^{(m;SH)} \Delta z], \exp[-i\omega q^{(m;TE)} \Delta z] \right). \quad (78)$$

With $q^{(m;P_f,P_s)} = \sqrt{\frac{1}{v_{P_f,P_s}^2} - p^2}$, $q^{(m;SV,SH)} = \sqrt{\frac{1}{v_{SV,SH}^2} - p^2}$, $q^{(m;TM,TE)} = \sqrt{\frac{1}{v_{TM,TE}^2} - p^2}$ the vertical slownesses for the compressional fast wave mode, the compressional slow wave mode, the vertical polarized shear wave mode, the horizontally polarized shear wave mode, the transverse magnetic EM mode, and the transverse electric EM mode. The choice of branch cuts is defined according to the computational methods used to perform the inverse Fourier and Hankel transforms:

$$Im(\omega q^{(m;P_f,P_s)}) > 0, Im(\omega q^{(m;SV,SH)}) > 0, Im(\omega q^{(m;TM,TE)}) > 0.$$

For $m \neq s$, see equation (60) and applying the linear transformation (69), the interface continuity condition can be written as,

$$\begin{bmatrix} M_{P,Q}^{(m;-)} & M_{P,Q}^{(m;+)} \\ N_{P,Q}^{(m;-)} & N_{P,Q}^{(m;+)} \end{bmatrix} \begin{bmatrix} E_{P,Q}^{(m)} & 0 \\ 0 & E_{P,Q}^{(m)-1} \end{bmatrix} \begin{bmatrix} \underline{p}^{(m;-)} \\ \underline{p}^{(m;+)} \end{bmatrix} - \begin{bmatrix} M_{P,Q}^{(m+1;-)} & M_{P,Q}^{(m+1;+)} \\ N_{P,Q}^{(m+1;-)} & N_{P,Q}^{(m+1;+)} \end{bmatrix} \begin{bmatrix} \underline{p}^{(m+1;-)} \\ \underline{p}^{(m+1;+)} \end{bmatrix} = \underline{0} \quad (79)$$

$$\tilde{W}_M^{(m;PSVTM)} = \begin{bmatrix} \underline{p}^{(m;PSVTM,-)} \\ \underline{p}^{(m;PSVTM,+)} \end{bmatrix}, \quad \tilde{W}_M^{(m;SHTE)} = \begin{bmatrix} \underline{p}^{(m;SHTE,-)} \\ \underline{p}^{(m;SHTE,+)} \end{bmatrix}. \quad (80)$$

1.4.2. Radiation Conditions

The radiation condition for halfspaces requires the conditions of no incoming waves from $|z| \rightarrow \infty$, implying $\underline{p}^{(1;-)} = 0$ and $\underline{p}^{(ND;+)} = 0$. The interface conditions at $z = z_1$ become

$$\begin{aligned} M_{P,Q}^{(1;+)} E_{P,Q}^{(1)-1} \underline{p}^{(1;+)} - M_{P,Q}^{(2;-)} \underline{p}^{(2;-)} - M_{P,Q}^{(2;+)} \underline{p}^{(2;+)} &= \underline{0} \\ N_{P,Q}^{(1;+)} E_{P,Q}^{(1)-1} \underline{p}^{(1;+)} - N_{P,Q}^{(2;-)} \underline{p}^{(2;-)} - N_{P,Q}^{(2;+)} \underline{p}^{(2;+)} &= \underline{0}. \end{aligned}$$

At $z = z_N$ the interface conditions are,

$$\begin{aligned} M_{P,Q}^{(ND-1;-)} E_{P,Q}^{(ND-1)} \underline{p}^{(ND-1;-)} + M_{P,Q}^{(ND-1;+)} E_{P,Q}^{(ND-1)-1} \underline{p}^{(ND-1;+)} - M_{P,Q}^{(ND;-)} \underline{p}^{(ND;-)} &= \underline{0} \\ N_{P,Q}^{(ND-1;-)} E_{P,Q}^{(ND-1)} \underline{p}^{(ND-1;-)} + N_{P,Q}^{(ND-1;+)} E_{P,Q}^{(ND-1)-1} \underline{p}^{(ND-1;+)} - N_{P,Q}^{(ND;-)} \underline{p}^{(ND;-)} &= \underline{0}. \end{aligned}$$

1.4.3. Free Surface Conditions

In this analysis the following matrices are used. The 3×3 matrix, $S_{33}^{(1;\mp)}$ which contains the mechanical eigenvector components that relate the mechanical wavefield amplitudes to the normal and shear bulk stresses, and to stresses in the fluid phase. The 2×3 matrix, $U_{23}^{(1;\mp)}$ which contains the mechanical eigenvector components that relate the mechanical wavefield amplitudes to the tangential magnetical and electrical wavefields. The 2×2 matrix, $V_{22}^{(1)}$ which contains the electromagnetic eigenvector components that relate the EM wavefield amplitudes to the tangential magnetical and electrical wavefields in the porous medium. The 2×1 matrix, $W_{21}^{(0;-)}$ which contains the electromagnetic eigenvector components that relate the EM wavefield amplitudes to the tangential magnetical and electrical wavefields in the upper halfspace (air). Where the superscript 1 denotes the first layer, 0 the halfspace above the free surface, and $\mp$ denotes eigenvector components relating to the

down or up going wavefields.

$$\begin{aligned}
S_{33}^{(1;\mp)} &= \begin{bmatrix} \pm 2i\omega p q v_{p_f} G & \pm 2i\omega p q v_{p_s} G & i\omega v_{sv} [q^2 - p^2] G \\ \frac{i\omega}{v_{p_f}} \left[H - 2G p^2 v_{p_f}^2 - \frac{\rho_B - \frac{H}{v_{p_f}^2}}{\rho_f - \frac{C}{v_{p_f}^2}} \right] & \frac{i\omega}{v_{p_s}} \left[H - 2G p^2 v_{p_s}^2 - \frac{\rho_B - \frac{H}{v_{p_s}^2}}{\rho_f - \frac{C}{v_{p_s}^2}} \right] & i\omega v_{sv} [q^2 - p^2] G \\ \frac{i\omega}{v_{p_f}} \left[C - \frac{\rho_B - \frac{H}{v_{p_f}^2}}{\rho_f - \frac{C}{v_{p_f}^2}} M \right] & \frac{i\omega}{v_{p_s}} \left[C - \frac{\rho_B - \frac{H}{v_{p_s}^2}}{\rho_f - \frac{C}{v_{p_s}^2}} M \right] & 0 \end{bmatrix} \\
U_{23}^{(1;\mp)} &= \begin{bmatrix} 0 & 0 & -\frac{i\omega L G \rho_E}{v_{sv} \rho_f} \frac{\left[\frac{1}{v_{sv}^2} - \frac{\rho_B}{G} \right]}{\left[\frac{1}{v_{sv}^2} - \epsilon \mu \right]} \\ -\frac{i\omega \rho_E L p v_{p_f}}{\epsilon} \frac{\left[\rho_B - \frac{H}{v_{p_f}^2} \right]}{\left[\rho_f - \frac{C}{v_{p_f}^2} \right]} & -\frac{i\omega \rho_E L p v_{p_s}}{\epsilon} \frac{\left[\rho_B - \frac{H}{v_{p_s}^2} \right]}{\left[\rho_f - \frac{C}{v_{p_s}^2} \right]} & \mp i\omega q v_{sv} L G \mu \frac{\rho_E}{\rho_f} \frac{\left[\frac{1}{v_{sv}^2} - \frac{\rho_B}{G} \right]}{\left[\frac{1}{v_{sv}^2} - \epsilon \mu \right]} \end{bmatrix} \\
V_{22}^{(1)} &= \begin{bmatrix} -\frac{i\omega L G \rho_E}{v_{tm} \rho_f} \frac{\left[\frac{1}{v_{tm}^2} - \frac{\rho_B}{G} \right]}{\left[\frac{1}{v_{tm}^2} - \epsilon \mu \right]} & -\frac{i\omega L G \rho_E}{v_{tm} \rho_f} \frac{\left[\frac{1}{v_{tm}^2} - \frac{\rho_B}{G} \right]}{\left[\frac{1}{v_{tm}^2} - \epsilon \mu \right]} \\ -i\omega q v_{tm} L G \mu \frac{\rho_E}{\rho_f} \frac{\left[\frac{1}{v_{tm}^2} - \frac{\rho_B}{G} \right]}{\left[\frac{1}{v_{tm}^2} - \epsilon \mu \right]} & i\omega q v_{tm} L G \mu \frac{\rho_E}{\rho_f} \frac{\left[\frac{1}{v_{tm}^2} - \frac{\rho_B}{G} \right]}{\left[\frac{1}{v_{tm}^2} - \epsilon \mu \right]} \end{bmatrix} \\
W_{21}^{(0;-)} &= \begin{bmatrix} -q v_{tm} \\ \sqrt{\frac{\epsilon}{\mu}} \end{bmatrix} \quad (81)
\end{aligned}$$

The up and downgoing mechanical wavefield amplitudes in the first layer are the fast wave amplitudes, $p_1^{(1,\pm)}$, the shear wave amplitudes, $p_2^{(1,\pm)}$, and the slow wave amplitudes, $p_3^{(1,\pm)}$, which can be related through the vanishing stress and fluid traction boundary conditions at the free surface.

$$p_{1-3}^{(1;-)} = - \left[S_{33}^{(1;-)} \right]^{-1} S_{33}^{(1;+)} p_{1-3}^{(1;+)}$$

The electromagnetic boundary conditions are the usual continuity of the magnetic, H_y , and electric, E_x , components of the electromagnetic TM mode. The radiation in the space above the free surface requires the condition of no incoming EM waves from $z \rightarrow -\infty$, implying $p_4^{(0;-)} = 0$, the TM wave downgoing amplitude is zero. The EM boundary conditions at the free surface are expressed as follows,

$$U_{23}^{(1;-)} p_{1-3}^{(1;-)} + U_{23}^{(1;+)} p_{1-3}^{(1;+)} + V_{22}^{(1)} \begin{bmatrix} p_4^{(1;-)} \\ p_4^{(1;+)} \end{bmatrix} = W_{21}^{(0;+)} \quad (82)$$

Using equation (82), the following relation between the up and downgoing *EM* wave amplitudes propagating in the first layer can be obtained:

$$\begin{bmatrix} p_4^{(1;-)} \\ p_4^{(1;+)} \end{bmatrix} = [V_{22}^{(1)}]^{-1} \left(W_{21}^{(0;-)} p_4^{(0;+)} - \left[-U_{23}^{(1;-)} [S_{33}^{(1;-)}]^{-1} S_{33}^{(1;+)} p_{1-3}^{(1;+)} + U_{23}^{(1;+)} p_{1-3}^{(1;+)} \right] \right). \quad (83)$$

Eliminating the $p_4^{(0;+)}$ amplitude, the downgoing *TM* amplitude in the first layer can be expressed in terms of the upgoing *TM* and mechanical wavefield amplitudes.

$$\begin{aligned} p_4^{(1;-)} &= \frac{1}{\det(V_{22}^{(1)})} \left[\Theta \left(\det(V_{22}^{(1)}) p_4^{(1;+)} + \Xi(2,1) p_1^{(1;+)} + \Xi(2,2) p_1^{(1;+)} + \Xi(2,3) p_3^{(1;+)} \right) \right. \\ &\quad \left. - \Xi(1,1) p_1^{(1;+)} - \Xi(1,2) p_2^{(1;+)} - \Xi(1,3) p_3^{(1;+)} \right] \end{aligned} \quad (84)$$

with,

$$\Theta = \frac{V_{22}^{(1)}(2,2)W_{21}^{(0;-)}(1) - V_{22}^{(1)}(1,2)W_{21}^{(0;-)}(2)}{-V_{22}^{(1)}(2,1)W_{21}^{(0;-)}(1) + V_{22}^{(1)}(1,1)W_{21}^{(0;-)}(2)} \quad (85)$$

$$\Xi_{23} = -U_{23}^{(1;-)} [S_{33}^{(1;-)}]^{-1} S_{33}^{(1;+)} p_{1-3}^{(1;+)} + U_{23}^{(1;+)} p_{1-3}^{(1;+)} \quad (86)$$

Equations (82) and (84) relate the upgoing wave amplitudes to the downgoing wave amplitudes satisfying the free surface conditions. The downgoing wavefields reflected from the free surface are phase delayed to the next interface and included in the global boundary condition matrix. The obtained global matrix is identical in shape to the matrix obtained for a radiating upper halfspace. Inverting the global matrix yields only the upgoing wave amplitudes in the first layer. The downgoing wavefields are calculated using equations (82) and (84).

To include a source, the source jump conditions are transferred to the bottom interface of the source layer. Packing all the the interface condition equations together in a matrix, in the *PSVTM* picture a $(ND-1)*8 \times (ND-1)*8$ dimensional and in the *SHTe* picture a $(ND-1)*4 \times (ND-1)*4$ dimensional "global" matrix is constructed (if the medium has two radiating halfspaces). Inverting this matrix equation obtains the amplitudes for the down and up going *PSVTM* or *SHTe* wave amplitudes.

The system that has to be solved for is,

$$\begin{bmatrix} \underline{\underline{M}}^{(1;+)} \underline{\underline{E}}^{(1)-1} & -\underline{\underline{M}}^{(2;-)} & -\underline{\underline{M}}^{(2;+)} \\ \underline{\underline{N}}^{(1;+)} \underline{\underline{E}}^{(1)-1} & -\underline{\underline{N}}^{(2;-)} & -\underline{\underline{N}}^{(2;+)} \\ & \underline{\underline{M}}^{(s;-)} \underline{\underline{E}}^{(s)} & \underline{\underline{M}}^{(s;+)} \underline{\underline{E}}^{(s)-1} & -\underline{\underline{M}}^{(s+1;-)} & -\underline{\underline{M}}^{(s+1;+)} \\ & \underline{\underline{N}}^{(s;-)} \underline{\underline{E}}^{(s)} & \underline{\underline{N}}^{(s;+)} \underline{\underline{E}}^{(s)-1} & -\underline{\underline{N}}^{(s+1;-)} & -\underline{\underline{N}}^{(s+1;+)} \\ & & \underline{\underline{M}}^{(nd-1;-)} \underline{\underline{E}}^{(nd-1)} & \underline{\underline{M}}^{(nd-1;+)} \underline{\underline{E}}^{(nd-1)-1} & -\underline{\underline{M}}^{(nd;-)} \\ & & \underline{\underline{N}}^{(nd-1;-)} \underline{\underline{E}}^{(nd-1)} & \underline{\underline{N}}^{(nd-1;+)} \underline{\underline{E}}^{(nd-1)-1} & -\underline{\underline{N}}^{(nd;-)} \end{bmatrix} \underline{\underline{W}} = \begin{bmatrix} \underline{\underline{0}} \\ \underline{\underline{0}} \\ \underline{\underline{S}}^- \\ \underline{\underline{S}}^+ \\ \underline{\underline{0}} \\ \underline{\underline{0}} \end{bmatrix}$$

Chin *et al.* (1984) show that the Thomson-Haskell method (Thomson, 1950; Haskell, 1953) can be viewed as treating the coefficient matrix of the linear matrix of the linear system in a block

diagonal form and applying a forward marching algorithm for its solution. This type of formulation is a shooting method for a two point boundary value problem. Gaussian elimination with partial pivoting yields stable solutions. To avoid time stability problems (exploding exponentials) the $\tilde{W}_M$ vector is multiplied with a diagonal phase matrix F_{IJ} that cancels the potentially unstable upgoing wave phase factors.

$$F_{IJ} = \text{diag} [E_{P,Q}^{(1)}, I_{P,Q}, E_{P,Q}^{(2)}, \dots, E_{P,Q}^{(ND)}, I_{P,Q}] \quad (87)$$

where $I_{P,Q}$ is the identity matrix. If we define $\tilde{W}_M = F_{MN} X_N$, X_N can be shown to satisfy,

$$\begin{bmatrix} \underline{\underline{M}}^{(1;+)} & -\underline{\underline{M}}^{(2;-)} & -\underline{\underline{M}}^{(2;+)} \underline{\underline{E}}^{(2)} \\ \underline{\underline{N}}^{(1;+)} & -\underline{\underline{N}}^{(2;-)} & -\underline{\underline{N}}^{(2;+)} \underline{\underline{E}} \\ & \underline{\underline{M}}^{(s;-)} \underline{\underline{E}}^{(s)} & \underline{\underline{M}}^{(s;+)} & -\underline{\underline{M}}^{(s+1;-)} & -\underline{\underline{M}}^{(s+1;+)} \underline{\underline{E}}^{(s+1)} \\ & \underline{\underline{N}}^{(s;-)} \underline{\underline{E}}^{(s)} & \underline{\underline{N}}^{(s;+)} & -\underline{\underline{N}}^{(s+1;-)} & -\underline{\underline{N}}^{(s+1;+)} \underline{\underline{E}}^{(s+1)} \\ & & \underline{\underline{M}}^{(nd-1;-)} \underline{\underline{E}}^{(nd-1)} & \underline{\underline{M}}^{(nd;+)} & -\underline{\underline{M}}^{(nd;-)} \\ & & \underline{\underline{N}}^{(nd-1;-)} \underline{\underline{E}}^{(nd-1)} & \underline{\underline{N}}^{(nd-1;+)} & -\underline{\underline{N}}^{(nd;-)} \end{bmatrix} X = \begin{bmatrix} \underline{\underline{0}} \\ \underline{\underline{0}} \\ \underline{\underline{S}}^- \\ \underline{\underline{S}}^+ \\ \underline{\underline{0}} \\ \underline{\underline{0}} \end{bmatrix}.$$

All terms that contained $\exp[-i\omega q^{(m;P_f,P_s)}\Delta z]$, $\exp[-i\omega q^{(m;SV,SH)}\Delta z]$, $\exp[-i\omega q^{(m;TM,TE)}\Delta z]$ are eliminated. These exponentials cause numerical instability when the wavefield goes evanescent (Dunkin, 1965; Kennett, 1983; Schmidt and Tango, 1986).

1.5. POINT FORCES IN A PORO-ELASTIC STRATIFIED MEDIUM

To model a general source, the moment tensor in the poro-elastic medium needs to be determined. The following equation has to be evaluated when a cylindrical coordinate system is used,

$$\underline{\underline{h}} = \underline{\underline{M}} \cdot \nabla \delta(\underline{\underline{r}} - \underline{\underline{r}}_s) = \begin{bmatrix} m_{rr} & m_{r\phi} & m_{rz} \\ m_{\phi r} & m_{\phi\phi} & m_{\phi z} \\ m_{zr} & m_{z\phi} & m_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} \end{bmatrix} \frac{\delta(r)}{r} \delta(\phi) \delta(z - z_s). \quad (88)$$

To simplify matters the explosion source description will be outlined. This type of source is given by,

$$\underline{\underline{h}} = m \nabla \delta(\underline{\underline{r}} - \underline{\underline{r}}_s) = m \nabla \left[\frac{\delta(r)}{r} \delta(\phi) \delta(z - z_s) \right]. \quad (89)$$

In component form the explosion source is given by,

$$h_r = m \frac{\partial}{\partial r} \left[\frac{\delta(r)}{r} \right] \delta(\phi) \delta(z - z_s) \quad (90)$$

$$h_\phi = m \frac{\delta(r)}{r^2} \frac{\partial}{\partial \phi} [\delta(\phi)] \delta(z - z_s) \quad (91)$$

$$h_z = m \frac{\delta(r)}{r} \delta(\phi) \frac{\partial}{\partial z} [\delta(z - z_s)]. \quad (92)$$

The horizontal component of the explosion source using definition (20) reads,

$$h_V = \frac{1}{r} \left[\frac{\partial}{\partial r} (r h_r) + \frac{\partial}{\partial \phi} h_\phi \right] = m \delta(z - z_s) \nabla_1^2 \left[\frac{\delta(r) \delta(\phi)}{r} \right] \quad (93)$$

The horizontal and vertical component of the explosion source after Hankel transform and mapping on a plane wave description reads,

$$\hat{h}_1 = \frac{\hat{h}_V}{i\omega p} = ikm\delta(z - z_s) \quad (94)$$

$$\hat{h}_z = m \frac{\partial}{\partial z} \delta(z - z_s). \quad (95)$$

The effect of a general point source in a stratified porous medium is accommodated by specifying a jump in the displacement stress vector B_J , across a horizontal plane containing the source (Hudson, 1969; Kennett, 1983).

To obtain this jump vector, the governing poro-elastic constitutive equations, (8) and (9), and the Fourier transformed equations of motion with an explosive point source acting on the solid frame and pore fluid in a poro-elastic medium,

$$-\omega^2 [\rho_B \underline{u} + \rho_f \underline{w}] = \nabla \cdot \underline{\tau} + m \nabla \delta(\underline{r} - \underline{r}_s) \quad (96)$$

$$-\omega^2 [\rho_f \underline{u} + \rho_E \underline{w}] = -\nabla P + m \nabla \delta(\underline{r} - \underline{r}_s) \quad (97)$$

are Hankel transformed and manipulated into a set of equations that have all derivatives with respect to z on the left hand side, and a right hand side identical to the mechanical submatrix of the electroseismic system matrix (41). The first set describes the change in z of the field quantities $[\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_1, \hat{\tau}_{zz}, \hat{T}_p]$, where the mapping given in equations (34), (35) is applied to describe the $P_f - P_s - SV$ case,

$$\frac{\partial}{\partial z} \begin{bmatrix} \hat{u}_1 \\ \hat{u}_z \\ \hat{w}_z \\ \hat{\tau}_1 \\ \hat{\tau}_{zz} \\ \hat{T}_p \end{bmatrix} = A_{IJ}^{(Biot;PSVTM)} \begin{bmatrix} \hat{u}_1 \\ \hat{u}_z \\ \hat{w}_z \\ \hat{\tau}_1 \\ \hat{\tau}_{zz} \\ \hat{T}_p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ m \\ m \end{bmatrix} \frac{\partial}{\partial z} \delta(z - z_s) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ i\omega p m \left(1 - \frac{\rho_f}{\rho_E}\right) \\ 0 \\ 0 \end{bmatrix} \delta(z - z_s) \quad (98)$$

where $A_{IJ}^{(Biot;PSVTM)}$ is defined in equation (41). The explosion source belongs to a class of sources that are confined to a plane and arise from the transformation of some point source leading to a dipolar contribution (Kennett, 1983). This description is given by,

$$X_I(z_s) = X_I^{(1)} \delta(z - z_s) + X_I^{(2)} \frac{\partial}{\partial z} \delta(z - z_s). \quad (99)$$

Across the source plane itself, the stress displacement vector suffers a discontinuity in the source level with a jump,

$$S_I(z_s) = \hat{\phi}(\omega) [B_I(z_s^+) - B_I(z_s^-)] = \hat{\phi}(\omega) [X_I^{(1)} + A_{I,J}^{(Biot;PSVTM)}(p, z_s) X_J^{(2)}]. \quad (100)$$

The dipolar contributions $X_I^{(1)}$ and $X_J^{(2)}$ are taken from equation (98) and $\hat{\phi}(\omega)$ is the Fourier transform of the source signature $\phi(t)$. The final jump representation of a mechanical explosive point source in a poro-elastic isotropic medium is,

$$S_I^{(P_f P_s SV)}(z_s) = \hat{\phi}(\omega) \left(\left[0, 0, 0, i\omega p m \left(1 - \frac{\rho_f}{\rho_E} \right), 0, 0 \right]^T + \left[0, \frac{m}{\Delta}(M - C), \frac{m}{\Delta} \left(H - C - \frac{\Delta p^2}{\rho_E} \right), i\omega p m(\beta - \alpha), 0, 0 \right]^T \right). \quad (101)$$

1.6. TRANSFORMATION BACK TO THE SPACE TIME DOMAIN

In this section, the back transformation to the space-time domain of $B_J(k, \omega, z)$, equation (69) is performed. An inverse Fourier transform is applied to go back to the time domain. An inverse Hankel transform is applied to obtain the 3D spatial dependence of the displacements, stresses, electric, and magnetic fields. The horizontal components of the displacements, stresses, electric, and magnetic fields require additional integration over r and ϕ to obtain, $u_r, u_\phi, \tau_{\phi z}, \tau_{rz}, H_r, H_\phi$ and E_r, E_ϕ due to the definitions of $u_V, u_H, \tau_{Vz}, \tau_{Hz}, H_V, H_H$ and E_V, E_H .

$$u_z(\omega, r, \phi, z) = \int_0^\infty k dk \sum_{n=-N}^N [J_n(kr) \hat{u}_z(\omega, k, n, z)] e^{in\phi}. \quad (102)$$

Identical relations exist for w_z, τ_{zz} and $T_p = -P$. The horizontal components may be recovered using the following relations:

$$u_r(\omega, r, \phi, z) = \int_0^\infty k dk \sum_{n=-N}^N \left[\frac{n}{kr} J_n(kr) \hat{u}_2(\omega, k, n, z) - i J'_n(kr) \hat{u}_1(\omega, k, n, z) \right] e^{in\phi} \quad (103)$$

$$u_\phi(\omega, r, \phi, z) = \int_0^\infty k dk \sum_{n=-N}^N \left[\frac{n}{kr} J_n(kr) \hat{u}_1(\omega, k, n, z) + i J'_n(kr) \hat{u}_2(\omega, k, n, z) \right] e^{in\phi}. \quad (104)$$

Identical relations exist for $\tau_{rz}, \tau_{\phi z}$ in terms of $\hat{\tau}_1$ and $\hat{\tau}_2$, for E_r, E_ϕ in terms of $\hat{E}_1$ and $\hat{E}_2$ and H_r, H_ϕ in terms of $\hat{H}_1$ and $\hat{H}_2$. The above representations may be regarded as a superposition of cylindrical waves whose order dictates the nature of their azimuthal modulation. At each frequency and angular order the radial contribution is obtained by superposing all horizontal wavenumbers k . This corresponds to including all propagating waves at the level z within the stratification, from vertically to purely horizontal traveling waves, including the evanescent waves. At any particular distance r the relative contributions of the wavenumbers are imposed by the radial phase functions $J_n(kr)$.

The integrals, (102), (103), (104) are evaluated by the discrete summation over many wavenumbers or the so-called Discrete Wavenumber Method (Bouchon and Aki, 1977).

The discretization of the radial wavenumber k in cylindrical coordinates introduces periodicity into the source distribution. The original single source problem changes after discretization into periodic concentric sources around the original source. The periodicity of these sources or the distance between two adjacent circular sources, L , is related to the discretization interval of the wavenumber, Δk , by the sampling relation,

$$L = \frac{2\pi}{\Delta k} \quad (105)$$

L and, therefore, Δk is determined by assuming a receiver located at $x_r = (r_0, z_0)$ and a source at $x_s = (0, z_s)$ on the symmetry axes of the medium configuration. Given the time window to record radiated waves from 0 to t_{max} , "pseudo" waves radiated from the periodic sources are not allowed to enter this time window. This requirement is,

$$\sqrt{(L - r_0)^2 + (z_0 - z_s)^2} > v_{fastest} t_{max} \quad (106)$$

or

$$L > r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2}. \quad (107)$$

The sampling equation becomes now,

$$\Delta k < \frac{2\pi}{r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2}}. \quad (108)$$

The above equation is the criterion for choosing the sample rate for the discrete wavenumber summation.

To perform the summation, the singularities of the integrands must be removed from the real k axes. This is done by adding a small imaginary part to the real frequency (analytic continuation into the complex plane), i.e. $\omega = \omega_R + i\omega_I$ with $\omega_I > 0$. If $e^{i\omega t}$ was used in the time to frequency transformation there would be a minus sign for the imaginary part of the frequency. The effect of complex frequency moves the singularities into the first and third quadrant of the complex k -plane. The use of complex frequency has the effect of smoothing the spectrum and enhancing the first motions relative to later arrivals. This effective attenuation is used to minimize the influence of the neighboring fictitious sources introduced by discrete k . The effect of the imaginary part of the frequency can be removed from the final time domain solution by inverse complex Fourier transform with the complex frequency with the same imaginary part used in the argument of the exponential function in the Fourier transform. The magnitude of the imaginary part is usually chosen to be,

$$\omega_I = \frac{\pi}{t_{max}}. \quad (109)$$

Larger ω_I increases the attenuation for later arrivals, but also magnifies the numerical noise for late times. If ω_I is chosen too small, the attenuation may not be large enough to damp out the arrivals from the fictitious sources.

1.7. NUMERICAL ELECTROSEISMOGRAM EXAMPLES

Four different numerical models are calculated in the *PSVTM* wavefield picture. The first example is a 100 *m* thick porous sand layer sandwiched in between two identical less porous halfspaces. The fluid chemistry is the same in all three layers. The source/receiver medium configuration is depicted in Figure 3. This example is meant to study electroseismic conversions as a result of a change in mechanical properties. Mechanical displacement seismograms and electroseismograms (*TM* mode components) are calculated and displayed in Figure 4. The converted magnetic and electric field amplitude behavior as function of antenna position are discussed for amplitudes determined at three different distances from the mechanical contrast, see Figure 5. In Figures 6 and 7, four time slice based snapshots are shown of the mechanical wavefront traversing an interface and the conversion of the mechanical wavefield into the electromagnetic *TM* mode components.

The second example studies the effect of a change in fluid salinity, which only affects the medium's electrical properties, on the conversion to electromagnetic waves. The source/receiver medium configuration is depicted in Figure 8. The mechanical displacement seismograms and electroseismograms (*TM* components) are calculated and displayed in Figure 9. The converted magnetic and electric field amplitude behavior as function of antenna position at three different distances from the electrical contrast are shown in Figure 10. The four snapshots, Figures 11 and 12 in time are calculated to trace in time the converted *TM* wavefield pattern generated by a mechanical wavefront traversing the electrical contrast.

The last two examples show the electroseismic effect in a Vertical Electro seismic Profiling (VESP) setting. The seismic source is located in the upper halfspace and the recording geophone/antennas are positioned vertically crossing two mechanical contrasts in the first VESP example (Figure 13) and an additional electrical contrast in the second VESP example (Figure 15).

The modeled mechanical source in all examples is a mechanical explosion source. To model true amplitudes, the diagonal elements in the seismic moment matrix, equation (88), need to be replaced by a realistic value. Using the relation $m = \frac{4\pi a^3}{3} K_G \Delta\Theta$, with a the radius of the of the nonlinear deformation zone around the source, K_G a coefficient from the deformation equation given in equation (16), $\Delta\Theta$ the fractional change in volume and $E = \frac{4\pi a^3}{3} K_G (\Delta\Theta)^2 = \epsilon [4.7 \times 10^6 \text{ J/Kg}] C$, with E the total energy, C the source weight in kilograms, and ϵ an efficiency factor expressing the fraction energy converted into sound, the following expression for m is obtained:

$$m = \sqrt{\frac{4\pi a^3}{3} K_G [4.7 \times 10^6 \text{ J/Kg}] C \epsilon}. \quad (110)$$

In the numerical calculation $m = 4.4 \times 10^6 \text{ J}$ is used, corresponding to $C = 1 \text{ Kg}$, $a = 1 \text{ m}$, $K_G = 10^9 \text{ Pa}$ and $\epsilon = 10^{-3}$.

1.7.1. The Electro seismic Conversion at Mechanical Contrasts

An explosion source is positioned 100 *m* above the first mechanical contrast in the upper halfspace. Fifteen receivers are positioned symmetrically in a straight horizontal line at both sides of the source 95 *m* above the interface. The receiver spacing is 10 *m*. The medium parameters of the two

halfspaces and the 100 m thick sand layer are given in Table I. Based on these medium properties the mechanical fast wave, slow wave, shear wave, and electromagnetic *TM* wave velocities are calculated. The complex velocities and calculated bulk conductivities are given in Table II. A slow wave and *TM* wave velocity range is given since these two wave phenomena are diffusive.

When an explosion source is set off, only *P* waves are generated in an isotropic poro-elastic medium. Numerically the 8×1 displacement-stress-EM wavefield component vector is calculated at each geophone/antenna position. In Figure 4 only the mechanical displacement seismograms (u_z and u_x components) and the electroseismograms (*TM* mode components) are shown. All plots are seismogram scaled. The amplitudes of the first 40 msec in the E_x and H_y electroseismograms are multiplied by a factor 50 to enhance the electroseismic converted field generated at the first interface.

At the mechanical contrast, the *P* wave reflects as a *P* wave, a converted *SV* wave, and a converted *TM* wave. The direct wave is omitted in all seismograms. The mechanical seismograms show the *PP* reflection and *PSV* conversion generated at the top and bottom of the thick sand layer. The converted *TM* wavefield components show up at all antennas at approximately half the two way *P* wave traveltime for normal incidence reflection. Since the *TM* wavefield velocity is at least two orders of magnitude faster than the *P* wavefield (Table II), the traveltime spent by the *TM* waves in traveling upward to the antennas is negligible and the total two way traveltime appears to coincide with the one way traveltime of the incoming *P* wave at normal incidence reflection. The hyperbolas arriving at a later time in the E_x and H_y electroseismograms have the same traveltime as the *PP* reflection (compare the mechanical seismogram with the E_x electroseismogram in Figure 4 and the same traveltime as the *PSV* conversion (compare mechanical seismogram with the H_y electroseismogram), respectively.

In the first case, the compressional waves traveling through a homogeneous porous medium cause pressure gradients which cause the charge to separate. This induces within the seismic pulse a system of electric fields that travel with the compressional wave speed. As explained before, the seismic pulse doesn't radiate electromagnetic fields away from the pulse. Therefore, when the reflected *P* wave pulse passes the antennas, an electric field is registered inside the *P* wave pulse. In the second case the vertical polarized rotational mechanical waves traveling through a homogeneous porous medium cause grain accelerations, setting up current sheets. This induces within the seismic pulse magnetic fields that travel with rotational mechanical wave speed. Therefore, when the converted *SV* wave pulse passes the antennas, a magnetic field is recorded inside the *SV* wave pulse. The electric and magnetic field strengths in the seismic pulse are in this case larger than the converted *TM* wavefield components strengths. The E_x hyperbolas arriving after the electroseismic conversion are associated with the electric fields inside the *P* wave pulse traveling with fast wave velocity. While the H_y hyperbolas arriving after the electroseismic conversion are associated with the magnetic fields inside the *SV* wave pulse traveling with *S* wavespeed velocity. The *TM* conversions from the bottom mechanical contrast have amplitudes which are too small to be identified in the electroseismogram with the scaling used.

In Figure 5, the converted electric and magnetic absolute field amplitudes versus antenna offsets are shown at 20, 50 and 95 m from the interface. The field amplitude at each antenna position is determined by calculating a root mean square amplitude inside an estimated pulse window in the

time domain. The converted *TM* wavefield, driven by the seismic pulse frequency, is diffusive, the real and imaginary parts of the *TM* velocity are almost equal, and therefore the largest signals are measured by antennas closest to the interface. The frequency content of the electromagnetic field is the same as the frequency content of the incident seismic wave. Since we are in the, $\frac{\sigma}{\omega\epsilon} \gg 1$, conducting medium regime, we can find a skindepth $\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$, with $\mu_0 = 4\pi \times 10^{-7}$ (Kong, 1990, p. 60), the permeability of the medium. If the propagation distance is much less than the skindepth, the near field of the radiating interface, then the frequency content in the mechanical and electromagnetic field are the same. The increase in amplitude with increasing source-antenna offset and a later decay in amplitude with offset show similarities with amplitudes that would be recorded if the interface was replaced by a seismically induced electric dipole right under the source.

To investigate the *TM* wavefield conversions in more detail, snapshots in time are calculated around the first mechanical contrast. In Figures 6 and 7, four successive time snapshots are displayed, showing the mechanical u_z wavefield component, the top figure, the electromagnetic H_y component, the middle figure, and the electromagnetic E_x component, bottom figure at 26.6 msec, 28.9 msec, 30.5 msec and 32.0 msec after the shot, respectively. The wavefields are determined at 120 by 60 receiver/antenna positions. The receiver/antenna spacings are 1 m in the horizontal direction and 3 m in the vertical direction. At $t = 26.6$ msec, the Ricker wavelet (Hosken, 1988) front has reached the interface at -90 m (Figure 6, left column). The $t = 26.6$ msec snapshot displaying the magnetic field component of the *TM* mode shows, with a position centered under the seismic source at distance 60 m, an amplitude radiation pattern of an equivalent magnetic current loop with field lines pointing at one side into the paper and at the other side out of the paper. The magnetic field diffuses with large velocity away from the interface. The $t = 26.6$ msec snapshot displaying the electric field component of the *TM* mode shows an amplitude radiation pattern of an electric dipole, positioned right under the seismic source at a distance 60 m. The largest electric fields are associated with the field inside the *P* wave pulse. At later times, when a larger part of the seismic pulse has traversed the interface, the magnetic current loop diameter increases and the *TM* wavespeed differences above and below the interface becomes visible. The current system imbalances across the interface change in direction in accordance with the pulse polarity, the magnetic and electric field polarities flip accordingly. The last two snapshots show complex patterns of mechanical field convergences into electromagnetic fields. More than one magnetic current loops appear with alternating field line directions and the inner loop diameter increases. Also more electric dipoles with opposite "dipole type moments" appear there where the wavefront passes. The combined E_x and H_y amplitude pattern away from the interface has a predominant effective electrical dipole character, or its dual, an effective magnetic current loop character away from the interface. In both cases the amplitudes decay fast with traveled distance.

1.7.2. The Electro seismic Conversion at Electrical Contrasts

The explosion source is again positioned 100 m above an electrical contrast in the upper halfspace. Fifteen receivers are positioned symmetrically in a straight horizontal line at both sides of the source 95 m above the interface to record the reflection from the contrast and 95 m below the interface to record the transmissions through the contrast. The receiver spacing is 10 m. The medium input

parameters describing the two halfspaces are given in Table I. The calculated medium velocities and bulk conductivities based on the medium parameters listed in Table I are shown in Table III.

In Figure 9, the mechanical displacement seismograms (u_z and u_x components) and the electroseismograms (TM mode components) are shown. All four plots are seismogram scaled. At the electrical contrast the P wave reflects only as converted TM waves. Since there is no mechanical contrast, the P wave does not reflect or convert into shear waves. The mechanical u_x and u_z seismograms show the transmitted wavefields through the electrical contrast at the geophones 95 m below the interface which are essentially the direct P wavefield from the explosion source in the upper halfspace. The TM wavefield components are recorded at antennas at approximately the one way P wave traveltime from source to interface. Since the direct wave in the upper halfspace is omitted, the E_x wavefield seismogram does not show induced electric fields within the seismic pulse that travel with the compressional wave speed.

In Figure 10, the converted electric and magnetic absolute field amplitudes versus antenna offsets are shown at 20, 50 and 95 m from the interface. An increase in amplitude with increasing source-antenna offset up to a global maximum is observed. When the source-antenna offset is further increased, the amplitudes decrease monotonically. The amplitude pattern shows similarities with amplitudes that would be recorded if the interface had been replaced by a seismically induced electric dipole or magnetic loop centered right under the seismic source. If Figures 5 and 10 are compared, the field amplitude curves have similar shape. The amplitudes, however, are a factor 10 bigger for the converted TM mode generated at the electrical contrast when compared to the converted TM mode amplitudes converted at the mechanical contrast.

In Figures 11 and 12, four successive snapshots in time are displayed, showing the mechanical u_z wavefield component, top figure, the electromagnetic H_y component, middle figure, and the electromagnetic E_x component, bottom figure at 26.6 $msec$, 28.9 $msec$, 30.5 $msec$ and 32.0 $msec$ after the shot, respectively. The number of geophones/receivers and their spacings in horizontal and vertical directions are identical to the mechanical contrast case. The top halfspace has a salinity of 1.0 mol/l and the lower halfspace has a salinity of 0.001 mol/l . This results in a larger conductivity and lower electromagnetic wavefield velocity in the upper halfspace than in the lower halfspace (Table III). Therefore, the electric fields within the incoming seismic P wave pulse are small in amplitude when compared to the electric fields within the mechanical P wave in the lower halfspace. This amplitude behavior can be clearly observed in the electromagnetic E_x component snapshots. Tracing the transient TM component wavefield pattern fronts in Figures 11 and 12, the TM wavefield velocity difference in both halfspaces is evident. The complex convergence patterns from mechanical wavefield into electromagnetic TM fields are similar to the patterns observed around the mechanical contrasts (compare Figures 6 and 7 with 11 and 12). The resultant E_x and H_y amplitude pattern away from the interface has again a predominant effective electrical dipole character or its dual an effective magnetic current loop character which are centered under the source at the contrast's depth. The amplitudes of the TM mode transients decay rapidly with traveled distance.

1.7.3. Vertical Electro seismic Profiling (VESP)

The converted TM mode amplitude behavior with distance, discussed when the surface electro seismic calculations were discussed, suggests that the VESP geometry setting can be an important one. Since with this technique the antennas are positioned close to the target of interest, larger converted electromagnetic signals can be recorded before they become too attenuated with distance. With the VESP technique the electro seismic method can be applied to targets, electrical and/or mechanical contrasts, at greater depths.

The first VESP example is a three layer model with the first interface at 300 m depth and the second interface at 400 m depth. The first receiver/antenna is positioned at a horizontal offset of 50 m and a depth of 155 m . The receiver/antenna depth spacing is 10 m . The medium description with its most important electro seismic medium parameters is shown in Figure 13. In Figure 14, the numerically calculated VESP's are shown. The top plot is the mechanical displacement response, and the middle and bottom plots show the E_x and H_y TM mode components, respectively. The top VESP in Figure 14 shows the P wave reflections and converted SV wave reflection at the top and bottom interface of the "sand reservoir." The direct P wave in the top halfspace is omitted. The other two vertical electro seismic profiles show the converted TM mode components at the two contrasts. The converted TM wavefield components show up at all antennas at virtually the same time. The high electrical conductivity in the middle layer attenuates the electric field amplitudes completely. The later arrivals in the E_x VESP are electric fields within the seismic pulse that travel with the compressional wave speed. The H_y component VESP shows the fast decay of the converted electro seismic fields with distance. The converted H_y component of the TM mode is much larger in amplitude than the induced magnetic fields inside the SV wave pulse. The main reason is the close antenna position to the target of interest.

The last numerical example has a new electrical contrast added to the previous model, see Figure 15. The 100 m thick "reservoir sand" is now divided into two sands with identical mechanical properties saturated with two fluids of different salinity. In Figure 16, the calculated VESP's are shown. The mechanical displacement response is identical to the previous calculation without the electrical contrast. But the TM mode component VESP's are modified with an extra electro seismic conversion at the electrical contrast.

1.8. CONCLUSIONS

A global matrix method is described which solves the macroscopic governing equations controlling the coupled electromagnetics and acoustics of porous media numerically in layered poro-elastic media driven by arbitrary seismic point sources. The coupled equations decouple into a $PSVTM$ and a $SHTE$ picture. The induced current motion plane determines to which electromagnetic mode the mechanical waves are coupled. Seismic motion which generates relative flow, induces a "streaming" electrical current due to cation motion. The driving force for the relative flow is a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain accelerations. The relative flow and therefore current can be both due to compressional and shear waves.

Compressional waves traveling through homogeneous porous medium cause pressure gradients

which cause the charge to separate. This induces within the seismic pulse a system of electric fields that travel with the compressional wavespeed. Rotational waves traveling through a homogeneous porous medium cause grain accelerations and set up current sheets. This induces within the seismic pulse magnetic fields that travel with the rotational wavespeed. Therefore, when the seismic pulse passes an antenna, an electric field is recorded inside the P wave pulse and a magnetic field is recorded inside the S wave pulse. The seismic pulse doesn't radiate electromagnetic waves away from the pulse.

Radiating electromagnetic wavefields are converted from seismic waves, however, when contrasts in mechanical and/or electrical properties are traversed. The principal features of the converted electromagnetic signals are: (1) contacts all antennas at approximately the same time; (2) arrives at the antennas at half of the seismic traveltime at normal incidence reflected P waves; and (3) changes sign on opposite sides of the shot. The frequency content of the converted electromagnetic field has the same frequency content of the driving incident seismic pulse, as long as the propagation distances are much less than the electromagnetic skin depth.

Root mean square converted electromagnetic amplitudes versus seismic point source-antenna offset are calculated for a mechanical porosity contrast and an electrical, salinity contrast at different depths. The amplitude curves are similar in shape, first a strong increase in amplitude to a global maximum is observed with increasing antenna offset and next a monotonic decrease in amplitude with increasing antenna offset. The amplitudes decrease rapidly with traveled distance.

Four snapshots in time show the wavefield there where the seismic wavefront passes the interface creating current imbalances across the interface. Equivalent sources can be identified with the conversion to electromagnetic waves. A magnetic current loop can be identified with the H_y component snapshots and electric current dipoles can be identified with the E_x snapshots. The total conversion to electromagnetic waves can be either represented by an effective current dipole or an effective magnetic loop since they are each others dual. The TM component amplitude radiation patterns recorded at some distance from the interface show similarities with the wavefield that would be obtained if the interface was replaced by an equivalent electric dipole or magnetic current loop positioned right beneath the seismic source at the contrast's depth. The amplitude versus offset results and the electroseismograms confirm such an effective conversion right beneath the source. The VESP modeling shows the rapid decay of the converted electroseismic signals with distance. The antennas close to the target of interest show larger amplitudes in the converted signal than electromagnetic signals inside the seismic pulse. With increasing distance to the antenna, the electromagnetic signal inside the seismic pulse totally dominates the converted fields in the electroseismograms.

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TABLE I. The mechanical contrast and the electrical contrast medium properties.

Property	Top and bottom halfspaces	Sand layer	Fresh water halfspace	Brine halfspace
porosities ($\phi[\%]$)	15	30	15	15
dc permeability ($k[m^2]$)	10^{-12}	10^{-12}	10^{-12}	10^{-12}
bulk modulus solid ($k_s[Pa]$)	3.6×10^{10}	3.6×10^{10}	3.6×10^{10}	3.6×10^{10}
bulk modulus fluid ($k_f[Pa]$)	2.2×10^9	2.2×10^9	2.2×10^9	2.2×10^9
frame bulk modulus ($k_{fr}[Pa]$)	9.9×10^9	9.9×10^9	9.9×10^9	9.9×10^9
frame shear modulus ($g_{fr}[Pa]$)	9.0×10^9	9.0×10^9	9.0×10^9	9.0×10^9
fluid viscosity ($\eta[Pa \cdot s]$)	1.0×10^{-3}	1.0×10^{-3}	1.0×10^{-3}	1.0×10^{-3}
density solid ($\rho_s[Kg/m^3]$)	2.7×10^3	2.7×10^3	2.7×10^3	2.7×10^3
density fluid ($\rho_f[Kg/m^3]$)	1.0×10^3	1.0×10^3	1.0×10^3	1.0×10^3
salinity ($C[mol/l]$)	1.0×10^{-3}	1.0	1.0×10^{-3}	1.0
temperature ($T[K]$)	298	298	298	298
permittivity (κ_f)	80	80	80	80
permittivity (κ_s)	4	4	4	4
tortuosity (α_∞)	3	3	3	3

TABLE II. Calculated wavefield velocities and bulk conductivities used in the mechanical contrast model.

Properties	Upper and lower halfspace	Sand reservoir layer
fast wave velocity[m/s]	(3282.62, -0.1037)	(3158.56, -0.7419)
slow wave velocity[m/s]	(21.2/146.2, -13.3/-119.1)	(16.7/111.6, -10.5/-100.3)
shear wave[m/s]	(1769.14, -1.39)	(1669.25, -1.53)
TM wave velocity[m/s]	(318890/2.04 10^6 , -201223/-2.02 10^6)	(7159/45665, -4517/ -45326)
conductivity ($\sigma[S/m]$)	0.000388	0.77

TABLE III. Calculated wavefield velocities and bulk conductivities used in the electrical contrast model.

Properties	Top fresh water saturated halfspace	Bottom brine saturated halfspace
fast wave velocity[m/s]	(3282.62, -0.1037)	(3282.62, -0.1037)
slow wave velocity[m/s]	(21.2/146.2, -13.3/-119.1)	(21.2/146.1, -13.3/-119.1)
shear wave[m/s]	(1769.14, -1.39)	(10124.4, -6388.66) - (64579.9, -64100.4)
TM wave velocity[m/s]	(318890/2.04 10^6 , -201223/-2.02 10^6)	(10124/64580, -6389/-64100)
conductivity ($\sigma[S/m]$)	0.000388	0.385

P waves: charge separation

However, no E-fields away from the pulse
in homogeneous media

Figure 1: Electric fields due to charge separation within a *P* wave seismic pulse.

S waves: current sheets

However, no EM fields away from the pulse
in homogeneous media

Figure 2: Magnetic fields due to current sheets within a *SV* wave seismic pulse.

"Thick" Permeable Sand Layer

Figure 3: The thick permeable sand medium configuration.

Figure 4: The mechanical displacement component seismograms and the *TM* mode component electroseismograms calculated for the thick permeable sand medium configuration.

Figure 5: Converted electric and magnetic root mean square wavefield amplitudes versus antenna offsets calculated at 20, 50 and 95 m from a mechanical contrast generated by an explosive point source.

Figure 6: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around a mechanical contrast at $t = 26.6$ msec and $t = 28.9$ msec after explosion. Top snapshot is u_z component of mechanical displacement wavefield, middle snapshot is H_y component of electromagnetic TM mode, bottom snapshot is E_x component of electromagnetic TM mode.

Time slice $t = 32.0$ msec, wavefield components U_z , H_y and E_x

Time slice $t = 30.5$ msec, wavefield components U_z , H_y and E_x

Figure 7: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around a mechanical contrast at $t = 30.5$ msec and $t = 32.0$ msec after explosion. Top snapshot is u_z component of mechanical displacement wavefield, middle snapshot is H_y component of electromagnetic TM mode, bottom snapshot is E_x component of electromagnetic TM mode.

Fresh Water / Brine

Figure 8: The fresh water-brine medium configuration.

Figure 9: The mechanical displacement component seismograms and the TM mode component electroseismograms calculated for the fresh water-brine medium configuration.

Figure 10: Converted electric and magnetic root mean square wavefield amplitudes versus antenna offsets calculated at 20, 50 and 95 m from an electrical contrast generated by an explosive point source.

Time slice $t = 28.9$ msec, wavelfield components U_z , H_y and E_x

Time slice $t = 26.6$ msec, wavelfield components U_z , H_y and E_x

Figure 11: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around an electrical contrast at $t = 26.6$ msec and $t = 28.9$ msec after explosion. Top snapshot is u_z component of mechanical displacement wavelfield, middle snapshot is H_y component of electromagnetic TM mode, bottom snapshot is E_x component of electromagnetic TM mode.

Time slice $t = 32.0$ msec, wavefield components U_z , H_y and E_x

Time slice $t = 30.5$ msec, wavefield components U_z , H_y and E_x

Figure 12: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around an electrical contrast at $t = 30.5$ msec and $t = 32.0$ msec after explosion. Top snapshot is u_z component of mechanical displacement wavefield, middle snapshot is H_y component of electromagnetic TM mode, bottom snapshot is E_x component of electromagnetic TM mode.

Vertical ElectroSeismic Profiling (VESP)

Figure 13: Vertical ElectroSeismic Profiling medium configuration.

Figure 14: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a Vertical Electrostatic Profiling medium configuration.

Vertical ElectroSeismic Profiling (VESP)

Figure 15: Vertical ElectroSeismic Profiling medium configuration with additional electrical contrast.

Figure 16: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a Vertical Electroseismic Profiling medium configuration with additional electrical contrast.

2. MODELING OF SEISMOELECTRIC EFFECTS IN A BOREHOLE

2.1. SUMMARY

We present a method to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered poroelastic medium. The electroseismic conversions occur at both the borehole wall and the layer boundaries. Most studies in electroseismic effects have been modelled and tested with seismic sources and detectors (geophones and antennas) at the surface. In this paper, we investigate the case of a seismic source in a borehole and receivers either at the surface or embedded in the medium. The method is formulated as a boundary element technique (where the poroelastic displacement and relative flow Green's functions are calculated by the discrete wavenumber method. The singular properties of the Green's functions are determined analytically using static Green's functions to regularize the integrals. This is necessary to calculate the element's self interaction. The borehole is cylindrical and its axis runs normal to the interfaces. The coupled electroseismic effects in the layered media are included by using the global matrix technique.

The developed method is an extension of the model of Biot-Rosenbaum, who applied the wavenumber integration technique to investigate the effect of formation permeability on Stoneley waves, using Biot's theory to model the wave propagation effects of a homogeneous permeable formation surrounding a borehole. We extend the Biot-Rosenbaum model by including the effect of a heterogeneous permeable formation surrounding the borehole. The effect of formation permeable zones (or fractured zones) on Stoneley waves can now be investigated. The other modification is the inclusion of conversions of mechanical into electromagnetic waves at mechanical and/or electrical contrasts in the poroelastic formation. The converted electromagnetic fields are sensitive to large permeability contrasts and fluid chemistry contrasts inside a reservoir. Using the electroseismic method downhole will provide more information about permeability/permeability contrasts in the formation, as well as additional lithological information (salinity of the fluids).

2.2. INTRODUCTION

Many rocks in the upper crust have porosity and are characterized as porous, permeable media. The determination of fluid-flow properties as well as fluid properties of sedimentary rocks is of great importance in groundwater hydrology studies, in environmental problems, and in the evaluation of hydraulic conductivity and fluid contrasts inside reservoirs in the petroleum industry. Full waveform acoustic logging offers an effective tool for characterizing permeable sand-shale sequences found in sedimentary formations. The stratigraphic permeable traps are important in the exploration and production of hydrocarbons.

The theoretical foundations of the study of acoustic wave propagation in a cylindrical borehole was laid by Biot (Bouchon, 1952), who presented a derivation of the period equation for borehole guided waves and their dispersion characteristics. The current technique to model borehole acoustic wave propagation through heterogeneous formation is the finite difference method (Cheng, 1994; Stephen and Cheng, 1985). The numerical difficulty of using explicit time marching FD , to model wave phenomena through porous media is caused by the dispersive nature of Biot's slow

wave (Biot, 1956) and the coupling of this wave with fast and shear waves. The wavenumber integration technique has been applied to calculate wave propagation in homogeneous porous formations. Rosenbaum (1974) investigated the effect of formation permeability using Biot's theory at high frequencies, combined with the open pore boundary conditions proposed by Deresiewicz and Skalak (1963) to model wave propagation effects of a homogeneous permeable formation surrounding a borehole. Schmitt and Bonnet (1988) investigated the case of a borehole surrounded by multiple radially layered isotropic poroelastic layers and modified later his theory to the more complicated case of a borehole surrounded by a radially transversely isotropic layered poroelastic medium (Schmitt, 1989).

In this paper, we generalize the Biot-Rosenbaum model with two modifications. We apply a *BEM* modelling technique to simulate wave propagation inside the borehole and outside the borehole in a heterogeneous poroelastic formation. At the mechanical and/or electrical boundaries in the formation, the conversions from mechanical into electromagnetical waves are included. Before we outline the method to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered poroelastic medium, we want to discuss the importance of measuring the acoustic fields and electromagnetic fields inside the borehole with respect to their dependence on permeability and fluid chemistry.

The fundamental mode of a guided wave travelling in a fluid column, the Stoneley wave, is an interface wave and it is therefore sensitive to formation properties, such as density, moduli and, most importantly, permeability. Field experiments clearly indicate the effect of formation permeability on borehole Stoneley waves (Williams *et al.* (1984). Theoretical models were developed to study the correlation between permeability and Stoneley wave propagation. White (1983) and Hsui and Toksöz, (1986) developed low frequency models of Stoneley wave propagation. Rosenbaum (1974) investigated the effects of permeability on high frequency borehole acoustic waves. Burns and Cheng (1986) went a step further and inverted for in-situ permeability from Stoneley wave velocity and attenuation using the Biot-Rosenbaum model. Winkler and Johnson (1989) performed laboratory model experiments on borehole Stoneley wave propagation to evaluate the applicability of the Biot theory to logging in porous formations. The theory was found to be in agreement with the experiments.

Others have tried to obtain fracture information from Stoneley wave measurements. Hornby and Plumb (1989) and Tang (1990) model both attenuation and reflection of Stoneley waves from a single plane fracture. They conclude that it takes a rather large fracture aperture of the order of centimeters, to get a significant Stoneley wave attenuation. Paillet and Tang (1989) suggest that in-situ fractures may consist of an array of flow passages or fracture layers instead of a single fluid layer. This suggests modeling fractures as a permeable zone in the formation. Key parameters used to characterize the permeable zone are thickness of the zone, permeability, porosity, and tortuosity. Since the last three parameters typically describe a porous model, we can use the Biot-Rosenbaum theory (Tang and Cheng, 1993) to model Stoneley wave characteristics in the permeable zone. Tang and Toksöz (1991) developed a simplified model for Stoneley wave propagation in a permeable formation, which is consistent with the Biot-Rosenbaum theory in the presence of a hard formation.

The approach we present in this chapter relies on the important conclusion that fractured zones

can be characterized using Biot's theory. The model has no real frequency constraints, as long as we are in the seismic/acoustic logging regime, and is applicable to heterogeneous porous formations. Winkler and Johnson (1989) predict in their theoretical calculations that in a $k = 10^{-14} m^2$ rock, the effect of permeability is to decrease the Stoneley velocity by less than 1 percent which is less than the uncertainty in Stoneley velocity measurements. Therefore they conclude that a $k = 10^{-14} m^2$ permeability is probably a lower limit to what can be expected to measure in the field using Stoneley velocities.

Additional information about very low permeability zones might be obtained, however, using the converted electric fields at high permeability contrasts. The amplitude of a converted electromagnetic field is sensitive to large permeability contrasts of the permeable sand impermeable shale type. The permeability of the formation can play an important role in the determination of the formation's electrical conductivity. The electrical conductivity increases monotonically with increasing porosity since the amount of bulk electrolyte increases linearly with porosity, (see Haartsen, 1995, chapter 2). This increase in conductivity is not linear with porosity due to the porosity dependence of the pore length parameter, which influences the conductivity enhancement due to surface conductances. The electrical conductivity decreases with increasing permeability. At very small permeabilities the surface conductances become important and dominate in value the conductivity contribution of the bulk conductivity in the fluid phase, (Haartsen, 1995, chapter 2). Physically this situation occurs when the pore space is filled with clay, reducing the permeability of the formation but not affecting the porosity, this greatly enhancing the conductances along the pore surfaces (Haartsen, 1995, chapter 2). When mechanical waves traverse such a permeability contrast, the induced electric fields at both sides of the interface experience a different electrical conductivity and produce a dynamic current imbalance which causes electromagnetic radiation.

Electroseismic conversions are also sensitive to fluid chemistry contrasts inside a homogeneous reservoir. The Stoneley waves in the borehole won't be affected by the different fluids, provided that the densities and viscosities of the two fluids are approximately the same. The body waves in the surrounding poroelastic formation are also not affected since the different saturating fluids cause only a negligible mechanical contrast.

2.3. STRATEGY TO SIMULATE COUPLED ACOUSTO-ELASTIC AND ELECTROMAGNETIC WAVES IN BOREHOLE GEOMETRY

The boundary element modelling technique (see Brebia and Dominguez, 1989, for general discussion on different types of BEM) is formulated to solve a vertical waveguide (borehole) which penetrates a horizontal flat layered poroelastic medium. We follow closely the boundary element based modelling technique proposed by Bouchon (1993). Bouchon used the method to solve the wavefield problem in an infinite open borehole and in the surrounding isotropic layered elastic formation. Dong (1993) added two extensions to Bouchon's modelling technique; he generalized the formation to include transversely isotropic layers and included casing in his formulation.

In our formulation at the layer boundaries, mechanical waves are partly converted into electromagnetic fields, satisfying all mechanical and electromagnetical boundary conditions. The conversion to electromagnetic fields induced by the mechanical tube waves is also included. But these electromagnetic fields don't satisfy the EM boundary conditions on the borehole wall. We first

describe the BEM implementation. Since four boundary conditions have to be complied with at the borehole wall, four fictitious ring sources need to be introduced. The P wave radiation in the borehole is uniquely described by a fluid volume injection ring source. The $P_f - SV - P_s$ wave radiation in the poroelastic formation are determined by the vertical, radial ring sources applied to the frame and by an explosive ring source on the fluid phase, respectively. The field vector components for each ring source are obtained by integrating the appropriate Green's functions directly over a ring. The poroelastic Green's functions are given in the appendix and derived in (Haartsen, 1995, chapter 5). The final fields are obtained by surface integrating the dynamic Green's functions. The integrals are first reduced to horizontal wavenumber integrals and solved by discrete wavenumber summation. BEM modelling requires the calculation of a square boundary condition matrix containing the wavenumber interaction results between all elements. To calculate the elements' self interaction, the singular properties of the Green's functions need to be determined. The stress field for the dynamic Green's functions with a force on the solid phase is singular, and the displacement for the dynamic Green's functions with a force on the fluid is also singular. The contribution of the stress and displacement singularities, when source and receiver coincide, are determined analytically by integration of the static Green's functions over a half spherical surface.

The last part of this paper discusses the inclusion of horizontal layers coupled to the borehole using the BEM modelling technique. At the horizontal interfaces, the electroseismic governing equations (Haartsen, 1995, chapter 2) with boundary conditions added are solved numerically. Biot's equations with an induced body force on Biot's equation of relative flow, and Maxwell's equations with an induced current source, and all the mechanical to electromagnetic coupling present in the transport equations, are solved simultaneously. A mechanical poroelastic displacement/stress and coupled electromagnetic wavefield component vector as a function of depth can be determined, which equals the so called system matrix, containing all medium properties and electrokinetic and osmotic coupling, times the same displacement/stress/EM wavefield component vector. The eigenvalues (slownesses) and corresponding up and down going fast, slow, SV and TM polarized wavefield eigenvectors of the coupled electromagnetic and acoustic-elastic system matrix are derived in Haartsen (1985, chapters 3 and 4).

To solve simultaneously the macroscopic electromagnetic and acoustic-elastic wavefields in layered media we use the global matrix method. We derive source jump representations for the introduced ring sources using the relative flow and displacement Green's functions together with the deformation equations in both phases. These mechanical sources generate fluid flow and electrical streaming current in the porous medium. Both current and fluid flow are related through the transport equations through which all electro-mechanical coupling occurs. At the borehole wall the mechanical waves are matched only. The effect of the borehole on the electromagnetic fields is assumed to be insignificant since the electromagnetic wavelengths are for typical circumstances (100 Hz seismic center frequency in the numerical modelling) on the order of a kilometer, while the borehole radius has typically a diameter of 20 cm.

In this paper, we will discuss the numerical results of the newly derived vertical, radial and volume injection ring sources in layered porous media. These calculation routines constitute the kernel of the BEM algorithm. The sum of the weighted ring sources determines the wavefields radiated into the formation. The weighting of each source is determined by solving a linear system

of equations that incorporate the mechanical boundary conditions on the borehole wall. The calculation of all element interactions for all wavenumbers for all frequencies is numerically feasible on a massive parallel computer (the BEM algorithm was implemented on ERL's nCUBE), but its long run time does not allow practical wave propagation study. Therefore, we have chosen to present the numerical comparison of the three ring sources in poroelastic layered media. The electromagnetic radiation induced by the mechanical vertical and radial ring sources is discussed. The conversion to electromagnetic disturbances generated by P and SV waves is studied for the three different ring sources.

2.4. BEM IMPLEMENTATION

To solve the mechanical boundary conditions at the borehole wall, we represent the borehole wall as a distribution of secondary sources. The borehole is discretized into small cylindrical elements and each element is represented by four sources: a volume source generating wavefields diffracted into the fluid, V_i^f , a vertical, F_i^v , and radial, F_i^r , ring source acting on the elastic frame, and a volume injection ring source, F_i^e , acting on the pore fluid. The last three sources give rise to the poroelastic wavefields radiated into the formation. The density of the fictitious sources' wavefields are assumed to be constant on each element. To calculate an element's self interaction, the singular properties of the Green's functions need to be determined. The stress field for the dynamic Green's functions with a force on the elastic frame are also singular, while the displacement/relative flow for the dynamic Green's functions with a force on the pore fluid are singular. The contribution of the stress and displacement/relative flow singularities are determined analytically by integration of the static Green's functions over a half spherical surface when source and receiver coincide. The strength of each source is obtained by solving a linear system of equations that incorporate the mechanical boundary conditions at the borehole wall.

2.5. BOUNDARY CONDITION MATRIX

Within each element, the fluid-porous boundary conditions have to be satisfied (Deresiewicz and Skalak, 1963; Lovera, 1987; de la Cruz and Spanos, 1989).

- Normal displacement in the fluid matches the net normal displacement in the poroelastic medium.
- The stress in the fluid matches the bulk normal stress in the poroelastic medium.
- The stress in the fluid matches the stress in the fluid phase in the poroelastic medium.
- The poroelastic tangential bulk stress vanishes.

$$\begin{aligned}
 [u_r + w_r] |_{r=r_0^+} - u_r |_{r=r_0^-} &= 0 \\
 \tau_{rr} |_{r=r_0^+} - \tau_{rr} |_{r=r_0^-} &= 0 \\
 T_p |_{r=r_0^+} - \tau_{rr} |_{r=r_0^-} &= 0 \\
 \tau_{rz} |_{r=r_0^+} &= 0
 \end{aligned} \tag{1}$$

The boundary conditions are satisfied at the center of each element. The displacements, stresses, and pressure at the center of each element are due to the four fictitious sources on all elements. The indirect formulation is used to calculate the displacement at the j -th element due to a source at the i -th element. At the j -th element, the boundary conditions become,

$$\begin{aligned}
\sum_{i=1}^{N_e} A_{ji}^f V_i^f + \sum_{i=1}^{N_e} A_{ji}^v F_i^v + \sum_{i=1}^{N_e} A_{ji}^r F_i^r + \sum_{i=1}^{N_e} A_{ji}^e F_i^e &= E_j^u \\
\sum_{i=1}^{N_e} B_{ji}^f V_i^f + \sum_{i=1}^{N_e} B_{ji}^v F_i^v + \sum_{i=1}^{N_e} B_{ji}^r F_i^r + \sum_{i=1}^{N_e} B_{ji}^e F_i^e &= E_j^{\tau_{rr}} \\
\sum_{i=1}^{N_e} C_{ji}^f V_i^f + \sum_{i=1}^{N_e} C_{ji}^v F_i^v + \sum_{i=1}^{N_e} C_{ji}^r F_i^r + \sum_{i=1}^{N_e} C_{ji}^e F_i^e &= E_j^{T_p} \\
\sum_{i=1}^{N_e} D_{ji}^v F_i^v + \sum_{i=1}^{N_e} D_{ji}^r F_i^r + \sum_{i=1}^{N_e} D_{ji}^e F_i^e &= E_j^{T_p}
\end{aligned} \tag{2}$$

where A_{ji}^f , A_{ji}^v and A_{ji}^r represent displacements at the j -th element due to the volume, vertical, radial, and explosive ring sources of unit strength at the i -th element, respectively. They are surface integrals of the appropriate Green's functions over the surfaces of the borehole. The B and C 's are the radial stresses and the stresses in the fluid phase of the poroelastic formation at the j -th element due to sources at the i -th element. The D 's are the tangential stresses at the j -th element due to sources at the i -th element. They are the surface integrals of the appropriate stress Green's functions. The E 's are the fields excited at the j -th element.

The number of elements depends on the time window and the fastest wave speed. the element heights dz satisfy,

$$dz \leq \frac{\min(c, \beta, \alpha)}{3f} \tag{3}$$

The number of elements, N_e is given by,

$$N_e = \frac{t_{max} \times \max(c, \beta, \alpha)}{dz} = \frac{3f \times t_{max} \times \max(c, \beta, \alpha)}{\min(c, \beta, \alpha)} \tag{4}$$

with j ranging from 1 to N_e , $4 \times N_e$ equations are obtained that need to be solved for $4 \times N_e$ unknowns. The fields inside and outside the borehole can be calculated knowing these fictitious sources. The surface integration of the Green's functions is accomplished in two steps. The integrals are first transformed into wavenumber integrals, where the Sommerfeld integral representation to the function $\frac{e^{-ikR}}{4\pi R}$ is used. The wavenumber integration is then evaluated by the discrete wavenumber method. The scalar Green's functions g_i with $i = \beta \vee \alpha_1 \vee \alpha_2 \vee f$, are expressed in a horizontal wavenumber integral using,

$$g_i = \frac{-i}{4\pi} \int_0^\infty \frac{k}{\nu_i} J_0(kD) e^{-i\nu_i|z-z'|} dk \tag{5}$$

where $\nu_i = \sqrt{k_i^2 - k^2}$ and $D = \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi - \phi_0)}$. The spherical wavefront is now represented by infinitely many cylindrical wavefronts. Using the addition theorem for the zeroth order Bessel function (Watson, 1966)

$$J_0(kD) = \sum_{m=0}^{\infty} \epsilon_m J_m(kr) J_m(kr_0) \cos m(\phi - \phi_0), \quad (6)$$

one obtains

$$g_i = \frac{-i}{4\pi} \sum_{m=0}^{\infty} \epsilon_m \cos m(\phi - \phi_0) \int_0^{\infty} \frac{k}{\nu_i} J_m(kr) J_m(kr_0) e^{-i\nu_i |z-z'|} dk \quad (7)$$

where $\epsilon_m = 2 - \delta_{m0}$, and J_m is the m -th order Bessel function of the first kind. The integrand of the above wavenumber integral simplifies when the surface integration $r_0 d\phi_0 dz$ is performed, due to the orthogonality of the sine and cosine functions over the integration range 0 to 2π . The final result of this integration represents the response of a circular ring source. The dz part of the surface integral can be carried out analytically through the integration of the ring source results over z' . Since the integral over element height depends on horizontal wavenumbers, its evaluation must precede the wavenumber integration. The integration over the source element coordinate z' is of two types, an integral of a complex exponential and an integral of a sign function multiplied by a complex exponential function. Both integral types can be performed analytically.

$$\int_{z_l}^{z_h} e^{-i\nu(z-z')} dz' = \begin{cases} \frac{1}{i\nu} [e^{-i\nu(z-z_h)} - e^{-i\nu(z-z_l)}] & \text{for } z > z_h; \\ \frac{-1}{i\nu} [e^{-i\nu(z_h-z)} - e^{-i\nu(z_l-z)}] & \text{for } z < z_l; \\ \frac{-1}{i\nu} [e^{-i\nu(z_h-z)} + e^{-i\nu(z-z_l)} - 2] & \text{for } z_l < z < z_h. \end{cases} \quad (8)$$

and,

$$\int_{z_l}^{z_h} \text{sgn}(z - z') e^{-i\nu(z-z')} dz' = \begin{cases} \frac{1}{i\nu} [e^{-i\nu(z-z_h)} - e^{-i\nu(z-z_l)}] & \text{for } z > z_h; \\ \frac{1}{i\nu} [e^{-i\nu(z_h-z)} - e^{-i\nu(z_l-z)}] & \text{for } z < z_l; \\ \frac{-1}{i\nu} [e^{-i\nu(z_h-z)} - e^{-i\nu(z-z_l)}] & z_l < z < z_h. \end{cases} \quad (9)$$

The element surface integration of the Green's function reduces to a horizontal wavenumber integral. The field integrals are evaluated by the discrete summation, the so-called Discrete Wavenumber Method (Bouchon and Aki, 1977). The discretization of the radial wavenumber k in cylindrical coordinates introduces periodicity into the source distribution. The original single source problem changes after discretization into periodic concentric sources around the original source. The periodicity of these sources, or the distance between two adjacent circular sources, L , is related to the discretization interval of the wavenumber, Δk , by the sampling relation,

$$L = \frac{2\pi}{\Delta k}. \quad (10)$$

L , and therefore Δk , is determined by assuming a receiver located at $x_r = (r_0, z_0)$ and a source at $x_s = (0, z_s)$ on the symmetry axes of the medium configuration. Given the time window to record

radiated waves from 0 to t_{max} , "pseudo" waves radiated from the periodic sources are not allowed to enter this time window. This requirement is,

$$\sqrt{(L - r_0)^2 + (z_0 - z_s)^2} > v_{fastest} t_{max} \quad (11)$$

or

$$L > r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2}. \quad (12)$$

The sampling equation becomes now,

$$\Delta k < \frac{2\pi}{r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2}}. \quad (13)$$

The above equation is the criterion for choosing the sample rate for the discrete wavenumber summation.

To perform the summation, the singularities of the integrands must be removed from the real k axes. This is done by adding a small imaginary part to the real frequency (analytic continuation into the complex plane), i.e. $\omega = \omega_R + i\omega_I$ with $\omega_I > 0$. If $e^{i\omega t}$ was used in the time to frequency transformation there would be a minus sign for the imaginary part of the frequency. The effect of complex frequency moves the singularities into the first and third quadrant of the complex k -plane. The use of complex frequency has the effect of smoothing the spectrum and enhancing the first motions relative to later arrivals. This effective attenuation is used to minimize the influence of the neighboring fictitious sources introduced by discrete k . The effect of the imaginary part of the frequency can be removed from the final time domain solution by inverse complex Fourier transform with complex frequency, using the same imaginary part in the argument of the exponential function in the Fourier transform. The magnitude of the imaginary part is usually chosen to be,

$$\omega_I = \frac{\pi}{t_{max}}. \quad (14)$$

Larger ω_I increases the attenuation for later arrivals, but also magnifies the numerical noise for late times. If ω_I is chosen too small, attenuation may not be large enough to damp out the arrivals from the fictitious sources.

2.6. SINGULARITIES WHEN SOURCE AND RECEIVER COINCIDE

In boundary integral or boundary element techniques, integration of the Green's function over boundary surfaces is necessary. When receiver and source position coincide, the Green's function becomes singular. The displacement and relative flow Green's function have first order singularities which are removable when integrated over the surface. The stress Green's function has a second order singularity and the surface integral is not defined. Here the principle value of the improper integral has to be calculated. The contribution of this second order singularity to the integral can be evaluated analytically. When the receiver and the source approach each other, the surface integral of the dynamic Green's functions can be regularized using the static Green's function (Kupradze, 1963). The improper integral of the dynamic Green's function is reduced to the integration of

the static Green's functions over a half spherical surface around the source/receiver point. First the displacement and stress field components are calculated using the static Green's functions, equations (A-23) and (A-29) for a vertical point force.

$$u_x = \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[\frac{(z - z')(x - x')}{R^3} \right] \quad (15)$$

$$u_y = \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[\frac{(z - z')(y - y')}{R^3} \right] \quad (16)$$

$$u_z = \frac{1}{4\pi GR} - \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[\frac{R^2 - (z - z')^2}{R^3} \right]. \quad (17)$$

Notice that the displacements have a first order singularity when $\mathbf{x} \rightarrow \mathbf{x}'$. The stress in a poroelastic medium along the $\hat{z}$ direction on a surface with the normal in the $\hat{x}$ direction is,

$$\begin{aligned} \tau_{xz} &= G \left[\frac{\partial}{\partial x} u_z + \frac{\partial}{\partial z} u_x \right] \\ &= \frac{x - x'}{4\pi GR^3} - \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[\frac{2R^3(x - x') - 3R(x - x')(z - z')^2}{R^6} \right]. \end{aligned} \quad (18)$$

The stress has a second order singularity when the source and receiver position coincide at $R = 0$. The stress integrated over a surface results in a finite value. The magnitude of this value can be obtained by integrating τ_{xz} over all possible $dydz$ surrounding the source. This integration can be replaced by integration over a half sphere around the source point. This surface is defined by $x - x' = R \sin(\theta) \cos(\phi)$, $y - y' = R \sin(\theta) \sin(\phi)$, $z - z' = R \cos(\theta)$. Then the surface mapping between $dydz$ and the differential spherical surface becomes, $dydz \sin(\theta) \cos(\phi) = R^2 \sin(\theta) d\theta d\phi$. Integrating the stress, τ_{xz} yields,

$$\begin{aligned} \int \tau_{xz} dydz &= \left[\frac{-1}{4\pi} + \frac{HM - C^2 - GM}{4\pi (HM - C^2)} \right] \int_{\pi/2}^{\pi/2} d\phi \int_0^\pi \sin(\theta) d\theta \\ &\quad - \frac{3(HM - C^2 - GM)}{4\pi (HM - C^2)} \int_{\pi/2}^{\pi/2} d\phi \int_0^\pi \sin(\theta) \cos^2(\theta) d\theta = -\frac{1}{2}. \end{aligned} \quad (19)$$

For a point force in the $\hat{x}$ direction, the displacement and relative flow components are,

$$u_x = \frac{1}{4\pi GR} - \frac{HM - C^2 - GM}{8\pi G (HM - C^2)} \left[\frac{R^2 - (x - x')^2}{R^3} \right] \quad (20)$$

$$u_y = \frac{HM - C^2 - GM}{8\pi G (HM - C^2)} \left[\frac{(x - x')(y - y')}{R^3} \right] \quad (21)$$

$$u_z = \frac{HM - C^2 - GM}{8\pi G (HM - C^2)} \left[\frac{(x - x')(z - z')}{R^3} \right] \quad (22)$$

$$w_x = \frac{-1}{4\pi GR} + \frac{HM - C^2 - GC}{8\pi G(HM - C^2)} \left[\frac{R^2 - (x - x')}{R^3} \right] \quad (23)$$

$$w_y = -\frac{HM - C^2 - GC}{8\pi G(HM - C^2)} \left[\frac{(x - x')(y - y')}{R^3} \right] \quad (24)$$

$$w_z = -\frac{HM - C^2 - GC}{8\pi G(HM - C^2)} \left[\frac{(x - x')(z - z')}{R^3} \right]. \quad (25)$$

The normal stress is given by,

$$\tau_{xx} = H \frac{\partial}{\partial x} u_x + (H - 2G) \frac{\partial}{\partial y} u_y + (H - 2G) \frac{\partial}{\partial z} u_z + C \left(\frac{\partial}{\partial x} w_x + \frac{\partial}{\partial y} w_y + \frac{\partial}{\partial z} w_z \right). \quad (26)$$

Integrating the stress, τ_{xx} yields,

$$\begin{aligned} \int \tau_{xx} dy dz &= \left[\frac{-1}{4\pi G} + \frac{3H(HM - C^2 - GM)}{8\pi G(HM - C^2)} + \frac{2(H - 2G)(HM - C^2 - GM)}{8\pi G(HM - C^2)} \right. \\ &+ \left. \frac{C}{4\pi G} - \frac{5C(HM - C^2 - GC)}{8\pi G(HM - C^2)} \right] \int_{\pi/2}^{\pi/2} d\phi \int_0^\pi \sin(\theta) d\theta \\ &+ \left[\frac{-3(H - 2G)(HM - C^2 - GM)}{8\pi G(HM - C^2)} + \frac{3C(HM - C^2 - GC)}{8\pi G(HM - C^2)} \right] \\ &\times \int_{-\pi/2}^{\pi/2} \int_0^\pi [\sin^3(\theta) \sin^2(\phi) + \sin^2(\theta) \cos(\theta)] d\phi d\theta \\ &+ \left[\frac{-3H(HM - C^2 - GM)}{8\pi G(HM - C^2)} + \frac{3C(HM - C^2 - GC)}{8\pi G(HM - C^2)} \right] \\ &\times \int_{-\pi/2}^{\pi/2} \int_0^\pi \sin^3(\theta) \cos^2(\phi) d\phi d\theta = -\frac{1}{2}. \end{aligned} \quad (27)$$

The point source potential and pressure, describing the acoustic behavior in the fluid phase of the poroelastic solid, have first order singularities when source and receiver coincide. These singularities, however, are removable; the integration of potential and pressure over an infinitesimal surface around $\mathbf{x} = \mathbf{x}'$ approaches zero when the surface shrinks to a point. The point displacement has a second order singularity, since the gradient of the Green's function is taken. Integration of the displacement over a spherical surface centered at $\mathbf{x} = \mathbf{x}'$ approaches a constant as the radius of the sphere shrinks to zero.

Using equations (A-16) and (A-17), the x component of the relative flow caused by a pressure source can be obtained. Integrating the relative flow, w_x , over all possible $dydz$ surrounding the source yields,

$$\int w_x dy dz = \lim_{R \rightarrow 0} \left[\int_{-\pi/2}^{\pi/2} d\phi \int_0^\pi \left(\Lambda_1(ik_{\alpha 1} R - 1) \frac{ik_{\alpha 1} R}{4\pi} + \right. \right.$$

$$\Lambda_2(ik_{\alpha 2}R - 1)\frac{ik_{\alpha 2}R}{4\pi}\sin(\theta)\Big]d\theta \quad (28)$$

$$= -\frac{1}{2}[\Lambda_1 + \Lambda_2] = \frac{1}{2}\frac{H}{HM - C^2}. \quad (29)$$

The x component of the bulk displacement caused by a pressure source is obtained by taking the divergence of equation (A-29). Integrating the displacement, u_x , over all possible $dydz$ surrounding the source yields,

$$\int u_x dydz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_0^\pi \left(\frac{C}{4\pi[HM - C^2]} \sin(\theta) \right) d\theta = -\frac{1}{2}\frac{C}{HM - C^2}. \quad (30)$$

To obtain the jump in the fluid displacement due to a **volume injection source**, the displacement potential Φ [dim m^2] will be introduced. The fluid displacement is related to the displacement potential as follows,

$$\underline{u}^{(f)} = \nabla\Phi = C\underline{u} + M\underline{w} \quad (31)$$

$$\int u_x^{(f)} dydz = C \int u_x dydz + M \int w_x dydz = \frac{1}{2}. \quad (32)$$

2.7. RING SOURCE RESULTS IN POROELASTIC MEDIA

In this section the displacements, relative flows and stresses, and pressures of the different ring sources are obtained by integrating the appropriate Green's functions directly over a ring.

2.7.1. Vertical Ring Source

For a vertical source located at (r_0, z') , $F^v(x) = \hat{z}\delta(z - z')\delta(r - r_0)$. The integration over the angle ϕ_0 is non zero only for $m = 0$, due to the orthogonality of the set $\{1, \cos \phi, \cos 2\phi, \dots\}$. Substituting equation (A-2) into displacement Green's function (A-1) and using equation (7) to rewrite the scalar Green's functions, the displacements at point (r, z) due to a ring of unit vertical forces at $x_0 = (r_0, z')$ are obtained. When the relative flow Green's function in equation (A-7) is used instead, the relative flows at point (r, z) due to a ring of unit vertical forces at $x_0 = (r_0, z')$ are obtained.

$$u_r = -\text{sgn}(z - z')\frac{r_0}{2} \int_0^\infty J_1(kr)J_0(kr_0) \times \left[U_\beta^r e^{-i\nu_\beta|z-z'|} + U_{\alpha 1}^r e^{-i\nu_{\alpha 1}|z-z'|} + U_{\alpha 2}^r e^{-i\nu_{\alpha 2}|z-z'|} \right] dk \quad (33)$$

$$u_z = -i\frac{r_0}{2} \int_0^\infty kJ_0(kr)J_0(kr_0) \left[U_\beta^z e^{-i\nu_\beta|z-z'|} + U_{\alpha 1}^z e^{-i\nu_{\alpha 1}|z-z'|} + U_{\alpha 2}^z e^{-i\nu_{\alpha 2}|z-z'|} \right] dk \quad (34)$$

$$w_r = -\text{sgn}(z - z')\frac{r_0}{2} \int_0^\infty J_1(kr)J_0(kr_0) \times \left[W_\beta^r e^{-i\nu_\beta|z-z'|} + W_{\alpha 1}^r e^{-i\nu_{\alpha 1}|z-z'|} + W_{\alpha 2}^r e^{-i\nu_{\alpha 2}|z-z'|} \right] dk \quad (35)$$

$$w_z = -i \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \left[W_\beta^z e^{-i\nu_\beta |z-z'|} + W_{\alpha_1}^z e^{-i\nu_{\alpha_1} |z-z'|} + W_{\alpha_2}^z e^{-i\nu_{\alpha_2} |z-z'|} \right] dk. \quad (36)$$

Substituting equations (33) - (36) into the stress equation in cylindrical coordinates and incorporating the static contribution to τ_{rz} , when source and receiver position coincide, the stresses are obtained at $x = (r, z)$.

$$\begin{aligned} \tau_{rr} &= -\operatorname{sgn}(z - z') \frac{r_0}{2} \int_0^\infty J_0(kr_0) I_1 dk \\ I_1 &= \sum_{p=\beta, \alpha_1, \alpha_2} \left[(HkU_p^r + CkW_p^r + (H - 2G)\nu_p kU_p^z + C\nu_p kW_p^z) J_0(kr) \right. \\ &\quad \left. - 2GU_p^r \frac{J_1(kr)}{r} \right] e^{-i\nu_p |z-z'|} \end{aligned} \quad (37)$$

$$\begin{aligned} \tau_{rz} &= -\frac{1}{2} \delta(z - z') + iG \frac{r_0}{2} \int_0^\infty k J_1(kr) J_0(kr_0) I_2 dk \\ I_2 &= 2k \left[U_\beta^z - \frac{1}{G\nu_\beta} \right] e^{-i\nu_\beta |z-z'|} + 2kU_{\alpha_1}^z e^{-i\nu_{\alpha_1} |z-z'|} + 2kU_{\alpha_2}^z e^{-i\nu_{\alpha_2} |z-z'|} \end{aligned} \quad (38)$$

$$\begin{aligned} \tau_{zz} &= -\operatorname{sgn}(z - z') \frac{r_0}{2} \int_0^\infty J_0(kr) J_0(kr_0) I_3 dk \\ I_3 &= \sum_{p=\beta, \alpha_1, \alpha_2} \left[((H - 2G)kU_p^r + CkW_p^r + H\nu_p kU_p^z + C\nu_p kW_p^z) \right] e^{-i\nu_p |z-z'|} \end{aligned} \quad (39)$$

$$\begin{aligned} T_p &= -\operatorname{sgn}(z - z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) I_4 dk \\ I_4 &= \sum_{p=\beta, \alpha_1, \alpha_2} \left[C(U_p^r + \nu_p U_p^z) + M(W_p^r + \nu_p W_p^z) \right] e^{-i\nu_p |z-z'|} \end{aligned} \quad (40)$$

where the introduced variables, $W_\beta^r, W_{\alpha_1}^r, W_{\alpha_2}^r, U_\beta^r, U_{\alpha_1}^r, U_{\alpha_2}^r, W_\beta^z, W_{\alpha_1}^z, W_{\alpha_2}^z, U_\beta^z, U_{\alpha_1}^z, U_{\alpha_2}^z$ are defined as,

$$W_\beta^r = [\Pi B^*(ws) - \Delta B(ws)] k^2, \quad U_\beta^r = [\Xi B^*(us) - \Gamma B(us)] k^2 \quad (41)$$

$$W_{\alpha_1}^r = [\Pi A_1^*(ws) - \Delta A_1(ws)] k^2, \quad U_{\alpha_1}^r = [\Xi A_1^*(us) - \Gamma A_1(us)] k^2 \quad (42)$$

$$W_{\alpha_2}^r = [\Pi A_2^*(ws) - \Delta A_2(ws)] k^2, \quad U_{\alpha_2}^r = [\Xi A_2^*(us) - \Gamma A_2(us)] k^2 \quad (43)$$

$$W_\beta^z = -\frac{\rho_f}{\rho_E} \frac{1}{G\nu_\beta} - \Delta B(ws)\nu_\beta + \Pi B^*(ws)\nu_\beta, \quad (44)$$

$$U_\beta^z = \frac{1}{G\nu_\beta} - \Gamma B(us)\nu_\beta + \Xi B^*(us)\nu_\beta \quad (45)$$

$$W_{\alpha_1}^z = -\Delta A_1(ws)\nu_{\alpha_1} + \Pi A_1^*(ws)\nu_{\alpha_1}, \quad U_{\alpha_1}^z = -\Gamma A_1(us)\nu_{\alpha_1} + \Xi A_1^*(us)\nu_{\alpha_1} \quad (46)$$

$$W_{\alpha_2}^z = -\Delta A_2(ws)\nu_{\alpha_2} + \Pi A_2^*(ws)\nu_{\alpha_2}, \quad U_{\alpha_2}^z = -\Gamma A_2(us)\nu_{\alpha_2} + \Xi A_2^*(us)\nu_{\alpha_2}. \quad (47)$$

Variables $A_q(ws), A_q^*(ws), B_q(ws), B_q^*(ws)$ are defined in equations (A-9) and (A-11) and variables $A_q(us), A_q^*(us), B_q(us), B_q^*(us)$ are defined in equations (A-3) and (A-5),

with $q = 1 \vee 2$, with,

$$\Gamma = G - \frac{1}{M} [HM - C^2] \quad (48)$$

$$\Xi = -2\omega^2 \rho_f C + M\omega^2 \frac{\rho_f^2}{\rho_E} + \frac{C^2}{M} \omega^2 \rho_E \quad (49)$$

$$\Delta = \frac{1}{C} [HM - C^2] - G \frac{\rho_E}{\rho_f} \quad (50)$$

$$\Pi = \frac{HM}{C} \omega^2 \rho_f + C\omega^2 \frac{\rho_B \rho_E}{\rho_f} - H\omega^2 \rho_E - M\omega^2 \rho_B \quad (51)$$

2.7.2. Radial Ring Source

For a radial ring source $F^r(x) = [\hat{r} \cos(\phi - \phi_0) - \hat{\phi} \sin(\phi - \phi_0)] \delta(r - r_0) \delta(z - z')$. Using the displacement Green's function, (A-1), and the scalar Green's functions, equations (A-2) and (A-6), which are rewritten using equation (7) and the orthogonality property in the integration over angle ϕ_0 , the displacements at (r, z) due to a ring of unit radial forces at $x' = (r_0, z')$ are obtained. When the relative flow Green's function, equation (A-7), and the scalar Green's functions, equations (A-8) and (A-12), are used instead, the relative flow vector components (r, z) due to a ring of unit radial forces at $x' = (r_0, z')$ are obtained.

$$u_r = i \frac{r_0}{2} \int_0^\infty J_1(kr) J_1(kr_0) \left[U_\beta^r e^{-i\nu_\beta |z-z'|} + U_{\alpha_1}^r e^{-i\nu_{\alpha_1} |z-z'|} + U_{\alpha_2}^r e^{-i\nu_{\alpha_2} |z-z'|} \right] dk \quad (52)$$

$$\begin{aligned} u_z &= \text{sgn}(z - z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_1(kr_0) \\ &\times \left[U_\beta^z e^{-i\nu_\beta |z-z'|} + U_{\alpha_1}^z e^{-i\nu_{\alpha_1} |z-z'|} + U_{\alpha_2}^z e^{-i\nu_{\alpha_2} |z-z'|} \right] dk \end{aligned} \quad (53)$$

$$w_r = i \frac{r_0}{2} \int_0^\infty J_1(kr) J_1(kr_0) \left[W_\beta^r e^{-i\nu_\beta |z-z'|} + W_{\alpha_1}^r e^{-i\nu_{\alpha_1} |z-z'|} + W_{\alpha_2}^r e^{-i\nu_{\alpha_2} |z-z'|} \right] dk \quad (54)$$

$$\begin{aligned} w_z &= \text{sgn}(z - z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_1(kr_0) \\ &\times \left[W_\beta^z e^{-i\nu_\beta |z-z'|} + W_{\alpha_1}^z e^{-i\nu_{\alpha_1} |z-z'|} + W_{\alpha_2}^z e^{-i\nu_{\alpha_2} |z-z'|} \right] dk \end{aligned} \quad (55)$$

Substituting equations (52)–(55) into the stress equation in cylindrical coordinates and incorporating the static contribution to τ_{rr} , when source and receiver position coincide, the stresses are obtained at $x = (r, z)$. When equations (52)–(55) are substituted into the pressure equation in cylindrical coordinates, the fluid traction is obtained at $x = (r, z)$.

$$\tau_{rr} = -\frac{1}{2} \delta(z - z') + i \frac{r_0}{2} \int_0^\infty J_1(kr_0) I_1 dk$$

$$I_1 = \sum_{p=\beta, \alpha_1, \alpha_2} \left[(HkU_p^r + CkW_p^r - (H - 2G)k\nu_p U_p^z - Ck\nu_p W_p^z) J_0(kr) - 2GU_p^r \frac{J_1(kr)}{r} \right] e^{-i\nu_p |z-z'|} \quad (56)$$

$$\tau_{rz} = \text{sgn}(z - z') \frac{r_0}{2} \int_0^\infty J_1(kr) J_1(kr_0) I_2 dk$$

$$I_2 = G \left[(\nu_\beta U_\beta^r - k^2 U_\beta^z) e^{-i\nu_\beta |z-z'|} + (\nu_{\alpha_1} U_{\alpha_1}^r - k^2 U_{\alpha_1}^z) e^{-i\nu_{\alpha_1} |z-z'|} + (\nu_{\alpha_2} U_{\alpha_2}^r - k^2 U_{\alpha_2}^z) e^{-i\nu_{\alpha_2} |z-z'|} \right] \quad (57)$$

$$\tau_{zz} = i \frac{r_0}{2} \int_0^\infty J_0(kr) J_1(kr_0) I_3 dk$$

$$I_3 = \sum_{p=\beta, \alpha_1, \alpha_2} \left[(H - 2G)kU_p^r + CkW_p^r - Hk\nu_p U_p^z - Ck\nu_p W_p^z \right] e^{-i\nu_p |z-z'|} \quad (58)$$

$$T_p = i \frac{r_0}{2} \int_0^\infty J_0(kr) J_1(kr_0) I_4 dk$$

$$I_4 = \sum_{i=\beta, \alpha_1, \alpha_2} \left[C(kU_p^r - \nu_p k U_p^z) + M(kW_p^r - \nu_p k W_p^z) \right] e^{-i\nu_p |z-z'|} \quad (59)$$

where the introduced variables, $W_\beta^r, W_{\alpha_1}^r, W_{\alpha_2}^r, U_\beta^r, U_{\alpha_1}^r, U_{\alpha_2}^r, W_\beta^z, W_{\alpha_1}^z, W_{\alpha_2}^z, U_\beta^z, U_{\alpha_1}^z, U_{\alpha_2}^z$ are defined as,

$$W_\beta^r = \frac{\rho_f k}{\rho_E G \nu_\beta} + B(ws) \Delta \frac{k^3}{\nu_\beta} - B^*(ws) \Pi \frac{k^3}{\nu_\beta}, \quad U_\beta^r = -\frac{k}{\nu_\beta G} \quad (60)$$

$$+ \Gamma B(us) \frac{k^3}{\nu_\beta} - \Xi B^*(us) \frac{k^3}{\nu_\beta} \quad (61)$$

$$W_{\alpha_1}^r = A_1(ws) \Delta \frac{k^3}{\nu_{\alpha_1}} - A_1^*(ws) \Pi \frac{k^3}{\nu_{\alpha_1}}, \quad U_{\alpha_1}^r = \Gamma A_1(us) \frac{k^3}{\nu_{\alpha_1}} - \Xi A_1^*(us) \frac{k^3}{\nu_{\alpha_1}} \quad (62)$$

$$W_{\alpha_2}^r = A_2(ws) \Delta \frac{k^3}{\nu_{\alpha_2}} - A_2^*(ws) \Pi \frac{k^3}{\nu_{\alpha_2}}, \quad U_{\alpha_2}^r = \Gamma A_2(us) \frac{k^3}{\nu_{\alpha_2}} - \Xi A_2^*(us) \frac{k^3}{\nu_{\alpha_2}} \quad (63)$$

$$W_\beta^z = [\Pi B^*(ws) - \Delta B(ws)] k, \quad U_\beta^z = [\Xi B^*(us) - \Gamma B(us)] k \quad (64)$$

$$W_{\alpha_1}^z = [\Pi A_1^*(ws) - \Delta A_1(ws)] k, \quad U_{\alpha_1}^z = [\Xi A_1^*(us) - \Gamma A_1(us)] k \quad (65)$$

$$W_{\alpha_2}^z = [\Pi A_2^*(ws) - \Delta A_2(ws)] k, \quad U_{\alpha_2}^z = [\Xi A_2^*(us) - \Gamma A_2(us)] k. \quad (66)$$

Variables $A_q(ws), A_q^*(ws), B_q(ws), B_q^*(ws)$ are defined in equations (A-9) and (A-11) and variables $A_q(us), A_q^*(us), B_q(us), B_q^*(us)$ are defined in equations (A-3) and (A-5), with $q = 1 \vee 2$, with variables, Γ, Ξ, Δ, Π defined in equation (48).

2.7.3. Explosive Ring Source in Porous Medium

For an explosive source located at (r_0, z') , $F^e(x) = V \delta(z - z') \delta(r - r_0)$. Using the displacement Green's function, (A-13), and the scalar Green's function, equation (A-14), which are rewritten

using equation (7), and the orthogonality property in the integration over angle ϕ_0 , the displacements at (r, z) due to a ring of explosive point forces at $x' = (r_0, z')$ are obtained. When the relative flow Green's function, equation (A-16), and the scalar Green's function, equation (A-17), are used instead, the relative flow vector components (r, z) due to a ring of explosive point forces at $x' = (r_0, z')$ are obtained.

$$u_r = -\frac{1}{2} \frac{C}{HM - C^2} \delta(z - z') + i \frac{r_0}{2} \int_0^\infty k^2 J_1(kr) J_0(kr_0) \left[U_{\alpha_1}^e e^{-i\nu_{\alpha_1}|z-z'|} + U_{\alpha_2}^e e^{-i\nu_{\alpha_2}|z-z'|} \right] dk \quad (67)$$

$$u_z = -\text{sgn}(z - z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \left[\nu_{\alpha_1} U_{\alpha_1}^e e^{-i\nu_{\alpha_1}|z-z'|} + \nu_{\alpha_2} U_{\alpha_2}^e e^{-i\nu_{\alpha_2}|z-z'|} \right] dk \quad (68)$$

$$w_r = \frac{1}{2} \frac{M}{HM - C^2} \delta(z - z') + i \frac{r_0}{2} \int_0^\infty J_1(kr) J_0(kr_0) \left[\Lambda_1 \frac{k^2}{\nu_{\alpha_1}} e^{-i\nu_{\alpha_1}|z-z'|} + \Lambda_2 \frac{k^2}{\nu_{\alpha_2}} e^{-i\nu_{\alpha_2}|z-z'|} \right] dk \quad (69)$$

$$w_z = -\text{sgn}(z - z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \left[\Lambda_1 e^{-i\nu_{\alpha_1}|z-z'|} + \Lambda_2 e^{-i\nu_{\alpha_2}|z-z'|} \right] dk. \quad (70)$$

Substituting equations (67)–(70) into the stress equation in cylindrical coordinates yields the stresses at $x = (r, z)$. When equations (52)–(70) are substituted into the pressure equation in cylindrical coordinates, the fluid traction is obtained at $x = (r, z)$.

$$\tau_{rr} = i \frac{r_0}{2} \int_0^\infty J_0(kr_0) I_1 dk$$

$$I_1 = \sum_{p=\alpha_1, \alpha_2} \left[\left(H k^2 U_p^e + (H - 2G) \nu_p^2 k U_p^e + C \Lambda_p \left(\frac{k^3}{\nu_p} + k \nu_p \right) \right) J_0(kr) - 2G U_p^e k^2 \frac{J_1(kr)}{r} \right] e^{-i\nu_p|z-z'|} \quad (71)$$

$$\tau_{rz} = \text{sgn}(z - z') r_0 \int_0^\infty k J_0(kr) J_0(kr_0) I_2 dk$$

$$I_2 = \sum_{p=\alpha_1, \alpha_2} G \nu_p k U_p^e e^{-i\nu_p|z-z'|} \quad (72)$$

$$\tau_{zz} = i \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) I_3 dk$$

$$I_3 = \left[H \nu_{\alpha_1}^2 U_{\alpha_1}^e + (H - 2G) k^2 U_{\alpha_1}^e + C \Lambda_1 \left(\frac{k^2}{\nu_{\alpha_1}} + \nu_{\alpha_1} \right) \right] e^{-i\nu_{\alpha_1}|z-z'|} + \left[H \nu_{\alpha_2}^2 U_{\alpha_2}^e + (H - 2G) k^2 U_{\alpha_2}^e + C \Lambda_2 \left(\frac{k^2}{\nu_{\alpha_2}} + \nu_{\alpha_2} \right) \right] e^{-i\nu_{\alpha_2}|z-z'|} \quad (73)$$

$$\begin{aligned}
T_p &= i \frac{r_0}{2} \int_0^\infty J_0(kr) J_0(kr_0) I_4 dk \\
I_4 &= \left[C(k^2 + \nu_{\alpha_1}^2) k U_{\alpha_1}^e + M \Lambda_1 \left(\frac{k^3}{\nu_{\alpha_1}} + k \nu_{\alpha_1} \right) \right] e^{-i \nu_{\alpha_1} |z-z'|} \\
&+ \left[C(k^2 + \nu_{\alpha_2}^2) k U_{\alpha_2}^e + M \Lambda_2 \left(\frac{k^3}{\nu_{\alpha_2}} + k \nu_{\alpha_2} \right) \right] e^{-i \nu_{\alpha_2} |z-z'|}
\end{aligned} \tag{74}$$

where the introduced variables $U_{\alpha_1}^e, U_{\alpha_2}^e$ are defined as,

$$U_{\alpha_1}^e = -[\Upsilon + \Omega] \frac{k_{\alpha_1}^2}{\nu_{\alpha_1}} (A_1(wf) + A_1^*(wf)) \tag{75}$$

$$U_{\alpha_2}^e = -[\Upsilon + \Omega] \frac{k_{\alpha_2}^2}{\nu_{\alpha_2}} (A_2(wf) + A_2^*(wf)). \tag{76}$$

Variables $A_q(wf), A_q^*(wf)$ are defined in equations (A-18) and (A-17), with $q = 1 \vee 2$, with,

$$\Upsilon = \frac{1}{C} \left[C^2 - HM + GC \frac{\rho_E}{\rho_f} \right] \tag{77}$$

$$\Omega = \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B. \tag{78}$$

2.7.4. Explosive Ring Source in Fluid

For an explosive source ring, the displacement potential is obtained by integrating $g(x, x') = e^{-ik_f |x-x'|} / (4\pi |x-x'|)$ with respect to angle ϕ ,

$$\phi = -\frac{ir_0}{2} \int_0^\infty \frac{k}{\nu_f} J_0(kr) J_0(kr_0) e^{-i \nu_f |z-z'|} dk. \tag{79}$$

The fluid displacement components are obtained by taking the derivative with respect of r and z of equation (79).

$$\begin{aligned}
u_r &= -\frac{1}{2} \delta(z-z') + \frac{ir_0}{2} \int_0^\infty \frac{k^2}{\nu_f} J_1(kr) J_0(kr_0) e^{-i \nu_f |z-z'|} dk \\
u_z &= -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) e^{-i \nu_f |z-z'|} dk
\end{aligned} \tag{80}$$

The stresses in the fluid, which are the negations of the fluid pressure, are,

$$\begin{aligned}
\tau_{rr} &= \rho \omega^2 \frac{ir_0}{2} \int_0^\infty \frac{k}{\nu_f} J_0(kr) J_0(kr_0) e^{-i \nu_f |z-z'|} dk \\
\tau_{zz} &= \tau_{rr} \\
\tau_{rz} &= 0.
\end{aligned} \tag{81}$$

2.7.5. Explosive Volume Source in an Open Borehole

The driving initial data for an explosive source at the center of a fluid filled borehole at the boundary wall are,

$$\begin{aligned} u_r &= \frac{V_s}{4\pi} i \int_0^\infty \frac{k^2}{\nu_f} J_1(kr_0) e^{-i\nu_f|z-z_s|} dk \\ \tau_{rr} &= \rho\omega^2 \frac{V_s}{4\pi} i \int_0^\infty \frac{k}{\nu_f} J_0(kr_0) e^{-i\nu_f|z-z_s|} dk \end{aligned} \quad (82)$$

$$\tau_{rz} = 0 \quad (83)$$

where the vertical position of the source and the element position are denoted by z_s and z , respectively. The strength of the volume source V_s [dim m^3] is taken 1600 cm^3 .

2.8. RING SOURCES IN HORIZONTALLY LAYERED POROELASTIC MEDIA

To include the ring sources into layered media, we first manipulate the wavefield components that will be continuous across the horizontal boundaries in the following form:

$$\begin{aligned} u_1(r, z) &= \int_0^\infty \hat{u}_1(k, z) k J_0(kr) dk \\ u_z(r, z) &= \int_0^\infty \hat{u}_z(k, z) k J_0(kr) dk \\ w_z(r, z) &= \int_0^\infty \hat{w}_z(k, z) k J_0(kr) dk \\ \tau_1(r, z) &= \int_0^\infty \hat{\tau}_1(k, z) k J_0(kr) dk \\ \tau_{zz}(r, z) &= \int_0^\infty \hat{\tau}_{zz}(k, z) k J_0(kr) dk \\ T_p(r, z) &= \int_0^\infty \hat{T}_p(k, z) k J_0(kr) dk \end{aligned} \quad (84)$$

with $\hat{u}_1 = \frac{\hat{u}_V}{ik}$ and $u_V = \frac{1}{r} \left[\frac{\partial}{\partial r}(ru_r) + \frac{\partial}{\partial \phi}(u_\phi) \right]$, $\hat{\tau}_1 = \frac{\hat{\tau}_{Vz}}{ik}$ and $\tau_{Vz} = \frac{1}{r} \left[\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial}{\partial \phi}(\tau_{\phi z}) \right]$.

The displacement stress wavefield vector for a vertical ring source is given by,

$$\begin{aligned} \left[u_1, u_z, w_z, \tau_1, \tau_{zz}, T_p \right]_v^T &= \sum_{p=\beta, \alpha_1, \alpha_2} \left(\left[\frac{1}{ik} \left(-\text{sgn}(z-z') U_p^r \right), -iU_p^z, -iW_p^z, \frac{1}{ik} (iGS_1^p), -\text{sgn}(z-z') S_2^p, \right] \right. \\ &\quad \left. -\text{sgn}(z-z') S_3^p \times \frac{r_0}{2} J_0(kr_0) e^{-i\nu_p|z-z'|} \right)^T \end{aligned} \quad (85)$$

with variables, $W_\beta^r, W_{\alpha_1}^r, W_{\alpha_2}^r, U_\beta^r, U_{\alpha_1}^r, U_{\alpha_2}^r, W_\beta^z, W_{\alpha_1}^z, W_{\alpha_2}^z, U_\beta^z, U_{\alpha_1}^z, U_{\alpha_2}^z$ defined in (41) and (44). Variables S_1^p, S_2^p, S_3^p with $p = \beta \vee \alpha_1 \vee \alpha_2$ are defined as,

$$S_1^\beta = 2k^2 \left[U_\beta^z - \frac{1}{G\nu_\beta} \right], \quad S_1^{\alpha_1} = 2k^2 U_{\alpha_1}^z, \quad S_1^{\alpha_2} = 2k^2 U_{\alpha_2}^z$$

$$\begin{aligned}
S_2^p &= (H - 2G)U_p^r + CW_p^r + H\nu_p U_p^z + C\nu_p W_p^z \\
S_3^p &= C[U_p^r + \nu_p U_p^z] + M[W_p^r + \nu_p W_p^z].
\end{aligned} \tag{86}$$

The displacement stress wavefield vector for a radial ring source is given by,

$$\begin{aligned}
\left[u_1, u_z, w_z, \tau_1, \tau_{zz}, T_p \right]_r^T &= \sum_{i=\beta, \alpha_1, \alpha_2} \\
&\left(\left[\frac{1}{ik} (iU_p^r), \operatorname{sgn}(z - z')U_p^z, \operatorname{sgn}(z - z')W_p^z, \frac{1}{ik} (\operatorname{sgn}(z - z')GS_1^p), iS_2^p, iS_3^p \right] \right. \\
&\left. \times \frac{r_0}{2} J_1(kr_0) e^{-i\nu_p |z - z'|} \right)^T
\end{aligned} \tag{87}$$

With variables, $W_\beta^r, W_{\alpha_1}^r, W_{\alpha_2}^r, U_\beta^r, U_{\alpha_1}^r, U_{\alpha_2}^r, W_\beta^z, W_{\alpha_1}^z, W_{\alpha_2}^z, U_\beta^z, U_{\alpha_1}^z, U_{\alpha_2}^z$ defined in (60) and (64). Variables S_1^p, S_2^p, S_3^p with $p = \beta \vee \alpha_1 \vee \alpha_2$ are defined as,

$$\begin{aligned}
S_1^p &= U_p^r \nu_p - k^2 U_p^z \\
S_2^p &= (H - 2G)U_p^r + CW_p^r - H\nu_p U_p^z - C\nu_p W_p^z \\
S_3^p &= C[U_p^r - \nu_p U_p^z] + M[W_p^r - \nu_p W_p^z]
\end{aligned} \tag{88}$$

The displacement stress wavefield vector for an explosive ring source is given by,

$$\begin{aligned}
\left[u_1, u_z, w_z, \tau_1, \tau_{zz}, T_p \right]_e^T &= \sum_{p=\alpha_1, \alpha_2} \\
&\left(\left[\frac{1}{ik} (ik^2 U_p^e), -\operatorname{sgn}(z - z')\nu_p U_p^e, -\operatorname{sgn}(z - z')\Lambda_p, \frac{1}{ik} \operatorname{sgn}(z - z')S_1^p, iS_2^p, iS_3^p \right] \right. \\
&\left. \times \frac{r_0}{2} J_0(kr_0) e^{-i\nu_p |z - z'|} \right)^T
\end{aligned} \tag{89}$$

with variables, $U_{\alpha_1}^e, U_{\alpha_2}^e$ defined in equation (77). Variables S_1^p, S_2^p, S_3^p with $p = \alpha_1 \vee \alpha_2$ are defined as,

$$S_1^i = 2Gk^2 \nu_i U_i^e \tag{90}$$

$$S_2^{\alpha_1} = H\nu_{\alpha_1}^2 / k U_{\alpha_1}^e + (H - 2G)k U_{\alpha_1}^e + C\Lambda_1 \left(\frac{k^2}{\nu_{\alpha_1}} + \nu_{\alpha_1} \right) \tag{91}$$

$$S_2^{\alpha_2} = H\nu_{\alpha_2}^2 / k U_{\alpha_2}^e + (H - 2G)k U_{\alpha_2}^e + C\Lambda_2 \left(\frac{k^2}{\nu_{\alpha_2}} + \nu_{\alpha_2} \right) \tag{92}$$

$$S_3^{\alpha_1} = C(k^2 + \nu_{\alpha_1}^2) U_{\alpha_1}^e + M\Lambda_1 \left(\frac{k^2}{\nu_{\alpha_1}} + \nu_{\alpha_1} \right) \tag{93}$$

$$S_3^{\alpha_2} = C(k^2 + \nu_{\alpha_2}^2) U_{\alpha_2}^e + M\Lambda_2 \left(\frac{k^2}{\nu_{\alpha_2}} + \nu_{\alpha_2} \right). \tag{94}$$

To match the boundary conditions at the borehole wall the w_r and τ_{rr} field components are needed, see equation (1). The radial relative flow component is obtained from the Fourier transformed transformation equation for the fluid expressed in cylindrical coordinates.

$$\hat{T}_p(k, z) = C \left[\hat{u}_1(k, z) + \frac{\partial}{\partial z} \hat{u}_z(k, z) \right] + M \left[\hat{w}_1(k, z) + \frac{\partial}{\partial z} \hat{w}_z(k, z) \right] \tag{95}$$

Therefore,

$$\hat{w}_1(k, z) = -\frac{\partial}{\partial z} \hat{w}_z(k, z) + \frac{1}{M} \left[\hat{T}_p(k, z) - C \left(\hat{u}_1(k, z) + \frac{\partial}{\partial z} \hat{u}_z(k, z) \right) \right]. \quad (96)$$

The radial component can be recovered using equation (103).

The radial stress component is obtained by using the deformation equation for the bulk in cylindrical coordinates.

$$\tau_{rr}(r, z) = H \frac{\partial}{\partial z} u_r(r, z) + [H - 2G] u_1(r, z) + C \left[w_1(r, z) + \frac{\partial}{\partial z} w_z(r, z) \right] \quad (97)$$

with $u_1(r, z)$, $w_z(r, z)$ obtained using equation (84) and $w_1(r, z)$ obtained using (56) and (84) and $u_r(r, z)$ obtained using (84) and (103).

2.9. MECHANICAL AND ELECTROMAGNETIC WAVES IN HORIZONTALLY LAYERED POROELASTIC MEDIA

We first transform the macroscopic governing equations that control the coupled electromagnetics and acoustics of porous media (Pride and Haartsen, 1995) into a field-vector-formalism that can be solved numerically. The global matrix method (Chin *et al.*, 1984) is employed to solve simultaneously the macroscopic electromagnetic and acoustic-elastic wavefield amplitudes which contribute to current and fluid flow. The current and fluid flow are related by the transport equations through which all electromagnetic and mechanical coupling occurs, i.e., currents generated by the acoustic-elastic wavefields, an electrokinetic phenomenon and fluid flow generated by the electromagnetic field, an osmosis phenomenon. In the global matrix method we propagate a wavefield vector which contains the mechanical stress/displacement and electromagnetic wavefield components that are continuous through the horizontal boundaries in the layered porous medium (Haartsen and Pride, 1995).

For isotropic media, the first order ordinary differential equations, the equations of state, relating the wavefields at all depths to the medium properties at depth, decompose into two equation sets. The first set describes the *PSVTM* picture, which is the situation where the compressional and vertical polarized mechanical waves drive currents in the *PSV* particle motion plane that couple to the electromagnetic wavefield components of the *TM* mode. The second set describes the *SHTE* picture where the horizontal polarized rotational waves drive currents in the *SH* particle motion plane that couples to the electromagnetic wavefield components of the *TE* mode. In this chapter we compute electroseismograms generated by three mechanical ring sources in the *PSVTM* polarization picture only.

The effect of a general mechanical ring source in a stratified porous medium is accommodated by specifying a jump in the displacement-stress vector across a horizontal plane, (see Hudson, 1969a,b , and Kennett, 1983). The wavefield components in the displacement-stress vector calculated for a vertical, radial, and explosive ring source, equations (85), (87) and (89) respectively, are the wavefields that are continuous at the horizontal boundaries in the stratified medium for the *PSVTM* picture (Pride and Haartsen, 1995; Haartsen and Pride, 1995). We define the total displacement-stress-EM wavefield vector as follows,

$$\underline{B} = [u_1, u_2, w_z, \tau_1, \tau_{zz}, T_p, H_2, E_1]^T. \quad (98)$$

At $z = z'$, with z' the source depth only the u_1 , τ_{zz} and T_p wavefields jump in the vertical point source representation across a dummy interface. Where a dummy interface denotes a boundary across which none of the medium properties change. The final jump representation across a source plane for a vertical ring source, using equations (85) and (98) reads,

$$\begin{aligned}\underline{S}(z') &= \left[\lim_{z \downarrow z'} \underline{B}(z) - \lim_{z \uparrow z'} \underline{B}(z) \right] \\ &= \sum_{p=\beta, \alpha_1, \alpha_2} \left[-\frac{r_0}{ik^2} U_p^r J_0(kr_0), 0, 0, 0, -r_0 S_2^p J_0(kr_0), \right. \\ &\quad \left. -r_0 S_3^p J_0(kr_0), 0, 0 \right]^T.\end{aligned}\quad (99)$$

At $z = z'$, with z' the source depth only the u_z , τ_1 and τ_{zz} wavefields jump in the radial ring source representation across a dummy interface. The final jump representation across a source plane for a radial ring source, using equations (87) and (98) reads,

$$\begin{aligned}\underline{S}(z') &= \left[\lim_{z \downarrow z'} \underline{B}(z) - \lim_{z \uparrow z'} \underline{B}(z) \right] \\ &= \sum_{p=\beta, \alpha_1, \alpha_2} \left[0, r_0 U_p^z J_1(kr_0), r_0 W_p^z J_1(kr_0), \right. \\ &\quad \left. r_0 \frac{1}{ik} G S_1^p J_1(kr_0), 0, 0, 0, 0 \right]^T.\end{aligned}\quad (100)$$

At $z = z'$, with z' the source depth only the u_z , τ_1 and τ_{zz} wavefields jump in the radial ring source representation across a dummy interface. The final jump representation across a source plane for an explosive ring source, using equations (89) and (98) reads,

$$\begin{aligned}\underline{S}(z') &= \left[\lim_{z \downarrow z'} \underline{B}(z) - \lim_{z \uparrow z'} \underline{B}(z) \right] \\ &= \sum_{p=\beta, \alpha_1, \alpha_2} \left[0, -r_0 \nu_p U_p^e J_0(kr_0), -r_0 \Lambda_p J_0(kr_0), \right. \\ &\quad \left. \frac{r_0}{ik} S_1^p J_0(kr_0), 0, 0, 0, 0 \right]^T.\end{aligned}\quad (101)$$

Note that the last two electromagnetic wavefield components in equations (99), (100) and (101) are zero since no electromagnetic source is applied.

2.10. TRANSFORMATION BACK TO THE SPACE TIME DOMAIN

An inverse Hankel transform is applied to obtain the 3-D spatial dependence of the displacements, stresses, electric, and magnetic fields. The horizontal components of the displacements, stresses, electric, and magnetic fields require additional integration over r and ϕ to obtain, $u_r, u_\phi, \tau_{\phi z}, \tau_{rz}, H_r, H_\phi$ and E_r, E_ϕ due to the definitions of $u_V, u_H, \tau_{Vz}, \tau_{Hz}, H_V, H_H$ and E_V, E_H .

$$u_z(\omega, r, \phi, z) = \int_0^\infty k dk \sum_{n=-N}^N [J_n(kr) \hat{u}_z(\omega, k, n, z)] e^{in\phi} \quad (102)$$

Identical relations exist for w_z, τ_{zz} and T_p .

$$u_r(\omega, r, \phi, z) = \int_0^\infty k dk \sum_{n=-N}^N \left[\frac{n}{kr} J_n(kr) \hat{u}_2(\omega, k, n, z) - i J'_n(kr) \hat{u}_1(\omega, k, n, z) \right] e^{in\phi} \quad (103)$$

The horizontal components may be recovered using the following relations:

$$u_\phi(\omega, r, \phi, z) = \int_0^\infty k dk \sum_{n=-N}^N \left[\frac{n}{kr} J_n(kr) \hat{u}_1(\omega, k, n, z) + i J'_n(kr) \hat{u}_2(\omega, k, n, z) \right] e^{in\phi}. \quad (104)$$

Identical relations exist for $\tau_{rz}, \tau_{\phi z}$ in terms of $\hat{\tau}_1$ and $\hat{\tau}_2$, for E_r, E_ϕ in terms of $\hat{E}_1$ and $\hat{E}_2$ and H_r, H_ϕ in terms of $\hat{H}_1$ and $\hat{H}_2$. The above representations may be regarded as a superposition of cylindrical waves whose order dictates the nature of their azimuthal modulation. At each frequency and angular order the radial contribution is obtained by superposing all horizontal wavenumbers k . This corresponds to including all propagating waves at the level z within the stratification, from vertically to purely horizontal travelling waves, including the evanescent waves. At any particular distance r the relative contributions of the wavenumbers are imposed by the radial phase functions $J_n(kr)$.

2.11. NUMERICAL RESULTS

In this section, we discuss the vertical, radial, and volume injection ring source results in layered media. The final *BEM* algorithm calls these vertical, radial, and volume injection ring sources in layered medium functions for each source/receiver element combination, including the self-interacting elements which are analytically included. The wavefield components, calculated at the receiving elements, that match the boundary conditions across the borehole wall, are calculated by discrete wavenumber integration and each element interaction fills an entry in the square boundary condition matrix. The wavenumber integral becomes computationally more and more expensive with decreasing vertical wavefield propagation, i.e. the interaction between neighboring elements are computationally more expensive than the interaction between further separated elements. Even with all the interacting element symmetries exploited, the *BEM* solution in layered media is prohibitively computationally expensive. Although numerically feasible on a massive parallel computer, implementing the *BEM* algorithm on the nCUBE did not have a practical runtime performance.

Therefore, in this section we will the secondary ring source results for three models in the *PSVTM* wavefield picture only. The first example is a simple surface electroseismic model example (Figure 1). The electroseismic response of the three different ring sources are modelled. In Figures 2, 4 and 6, the mechanical and electroseismograms (*TM* mode components) are depicted for a volume injection ring source, a vertical and radial ring source. Only the reflection/conversion results are shown, i.e., the direct wavefields are not included. The last two examples show the electroseismic effect in a Vertical Electroseismic Profiling (*VESP*) setting. The seismic ring source is located in the upper halfspace and the recording geophone/antenna are positioned vertically crossing two mechanical/electrical contrasts in the first *VESP* example (Figure 7) and an additional fluid chemistry contrast in the second *VESP* example (Figure 11). In Figures 8, 9 and 10 the

calculated results are shown for the volume injection ring source, the vertical ring source, and the radial ring source, respectively. Figures 12, 13 and 14 show the additional electroseismic response of the included fluid chemistry contrast within the sand reservoir generated by the three ring sources.

2.11.1. Electroseismics in Surface Geometry

The ring source is positioned 100 *m* above the contrast and has a 100 *Hz* center frequency. Fifteen antennas/geophones are positioned symmetrically in a straight horizontal line at both sides of the source, 95 *m* above the interface. The receiver spacing is 10 *m*. The medium properties of the two halfspaces and the calculated wave velocities and bulk conductivities, calculated at source center frequency, are given in Tables I, II and III. All plots in Figures 2, 4 and 6 are seismogram scaled. When a volume injection ring source is set off, only *P* waves are generated in an isotropic poroelastic medium while the vertical and radial ring sources generate *P* as well as *SV* waves. The mechanical displacement seismograms (u_z and u_x components) in Figures 4 and 6 show clearly the difference in amplitude variability versus offset to be expected for vertical and radial ring sources. For example, the u_z component in the *PP* reflection is maximum at zero offset reflection for a vertical ring source, while the same wavefield component is minimum at zero offset reflection for a radial ring source.

The converted *TM* wavefield components show up at all antennas at approximately half the two-way *P* wave traveltime for normal incidence reflection. The hyperbolas arriving at later time in the E_x and H_y electroseismograms appear at the same traveltime as the *PP* reflection, *PSV* conversion and *SVSV* reflection, *SVP* conversion for the vertical and radial ring sources only. The electric fields travelling with compressional wavespeed through a homogeneous porous medium are caused by a charge separation generated by pressure gradients inside the seismic pulse. The magnetic fields travelling with rotational wavespeed through homogeneous porous medium are induced by current sheets generated by the divergence free grain accelerations.

The vertical and radial ring sources generate radiating *EM* wavefields. These fields show up in the electroseismograms (Figures 4 and 6) at approximately zero traveltime. The electroseismograms are slightly shifted in time to avoid signal wrapping in time, which will be introduced by the convolution of the impulse response with a non-causal Ricker wavelet (Hosken, 1988). Physically, a vertical and radial ring source induce a vertical and radial streaming current, respectively, at a hypothetical source plane, due to a mechanically driven vertical and radial relative flow. The electromagnetic fields associated with these mechanically induced streaming currents at the source plane need to be balanced by electromagnetic fields that make the E_x and H_y field component entries in the vertical and radial jump representations across the source plane vanish, see equations (99) and (100), respectively. Due to symmetry properties, the volume injection ring source doesn't radiate *EM* fields away from the mechanical source. In Figures 3 and 5 the expected radiation pattern for a vertical and radial ring source above a contrast is outlined. The amplitude behavior of the reflected electromagnetic signal is depicted for a hypothetical perfect reflector case, using the method of images. The horizontal component of the reflected electromagnetic field is zero right at the center of the ring sources and increases with horizontal distance. This amplitude behavior can be identified in the electroseismograms in Figures 4 and 6 and the cartoons in Figures 3 and 5. The length of the arrows in Figures 3 and 5 are an indication of the the absolute field amplitudes.

The reflected *EM* wavefields generated by the radial ring source are much smaller than the fields generated by a vertical ring source.

The electroseismic conversion of an incoming *SV* wave generated by the vertical and radial ring sources is insignificant to the pressure gradient, *P* wave induced converted electromagnetic signals. The *SV* to *EM* conversion is not discernible at the the used electroseismogram scaling in Figures 4 and 6. The importance of a dynamic vertical pressure gradient imbalance in the conversion to electromagnetic signals is evident when we compare the volume injection and vertical ring source results with the radial ring source result. The *P* wave conversion into electromagnetic signal is orders of magnitude smaller in the radial ring source synthetics, since its *P* wave radiation pattern points mainly to the horizontal direction parallel to the contrast, as opposed to the volume injection and vertical ring source radiation pattern that point their *P* wave energy mainly towards the contrast.

2.11.2. Electroreismograms in VESP Geometry

In the *VESP* geometry the antennas are positioned close to the target of interest and therefore larger converted electromagnetic signals can be recorded before they become too attenuated with travelled distance. With the *VESP* technique, the electroseismic method can be extended to targets (electrical and/or mechanical contrasts) at greater depths. The first *VESP* example is a sandwiched reservoir sand in between two low permeability halfspaces with smaller porosity. The first receiver/antenna is positioned at 20 *m* horizontal offset and a depth of 40 *m*. The receiver/antenna spacing is 5 *m*. The model is depicted in Figure 7. The complete set of medium properties and the at source center frequency calculated wavefield velocities and bulk conductivities are given in Tables I, II and III. In Figures 8, 9 and 10, the numerically calculated *VESP*'s are shown for a volume injection source, a vertical, and radial ring source. The top plot shows the mechanical displacement response, the center and bottom plots show the E_x and H_y *TM* medium components, respectively.

The converted *TM* wavefield components show up at all receiving antennas at approximately the same time. These events can be recognized as the straight vertical lines in the figures. All later arrivals in the E_x *VESP* are the electric fields within the seismic pulse that travel with the compressional wavespeed. The later arrivals in the H_y *VESP* are the magnetic fields within the seismic pulse that travel with the rotational wavespeed. When we compare the electroseismograms generated by a vertical and radial ring source, we notice the very strong *SV* and *SV* induced signals in the radial ring source seismogram and electroseismograms. The vertical ring source seismogram shows strong *P* wave conversions into *TM* signal. The conversion from *SV* into *TM* signal is very weak for the shear waves generated by the vertical ring source. The radial ring source on the other hand shows strong *SV* to electromagnetic wave conversions. The *SV* wave converted signals are in this case larger in amplitude than the *P* wave converted signals.

The last numerical example has an additional electrical contrast added to the previous model (Figure 11). The 100 *m* thick "reservoir sand" is divided into two sands with identical mechanical properties saturated with two fluids of different salinity. In Figures 12, 13 and 14 the calculated *VESP*'s for a volume injection ring source, a vertical and a radial ring source are shown, respectively. The mechanical response is identical to the previous calculations without the electrical

fluid contrast (Figures 8, 9 and 10). But the *TM* mode component *VESP*'s show an additional electroseismic conversion generated at the electrical contrast.

2.12. CONCLUSIONS AND DISCUSSION

The Biot-Rosenbaum model is extended by including the effect of a heterogeneous formation surrounding the borehole and by including the conversions of mechanical into electromagnetic waves at the mechanical and/or electrical contrasts in the poroelastic formation. The method is formulated as a boundary element technique where the poroelastic mechanical wavefield components to be matched across the borehole wall are calculated by the discrete wavenumber method, integrating the dynamic poroelastic Green's function wavefield solutions. The coupled electromagnetic and acoustic wavefields in layered media are included by employing the global matrix technique.

The singular properties of the stress field for the dynamic Green's functions with a force applied on the solid phase and the singular property of the displacement for the dynamic Green's function with a force applied on the fluid phase, when source and receiving element coincide, are determined analytically by integration of the derived static Green's functions over a half spherical surface.

Four secondary ring sources are introduced: a volume injection ring source (generating wavefields diffracted into the fluid), a vertical and radial ring source acting on the elastic frame, and a volume injection ring source acting on the pore fluid. The last three sources give rise to the wavefields radiated into the poroelastic formation.

The developed *BEM* algorithm needs to be optimized to decrease its runtime on the nCUBE to allow full waveform modelling of seismic and converted electromagnetic waves in a borehole embedded in a layered porous medium. The Stoneley waves and converted electromagnetic signals both contain information about permeability. The converted electromagnetic signals are also sensitive to fluid chemistry contrasts (i.e. oil-water contact) in a homogeneous reservoir. The study of these effects on the full waveforms awaits the optimization of the developed algorithm.

In this paper, three mechanical source jump representations for the three ring sources in the poroelastic formation are derived. These jump conditions are included in the global matrix technique and constitute the kernel program functions of the *BEM* algorithm. The numerical results show that the vertical and radial ring sources generate radiating electromagnetic fields. The induced step in electrical streaming current imposes a jump in electromagnetic fields to accommodate for the predefined zero magnetic and electric field components (since no electromagnetic source is applied) in the vertical and radial ring source jump representations across the source plane.

The conversion to electromagnetic disturbances is found to be mainly due to pressure gradients generated by the *P* waves across the contrasts for the vertical and volume injection ring source. Even though rotational waves generated by the vertical ring source induce larger streaming currents, they do not necessarily create larger current imbalances across the interface and therefore no larger converted electromagnetic disturbances, whereas in the radial ring source case, the *SV* wave induced current imbalances across the contrasts account for the main conversions to electromagnetic disturbances. This illustrates the magnitude dependence of the electroseismic conversions from *SV* waves, on the *SV* radiation pattern orientation with respect to the interface.

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TABLE I. The medium properties in the layers used in the numerical calculations.

Property	Top and bottom halfspaces	Salinity I layer	Salinity II layer	Salinity III halfspace
porosities ($\phi[\%]$)	15	30	30	30
dc permeability ($k[m^2]$)	10^{-14}	10^{-12}	10^{-12}	10^{-12}
bulk modulus solid ($k_s[Pa]$)	3.0×10^{10}	1.0×10^{10}	1.0×10^{10}	1.0×10^{10}
bulk modulus fluid ($k_f[Pa]$)	2.2×10^9	2.2×10^9	2.2×10^9	2.2×10^9
frame bulk modulus ($k_{fr}[Pa]$)	8.0×10^9	3.0×10^9	3.0×10^9	3.0×10^9
frame shear modulus ($g_{fr}[Pa]$)	9.0×10^9	8.5×10^9	8.5×10^9	8.5×10^9
fluid viscosity ($\eta[Pa\cdot s]$)	1.0×10^{-3}	1.0×10^{-3}	1.0×10^{-3}	1.0×10^{-3}
density solid ($\rho_s[Kg/m^3]$)	2.7×10^3	2.7×10^3	2.7×10^3	2.7×10^3
density fluid ($\rho_f[Kg/m^3]$)	1.0×10^3	1.0×10^3	1.0×10^3	1.0×10^3
salinity ($C[mol/l]$)	1.0×10^{-3}	1.0×10^{-2}	1.0×10^{-1}	1.0
temperature ($T[K]$)	298	298	298	298
permittivity (κ_f)	80	80	80	80
permittivity (κ_s)	4	4	4	4
tortuosity (α_∞)	3	3	3	3

TABLE IIa. The calculated (at center frequency) wavefield velocities and bulk conductivities in the surface seismic and VESP geometry models.

Properties	Upper and lower halfspace	Salinity I layer
fast wave velocity[m/s]	(3154.97, -0.00011)	(2502.21, -0.0524)
slow wave velocity[m/s]	(5.52, -5.28)	(40.54, -38.17)
shear wave[m/s]	(1768.85, -0.00248)	(1648.32, -0.258)
TM wave velocity[m/s]	(827466.0/-792166.0)	(192248.0/-184061.0)
conductivity ($\sigma[S/m]$)	0.000416	0.0771

TABLE IIb. The calculated (at center frequency) wavefield velocities and bulk conductivities in the surface seismic and VESP geometry models.

Properties	Salinity III halfspace
fast wave velocity[m/s]	(2502.21, -0.0524)
slow wave velocity[m/s]	(40.54, -38.17)
shear wave[m/s]	(1648.32, -0.258)
TM wave velocity[m/s]	(19238.2, -18419.1)
conductivity ($\sigma[S/m]$)	0.77

APPENDIX A. DYNAMIC AND STATIC GREEN'S FUNCTIONS IN POROELASTIC MEDIA

Displacement Green's Function with Source on Frame

To evaluate the element surface integrals, the following poroelastic Green's functions are used. The detailed derivation of the dynamic Green's functions is outlined in (Haartsen, 1995, chapter 5).

$$\underline{\underline{G}}_s^u = g^u \underline{\underline{I}} + \left[G - \frac{1}{M}(HM - C^2) \right] \nabla \nabla \phi - \left[-2\omega^2 \rho_f C + M\omega^2 \frac{\rho_f^2}{\rho_E} + \frac{C^2}{M} \omega^2 \rho_E \right] \nabla \nabla \phi^* \quad (\text{A-1})$$

where the superscript u denotes the Green's function to be a displacement Green's function and subscript s denotes a point force applied to the solid matrix. With,

$$\phi = B(us)g_\beta + A_1(us)g_{\alpha 1} + A_2(us)g_{\alpha 2} = B(us)\frac{e^{ik_\beta R}}{4\pi R} + A_1(us)\frac{e^{ik_{\alpha 1} R}}{4\pi R} + A_2(us)\frac{e^{ik_{\alpha 2} R}}{4\pi R}. \quad (\text{A-2})$$

With the introduced variables $B(us)$, $A_1(us)$, $A_2(us)$, where (us) denotes scalar Green's function amplitudes belonging to the displacement Green's function with a source applied on the solid frame defined as,

$$\begin{aligned} B(us) &= \frac{1}{G[HM - C^2]} \frac{-k_\beta^2 M + \omega^2 \rho_E}{(k_\beta^2 - k_{\alpha 1}^2)(k_\beta^2 - k_{\alpha 2}^2)} \\ A_1(us) &= \frac{1}{G[HM - C^2]} \frac{-k_{\alpha 1}^2 M + \omega^2 \rho_E}{(k_{\alpha 1}^2 - k_\beta^2)(k_{\alpha 1}^2 - k_{\alpha 2}^2)} \\ A_2(us) &= \frac{1}{G[HM - C^2]} \frac{-k_{\alpha 2}^2 M + \omega^2 \rho_E}{(k_{\alpha 2}^2 - k_\beta^2)(k_{\alpha 2}^2 - k_{\alpha 1}^2)} \end{aligned} \quad (\text{A-3})$$

where ϕ^* is defined as,

$$\phi^* = B^*(us)g_\beta + A_1^*(us)g_{\alpha 1} + A_2^*(us)g_{\alpha 2}. \quad (\text{A-4})$$

With the variables $B^*(us)$, $A_1^*(us)$, $A_2^*(us)$ (variables $B(us)$, $A_1(us)$, $A_2(us)$ divided by Ξ) defined as,

$$B^*(us) = \frac{B(us)}{\omega^2 \rho_E - M k_\beta^2}, \quad A_1^*(us) = \frac{A_1(us)}{\omega^2 \rho_E - M k_{\alpha 1}^2}, \quad A_2^*(us) = \frac{A_2(us)}{\omega^2 \rho_E - M k_{\alpha 2}^2}. \quad (\text{A-5})$$

With the displacement scalar shear Green's function,

$$g^u(\mathbf{x}, \mathbf{x}') = \frac{e^{ik_0 R}}{4\pi G R} \quad (\text{A-6})$$

where $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$ is the distance from the source to the receiver and $k_0 = \omega / \sqrt{G/(\rho_B - \frac{\rho_f^2}{\rho_E})}$ is the wavenumber.

Relative Flow Green's Function with Source on Frame

$$\begin{aligned} \underline{\underline{G}}_s^w &= g^w \underline{\underline{I}} + \left[\frac{1}{C} (HM - C^2) - G \frac{\rho_E}{\rho_f} \right] \nabla \nabla \Phi \\ &- \left[\frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \nabla \Phi^* \end{aligned} \quad (\text{A-7})$$

where the superscript w denotes the Greens's function to be a relative flow Green's function and the subscript s denotes the point force applied to the solid matrix. With,

$$\Phi = B(ws)g_\beta + A_1(ws)g_{\alpha 1} + A_2(ws)g_{\alpha 2} = B(ws)\frac{e^{ik_\beta R}}{4\pi R} + A_1(ws)\frac{e^{ik_{\alpha 1} R}}{4\pi R} + A_2(ws)\frac{e^{ik_{\alpha 2} R}}{4\pi R}. \quad (\text{A-8})$$

With the introduced variables $B(ws)$, $A_1(ws)$, $A_2(ws)$, where (ws) denotes scalar Green's function amplitudes belonging to the relative flow Green's function with a source applied on the solid frame defined as,

$$\begin{aligned} B(ws) &= \frac{1}{G[HM - C^2] \frac{\rho_E}{\rho_f}} \frac{-k_\beta^2 C + \omega^2 \rho_f}{(k_\beta^2 - k_{\alpha 1}^2)(k_\beta^2 - k_{\alpha 2}^2)} \\ A_1(ws) &= \frac{1}{G[HM - C^2] \frac{\rho_E}{\rho_f}} \frac{-k_{\alpha 1}^2 C + \omega^2 \rho_f}{(k_{\alpha 1}^2 - k_\beta^2)(k_{\alpha 1}^2 - k_{\alpha 2}^2)} \\ A_2(ws) &= \frac{1}{G[HM - C^2] \frac{\rho_E}{\rho_f}} \frac{-k_{\alpha 2}^2 C + \omega^2 \rho_f}{(k_{\alpha 2}^2 - k_\beta^2)(k_{\alpha 2}^2 - k_{\alpha 1}^2)} \end{aligned} \quad (\text{A-9})$$

where Φ^* is defined as,

$$\Phi^* = B^*(ws)g_\beta + A_1^*(ws)g_{\alpha 1} + A_2^*(ws)g_{\alpha 2}. \quad (\text{A-10})$$

With the variables $B^*(ws)$, $A_1^*(ws)$, $A_2^*(ws)$ (variables $B(ws)$, $A_1(ws)$, $A_2(ws)$, divided by ξ) defined as,

$$B^*(ws) = \frac{B(ws)}{\omega^2 \rho_f - C k_\beta^2}, \quad A_1^*(ws) = \frac{A_1(ws)}{\omega^2 \rho_f - C k_{\alpha 1}^2}, \quad A_2^*(ws) = \frac{A_2(ws)}{\omega^2 \rho_f - C k_{\alpha 2}^2}. \quad (\text{A-11})$$

The relative flow scalar shear Green's function is given by,

$$g^w(\mathbf{x}, \mathbf{x}') = -\frac{\rho_f}{\rho_E} \frac{e^{ik_0 R}}{4\pi G R} \quad (\text{A-12})$$

where $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$ is the distance from the source to the receiver and $k_0 = \omega / \sqrt{G/(\rho_B - \frac{\rho_f^2}{\rho_E})}$ is the wavenumber.

Displacement Green's Function with Source on Fluid

$$\begin{aligned}
 \underline{G}_f^u &= \nabla \cdot \underline{G}_s^w \\
 &= \frac{1}{C} \left[C^2 - HM + GC \frac{\rho_E}{\rho_f} \right] \nabla \Pi + \left[\frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E \right. \\
 &\quad \left. - M \omega^2 \rho_B \right] \nabla \Pi^*
 \end{aligned} \tag{A-13}$$

where Π and Π^* are defined as,

$$\begin{aligned}
 \Pi &= - \left[A_1(ws) k_{\alpha_1}^2 g_{\alpha_1} + A_2(ws) k_{\alpha_2}^2 g_{\alpha_2} \right] = -A_1(ws) k_{\alpha_1}^2 \frac{e^{ik_{\alpha_1} R}}{4\pi R} \\
 &\quad - A_2(ws) k_{\alpha_2}^2 \frac{e^{ik_{\alpha_2} R}}{4\pi R}
 \end{aligned} \tag{A-14}$$

$$\begin{aligned}
 \Pi^* &= - \left[A_1^*(ws) k_{\alpha_1}^2 g_{\alpha_1} + A_2^*(ws) k_{\alpha_2}^2 g_{\alpha_2} \right] = -A_1^*(ws) k_{\alpha_1}^2 \frac{e^{ik_{\alpha_1} R}}{4\pi R} \\
 &\quad - A_2^*(ws) k_{\alpha_2}^2 \frac{e^{ik_{\alpha_2} R}}{4\pi R}.
 \end{aligned} \tag{A-15}$$

The variables $A_1(ws)$, $A_2(ws)$ and $A_1^*(ws)$, $A_2^*(ws)$ are defined in equations (A-8) and (A-10), respectively.

Relative Flow Green's Function with Source on Fluid

$$\underline{G}_f^w = \nabla \Pi^*, \quad \Pi^* = \Lambda_1 g_{\alpha_1} + \Lambda_2 g_{\alpha_2} \tag{A-16}$$

where,

$$\begin{aligned}
 \Lambda_1 &= \left[HG \omega^2 \rho_E (k_{\alpha_1}^2 k_{\beta}^2 - k_{\alpha_1}^4) + \omega^4 \rho_E \rho_B G (k_{\alpha_1}^2 - k_{\beta}^2) \right] A_1^*(wf) \\
 \Lambda_2 &= \left[HG \omega^2 \rho_E (k_{\alpha_2}^2 k_{\beta}^2 - k_{\alpha_2}^4) + \omega^4 \rho_E \rho_B G (k_{\alpha_2}^2 - k_{\beta}^2) \right] A_2^*(wf) \\
 A_1^*(wf) &= \frac{A_1(wf)}{(\omega^2 \rho_B - k_{\alpha_1}^2 H) (\omega^2 \rho_B - k_{\alpha_1}^2 G)}, \\
 A_2^*(wf) &= \frac{A_2(wf)}{(\omega^2 \rho_B - k_{\alpha_1}^2 H) (\omega^2 \rho_B - k_{\alpha_1}^2 G)}.
 \end{aligned} \tag{A-17}$$

With the introduced variabls $A_1(wf)$, $A_2(wf)$, where (wf) denotes scalar Green's function amplitudes belonging to the relative flow Green's function with a source applied on the fluid phase defined as,

$$\begin{aligned}
 A_1(wf) &= \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{(\omega^2 \rho_B - k_{\alpha_1}^2 H) (\omega^2 \rho_B - k_{\alpha_1}^2 G)}{(k_{\alpha_1}^2 - k_{\beta}^2) (k_{\alpha_1}^2 - k_{\alpha_2}^2)} \\
 A_2(wf) &= \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{(\omega^2 \rho_B - k_{\alpha_2}^2 H) (\omega^2 \rho_B - k_{\alpha_2}^2 G)}{(k_{\alpha_2}^2 - k_{\beta}^2) (k_{\alpha_2}^2 - k_{\alpha_1}^2)}.
 \end{aligned} \tag{A-18}$$

Static Green's Functions in Poroelastic Media

Static Displacement Green's Function

To determine the singular behavior of the dynamic Green's function when a field point approaches the source point, the static case must be considered. This regularizes the surface integrals of the dynamic Green's function (Kupradze, 1963). When ω is set to zero, a static Green's function is obtained. To obtain the g^u and ϕ equations in the static limit we have to solve,

$$G\nabla_t^2 g^u + G \frac{\partial^2}{\partial z^2} g^u = -\delta(\mathbf{x} - \mathbf{x}') \quad (\text{A-19})$$

$$\frac{G}{M} [HM - C^2] \left[\nabla_t^2 + \frac{\partial^2}{\partial z^2} \right]^2 \phi = -\delta(\mathbf{x} - \mathbf{x}'). \quad (\text{A-20})$$

The solution of equation (A-19) is

$$g^u = \frac{1}{4\pi G} \frac{1}{R}. \quad (\text{A-21})$$

The solution of equation (A-20), using Poisson's equation $\nabla^2 \frac{1}{4\pi R} = -\delta(\mathbf{x} - \mathbf{x}')$ and the identity $\nabla^2 R = \frac{2}{R}$ is,

$$\phi = \frac{M}{8\pi G [HM - C^2]} R \quad (\text{A-22})$$

With $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$. Substituting ϕ and g into equation (A-1) in the static limit yields,

$$\underline{\underline{G}} = \frac{1}{4\pi G} \frac{1}{R} \underline{\underline{I}} - \frac{\frac{1}{M} [HM - C^2 - GM]}{8\pi \frac{G}{M} [HM - C^2]} \nabla \nabla R. \quad (\text{A-23})$$

Taking the isotropic elastic limit, physically meaning the collapse of the pore space ($\phi \rightarrow 0$), of equation (A-23) gives,

$$\underline{\underline{G}} = \frac{1}{4\pi\mu} \frac{1}{R} \underline{\underline{I}} - \frac{\lambda + \mu}{8\pi\mu(\lambda + 2\mu)} \nabla \nabla R \quad (\text{A-24})$$

which is the isotropic elastic static Green's function (Love, 1944).

Static Relative Flow Green's Function

To obtain the g^w and Φ equations in the static limit we have to solve,

$$-G \frac{\rho_E}{\rho_f} \nabla_t^2 g^w - G \frac{\rho_E}{\rho_f} \frac{\partial^2}{\partial z^2} g^w = -\delta(\mathbf{x} - \mathbf{x}') \quad (\text{A-25})$$

$$\frac{G}{C} \frac{\rho_E}{\rho_f} [HM - C^2] \left[\nabla_t^2 + \frac{\partial^2}{\partial z^2} \right]^2 \Phi = -\delta(\mathbf{x} - \mathbf{x}'). \quad (\text{A-26})$$

The solution of equation (A-25) is

$$g^w = -\frac{1}{4\pi G} \frac{1}{R} \frac{\rho_E}{\rho_f}. \quad (\text{A-27})$$

The solution of equation (A-26), using Poisson's equation $\nabla^2 \frac{1}{4\pi R} = -\delta(\mathbf{x} - \mathbf{x}')$ and the identity $\nabla^2 R = \frac{2}{R}$ is,

$$\Phi = \frac{C \rho_f}{8\pi G \rho_E [HM - C^2]} R \quad (\text{A-28})$$

with $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$. Substituting ϕ and g into equation (A-7) in the static limit yields,

$$\underline{\underline{G}} = -\frac{1}{4\pi G} \frac{1}{\frac{\rho_E}{\rho_f} R} \underline{\underline{I}} + \frac{\frac{1}{C} [HM - C^2 - GC \frac{\rho_E}{\rho_f}]}{8\pi \frac{G}{C} \frac{\rho_E}{\rho_f} [HM - C^2]} \nabla \nabla R. \quad (\text{A-29})$$

In the static limit, $\omega \rightarrow 0$, the effective fluid density reduces to the fluid density in the pore space, $\rho_E \rightarrow \rho_f$.

Surface ElectroSeismic Geometry

Figure 1: An explosive and vertical point source 100 m above an interface in a surface electroseismic geometry setting.

Figure 2: The mechanical displacement component seismograms and the *TM* mode component electroseismograms, without direct waves, calculated for an explode ring source above a one interface model.

Vertical Ring Source above Contrast

Figure 3: Vertical ring source above a contrast.

Figure 4: The mechanical displacement component seismograms and the *TM* mode component electroseismograms, without direct waves, calculated for a vertical ring source above a one interface model.

Radial Ring Source above Contrast

Figure 5: Radial ring source above a contrast.

Figure 6: The mechanical displacement component seismograms and the TM mode component electroseismograms, without direct waves, calculated for a radial ring source above a one interface model.

Vertical ElectroSeismic Profiling (VESP)

Figure 7: An explosive and vertical point source in a Vertical ElectroSeismic Profiling geometry (reservoir sand saturated with one fluid type).

Figure 8: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for an explosive ring source without direct waves in a Vertical Electro seismic Profiling geometry.

Figure 9: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a vertical ring source without direct waves in a Vertical Electro seismic Profiling geometry.

Figure 10: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a radial ring source without direct waves in a Vertical Electro-seismic Profiling geometry.

Vertical ElectroSeismic Profiling (VESP)

Figure 11: An explosive and vertical point source in a Vertical ElectroSeismic Profiling geometry (reservoir sand saturated with two fluid types).

Figure 12: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for an explosive ring source without direct waves in a Vertical Electro seismic Profiling geometry with an additional electrical contrast.

Figure 13: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a vertical ring source without direct waves in a Vertical Electro-seismic Profiling geometry with an additional electrical contrast.

Figure 14: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a radial ring source without direct waves in a Vertical Electro-seismic Profiling geometry with an additional electrical contrast.

DYNAMIC STREAMING CURRENTS FROM SEISMIC POINT SOURCES IN HOMOGENEOUS POROELASTIC MEDIA

3.1. SUMMARY

In a porous medium saturated with a fluid electrolyte, mechanical and electromagnetic disturbances are coupled. The coupling is electrokinetic in nature since it is due to an excess of electrolyte ions that exist in an electric double layer near the grain surfaces within the material. Mechanically-induced streaming currents generated by point sources in homogeneous, isotropic porous media are presented. The electrically-induced streaming current is shown to be second-order in the electrokinetic coupling coefficient and can be neglected. This decouples the mechanical behavior from the electromagnetic behavior with respect to the induced fluxes and simplifies the analysis of the relative fluid flow and dynamic streaming current.

We used Biot theory to calculate the amount of induced relative flow by the solution to Green's function. The transport coefficients—conductivity, dynamic permeability, and the electrokinetic coupling coefficient—and their sensitivity with respect to porosity, dc permeability, and frequency changes are evaluated. Conductivity decreases with increasing dc permeability. It has a $k_0^{-1/2}$ dependence when grain surface conductances are more important than the bulk fluid phase conductivity.

Stationary phase relative flow and streaming current solutions are calculated for an explosive and vertical point source acting on the bulk and a volume injection source acting on the fluid. The streaming currents are induced both by P and S waves. The streaming current decreases with increasing fluid conductivity. This is consistent with the decrease of the diffuse double layer thickness and ζ -potential.

The porosity effect on the streaming current induced by S waves is different from the currents induced by the P waves. The porosity affects the bulk moduli of the solid. Its effect, combined with the frame bulk modulus and compressibility of the saturating fluid, determines the streaming current amplitude induced by a P wave versus porosity. The increase in streaming current amplitude induced by S waves with increasing porosity is due to the decrease of the shear frame modulus with increasing porosity. The streaming current behavior with respect to dc permeability is found to differ for sources applied to the elastic frame and volume injection sources.

3.2. INTRODUCTION

Electrokinetic phenomena are usually studied with respect to solid and liquid interfaces. When two phases of different chemical composition are in contact, an electric potential difference develops between the two interfaces. The anions from the electrolyte are adsorbed to the solid interface (i.e., a quartz matrix), leaving behind a net excess of cations distributed near the wall, and producing an electric double layer. There are several models of the electric double layer. One is the Gouy-Chapman type model (Bockris 1970; Dukhin and Derjaguin, 1974), a diffuse layer model where the counter ions are attracted to the surface only by electrostatic forces. Another model of the solid/fluid interface which is currently favored is largely due to Stern (1924). He argued that the

electrolyte ions and structured (hydrogen bonded) water molecules are not only electrostatically but also chemically adsorbed to the surface of the solid. This adsorbed Helmholtz layer (Overbeek, 1952; Bockris, 1970), contains immobile adsorbed ions and can also include ionized surface sites present on the grain surfaces, as shown in Figure 1. Beyond this first layer of bound ions, there is a diffuse distribution of mobile ions whose position is determined by the Poisson-Boltzman statistics. In the diffuse layer the ions are under the continued influence of the ordering chemical and disordering thermal forces. The Stern model is therefore a composite of a Helmholtz layer and a Gouy-Chapman diffusion zone. The separation plane between the mobile and immobile charge is the hydrodynamic slipping or shear plane. The potential at this shear plane is called the ζ -potential, the value of which is often used to characterize the double layer. The electric potential in the neutral electrolyte (no excess charge) is defined to be zero. Modifications to the Stern model have been proposed. In particular, Grahame (1947) suggested splitting the Helmholtz layer into an inner and outer region. The inner region is populated by dehydrated ions stuck to the bare solid with specific adsorption forces, while the outer region would contain the partly hydrated ions and structured (hydrogen bonded) water molecules that touch a hydrated solid rather than stick to a bare solid. There is experimental evidence suggesting that long-range chemical forces from adsorption sites on the quartz surface may align water molecules for up to several hundred molecular distances away from the wall, as discussed in Derjaguin *et al.* (1987).

An important aspect of the adsorbed layer is the fact that chemical adsorption of dissociated OH^- and H^+ ions at the quartz surface is more dominant than adsorption of the electrolyte anions in determining the potential (Gaudin and Fuerstenau 1955; Fuerstenau and Modi 1959). Interesting discussions, which show the performance of the theoretical Stern model with respect to aqueous organic electrolyte solutions, are in Anderson (1959) and Fuerstenau and Modi (1959). The adsorption mechanism involves a combination of electrostatic attraction and long organic (hydrocarbon) chain association (Van der Waals attraction) which can be incorporated into the Stern framework of a double layer description.

In this paper, we study the effects of dynamic relative motion between a solid and monovalent anorganic electrolytes. The Stern electrical double layer description is used to characterize the grain-electrolyte interfaces. The fluid is assumed to be an ideal electrolyte, thus restricting salt concentrations to be less than 1.0 mol/l. At this molarity the ion separation distance becomes such that electrostatic interaction energy becomes significant compared to thermal energy. There are numerous phenomena at the shear plane defined by its ζ -potential. Examples of these 'electrokinetic phenomena' are electroosmosis, electrophoresis, and the streaming potential (Bockris, 1970). The ζ -potential is fundamental in all these kinetic phenomena and will be discussed later in this paper.

Our overall goal is to provide theoretical/fast numerical tools to analyze future experimental streaming current data as a function of fluid chemistry and permeability in porous material. An advantage of measuring streaming currents is not to have to know the value of the grain surface conductance if one wants to determine the ζ -potential. In inhomogeneous media (i.e., layered media), an imbalance of dynamic streaming currents across an interface causes electromagnetic radiation. A so-called seismoelectric conversion has occurred (Haartsen *et al.*, 1996). To quantify such a conversion it is important to understand the effect of fluid chemistry and the permeability of the porous material on the dynamic streaming current.

The first part of this paper discusses the macroscopic transport coefficients and their sensitivity to porosity and dc permeability. The second part derives descriptions of dynamic streaming currents induced by point forces (vertical and explosive point forces). The last part shows the streaming current behavior with respect to important lithological parameters.

3.3. MACROSCOPIC TRANSPORT COEFFICIENTS

Transport equations to describe electrokinetic phenomena in a porous medium can be treated within the framework of nonequilibrium thermodynamics (Groot and Mazur, 1984). For linear processes the flux/force equations relating the conservation equations for fluid flow and current flow, when there are electric potential and fluid pressure gradients, are:

$$\underline{J} = \sigma(\omega)\underline{E} + L(\omega)[- \nabla P + i\omega\rho_f\dot{\underline{u}}] \quad (1)$$

$$\dot{\underline{w}} = \frac{k(\omega)}{\eta} [- \nabla P + i\omega\rho_f\dot{\underline{u}}] + L(\omega)\underline{E}, \quad (2)$$

where $\underline{J}$ is the electric current density, $\sigma(\omega)$ is the frequency dependent conductivity of the two-phase medium, $k(\omega)$ is the dynamic permeability, $L(\omega)$ is the dynamic electrokinetic coupling coefficient, and η the fluid viscosity. The electric field strength $\underline{E}$ can be expressed as the gradient of a potential, $\underline{E} = -\nabla\Phi$. The pressure P in the pore fluid which has a density ρ_f , $\underline{u}$ is the displacement in the bulk material and $\dot{\underline{u}}$ is the relative fluid-solid motion. The dots above the $\underline{u}$ and $\underline{w}$ vectors denote a derivative with respect to time. The first term in equation (1) is the conduction current contribution and the second term is the streaming current contribution to the total current. The forces which drive the relative flow are a combination of pressure gradients set up by a compressional wave and by grain acceleration. The relative flow caused by grain acceleration can therefore be due both to compressional and shear waves. The first term in equation (2) represents Darcy's law. The second term in the same equation describes the amount of fluid flow caused by the electric field that drives the hydrated mobile excess charge in the diffuse boundary layer—an osmosis phenomenon.

The fact that the cross-terms have the same coupling coefficients $L(\omega)$, a symmetry which is one of the fundamental results in the theory of irreversible processes, is a statement of Onsager reciprocity (Onsager 1931a,b). The coefficient equality implies that the action of the electric field on the charge density is reciprocal to the action of the fluid pressure on the charge. In other words, the mobility of the ion is independent of the charge moving through the fluid or the fluid moving around the charge.

In order to give explicit expressions for electrokinetic coefficient L for porous media, the capillary model is often taken. The porous medium is assumed to be a composite of a bundle of capillaries. This approach—to obtain expressions for the electrokinetic coupling coefficient—is taken by Levine *et al.* (1975), Broz and Epstein (1976), Anderson and Koh (1977), Donath and Voigt (1986), Manzanares *et al.* (1991), and others. Another approach is to study the behavior of these coefficients numerically. Jin and Sharma (1991) developed a numerical model for electrochemical and electrokinetic coupling in an inhomogeneous porous medium. Their network approach solves the coupled

transport processes in inhomogeneous charged membranes/pores. The numerical results provide insight into the relationships between pore structure, flow properties, and coupling coefficients. They conclude that several obvious characteristics of real porous media are not present in capillary-based network models. These differences between the model and experiment are discussed systematically in Sharma (1988) and Kuo *et al.* (1988). Their streaming potential measurements conducted on mixtures of alumina and silica yield results that differ significantly from that predicted by a simple, weighted average capillary tube model. The streaming potential was found to be a function of the grain sizes of the individual components. This grain size effect on the streaming potential in artificial spherical grain sands is also observed in data collected by Schriever and Bleil (1957).

Wurmstich and Morgan (1994) solved 3-D static transport equations by a finite difference scheme to investigate whether streaming potential measurements in boreholes and at the surface can be used to monitor subsurface flow and to detect subsurface flow patterns in oil reservoirs. Their model was limited by inadequate knowledge of the electrokinetic cross-coupling coefficients, which are important for determining streaming current/streaming potential magnitudes as a function of medium properties.

The coupling coefficients used in this paper, which are frequency dependent, are explicitly obtained by a volume average procedure of Maxwell's equations coupled to Biot's equations (Pride, 1994). Two possible functions that smoothly connect the low and high frequency regimes of the transport coefficients are:

$$\frac{k(\omega)}{k_0} = \left[\left[1 - i \frac{\omega}{\omega_t} \frac{4}{m} \right]^{1/2} - i \frac{\omega}{\omega_t} \right]^{-1} \quad (3)$$

$$\frac{L(\omega)}{L_0} = \left[1 - i \frac{\omega}{\omega_t} \frac{m}{4\alpha_\infty^2} \left(1 - 2\alpha_\infty \frac{\tilde{d}}{\Lambda} \right)^2 \left(1 - i^{3/2} \tilde{d} \sqrt{\frac{\omega \rho_f}{\eta}} \right)^2 \right]^{-1/2}. \quad (4)$$

The transition frequency ω_t separates the low frequency viscous flow behavior from the high frequency inertial flow and is defined by,

$$\omega_t = \frac{\phi}{\alpha_\infty k_0} \frac{\eta}{\rho_f} \quad (5)$$

and the dimensionless number m by,

$$m \equiv \frac{\phi \Lambda^2}{\alpha_\infty k_0}. \quad (6)$$

This number consists only of geometry terms and is model-dependent.

The static coupling coefficient L_0 is given by,

$$L_0 = -\frac{\phi}{\alpha_\infty} \frac{\epsilon_0 \kappa_f \zeta}{\eta} \left[1 - 2\alpha_\infty \frac{\tilde{d}}{\Lambda} \right]. \quad (7)$$

where κ_f is the relative fluid permittivity, ζ is the zeta potential, α_∞ is the tortuosity (normally a number between 3 and 10) and ϕ is the porosity. The $\sqrt{\frac{\omega \rho_f}{\eta}}$ factor in equation (4) determines

the viscous skin depth and $\tilde{d}$ is a measure of double layer thickness and depends on the electrolyte concentration, C . The pore length parameter, Λ , is the same as defined in Johnson *et al.* (1987).

An important factor affecting the electrokinetic coefficient is the ζ -potential, as shown in equations (4) and (7). Theoretical determination of ζ remains problematic due to the complexity of the adsorbed layer. The ζ -potential employed in this paper is taken from experimental studies. A variety of researchers have determined the ζ -potential as a function of electrolyte concentration from streaming potential data. The ζ -potential values we used are based on saturated NaCl, KCl quartz samples at $T = 25^\circ\text{C}$ and $pH = 7$, determined by Gaudin and Fuerstenau (1955), Sidorova *et al.* (1975) and Hidalgo-Alvarez *et al.* (1985). Pride and Morgan (1991) applied regression analysis to their combined data and obtained the following expression for the ζ -potential as a function of electrolyte concentration;

$$\zeta(V) = 0.008 + 0.026 \log_{10}(C). \quad (8)$$

There have been few studies determining the ζ -potential for naturally occurring geological samples such as sandstones. Sharma *et al.* (1987) determined the ζ -potential from streaming potential data for unclean samples of Berea sandstones. They obtained ζ -potential values at $pH = 7$ and $T = 25^\circ\text{C}$ for a $10^{-4} - 10^{-3} M$ NaCl electrolyte which are comparable to the values obtained using equation (8) for a specific concentration. Jouniaux and Pozzi (1995) determined ζ -potentials from streaming potential data for Fontainebleau sandstones under triaxial stress up to failure.

Morgan *et al.* (1989) and Ishido and Mizutani (1981) determined ζ -potentials as a function of temperature experimentally from streaming potential data. Morgan found that ζ was independent of temperature for crushed Westerly granite in $10^{-2} M$ NaCl at $pH = 5.5$, while Ishido and Mizutani found ζ to change in magnitude by about $-65 \text{ mV}/^\circ\text{C}$ for quartz in $10^{-3} M \text{ KNO}_3$ at $pH = 6.1$. Streaming potential data collected in artificial spherical-grain sands (Schriever and Bleil, 1957), show that the ζ -potential has a positive temperature coefficient whose approximate value is $0.039 /^\circ\text{C}$, and the streaming potential has a positive temperature coefficient of $0.0535 /^\circ\text{C}$. More experimental work is needed to understand the ζ -potential versus T relation. In our numerical calculations we used a constant 298 K temperature.

3.3.1. Sensitivity of Transport Coefficients to Porosity and DC Permeability

To investigate the transport coefficient behavior with respect to porosity and permeability changes, we calculate the $k_0/k(\omega)$ and $L_0/L(\omega)$ ratios as a function of frequency using equations (3) and (4). The real and imaginary parts of the $k_0/k(\omega)$ and $L_0/L(\omega)$ ratios versus porosity are shown in Figures 2 and 3; the same ratios versus permeability are shown in Figures 4 and 5.

There is a distinct frequency relaxation present in the $k(\omega)$ and $L(\omega)$ transport coefficients. This relaxation frequency demarcates the transition from viscous flow to inertial flow. The $k(\omega)$ relaxes at frequency ω_t , with ω_t given by equation (5), while $L(\omega)$ relaxes at a frequency ω_t/m , with m given in equation (6). The $k(\omega)$ amplitude decreases as ω when $\omega > \omega_t$, while $L(\omega)$ decreases as $\omega^{1/2}$ when $\omega > \frac{4\alpha_\infty^2 \omega_t}{m}$ [see $\text{Re}[k_0/k(\omega)]$ and $\text{Re}[L_0/L(\omega)]$ in Figures 2 and 3 and equations (3) and (4)].

The conductivity $\sigma(\omega)$, which controls the conduction current, relates current density to electric field strength and is defined in Pride (1994),

$$\sigma(\omega) = \phi \left[\frac{\sigma_f}{\alpha_\infty} + \frac{2}{\Lambda} (C_{em} + C_{os}(\omega)) \right]. \quad (9)$$

With C_{em} and C_{os} , the conductances due to electromigration and osmosis along the grain-pore fluid interfaces are, respectively,

$$C_{em} = 4\tilde{d}q^2 6 \times 10^{26} C \left[\cosh\left(\frac{q\zeta}{2kT}\right) - 1 \right] \quad (10)$$

$$C_{os}(\omega) = \frac{(\epsilon_0 \kappa_f \zeta)^2}{2\tilde{d}\eta} \left[1 - i^{3/2} \sqrt{\frac{\omega \rho_f}{\eta}} \right]^{-2}, \quad (11)$$

with q , the elementary electron charge; C , salinity; k , Boltzmann's constant; T , temperature and $\tilde{d}$, the Debye length, which is a measure of the thickness of the diffuse double layer and defined as,

$$\tilde{d} = \sqrt{\frac{q^2 z^2 n}{\epsilon_0 \kappa_f k T}} \quad (12)$$

with z the ion valence and n the ionic concentration. σ_f is the conductivity of the fluid.

Figures 6 and 7 show the conductivity versus porosity and permeability. Conductivity increases monotonically with increasing porosity. Conductivity does not change linearly with porosity due to porosity dependence in the $\Lambda[m]$, pore length parameter.

Conductivity decreases with increasing permeability. At a 10^{-16} m^2 permeability, the Λ parameter becomes comparable in value with the sum of osmotic and electromigration conductances. At a constant porosity and permeabilities less than 10^{-16} m^2 , the surface conductances become more important than the fluid phase conductivity. In Figure 7, this behavior shows itself as a sharp increase in conductivity at the low permeability end of the figure. Physically this situation occurs when the pore space is filled with clays which do not decrease the porosity but greatly enhance the surface-to-pore-volume ratio and therefore enhance the conduction along the pore surfaces.

3.3.2. Mechanically-Induced Streaming Currents

The macroscopic current density in equation (1) has three distinct contributions: diffusion current density, conduction current density, and streaming current density. If frequencies are in the seismic frequency range of interest macroscopic diffusion currents can be neglected. We will concentrate on the dynamic mechanical streaming current contribution to the total macroscopic current. From the transport relations we deduce the streaming current induced by relative flow. Since there is an electrically- and a mechanically-induced relative flow, we also have electrically- and mechanically-induced streaming currents. Substituting equation (2) into equation (1), the total streaming current can be written as,

$$\underline{J}_s = -i\omega \frac{\eta}{k(\omega)} L(\omega) \underline{w} - L(\omega)^2 \underline{E}. \quad (13)$$

We neglect the electrically-induced part of the streaming current since it is second-order in electrokinetic coupling coefficient $L[Cs/kg]$, which was also derived by Neev and Yeatts (1989). Defining an effective fluid density operator (Pride *et al.*, 1993) that controls the magnitude of the wave-induced relative flow $\rho_E[kg/m^3] = -\frac{\eta}{\omega k(\omega)}$, the mechanical part of the streaming current can be written as,

$$\underline{J}_{sm} = -\omega^2 \rho_E L(\omega) \underline{w}. \quad (14)$$

The relative flow vector is obtained by a Green's function solution (see appendix). This relative fluid flow, driven by pressure gradients and grain acceleration, is determined from linearized force balances. In Pride *et al.* (1993) a geometric condition on nonlinear flow in a homogeneous poroelastic medium is obtained. The nonlinear convection term of fluid flow is considered negligible if the pore radii are less than $100 \mu m$ for water solution-saturated media under a maximum allowed strain of 10^{-6} . It is assumed that the above conditions are satisfied in the numerical modeling. Therefore, the effect of turbulence upon electrokinetic phenomena (i.e., streaming potentials/currents) can be ignored in this study (Rutgers *et al.*, 1957; Kurtz *et al.*, 1957).

Measurements to determine the electrokinetic properties of capillary surfaces or porous media have been traditionally performed under a static electric potential gradient or pressure gradient. Transient streaming potential modeling using Biot's consolidation work in the quasi-static limit to obtain quasi-static flow, which drives a streaming current through a porous medium, was studied by Chandler (1981) and Chandler and Johnson (1981). The performed experimental studies of diffusive flow from transient pressure pulses in various porous structures, where streaming potentials were used to monitor fluid pressure, show that the Biot slow wave can be detected with the streaming potential. Chandler (1981) proposed a streaming potential-based logging tool that would indicate permeability variations.

Dynamic measurements of streaming potentials and streaming currents are discussed by Packard (1952), Groves and Sears (1975), Sears and Groves (1978), and Cerda and Non-Chom (1981). Their experiments are based on sinusoidally alternating fluid flow through capillaries and porous media to measure electrokinetic phenomena.

Direct measurements of streaming currents can be found in Hurd and Hackerman (1955) and Rutgers *et al.* (1957). They argue that investigations of electrokinetic phenomena traditionally have been restricted almost exclusively to measurements of the streaming potential because of the exceedingly low streaming currents. The advantage of measuring streaming currents is to not have to know the value of the surface conductance to determine the ζ -potential, which needs to be determined when streaming potential measurements are made. An additional disadvantage of streaming potential measurements is their sensitivity to temperature due to the temperature-dependence of resistivity. Groves and Sears designed an experiment to measure alternating streaming currents resulting from monochromatic sinusoidally alternating fluid flow.

In this paper, we investigate numerically the effect of porosity, permeability and fluid chemistry on the dynamic streaming currents caused by point forces in porous media. The numerical results are useful in designing an experiment to measure dynamic streaming currents generated by point sources/transducers in poroelastic media. The theoretical streaming current prediction could possibly be used to invert for a medium parameter, such as permeability, from dynamic streaming current data, provided the data is sensitive to the particular medium parameter.

3.4. DYNAMIC STREAMING CURRENTS MECHANICALLY INDUCED BY POINT FORCES

We obtain the relative flow vector generated by vertical point forces and explosive point forces (volume injection sources) by using the relative flow Green's function solution for a point source applied to the frame and fluid [equations (A-1) and (A-7)]. The dynamic streaming current components can be determined by substituting this relative flow vector into the mechanical streaming current equation, (14), and using electrokinetic coefficient $L(\omega)$ defined in equation (4). We first calculate the streaming current components generated by a vertical point source applied on the frame. Then the streaming currents for an explosive point source applied on either the solid or fluid phase are given.

Vertical Point Force on Solid. The mechanical streaming current is calculated for a vertical point force (parallel to the symmetry axis) at the origin, $\mathbf{F}(\mathbf{x}) = \hat{z}\delta(\mathbf{x})$. Using Green's theorem, we obtain,

$$\underline{w} = \int_V \underline{G}_s^w \cdot \mathbf{F}(\mathbf{x}') d\mathbf{x}' = g^w \hat{z} - \psi \nabla \frac{\partial}{\partial z} \Phi - \gamma \nabla \frac{\partial}{\partial z} \Phi^* \quad (15)$$

with,

$$\psi = \frac{1}{C} \left[C^2 - HM + GC \frac{\rho_E}{\rho_f} \right] \quad (16)$$

$$\gamma = \omega^2 \rho_f \frac{HM}{C} + \omega^2 C \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \quad (17)$$

where $\underline{G}_s^w, g^w, \Phi, \Phi^*$ are defined in equations (A-1), (A-6), (A-2) and (A-4), respectively. Writing out in component form the following equations are obtained,

$$J_r^s = \frac{1}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} \left[l W_{\beta}^r H_1^{(2)}(lr) + m W_{\alpha_1}^r H_1^{(2)}(mr) + n W_{\alpha_2}^r H_1^{(2)}(nr) \right] k_z e^{-ik_z z} dk_z \quad (18)$$

$$J_z^s = \frac{i}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} k_z \left[W_{\beta}^z H_0^{(2)}(lr) + W_{\alpha_1}^z H_0^{(2)}(mr) + W_{\alpha_2}^z H_0^{(2)}(nr) \right] e^{-ik_z z} dk_z \quad (19)$$

where the introduced variables, W_p^r and W_p^z , with $p = \beta \vee \alpha_1 \vee \alpha_2$ are defined as,

$$\begin{aligned} W_{\beta}^r &= \psi B(ws) + \gamma B^*(ws), & W_{\beta}^z &= \frac{-1}{G \frac{\rho_E}{\rho_f}} \frac{1}{k_z^2} + \psi B(ws) + \gamma B^*(ws) \\ W_{\alpha_1}^r &= \psi A_1(ws) + \gamma A_1^*(ws), & W_{\alpha_1}^z &= \psi A_1(ws) + \gamma A_1^*(ws) \\ W_{\alpha_2}^r &= \psi A_2(ws) + \gamma A_2^*(ws), & W_{\alpha_2}^z &= \psi A_2(ws) + \gamma A_2^*(ws), \end{aligned} \quad (20)$$

with $B(ws), A_q(ws)$ defined in equation (A-3) and $B^*(ws), A_q^*(ws)$ defined in equation (A-5), with $q = 1 \vee 2$.

Explosive Point Source on Solid and Fluid. The relative flow vector for an explosive point source at the origin can be obtained by taking the divergence of the relative flow Green's function with respect to the source coordinates.

$$\underline{w} = \nabla g^w - \psi \nabla \nabla^2 \Phi - \gamma \nabla \nabla^2 \Phi^* \quad (21)$$

where $\underline{G}^w, g^w, \Phi, \Phi^*, \psi, \gamma$ are defined in equations (A-1), (A-6), (A-2), (A-4), (16) and (17), respectively. The two relative flow components are,

$$J_r^s = \frac{-i}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} \left[m W_{\alpha_1}^r H_1^{(2)}(mr) + n W_{\alpha_2}^r H_1^{(2)}(nr) \right] e^{-ik_z z} dk_z \quad (22)$$

$$J_z^s = \frac{1}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} k_z \left[W_{\alpha_1}^z H_0^{(2)}(mr) + W_{\alpha_2}^z H_0^{(2)}(nr) \right] e^{-ik_z z} dk_z \quad (23)$$

where the introduced variables, W_p^r and W_p^z , with $p = \alpha_1 \vee \alpha_2$ are defined as,

$$\begin{aligned} W_{\alpha_1}^z &= W_{\alpha_1}^r = (\psi A_1 + \gamma A_1^*) k_{\alpha_1}^2 \\ W_{\alpha_1}^r &= W_{\alpha_2}^r = (\psi A_2 + \gamma A_2^*) k_{\alpha_2}^2 \end{aligned} \quad (24)$$

with $A_q(ws)$ defined in equation (A-3) and $A_q^*(ws)$ defined in equation (A-5), with $q = 1 \vee 2$. In isotropic media only compressional waves are generated by an explosive point source.

The relative flow vector caused by a volume injection source is,

$$\underline{w} = \nabla \Upsilon_f^w \quad (25)$$

with Υ_f^w defined in (A-7).

The induced mechanical streaming current components are,

$$J_r^s = -\frac{i}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} \left[m W_{\alpha_1}^r H_1^{(2)}(mr) + n W_{\alpha_2}^r H_1^{(2)}(nr) \right] e^{-ik_z z} dk_z \quad (26)$$

$$J_z^s = \frac{1}{4\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} k_z \left[W_{\alpha_1}^z H_0^{(2)}(mr) + W_{\alpha_2}^z H_0^{(2)}(nr) \right] e^{-ik_z z} dk_z \quad (27)$$

where the introduced variables, W_p^r and W_p^z , with $p = \alpha_1 \vee \alpha_2$ are defined as,

$$\begin{aligned} W_{\alpha_1}^r &= W_{\alpha_1}^z = \Lambda_1 \\ W_{\alpha_2}^r &= W_{\alpha_2}^z = \Lambda_2 \end{aligned} \quad (28)$$

with Λ_1 and Λ_2 defined in equations (A-8).

The following horizontal wavenumbers were used in the above, $l = \left[\frac{\omega^2}{\beta^2} - k_z^2 \right]^{1/2}$, $m = \left[\frac{\omega^2}{\alpha_1^2} - k_z^2 \right]^{1/2}$ and $n = \left[\frac{\omega^2}{\alpha_2^2} - k_z^2 \right]^{1/2}$.

3.4.1. Stationary-Phase Solutions

The method of stationary-phase yields an asymptotic solution to integrals of the form (Ben-Menahem and Singh, 1981; Bender and Orzag, 1978; Bleistein and Handelsman, 1986)

$$K(\lambda) = \int_{-\infty}^{\infty} g(\zeta) e^{i\lambda f(\zeta)} d\zeta. \quad (29)$$

In the above expression, λ is a large valued, positive constant. Because λ is large, the integrand oscillates rapidly and the contributions to the integral tend to cancel except near the stationary value of $f(\zeta)$, the value ζ_0 where $f'(\zeta_0) = 0$ (the prime indicates differentiation with respect to ζ). The leading order asymptotic behavior of the integral is then controlled by contributions from a small region around this stationary point. The desired asymptotic approximation to this equation is

$$K(\lambda) \sim \left[\frac{2\pi}{\lambda |f''(\zeta_0)|} \right]^{1/2} g(\zeta_0) e^{i\lambda f(\zeta_0) + (i\pi/4) \text{sgn} f''(\zeta_0)}. \quad (30)$$

The solution will be inaccurate, and the method of stationary phase invalid, if the integrand is not regular (Bleistein and Handelsman, 1986).

An additional simplification can be made by applying the large-argument asymptotic forms for the Hankel functions (Abramowitz and Stegun 1964),

$$\begin{aligned} H_0^{(2)}(z) &\sim \sqrt{\frac{2}{\pi z}} e^{-i(z-\pi/4)} \\ H_1^{(2)}(z) &\sim \sqrt{\frac{2}{\pi z}} e^{-i(z-3\pi/4)}. \end{aligned} \quad (31)$$

Substituting equations (31) into equations (18) and (19), and into equations (22) and (23), and into equations (26) and (27), show that these far-field streaming current fields are of the form $K(\lambda)$ suitable for application of the stationary-phase approach. The terms in each integral represent the contribution of shear, fast, and slow waves to the dynamic streaming current components. The stationary phase value k_0 is different for the three wave types. Considering the first term in the integral for the radial component in equation (18), the quantity analogous to $i\lambda f(\zeta)$ in equation (29) is

$$-i[k_z z + lr] = iR[k_z \cos(\gamma) - l \sin(\gamma)] \quad (32)$$

where $z = -R \cos(\gamma)$ and $r = R \sin(\gamma)$. Hence, $\zeta = k_z$ and $\lambda = R$.

Setting,

$$\frac{df}{d\zeta} = \cos(\gamma) + \frac{k_z \sin(\gamma)}{(\frac{\omega^2}{\beta^2} - k_z^2)^{1/2}} = 0 \quad (33)$$

and solving for k_z gives the stationary wavenumber for the S-waves, $k_0^\beta = (-\omega \cos(\gamma))/\beta$. Similarly, the analogous quantity for the compressional waves are $k_0^{\alpha_1} = (-\omega \cos(\gamma))/\alpha_1$ and $k_0^{\alpha_2} = (-\omega \cos(\gamma))/\alpha_2$. Using these values for the wavenumbers k_0 and applying the stationary-phase asymptotic solution in equation (30) gives the following far-field dynamic streaming current fields.

Vertical Point Force on Fluid.

$$J_r^s = -\omega^2 \rho_E L(\omega) \frac{\sin(\gamma)}{4\pi R} \left[\frac{\omega}{\beta} W_\beta^r(k_0^\beta) e^{-iR\frac{\omega}{\beta}} + \frac{\omega}{\alpha_1} W_{\alpha_1}^r(k_0^{\alpha_1}) e^{-iR\frac{\omega}{\alpha_1}} + \frac{\omega}{\alpha_2} W_{\alpha_2}^r(k_0^{\alpha_2}) e^{-iR\frac{\omega}{\alpha_2}} \right] \quad (34)$$

$$J_z^s = \omega^2 \rho_E L(\omega) \frac{\cos(\gamma)}{4\pi R} \left[\frac{\omega}{\beta} W_\beta^z(k_0^\beta) e^{-iR\frac{\omega}{\beta}} + \frac{\omega}{\alpha_1} W_{\alpha_1}^z(k_0^{\alpha_1}) e^{-iR\frac{\omega}{\alpha_1}} + \frac{\omega}{\alpha_2} W_{\alpha_2}^z(k_0^{\alpha_2}) e^{-iR\frac{\omega}{\alpha_2}} \right] \quad (35)$$

where W_p^r and W_p^z , with $p = \beta \vee \alpha_1 \vee \alpha_2$ are defined in equations (20).

Explosive Point Source on Solid and Fluid.

$$J_r^s = \omega^2 \rho_E L(\omega) \frac{i \sin(\gamma)}{4\pi R} \left[\frac{\omega}{\alpha_1} W_{\alpha_1}^r(k_0^{\alpha_1}) e^{-iR\frac{\omega}{\alpha_1}} + \frac{\omega}{\alpha_2} W_{\alpha_2}^r(k_0^{\alpha_2}) e^{-iR\frac{\omega}{\alpha_2}} \right] \quad (36)$$

$$J_z^s = -\omega^2 \rho_E L(\omega) \frac{i \cos(\gamma)}{4\pi R} \left[\frac{\omega}{\alpha_1} W_{\alpha_1}^z(k_0^{\alpha_1}) e^{-iR\frac{\omega}{\alpha_1}} + \frac{\omega}{\alpha_2} W_{\alpha_2}^z(k_0^{\alpha_2}) e^{-iR\frac{\omega}{\alpha_2}} \right] \quad (37)$$

where for the force applied on the solid W_p^r and W_p^z , with $p = \alpha_1 \vee \alpha_2$ are defined in equations (24) and for the force applied on the fluid W_p^r and W_p^z , with $p = \alpha_1 \vee \alpha_2$ are defined in equations (28). The "W" coefficients are now written with argument k_0 to emphasize that they are computed at the appropriate stationary-phase wavenumber. Since there is no low-frequency approximation, the solutions can be used to explore the changes in radiation pattern as the frequency of the source increases. To compute the total radiation pattern of a given source, it is more convenient to reorganize the terms in equations (34), (35), (36) and (37) to express separately the P_{fast} -, P_{slow} - and S - wave amplitudes. By simple arrangements the following radiation patterns are obtained.

Vertical Point Force

$$M_\beta = \omega^2 \rho_E L(\omega) W_\beta^r(k_0^\beta) \frac{\omega^2}{\beta^2} \frac{e^{-iR\omega/\beta}}{4\pi R} \sin(\gamma) \quad (38)$$

$$M_{\alpha_1} = \omega^2 \rho_E L(\omega) W_{\alpha_1}^r(k_0^{\alpha_1}) \frac{\omega^2}{\alpha_1^2} \frac{e^{-iR\omega/\alpha_1}}{4\pi R} \cos(\gamma) \quad (39)$$

$$M_{\alpha_2} = \omega^2 \rho_E L(\omega) W_{\alpha_2}^r(k_0^{\alpha_2}) \frac{\omega^2}{\alpha_2^2} \frac{e^{-iR\omega/\alpha_2}}{4\pi R} \cos(\gamma) \quad (40)$$

where W_p^r and W_p^z , with $p = \beta \vee \alpha_1 \vee \alpha_2$ are defined in equations (20).

Explosive Point Source on Solid and Fluid

$$M_{\alpha_1} = i\omega^2 \rho_E L(\omega) W_{\alpha_1}(k_0^{\alpha_1}) \frac{\omega}{\alpha_1} \frac{e^{-iR\omega/\alpha_1}}{4\pi R} \quad (41)$$

$$M_{\alpha_2} = i\omega^2 \rho_E L(\omega) W_{\alpha_2}(k_0^{\alpha_2}) \frac{\omega}{\alpha_2} \frac{e^{-iR\omega/\alpha_2}}{4\pi R} \quad (42)$$

where for the force applied on the solid W_p^r and W_p^z , with $p = \alpha_1 \vee \alpha_2$ are defined in equations (24) and for the force applied on the fluid W_p^r and W_p^z , with $p = \alpha_1 \vee \alpha_2$ are defined in equations (28).

3.5. DYNAMIC STREAMING CURRENTS FROM POINT FORCES

Streaming currents are generated by both compressional and rotational waves. We will discuss these mechanically-induced streaming current responses generated by an unit strength vertical point force, by an explosive point force acting on the solid frame and a volume injection source acting on the fluid.

In Figure 8, the stationary-phase radiation patterns at 200 *Hz* are shown for a vertical point force where equations (38) and (39) are used, and an explosive point source on the solid and a volume injection source where equation (41) is used. The correctness of the stationary-phase solutions were checked against synthetic seismograms computed using the discrete-wavenumber method. This method evaluates the integral expressions by discretizing the integral over the wavenumber using FFT methods to perform the frequency integrals (Bouchon and Aki, 1977).

The streaming current radiation pattern parameters are calculated at a permeability, $k = 10^{-12} m^2$, salinity, $C = 10^{-3} mol/l$ and a 20 % porosity (see Figure 9). The unit force strength vertical point force induces the largest streaming current magnitudes compared to the other point sources, as shown in Figure 8. The rotational mechanical waves induce in this case approximately 10 times larger streaming currents compared to the streaming currents induced by the compressional waves in a medium with 20% porosity.

In Figure 9, the *P* and *SV* relative flow amplitudes, W_β^r and W_{α_1} , defined in equations (38) and (39) are shown. The increase in the W_β^r coefficient is largely due to the decrease in shear frame modulus with increasing porosity (a squishier material). The combined porosity effect on the medium and Biot's moduli result in the more complicated W_{α_1} behavior. The relative flow amplitude is zero at exactly zero porosity, resulting in a zero streaming current amplitude as well. Approaching zero porosity, i.e., the elastic limit, the W_β^r and W_{α_1} coefficients become equal in amplitude and are normalized to one in Figure 9. At zero porosity the difference in induced streaming current amplitude is solely due to the difference in *P* and *SV* wavefield velocity. For nonzero porosities the W_β^r and W_{α_1} amplitudes are different from each other. The *P* wave and *SV* wave-induced streaming current amplitudes are now not only determined by the wavefield velocities but also by the relative flow amplitudes. At a 20% porosity the combined effect of wave type velocity, and W_β^r/W_{α_1} coefficient difference, results in a ten times larger streaming current induced by the rotational waves than by the fast compressional waves.

Figures 10, 11 and 12 show the streaming current amplitude behavior with respect to fluid salinity, porosity and permeability, calculated at a 100 Hz center frequency. Figure 13 shows the streaming current amplitude behavior with respect to permeability, calculated at 100 kHz center frequency. The rows indicated by *a*, *b*, and *c* in Figures 10, 11, 12 and 13 denote relative flow/streaming currents induced by a vertical point force, an explosive point force acting on the frame, and a volume injection source, respectively. The left column shows the relative flow amplitude behavior versus medium property and the right column shows the streaming current behavior versus medium property. In Figure 14, the *S* wave-induced relative flow/streaming currents are plotted against salinity (row *a*), permeability (row *b*), and porosity (row *c*), at a 100 Hz center frequency. The maximum relative flow/streaming current amplitudes are plotted in the case of a vertical point force. The amplitudes at frequency 100 Hz are calculated at $R = 200\text{ m}$ from the source, while the amplitudes at frequency 100 kHz are calculated at $R = 0.2\text{ m}$, corresponding to about 6 wavelengths from the source.

3.5.1. Streaming Current Behavior with Respect to Salinity

Figure 10 shows the relative flow/streaming current amplitude behavior with respect to changes in fluid salinity for the three point forces. The relative flow amplitudes are constant over five orders of fluid salinity change. This comes as no surprise, since we have neglected the electrically-induced part of the relative flow. With decreasing salinity the electrical double layer and the ζ -potential become larger. Therefore, with decreasing salinity a bigger osmotic drag that opposes the relative flow is expected. The streaming current decreases with increasing salinity. This is consistent with the decrease of the diffuse double layer thickness and the decrease of the zeta potential. At a salinity of $C = 0.5\text{ mol/l}$, the ζ -potential changes sign, [see equation (8)], whose effect on the streaming currents can be observed in Figure 10.

3.5.2. Streaming Current Behavior with Respect to Porosity

Figure 11 shows the relative flow/streaming current behavior with respect to changes in porosity for the different point sources. The *P* and *SV* wavefield velocities change with porosity and therefore scale the streaming current amplitudes accordingly. In the elastic limit (zero porosity), both the relative flow and streaming current are zero. The local minimum response at 10% porosity is a combined effect of the bulk moduli of the solid and fluid phase and the frame bulk and shear modulus. A smaller bulk modulus of the solid phase and a smaller frame bulk and shear moduli shift the minimum to a bigger porosity. A monotonic increase of the relative flow with increasing porosity is obtained when the pore space is saturated with a very compressible fluid (i.e., a gas). The streaming current trends with increasing porosity are similar for all calculated source types and show a corresponding functional shape with the relative flow curves.

3.5.3. Streaming Current Behavior with Respect to Permeability

Figures 12 and 13 show the streaming current behavior with respect to changes in permeability for three different sources at a 100 Hz and 100 kHz source center frequency. The streaming

current behavior with respect to permeability is different when induced by a force applied on the solid or induced by a source applied on the fluid. In Figure 12, a transition frequency of $\omega_t = 100$ Hz corresponds to a permeability of 10^{-9} m^2 when the porosity = 20 %, viscosity = 10^{-3} Pas and $\rho_f = 10^{-3} \text{ kg/m}^3$ are the other medium properties [see equation (5)]. All curves in Figure 12 are in the viscous-dominated flow regime.

In Figure 13 however, a transition frequency of $\omega_t = 100 \text{ kHz}$ corresponds to a permeability of 10^{-13} m^2 . We distinguish two flow type regimes—a viscous flow-dominated regime for permeabilities less than 10^{-13} m^2 and an inertial flow regime for permeabilities larger than 10^{-13} m^2 . Above the transition frequency in the inertial flow regime, the $L(\omega)/k(\omega)$ ratio decreases with increasing permeability, since the permeability decreases as ω while L decreases as $\omega^{1/2}$. This results in a drop-off in streaming current for permeabilities larger than 10^{-13} m^2 in Figure 13.

The decrease in streaming current at the permeabilities corresponding to the viscous flow regime causes a decrease of the $L(\omega)/k(\omega)$ ratio ($k(\omega)$ increase is stronger than the increase in $L(\omega)$) with decreasing permeability, as shown in Figures 4 and 5.

3.5.4. Streaming Current Sheet Behavior with Respect to Salinity, Permeability and Porosity

In Figure 14, we show the relative flow and streaming current responses induced by S waves generated by a vertical point force at a 100 Hz center frequency. Shear waves are divergence-free and cannot induce relative fluid motion due to pressure gradients, as within a P wave, but can induce relative fluid flow due to grain accelerations. Therefore, both P and S waves generate streaming currents. The relative flow and streaming currents are larger when induced by the S waves than the P waves since they depend on the factors $1/v_s^2$ and $1/v_p^2$, respectively.

The relative flow and streaming current behavior for S waves is similar to its response for P waves. The streaming current response versus permeability shows a local minimum at $10^{-11} < k < 10^{-10} \text{ m}^2$, but will fall off at permeabilities larger than $k = 10^{-9} \text{ m}^2$. The relative flow and streaming current behavior versus porosity is different from its response for P waves, as shown in Figure 11. The relative flow increases with increasing porosity. Its increase is largely due to the decrease in shear frame modulus with increasing porosity.

3.6. DISCUSSION AND CONCLUSIONS

In this section, we summarize the results of this study. The dynamic streaming current analysis is simplified by decoupling the mechanically- and electrically-induced parts. The electrically-induced streaming current is shown to be second-order in electrokinetic coupling coefficient and is neglected. We have used Biot theory to calculate the amount of induced relative flow by the Green's function solution.

The sensitivities of the dynamic transport coefficients, $\sigma(\omega)$, $k(\omega)$ and $L(\omega)$, to porosity and dc permeability are determined. The conductivity decreases with increasing permeability. When the pore length parameter becomes comparable to the grain surface conductances, the surface conductances become more important than the fluid phase conductivity. This occurs when the

pore space is filled with clays which don't decrease the porosity much but greatly enhance the conduction along the pore surfaces.

The streaming current amplitude variability is calculated versus fluid salinity, porosity, and permeability induced by P and S waves generated by three different point sources. Streaming currents are induced both by P waves, through pressure gradients, and divergence-free S waves, through grain accelerations. The streaming current decreases with increasing salinity. This is consistent with the decrease of the diffuse double-layer thickness and a decrease of the ζ -potential. The streaming current response versus porosity induced by the P waves differs from the response induced by the S waves. The porosity affects the bulk moduli of the solid. Its effect, combined with the frame bulk modulus and the compressibility of the saturating fluid, determines the streaming current amplitude induced by a P wave versus porosity. The increase in streaming current amplitude induced by S waves with increasing porosity is due to the decrease of the shear frame modulus with increasing porosity. The streaming current with respect to the dc permeability response differs for sources applied to the elastic frame and volume injection sources. The $L(\omega)/k(\omega)$ amplitude with respect to permeability above and below the transition frequency determines the streaming current behavior with respect to permeability.

The numerical results can be used to design an experiment to measure dynamic streaming currents generated by transducers or volume injection sources. Experimental data analyzed with the stationary-phase radiation approach allows quick computation of the streaming currents as a function of the transport and fluid chemistry parameters. In addition, no frequency restrictions are required for this approach, making the method useful for investigating the amplitude variability at different source center frequencies (above and below the transition frequency). The advantage of measuring streaming currents is one does not need to know the value of the (surface) conductance if one wants to determine the ζ -potential. Since a dynamical measurement is made, electrode polarization might be less of a problem.

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APPENDIX: Dynamic Relative Flow Green's Functions in Poro-Elastic Media

To obtain the relative flow vector generated by vertical and explosive (volume injection) point forces, the relative flow Green's function solutions for a point source applied to either the frame or the pore fluid are used. Substitution of this relative flow vector into the mechanical streaming current equation (14) yields the dynamic streaming current components. The relative flow Green's functions are derived in Haartsen *et al.* (1996).

Relative Flow Green's Function with the Source on the Frame

$$\begin{aligned} \underline{\underline{G}}_s^w &= g^w \underline{\underline{I}} + \left[\frac{1}{C} (HM - C^2) - G \frac{\rho_E}{\rho_f} \right] \nabla \nabla \Phi \\ &- \left[\frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \nabla \Phi^* \end{aligned} \quad (\text{A-1})$$

where the superscript w denotes the Greens's function to be a relative flow Green's function and the subscript s denotes the point force applied to the solid matrix.

$$\Phi = B(ws) \frac{e^{ik_\beta R}}{4\pi R} + A_1(ws) \frac{e^{ik_{\alpha 1} R}}{4\pi R} + A_2(ws) \frac{e^{ik_{\alpha 2} R}}{4\pi R} \quad (\text{A-2})$$

with the introduced variables $B(ws)$, $A_1(ws)$, $A_2(ws)$, where (ws) denotes scalar Green's function amplitudes belonging to the relative flow Green's function with a source applied on the solid frame defined as,

$$\begin{aligned} B(ws) &= \frac{1}{G [HM - C^2] \frac{\rho_E}{\rho_f}} \frac{-k_\beta^2 C + \omega^2 \rho_f}{(k_\beta^2 - k_{\alpha 1}^2) (k_\beta^2 - k_{\alpha 2}^2)} \\ A_1(ws) &= \frac{1}{G [HM - C^2] \frac{\rho_E}{\rho_f}} \frac{-k_{\alpha 1}^2 C + \omega^2 \rho_f}{(k_{\alpha 1}^2 - k_\beta^2) (k_{\alpha 1}^2 - k_{\alpha 2}^2)} \\ A_2(ws) &= \frac{1}{G [HM - C^2] \frac{\rho_E}{\rho_f}} \frac{-k_{\alpha 2}^2 C + \omega^2 \rho_f}{(k_{\alpha 2}^2 - k_\beta^2) (k_{\alpha 2}^2 - k_{\alpha 1}^2)} \end{aligned} \quad (\text{A-3})$$

Φ^* is defined as,

$$\Phi^* = B^*(ws) g_\beta + A_1^*(ws) g_{\alpha 1} + A_2^*(ws) g_{\alpha 2} \quad (\text{A-4})$$

with the variables $B^*(ws)$, $A_1^*(ws)$, $A_2^*(ws)$ defined as,

$$B^*(ws) = \frac{B(ws)}{\omega^2 \rho_f - C k_\beta^2}, \quad A_1^*(ws) = \frac{A_1(ws)}{\omega^2 \rho_f - C k_{\alpha 1}^2}, \quad A_2^*(ws) = \frac{A_2(ws)}{\omega^2 \rho_f - C k_{\alpha 2}^2}. \quad (\text{A-5})$$

The relative flow scalar shear Green's function is given by,

$$g^w(\mathbf{x}, \mathbf{x}') = -\frac{\rho_f}{\rho_E} \frac{e^{ik_0 R}}{4\pi G R} \quad (\text{A-6})$$

where $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$ is the distance from the source to the receiver and $k_0 = \omega / \sqrt{G / (\rho_B - \frac{\rho_f^2}{\rho_E})}$ is the wave number.

Relative Flow Green's Function with the Source on the Fluid

$$\underline{\underline{G}}_f^w = \nabla \Pi^*, \quad \Pi^* = \Lambda_1 g_{\alpha_1} + \Lambda_2 g_{\alpha_2} \quad (A-7)$$

where,

$$\begin{aligned} \Lambda_1 &= \left[HG\omega^2 \rho_E (k_{\alpha_1}^2 k_{\beta}^2 - k_{\alpha_1}^4) + \omega^4 \rho_E \rho_B G (k_{\alpha_1}^2 - k_{\beta}^2) \right] A_1^*(wf) \\ \Lambda_2 &= \left[HG\omega^2 \rho_E (k_{\alpha_2}^2 k_{\beta}^2 - k_{\alpha_2}^4) + \omega^4 \rho_E \rho_B G (k_{\alpha_2}^2 - k_{\beta}^2) \right] A_2^*(wf) \\ A_1^*(wf) &= \frac{A_1(wf)}{(\omega^2 \rho_B - k_{\alpha_1}^2 H) (\omega^2 \rho_B - k_{\alpha_1}^2 G)} \\ A_2^*(wf) &= \frac{A_2(wf)}{(\omega^2 \rho_B - k_{\alpha_1}^2 H) (\omega^2 \rho_B - k_{\alpha_1}^2 G)} \end{aligned} \quad (A-8)$$

with the introduced variables $A_1(wf), A_2(wf)$, where (wf) denotes scalar Green's function amplitudes belonging to the relative flow Green's function with a source applied on the fluid phase defined as,

$$\begin{aligned} A_1(wf) &= \frac{1}{G\omega^2 \rho_E [HM - C^2]} \frac{(\omega^2 \rho_B - k_{\alpha_1}^2 H) (\omega^2 \rho_B - k_{\alpha_1}^2 G)}{(k_{\alpha_1}^2 - k_{\beta}^2) (k_{\alpha_1}^2 - k_{\alpha_2}^2)} \\ A_2(wf) &= \frac{1}{G\omega^2 \rho_E [HM - C^2]} \frac{(\omega^2 \rho_B - k_{\alpha_2}^2 H) (\omega^2 \rho_B - k_{\alpha_2}^2 G)}{(k_{\alpha_2}^2 - k_{\beta}^2) (k_{\alpha_2}^2 - k_{\alpha_1}^2)}. \end{aligned} \quad (A-9)$$

The following variables have been used in the relative flow Green's function description: $\underline{\underline{I}}$ denotes the identity matrix, ρ_B is the bulk density and is defined as the sum of the phase volume weighted fluid and solid densities, ρ_f is the fluid density, $\rho_E[kg/m^3] = \frac{-\eta}{i\omega k(\omega)}$ with η the fluid viscosity and $k(\omega)$ the frequency dependent permeability, and k_{α_1} , k_{α_2} and k_{β} are the fast, slow and shear wavenumbers, respectively.

The H , M , C and G moduli describe the stress-strain behavior of the two phases and are related by the following equations.

$$K_G = H - \frac{4}{3}G = \frac{K_{fr} + \phi K_{fr} + (1 - \phi)K_s \Delta}{1 + \Delta} \quad (A-10)$$

$$C = \frac{K_f + K_s \Delta}{1 + \Delta} \quad (A-11)$$

$$M = \frac{1}{\phi} \frac{K_f}{1 + \Delta} \quad (A-12)$$

where the parameter Δ is defined as,

$$\Delta = \frac{K_f}{\phi K_s^2} [(1 - \phi)K_s - K_{fr}] \quad (\text{A-13})$$

The moduli K_{fr} and G_{fr} are the bulk and shear moduli of the framework of the grains, when the fluid is absent. The frame moduli may either be considered experimentally determined or may be obtained from approximate theoretical models for specific pore grain geometries. C and M are the incompressibilities used by Biot (1962,a,b). They are complex and frequency dependent, allowing for losses in addition to those associated with relative flow.

The Quartz / Electrolyte Interface

Figure 1: The Quartz/electrolyte interface according to Stern (1924). In the compact layer the electrolyte ions and structured water dipole molecules are electrostatically and chemically adsorbed to the surface of the solid. Beyond the compact layer of bound ions there is a diffuse distribution of mobile ions whose position is determined by Poisson-Boltzman statistics. The potential at the separation plane between the mobile and immobile charge is called the ζ -potential.

dc permeability/dynamic permeability versus porosity versus frequency

Figure 2: Dependence of the ratio of dc permeability to dynamic permeability on porosity and frequency. Top: the real part. Bottom: the imaginary part.

dc L/dynamic L versus porosity versus frequency

Figure 3: Dependence of the ratio of dc electrokinetic coupling coefficient to dynamic electrokinetic coupling coefficient on porosity and frequency. Top: the real part. Bottom: the imaginary part.

dc permeability/dynamic permeability versus dc permeability versus frequency

Figure 4: Dependence of the ratio of dc permeability to dynamic permeability on permeability and frequency. Top: the real part. Bottom: the imaginary part.

dc L/dynamic L versus dc permeability versus frequency

Figure 5: Dependence of the ratio of dc electrokinetic coupling coefficient to dynamic electrokinetic coupling coefficient ratio on permeability and frequency. Top: the real part. Bottom: the imaginary part.

Figure 6: Conductivity versus porosity in a porous medium with $k = 10^{-12}m^2$ and $C = 10^{-3}mol/l$. The conductivity doesn't change linearly with porosity due to the porosity dependence of the pore length parameter, $\Lambda[m]$.

Figure 7: Conductivity versus permeability in a porous medium with $\phi = 20\%$ and $C = 10^{-3} mol/l$. At $k_0 < 10^{-16} m^2$ the pore length parameter $\Lambda[m]$ becomes comparable to the sum of the grain surface conductances. Physically this situation occurs when the pore space is filled with clays, which do not decrease the porosity but greatly enhance the surface to pore volume ratio and therefore enhance the conduction along the pore surfaces.

Figure 8: Streaming current radiation patterns for different unit strength seismic sources calculated at a 200 Hz center frequency. (a) vertical point force; (b) an explosive point force acting on the solid frame; (c) volume injection source.

Figure 9: Relative fluid-solid motion generated by P (line with circles) and S (line with squares) waves as a function of porosity. The relative flow amplitude generated by the P -waves are affected by the porosity effect on the bulk and frame moduli of the solid and the compressibility of the saturating fluid. The increase in relative fluid-solid motion generated by S -waves with increasing porosity is due to the decrease of the shear frame modulus with increasing porosity.

Figure 10: Relative flow (left column) and streaming current density (right column) dependence on salinity, induced by P waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 100 Hz center frequency.

Figure 11: Relative flow (left column) and streaming current density (right column) dependence on porosity, induced by P waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 100 Hz center frequency.

Figure 12: Relative flow (left column) and streaming current density (right column) dependence on permeability, induced by P waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 100 Hz center frequency.

Figure 13: Relative flow (left column) and streaming current density (right column) dependence on permeability, induced by P waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 100 kHz center frequency.

Figure 14: Relative flow (left column) and streaming current density sheets (right column) induced by S waves generated by a vertical point force as function of (a) salinity, (b) permeability and (c) porosity at a 100 Hz center frequency.

4. EXPERIMENTAL STUDIES OF ELECTROKINETIC CONVERSIONS IN A FLUID-SATURATED POROUS MEDIUM

4.1. SUMMARY

The electrokinetic effect in a fluid-saturated porous rock is defined as the coupling and conversion between seismic and electric energies. When seismic waves propagate through a fluid-saturated formation and cause a pore fluid-flow relative to the solid matrix, the motion of the cations in the fluid-flow forms a streaming electrical current and induces an electromagnetic wave at any discontinuous interface of the formation or stationary electric potential inside the homogeneous formation. Another conversion of energies opposite to the seismoelectric conversion is when an alternating electric field induces a relative fluid-flow in a fluid-saturated porous rock where fluid-flow can generate an electroseismic wave in the rock. In this paper we study the electrokinetics in porous sandstone and man-made porous models at high frequencies. A P-wave or S-wave transducer excites different acoustic wave modes in a cylinder, layer, or borehole model. Our experiments observe and record the radial or stationary seismoelectric signals induced at the interface or inside the formation. Some relative experiments have confirmed the reliability of the electrokinetic phenomenon observed in our experiments and the mechanism that is different from the piezoelectric effect. The results show that the seismoelectric signal induced by the extensional or flexural wave in the sandstone cylinder is a stationary local electric potential. The seismoelectric signal induced at the interface of the layer model is an electromagnetic wave which can be received within the fluid-filled porous medium. Experimental measurements performed in a borehole model by means of vertical seismic profiling (VSP) and single borehole logging show it is possible to conduct seismoelectric measurements in a deep borehole of petroleum formation. Measurements of electrokinetics can thus provide an effective means for estimating parameters in a fluid-saturated porous formation.

4.2. INTRODUCTION

The electrokinetic phenomenon in porous media is intimately related to the electrochemistry of the interface region between a solid and a fluid. When a fluid electrolyte comes into contact with a neutral solid surface (like a quartz wall), anions from the electrolyte are chemically adsorbed to the surface. An excess of cations is left behind the negative layer and distributed near the surface. Some are bound so strongly that they are immobilized. Beyond this layer there are mobile cations (Pride and Morgan, 1991). When a fluid electrolyte moves related to a solid, the movement of the mobile cation forms an electric field. The potential associated with this electric field is called the streaming potential. In contrast, when an electric field drives a cation motion, the generated relative movement between the fluid and solid excites mechanic movement or an acoustic wave in a porous medium. Many laboratory experiments (Broz and Epstein, 1976; Ishido and Mizutani, 1981; Kuin and Stein, 1987; Cerda and Kiry, 1989; and Morgan *et al.*, 1989) were performed to investigate the streaming potential generated by the flowing of different fluids at different pressures through porous sands or man-made porous material. In these experiments the electrolyte moves forward a certain direction or moves back and forth at low frequency range (usually under 1000 Hz).

The electrokinetic phenomenon of the conversion between seismic and electromagnetic energies was discovered and proposed to be used as an exploratory tool in the 1930s (Thompson, 1936; Belluigi, 1937). When seismic waves propagate through a fluid-saturated formation, the vibration and displacement of the grain and the pore fluid cause their relative motion. The mobile cations in the fluid act as a current source and induce an electric field in the formation. The current flows generate electromagnetic waves at the interface where the electric or acoustic properties are different on both sides. On the other hand, when the pore fluid with cations has a relative motion enforced by an alternating electric field, the relative action between the grain and pore fluid generates seismic waves. Theoretical studies have demonstrated electrokinetic effects in porous media (Kozak and Davis, 1987; Neev and Yeatts, 1989; Thompson and Gist, 1991; Pride and Morgan, 1991; Pride *et al.*, 1993; Haartsen, 1995). Field tests have also observed seismoelectric effects (Martner and Sparks, 1959; Sobolev *et al.*, 1984; Russell and Barker, 1991; Thompson and Gist, 1993). Because of the low signal-to-noise ratio in field tests, intensive data processing is needed to extract the electric signals; it is difficult to compare the field results to the theoretical calculations.

To investigate electrokinetic behavior and compare it with theoretical predictions, we studied the electrokinetic effect of a fluid-saturated sandstone or man-made porous medium in the M.I.T. Earth Resources Laboratory's ultrasonic laboratory, observed the seismoelectric and electroseismic conversions, and studied their characteristics and radiation. We constructed cylinder, layer, and borehole models to excite different acoustic wave modes. The seismoelectric potentials generated by the acoustic waves were measured with an electrode and compared with the acoustic waveforms. Relative experiments have eliminated the nonreliability in our experiments and confirmed the basic principle that fluid-flow induces the seismoelectric effect.

4.3. EXPERIMENTS IN A CYLINDER MODEL

To excite a simple and pure acoustic mode, a cylinder model was made of Berea sandstone or Coconino sandstone 1.24 cm in diameter and 10 cm in length. Table 1 lists the properties of the sandstones. Figure 1 shows a diagram of the experimental setup. A water-saturated cylinder sample is placed between the acoustic source and receiver transducers. The measurement points of 0.2 cm in diameter and 1 cm in spacing are made of conducting glue on the sample surface along its axis. An electrode with a sharp pin-head touches these points consecutively and receives the electric potentials on the surface. The electric signals received through an amplifier and a filter are digitly recorded. A metal film shields the whole sample to prevent outside electric influence. If the source is a compressional wave transducer, an extensional wave is excited in the cylinder sample; if it is a shear transducer, a flexural wave can be excited. The electric pulse for exciting the acoustic transducer is a square wave with a high voltage of 500 volts. The width of the pulse is variable and is usually equal to the half cycle of the acoustic wave.

Figure 2 shows the seismoelectric signals generated by an acoustic compressional wave in the water-saturated Berea sandstone cylinder. The top trace (dashed line) in the plot is the acoustic wave received by the compressional transducer on the other end of the cylinder. The center frequency of these waveforms is approximately 18 kHz. The phase velocity determined by the slope of the waveforms is about 1700 m/s.

From the traveltimes of the first arrivals, we know that the signals are not due to electric

interference. Comparing these signals with the acoustic waveform received on the other end of the sample, we can see that the shape and the arrival time of the acoustic wave coincide with those of the electric signals. This means that the electric signals are generated by the extensional wave propagating along the cylinder sample.

A shear source transducer can excite a flexural wave in a cylinder model shown in Figure 1. Figure 3 shows electric signals generated by an acoustic shear wave and received by an electrode on the surface of the Coconino cylinder sample. The center frequency and the phase velocity of the signals recorded in Figure 3 are 27 kHz and about 1150 m/s, respectively. Despite the different orientations in particle motions of the extensional and flexural waves, once the relative motion between the pore fluid and the solid matrix is excited, both modes of the waves can generate seismoelectric signals in a fluid-saturated porous sandstone.

The received electric signals are related only to the acoustic wave arriving at the measurement point. In other words, the seismoelectric signals cannot be received at other points that are not disturbed by the acoustic wave, although the water-saturated sandstone cylinder is not an electric isolator. This means that the received electric signal is a stationary, local potential, which cannot propagate at electromagnetic speed within the sandstone cylinder. Basic principles of seismokinetics explain the reason: an acoustic wave excites the charge motion in the pore fluid and induces the streaming potential in the disturbed area. If the porous rock is homogeneous, the sum of the streaming currents within the disturbed area should be equal to zero; therefore, they do not induce any electric potential outside the area. If the rock is mechanically or electrically nonhomogeneous (e.g., there is an interface with mechanical or electrical difference) the nonsymmetric streaming current will induce an electromagnetic wave which propagates at high speed and can be received anywhere. We made a layer rock model to investigate the electrokinetics at an interface.

The relationship between the acoustic wave and the seismoelectric signal and the effect of the conductivity of the saturant on the seismoelectric signal have been investigated experimentally using the measurement setup shown in Figure 1. We have also studied the opposite energy conversion from electric into acoustic energy (Zhu *et al.*, 1994).

4.4. EXPERIMENTS IN A LAYER MODEL

Layer models are made of natural sandstones or a man-made porous medium to investigate the seismoelectric field generated at an interface and within the porous medium.

4.4.1. Sandstone-Layer Model

The layer model is composed of a Lucite block and a piece of a sandstone rock (Figure 4). A compressional transducer, as an acoustic source, excites the P-wave which radiates at 45 degrees on the interface between the Lucite and the sandstone block. The center frequency of the acoustic wave is about 100 kHz. The Lucite block serves to delay the arrival time of the acoustic wave on the interface and eliminates the electric interference of the exciting pulse (500 V) on the seismoelectric signals. On the interface between the Lucite and the rock ten measurement points on 2 cm centers are made of conducting glue to measure the electric field. The rock sample is water-saturated Berea or Coconino sandstone. There is no air bubble entrapped at the interface and all of the possible

gaps between the Lucite and the sandstone are saturated with water. The instrumentation of the system is depicted in Figure 1.

Measurements begin with the acquisition of acoustic waves by a small standard P-wave transducer at each measurement point on the Lucite. Figure 5 shows the received acoustic waveforms which are normalized by their maximum amplitude. Figures 6 and 7 show the electric signals which are generated by the acoustic wave and received by the electrode connecting the measurement points on the interface between the Lucite block and Berea or Coconino sandstone. The amplitude in Figure 5 and the following plots is normalized by the maximum of the amplitudes in each plot. Comparing these signals with the acoustic waveforms shown in Figure 5, we see that the arrival times, the shape of the waveforms, and the amplitude variation of the electric signals are in good agreement with the acoustic waves. Therefore, we conclude that the electric signals are indeed the seismoelectric ones generated on the interface between the nonpermeable Lucite and water-saturated sandstone; the conversion between the acoustic and electric energies occurs here.

4.4.2. Man-Made Layer Model

In order to investigate the electrokinetics inside a homogeneous porous medium and to compare them with those at an interface, we made a layer model with a Lucite block and sand. The sand is glued with epoxy and poured onto the surface of the Lucite block. The physical properties are shown in Table 1. Four sets of electrodes made of fine wires are buried on the interface and in the glued sand with a spacing of 1 cm (Figure 8). The conductivity and permeability of the water-saturated glued sand are $280 \Omega \cdot m$ and about 200 darcy, respectively. A P-wave transducer, as a source, is placed at the middle of the model.

Figure 9 shows the acoustic waveforms received by a small P-wave transducer on the Lucite bottom surface. The first arrivals in the waveforms are the compressional waves and the secondary arrivals are those reflected from the boundaries of the Lucite block. Figure 10 shows the waveforms of the electric signals received by electrode sets I, II, III, and IV. The arrival times and the amplitude variation of the electric signals shown in Figure 10a are in good agreement with the acoustic waves shown in Figure 9. This means the acoustic wave induces seismoelectric signals at the interface. From Figure 10b we see that the acoustic wave passing through the interface induces the seismoelectric signals inside the glued sand, and some electric signals at higher frequency and with lower amplitude are received before the acoustic wave reaches electrode set II. Comparing these waveforms with the first arrival of the electric signals in Figure 10a, we confirm that the first arrivals of the electric signals in Figure 10b are generated at the interface, then propagated in the medium with the electromagnetic wave velocity and finally received by electrode set II. Because the electromagnetic velocity is much faster than the acoustic velocity, there is virtually no time delay between the first arrivals of the signals shown in Figures 10a and 10b. In the signals received by electrode sets III and IV (Figures 10c and 10d), we can also observe the electric signals which are generated at the interface and then propagated into the glued sand.

Figure 11 shows the electric signals received by four electrodes at the same horizontal but different vertical locations. The distance of the electrodes from the interface is 0 cm, 1 cm, 2 cm and 3 cm, respectively. From these waveforms we see that there are two sets of the electric signals. One set is the first arrivals with the same arrival time. The other one is related to the acoustic

wave propagating in the medium. This means that only the seismoelectric signal generated at the interface can be received by the other electrodes inside the homogeneous medium. The electric signal induced inside the medium cannot be received by the electrodes outside the area disturbed by the acoustic wave. We conclude that the seismoelectric signal generated at the interface is an electromagnetic wave. The experiment results have been compared with the theoretical calculation (Haartsen *et al.*, 1995).

4.5. EXPERIMENTS IN A BOREHOLE MODEL

Due to the low efficiency of the seismoelectric conversion, the field measurement is only applied in exploring the shallow formation. To investigate the possibility of the direct seismoelectric measurement in a fluid-filled borehole, we made a borehole model (Figure 12) by pouring glued sand onto one side of a Lucite cylinder with a borehole along its axis and burying wire electrodes inside the sand. We have measured seismoelectric signals on the borehole wall and inside the porous medium by means of two methods similar to the VSP and single borehole logging, respectively. The acoustic sources are located on the side of the cylinder model and in the water-filled borehole, respectively.

Figure 13 shows a diagram of VSP measurement. A compressional wave transducer moves around the borehole on the side of the model. When a high voltage pulse excites the transducer and generates an acoustic wave in the model, we record the electric signals received by the electrodes on the borehole wall with the transducer movement of 30° /trace (Figure 14). Figure 15 shows the electric signals received by the electrodes at the same azimuth and different depth when the acoustic source is fixed at a certain position (180°). The acoustic wave induces a stronger seismoelectric (electromagnetic) wave (Figure 14a) at the interface between the Lucite and the porous medium, which propagates at the velocity of an electromagnetic wave in the medium and is received by the electrodes at a different depth of the borehole (Figures 14b-c). This method is useful for determining the formation interface by directly measuring the seismoelectric signals on the borehole wall. The signal-to-noise ratio of the recorded signals increases due to the direct measurement on the wall.

Figure 16 shows a diagram for measurements in a single water-filled borehole. Figure 17 shows the acoustic waves measured by an acoustic receiver in the borehole section of the porous medium when the acoustic source moves up in the Lucite section of the borehole. At the same time the electrodes on the borehole wall at different depths record the seismoelectric signals (Figure 18). It is clear that there are two kinds of seismoelectric signals induced by the P- and Stoneley wave in Figure 18. In this case, the electromagnetic wave induced at the interface between the Lucite and the porous medium is very weak because the main energy of the acoustic wave is concentrated in the borehole. The received seismoelectric wave is mainly the local electric potential. We may apply this method to measure seismoelectric signals in a deep petroleum formation. Because the acoustic source is close to the electrodes, it is possible to measure the borehole formation with higher resolution.

4.5.1. Relative Experiments

We have performed some relative experiments to confirm the seismoelectric phenomena observed in the previous experiments and to rule out the possibility of experimental errors. The measurement

with a dry sandstone sample is performed using the setup shown in Figure 1. In other experiments with a layer model we have checked to see if the seismoelectric signals can be generated on the interface between the Lucite and air, water, dry rock or unconsolidated sand, respectively.

The piezoelectric effect is a special phenomenon that needs to be distinguished from electrokinetics. The quartz grain in a sandstone is a piezoelectric crystal. When quartz is strained, an electric potential of a dipole moment is induced along its electric axis. The direction of the piezoelectric field depends on the orientation of the quartz grain which is random in sandstone. Therefore, when an acoustic wave propagates along a certain direction, both the amplitude and the polarization of the piezoelectric field generated by the acoustic wave depend on the orientations of the quartz grains.

Figure 19 shows the electric signals generated by a P-wave transducer and received on a dry Berea sandstone cylinder sample (Figure 1) that has been heated in a vacuum vessel for eight hours. From the time delay of the first arrivals in the signals we know that these electric signals are generated by the acoustic wave. On the other hand, not only the amplitude but also the phases of the first arrivals vary randomly with the measurement points. The maximum amplitude is about $20 \mu V$ and much less than that of the seismoelectric signals (Figure 2). Therefore, we conclude that the electric potentials generated by the acoustic wave in a dry rock sample are due to the piezoelectric phenomenon. In contrast to the piezoelectric case, the electric fields generated by the same acoustic wave in a water-saturated rock sample are all in the direction of the pressure gradient (see Figure 2) because seismokinetic coupling depends on the fluid flow and is significantly different from piezoelectric coupling.

A Lucite block is used in the layer models to eliminate the interference of the source pulse (Figure 4). We have checked the electric field generated by the P-wave on the Lucite surface. No electric signal can be received on the interface between the Lucite and the air or water (free surface) at the same experimental scale as that in Figure 6. We also measured the electric fields on the interfaces where the Lucite block contacts dry Berea sandstone or water-saturated unconsolidated sand. In all of these cases, we cannot receive any electric signal at the same recording scale. The baseline experimental results confirm that the electrokinetics predicted and observed are quite separate from the piezoelectric effect in a crystal. The experiment with the unconsolidated sand shows that the electrokinetics depends on the relative fluid flow. Because in this case the sand is not consolidated and moves together with the fluid under acoustic pressure gradient, the relative motion between the fluid and the sand decreases and the amplitude of the seismoelectric signals is small.

4.6. CONCLUSIONS

Our experiments demonstrate seismoelectric conversion at a high frequency region in a fluid-saturated porous medium. The experimental results confirm that the mechanism of electrokinetics is different from that of piezoelectrics in a crystal. The electrokinetic phenomenon is closely related to the relative flow of the pore fluid to the solid matrix. The seismic waves with various modes, such as compressional, shear, extensional, flexural, and Stoneley waves, can generate seismoelectric signals in a fluid-saturated porous formation.

The mechanism of the seismoelectric signals generated at an interface or inside a porous medium is the same, but there are two kinds of streaming potential—an electromagnetic wave that can

propagate in the medium, and a stationary local potential. If there is a mechanic or an electric discontinuity in the porous medium (interface), the acoustic wave will induce an electromagnetic wave due to streaming current imbalances at the interface. The seismoelectric signal induced inside a homogeneous formation or on a free surface is a stationary local streaming potential, which can be measured by an electrode, but does not propagate within the medium nor can it be measured in an undisturbed area. The experiments with the borehole model investigate the possibility of seismoelectric measurement which is similar to vertical seismic profiling or acoustic well logging. Therefore, it would be a very useful means for exploring a fluid-saturated porous formation in geophysical prospecting.

The results of the experiment with complex models (layer and borehole models) only demonstrate the seismoelectric phenomena qualitatively. In future work, we will study the quantitative relationship between the seismoelectric signal and the formation parameters.

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Table 1: Physical Parameters of Water-Saturated Samples

Sample	Density (g/cm^3)	Porosity (%)	Vp (m/s)	Vs (m/s)
Berea Sandstone	2.08	18.9	3050	1950
Coconino Sandstone	2.29	13.2	3680	2290
Glued Sand	1.71	25.6	2540	1200
Lucite	1.18	0.0	2700	1290

Figure 1: Experimental diagram for measuring the electric field generated by a transducer in a cylinder rock sample.

Figure 2: Electric signals generated by a P-wave transducer in a water-saturated Berea sandstone cylinder sample and the compressional wave (dotted line) received on the other end of the cylinder.

Figure 3: Electric signals generated by an S-wave transducer in a water-saturated Coconino sandstone cylinder sample and the shear wave (dotted line) received on the other end of the cylinder.

Figure 4: A layer model for measuring the electric signals generated by a P-wave at the interface between a Lucite block and a piece of sandstone.

Figure 5: Normalized acoustic waveforms received by a P-wave transducer on the bottom surface of the Lucite block (Figure 4).

Figure 6: Electric signals received at 10 measurement points on the interface between the Lucite block and the water-saturated Berea sandstone sample.

Figure 7: Electric signals received at 10 measurement points on the interface between the Lucite block and the water-saturated Coconino sandstone sample.

Figure 8: Man-made layer model for measuring electric signals. The parameters of the glued sand as shown in Table 1. Four sets of the electrodes are buried in the sand. The conductivity and the permeability of the water-saturated glued sand is about $280\Omega \cdot m$ and 200 darcy, respectively.

Figure 9: Normalized compressional waveforms generated by a P-wave source and received by a P-wave receiver on the bottom surface of the Lucite block (Figure 8).

Figure 10: Electric signals generated by a P-wave source (Figure 8) and received by the electrode sets I (a), II (b), III (c), and IV (d). The amplitude is normalized by the maximum of 11 traces in each plot.

Figure 11: Electric signals received by the electrodes at the same position but different depth. The distance of the electrodes from the interface is 0 cm, 1 cm, 2 cm, and 3 cm for trace 1 to trace 4, respectively.

Figure 12: A borehole model made of Lucite and glued sand. The diameter of the borehole is 1.27 cm.

Figure 13: A diagram for VSP measurement. A compressional transducer moves around the borehole on the top side of the Lucite. The electrodes are buried in the borehole wall to receive the electric signals.

Figure 14: Electric signals received by electrodes #1 (a), #2 (b), #3 (c), and #4 (d) when the P-wave transducer moves around with 30° /trace on the top side of the Lucite. The amplitude is normalized by the maximum of 13 traces in each plot.

Figure 15: Electric signals received by electrodes #2, #3, and #4 when the transducer is fixed at 270° on the top side of the Lucite.

Figure 16: A diagram for borehole measurement. When an acoustic source moves up in the water-filled borehole with 1 cm spacing, the acoustic wave in the borehole and the electric signal on the borehole wall can be received and recorded by a fixed acoustic receiver and the electrodes, respectively.

Figure 17: Acoustic wave received by an acoustic receiver fixed in the borehole section of the glued sand when the acoustic source moves up with 1 cm/trace along the water-filled borehole.

Figure 18: Electric signals received by electrodes #1 (a), #2 (b), #3 (c), and #4 (d) when the acoustic source moves up with 1 cm/trace along the water-filled borehole. The amplitude is normalized by the maximum of 8 traces in each plot.

Figure 19: Electric signals generated by a P-wave transducer with a dry Berea cylinder sample (Figure 1).

5. EXPERIMENTAL STUDIES OF SEISMOELECTRIC MEASUREMENTS IN A BOREHOLE

5.1. SUMMARY

Experimental and theoretical studies show that there are two kinds of electromagnetic (EM) fields generated by seismic waves in a fluid-saturated porous medium. First, at an interface where the formation properties are different, the generated seismoelectric wave is a propagating electromagnetic wave that can be received anywhere. The second kind of field occurs inside a homogeneous formation where the seismic wave generates an electromagnetic field which exists only in the area disturbed by the seismic wave and whose apparent velocity is that of the seismic wave. An electrode, used as a receiver located on the ground surface, can only receive the propagating EM wave. However, when an electrode is in a borehole and close to the porous formation, it can receive both of the above EM waves. In this study, electrokinetic measurements are performed with borehole models made of natural rocks or artificial materials. The results of the experiment show that the Stoneley wave and other acoustic modes excited by a monopole source in the borehole models generate seismoelectric waves in fluid-saturated formations. The electrical components of the seismoelectric waves can be received by an electrode in the borehole or on the borehole wall. The amplitude and frequency of the seismoelectric wave are related not only to the seismic wave, but also to the formation properties, such as permeability, conductivity, etc. Therefore, seismoelectric logging may explore different properties of the formation than those investigated by standard acoustic logging. Electroseismic measurements are also performed with these borehole models. The electric pulse introduced through the electrode in the borehole or on the borehole wall induces a Stoneley wave in the fluid-saturated model which can be received by a monopole transducer in the same borehole. These measurement methods, seismoelectric logging or electroseismic logging, can be applied to field borehole logging to investigate formation properties relating to pore fluid flow.

5.2. INTRODUCTION

Because of the electro-chemical effects on the interfaces between solid and fluid in a fluid-filled porous medium, there is a phenomenon known as the electric double layer on the solid surface and free charges in the pore fluid (Belluigi, 1937; Martner and Sparks, 1959; Broz and Epstein, 1976; Neev and Yeatts, 1989; Haartsen, 1995; Haartsen and Toksöz, 1996). When a seismic wave propagates in the medium, the charge motion generates an electromagnetic field. This is the seismoelectric conversion (Chandler, 1981; Ishido and Mizutani, 1981; Morgan *et al.*, 1989; Zhu *et al.*, 1994; Jouniaux and Pozzi, 1995). On the other hand, when an electric field induces a relative motion of the free charges in the pore fluid against a solid matrix, the interaction between the pore fluid and the solid generates an electroseismic wave. This is the electroseismic conversion (Kozak and Davis, 1989; Pride and Haartsen, 1996; Zhu *et al.*, 1997).

The generated seismoelectric signals relate not only to the seismic waves, but also to the conductivity (or resistivity) of the pore fluid and the permeability of the formation (Cerdeña and Kiry, 1989; Kuin and Stein, 1987; Pride, 1994). When the properties of the contacting formations, such as conductivity, permeability, rock grains, etc., are different from each other, a physical interface is

formed between the two formations. Because of the imbalance of the free charges in the pore fluid at the interface, the seismic wave generates a free electromagnetic wave propagating at light speed, which is much faster than that of any acoustic wave. Inside the homogeneous medium, the charges are balanced in the areas disturbed by the seismic wave, which generates a seismoelectric field in the disturbed area, but does not generate a seismoelectric wave outside the area. This seismoelectric wave is a local EM wave and does not propagate in the medium. Its apparent velocity is the same as the seismic velocity (Pride and Haartsen, 1996; Zhu and Toksöz, 1996; Zhu *et al.*, 1996).

Many previous measurements were conducted on the earth's surface or in shallow boreholes (Thompson and Gist, 1993; Butler *et al.*, 1994; Mikhailov *et al.*, 1997). Only seismoelectric signals generated at an interface can be received. It is difficult to apply this measurement to explore deep formations due to the low efficiency of the seismoelectric conversion and the high attenuation in the formations.

In this paper, we investigate a logging method that can measure both the propagating and the local seismoelectric waves by setting the electric receiver in a borehole. Since the electrode is close to the borehole formation, stronger signals can be received with high measurement accuracy. If both the seismic source and the electrode are in a borehole, this method can explore deep formations and can be applied to petroleum geophysics.

In our study, several ultrasonic borehole models are made of natural rocks and artificial materials. We measured the seismoelectric signals with different logging methods, investigated the seismoelectric fields at different formations, and compared them with the acoustic waves in the boreholes. We also measured and compared the electroseismic waves generated by an electric pulse with high voltage at the different sections of the borehole models.

5.3. BOREHOLE MODELS AND MEASUREMENT SYSTEM

The simplest borehole model is a sandstone cylinder with a borehole of 1.27 cm in diameter along its axis. Other models are made of two different materials to simulate different formations and an interface (Figure 1). The sizes and materials of the five borehole models we used are listed in Table 1. The glued sand with large permeability is made of fine quartz sand glued with epoxy. Some electrodes are buried on the borehole wall inside the glued sand. The properties of the materials are listed in Table 2.

For the measurements of the seismoelectric conversion, a PZT transducer 0.9 cm in diameter, is the seismic source which is excited by an electric square pulse of 0.01 ms in width and 750 V in amplitude. The electrode is a point receiver, 0.05 cm in diameter and 0.1 cm in length, made of a shielding cable wire. The received seismoelectric signals are displayed and recorded on a digital oscilloscope after going through an amplifier and a filter (Zhu *et al.*, 1996). The borehole models are put in a vacuum system and saturated with water, and then the models and the source transducer, as well as the point electrode, are put in a water tank. To compare the seismoelectric field with the acoustic field in the boreholes, the experiments are repeated with a hydrophone (B & K 8103) replacing the electrode to measure the acoustic fields.

For the measurements of the electroseismic conversion, an electrode emitting a high-voltage pulse is the source. A hydrophone is used to receive and record the electroseismic waves.

All received waveforms are recorded with a certain time delay to avoid the huge electric influence of the high voltage pulse. The amplitudes of the waveforms are normalized by the maximum amplitude in each plot. The maximum amplitude of the seismoelectric signals in our experiments is around 0.1 mV.

5.4. SEISMOELECTRIC CONVERSION

The goal of the experiments described in this section is to confirm that seismic waves in a borehole, particularly the Stoneley wave, generate seismoelectric signals. Later we will investigate the seismoelectric field in a borehole. In the other words, first we fix the electrode and move the seismic source in the boreholes (Figure 2). Then we investigate the seismoelectric field in a borehole. We will move the electrode to measure the seismoelectric signals in the boreholes when the seismic source is fixed. To compare this with the acoustic field in the borehole, we replace the electrode with a hydrophone to record the acoustic waves under the same conditions.

Figure 3 shows the acoustic waves (Figure 3a) and the seismoelectric waves (Figure 3b) in the sandstone model (Model 1), when the electrode is fixed in the borehole center and the acoustic source is moved along the borehole axis. Because of the excitation and the attenuation in the sandstone borehole, the frequency of the Stoneley wave is very low (about 20 kHz), but the frequency of the P-wave and pseudo-Rayleigh wave are much higher (about 150 kHz). From the slope of the seismoelectric signals in Figure 3b, we know that the Stoneley wave and the low-frequency component of the P-wave generate the seismoelectric waves.

Figure 4 shows the acoustic waves (Figure 4a) and the seismoelectric signals (Figure 4b) in Model 2. Because the seismic source is located in the Lucite section, only P-wave and Stoneley waves can be excited in the borehole. We can observe the corresponding seismoelectric signals from the electric waveforms (Figure 4b) received by the electrode fixed in the sandstone section.

In Model 3, shown in Figure 2, a set of point electrodes made of wires is buried in the borehole wall. Both the electrodes in the borehole and on the wall receive the seismoelectric signals. When the acoustic source moves in the Lucite section of the borehole, the received signals are as shown in Figure 5a and 5b, respectively.

The above experiments confirm that seismic waves in a borehole generate seismoelectric signals in the borehole formation which can be received by an electrode in the borehole center or the borehole wall. The results show that the seismoelectric signals are generated in the wall formation and their apparent velocities are the same as those of the seismic waves. Therefore, they are local, nonpropagating seismoelectric waves.

We replace the monopole transducer in the borehole of Model 3 with a plane transducer located on the upper side of the Lucite cylinder (Figure 2). Figure 6 show the seismoelectric signals received by electrodes #1, #2, #3, and #4 buried in the borehole wall. From the recorded waveforms, we see that the arrival times of the first maximum wave vary with the depth of the electrodes. Therefore the first maximum arrivals are the local seismoelectric signals generated by the seismic wave which arrives at the electrode. We can also see waves with small amplitude before the first maximum arrivals in the waveforms received by electrode #3, and #4 (Figure 6). Their arrival times are the same as the first arrivals of the seismoelectric wave on the electrode #1. Thus, we conclude that the small wavelets are the seismoelectric signals generated at the interface between the Lucite and

the glued sand. They propagate in the medium with the electromagnetic wave velocity and decay according to the attenuation of the electromagnetic wave.

5.5. SEISMOELECTRIC FIELD

The following experiments show the whole seismoelectric field generated by the seismic source in a borehole (Figure 7). The seismoelectric waves are recorded in Figure 8 when a source is fixed in material 1 and the electrode moves from material 2 across the interface to material 1. The measurements are conducted using Models 3, 4 and 5; both the seismic field and seismoelectric field are shown in Figure 8. The electrode (or the hydrophone) for the fifth trace in the plots is located at the interface.

The seismic waves in material 1 are the strongest and decrease with the distance from the source. If the impedances of the two materials are close to each other (such as in Model 3), the seismic waves have no obvious change when passing from one to the other (Figure 8a).

The strength of the seismoelectric field depends not only on the strength of the seismic waves, but also on the properties of the material. In materials with low porosity and permeability, such as Lucite and slate, only very weak seismoelectric signals are generated, even though the seismic waves are strong. In the opposite case, in porous materials such as sandstone and glued sand, the generated seismoelectric signals are much stronger. Note that the seismoelectric signals with high frequency are generated at the interface, and received by the near electrodes (Figures 8b, 8d, and 8f). Comparing the waveforms of the 4th, 5th, and 6th traces in Figures 8b, 8d, and 8f, we know that the first arrival times are the same due to the high velocity in the borehole. This means that seismic waves generate the propagating seismoelectric waves at the interface between the materials and generate the local, nonpropagating seismoelectric signals inside the porous materials.

Comparing the seismoelectric waves with seismic waves, we know that they are based on a different mechanism and relate to different properties of the wall formation. Therefore, seismoelectric logging could be a new logging method to explore the physical properties related to fluid flow in the wall formation.

5.6. ELECTROSEISMIC CONVERSION

The above experiments have examined the seismoelectric signals generated by seismic waves in a borehole. In this section, we investigate the electroseismic conversion in a borehole. In this case we receive the seismic waves induced by an electric signal. The source is the electrode emitting a high-voltage electric pulse and the receiver is a seismic transducer in the borehole. We investigate the electroseismic waves induced by the electrode in the borehole or on the borehole wall, and the variation of the electroseismic waves when the electrode is moved along the borehole (Figure 9). Figure 10 shows the electroseismic waves received by an acoustic transducer moving in the Lucite section with an increase of 1 cm/trace when the positive pole of the electrode is located in the glued sand section. The amplitude of the square pulse coming from the electrode is about 500 V. When the width of the square pulse is matched to the half period of the received seismic wave, the amplitude of the seismic wave is at maximum. From the moveout of the received electroseismic

waves, we calculate the phase velocity (1050m/s) and know that they are Stoneley waves in the borehole.

When the electrodes are on the borehole wall (electrodes 2-2 in Figure 9), the electric pulse also generates a Stoneley wave (Figure 11). The waveforms in Figure 10 and Figure 11 are slightly different, but they propagate with the same velocity.

In the above experiments we fixed the electrodes and moved the acoustic receiver. If we fix the acoustic receiver and move the electrode, a new set of seismic waves can be recorded (Figure 12). From the seismic diagram we know that the electric pulse generates electroseismic waves in the porous formation section that propagate with the Stoneley wave velocity along the borehole. The closer the electrode is to the acoustic receiver, the stronger the amplitude of the electroseismic waves and the earlier the arrival times. If the electrode is located at the Lucite section without permeability, no seismic wave can be generated. The results show that the electric pulse can generate the electroseismic wave through the electrode near the porous borehole wall and the wave propagates in the borehole with Stoneley wave velocity due to its low frequency.

5.7. CONCLUSIONS

We studied both seismoelectric and electroseismic conversions experimentally with various borehole models. The seismic waves generate seismoelectric signals and the electric pulses induce electroseismic waves in a fluid-saturated porous borehole due to the conversions. The results of this experiment show that the seismoelectric signals or the electroseismic waves can be generated only in the sections of the sandstone or the artificial porous materials where the source and the receiver are in the same fluid-filled borehole. In the borehole sections surrounded by a homogeneous porous formation, the generated seismoelectric signal is a stationary, local electromagnetic wave whose apparent velocity is the seismic wave velocity. At the interface between the formations with different physical properties, the seismic wave in the borehole generates a seismoelectric signal propagating with light speed in the borehole. The seismoelectric field is different from the acoustic field in the borehole due to the mechanism of the seismoelectric conversion. The seismoelectric signals depend on the pore fluid properties such as conductivity, porosity, permeability, etc. The experimental results confirm the possibility of a new borehole logging technique in a fluid-filled borehole to explore more physical properties related to the fluid and its flow in the porous formations. In this paper we do not investigate the frequency response of the seismoelectric conversion in a borehole in detail. More experiments are needed in the future.

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Table 1: Configurations of the borehole models					
Models	Model 1	Model 2	Model 3	Model 4	Model 5
Material 1	sandstone	Lucite	Lucite	slate	slate
Material 2	sandstone	sandstone	glued sand	sandstone	glued sand
D (cm)	1.27	1.5	1.27	1.27	1.27
L1 (cm)	12	6.5	14	20	20
L2 (cm)	12	7.5	5.5	9.0	5.5

Table 2: Physical parameters of the water-saturated samples					
Sample	Density (g/cm^3)	Porosity (%)	Vp (m/s)	Vs (m/s)	Permeability (Darcy)
Sandstone	2.08	18.9	2900	1480	0.3
Glued Sand	1.71	25.6	2540	1200	190
Slate	2.65	0.5	4750	2650	0
Lucite	1.18	0.0	2700	1290	0

Figure 1: Diagram of ultrasonic borehole model. The sizes and materials are listed in Table 1. The properties of materials are listed in Table 2.

Figure 2: Diagram for measuring seismoelectric conversion in borehole models. The electric receiver is fixed in the section of Material 2 and the acoustic source is moved along the borehole or a plane transducer is fixed on the top side.

Figure 3: Acoustic waveforms (a) and the seismoelectric signals (b) in the sandstone borehole model (Model 1) recorded by fixing the acoustic or electric receiver and moving the acoustic source along the borehole.

Figure 4: Acoustic waveforms (a) and electric signals (b) in Model 2 recorded by fixing the acoustic or the electric receiver in the sandstone section and moving the acoustic source in the Lucite section.

Figure 5: Seismoelectric signals received by the electrode in the borehole (a) or by the electrodes in the borehole wall (b) when the acoustic source moves in the Lucite section of Model 3.

Figure 6: Seismoelectric signals received by the wall electrodes #1, #2, #3, and #4 of Model 3 when a plane acoustic source is fixed on the top side of Model 3.

Figure 7: Diagram for measuring seismoelectric field in the borehole models with fixing the acoustic source and moving the electrode along the borehole.

Figure 8: Acoustic waveforms (a) and seismoelectric signals (b) recorded in Model 3 when the acoustic source is fixed in the section of material 1 and the acoustic receiver or electrode moves from the section of material 2 to material 1. The fifth trace is located at the interface.

Figure 8, continued: Acoustic waveforms (c) and seismoelectric signals (d) recorded in Model 4 when the acoustic source is fixed in the section of material 1 and the acoustic receiver or electrode moves from the section of material 2 to material 1. The fifth trace is located at the interface.

Figure 8, continued: Acoustic waveforms (e) and seismoelectric signals (f) recorded in Model 5 when the acoustic source is fixed in the section of material 1 and the acoustic receiver or electrode moves from the section of material 2 to material 1. The fifth trace is located at the interface.

Figure 9: Diagram for measuring the electroseismic conversion in Model 3. The source electrodes are in the borehole (1-1) or in the borehole wall (2-2) and the acoustic receiver moves along the borehole.

Figure 10: Electroseismic waveforms generated by the electrodes in the borehole (a) and the electrodes in the borehole wall (b) of Model 3 when the acoustic receiver moves in the Lucite section of the borehole.

Figure 11: Electro seismic waveforms generated by an electrode moving from the glued-sand section to the Lucite section of Model 3 when the acoustic receiver is fixed in the Lucite section. The eighth trace is located at the interface.

6. EXPERIMENTAL STUDIES OF ELECTROSEISMIC CONVERSION IN A FLUID-SATURATED POROUS MEDIUM

6.1. SUMMARY

The coupling between seismic and electromagnetic waves in a fluid-saturated porous medium is essentially controlled by the electrokinetic effect. The inverse effect of the seismoelectric conversion, the electroseismic conversion, is investigated experimentally in the laboratory. The electric field induces movement of the ions in a pore fluid relative to the solid matrix. The interaction between the pore fluid and the solid matrix generates acoustic waves known as electroseismic waves. Our studies confirm experimentally that the electroseismic conversion at ultrasonic frequencies is the electrokinetic effect in nature. In measurements with a homogeneous rock cylinder, P- and S-wave transducers receive, respectively, the P- and S- components of the extensional and flexural waves generated by an electric pulse with ring or parallel electrodes when the electrodes are on the surface or inside a porous medium. The electroseismic waves are measured in layered models, made of sandstone or artificial materials, to determine the area where the electroseismic waves are generated. Further experiments with the layered model investigate the relationship between electroseismic conversion and the conductivity of the fluid-saturated medium or the pore fluid. When fluid conductivity increases, the amplitude of the electroseismic wave increases. Experimental results show that electroseismic conversion is different from the piezoelectric effect of quartz grains in sandstone and is closely related to the relative motion between the fluid and the solid. The results also eliminate the possibility that the electroseismic wave is generated by a spark of a high-voltage pulse. Electroseismic waves can be generated at low voltage and increased continually with the voltage, without a big voltage jump similar to the spark in an isolated material. Our results confirm the existence and measurability of electroseismic conversion in porous formation at ultrasonic frequency ranges. Therefore, electroseismic measurements may be an effective means to investigate the pore fluid flow and rock properties.

6.2. INTRODUCTION

In a porous medium most grain surfaces that are in contact with a fluid electrolyte acquire a chemically-bound surface charge that is balanced by a diffuse distribution of mobile counter ions in a thin fluid layer surrounding each grain. The bound- and diffuse-charged layers together are known as the electric double layer (Broz and Epstein, 1976; Cerda and Kiri, 1989; Chandler, 1981; Ishido and Mizutani, 1981; Kozak and Davis, 1989). If the counter ions of the diffuse layer move by an electric force, the fluid will move also. The relative movement and interaction between the fluid and the solid matrix generate seismic vibration and mechanic waves (Haartsen, 1995; Haartsen and Pride, 1996; Morgan *et al.*, 1989).

In this paper, we study the electroseismic waves generated by an electric pulse in Berea sandstone cylinder models, confirming that electroseismic conversion exists in natural, fluid-saturated porous rocks. We investigate the electroseismic waves generated by the electrodes on the surface of or inside the porous medium with a layered model made of Lucite and epoxy-glued sand. The relationships between the saturant conductivity and the electroseismic conversion, and the effects

of the amplitude and polarization of the electric pulse on the electroseismic conversion, are studied experimentally.

6.3. EXPERIMENT IN A CYLINDER MODEL

In our experiments, a high-voltage (50–1000 V) electric pulse, as an exciting source, is applied to a fluid-saturated porous medium through electrodes with different shapes. The round or square cylinder model is made of natural Berea sandstone. Ring or parallel electrodes are attached on one side of the cylinder and connect to the high-voltage pulse. A compressional (P-) or shear (S-) transducer is fixed on the other side and receives the seismic waves.

Figure 1a is a diagram of the measurement system and the rock cylinder with ring electrodes. When the high-voltage pulse connects with the nearest electrodes, the transducer measures and records the compressional waves (Figure 1b) on the other side of the cylinder. From the acoustic waveforms in Figure 1b, we see that the closer the exciting area is to the transducer, the earlier the arrival time and the bigger the amplitude of the wave. The frequencies of the waves are almost the same. This means that the electroseismic waves are generated by the electric field along the axis in the area between the two electrodes. This wave is an extensional wave propagating along the axis. Because the length of the exciting area is the same during the course of the experiments, the frequency of the electroseismic waves will be constant.

Figure 2a shows the Berea sandstone cylinder model with parallel electrodes. Two electrodes of the conducting glue are on the sides of the square cylinder. A shear wave transducer is fixed with shear wave coupling on the other side; its polarization can be changed in the horizontal plane. When the electrodes are connected to a high-voltage pulse generator, an alternate electric field is excited in the direction perpendicular to the axis of the cylinder, and the shear components of the generated electroseismic waves are received by the S-wave transducer at three polarizations (Figure 2b). To check the phases of the waveforms, the influences of the high-voltage pulse (the first one-and-a-half circles) remain in the plot and have the same arrival times, amplitudes and phases. Not only the amplitude, but also the phase of the electroseismic waves vary with the polarization of the shear transducer. This means that the electroseismic wave is generated in the area between the electrodes and propagates as a flexural wave along the axis of the cylinder. The particle motion is perpendicular to the axis and parallel to the exciting electric field.

6.4. EXPERIMENT IN A LAYERED MODEL

To study the electroseismic wave generated by the electrodes inside a porous medium, we made a layered model with a Lucite block and epoxy-glued fine sand where some electrodes are buried. Some parameters of the glued sand and Lucite were listed in Zhu *et al.* (1997). The reason for using the Lucite block is to delay the arrival time of the electroseismic wave and to avoid the strong electronic influence of the high-voltage pulse.

When the electrodes are on the surface of the glued sand and excited by an electric pulse (Figure 3a), and the P-wave transducer moves along the the Lucite surface with an increase of 2 cm/trace, the transducer receives the electroseismic waves shown in Figure 3b. We replace the electrodes with a P-wave source transducer and record the acoustic field on the Lucite (Figure 4a). Comparing

the electroseismic waves (Figure 3b) with the acoustic waves (Figure 4b), we see that they have the same arrival time and similar amplitude variation. This means that the electroseismic wave is generated in the fluid-saturated porous medium near the electrodes and propagates as an acoustic wave in the model.

If the electrodes are at the same horizontal level (Figure 5a), but the spacing between the negative and positive electrodes varies, the P-wave transducer generates electroseismic waves (shown in Figure 5b) when the spacings are 14 cm, 10 cm, 8 cm, 6 cm, and 4 cm, respectively. When the spacing decreases, the strength of the electric field between the two electrodes increases and the generated electroseismic wave becomes stronger. Although the distances between the center of the electrodes and the transducer are the same, the arrival times of the electroseismic waves (Figure 5b) are different. This means that the depth or area where the electroseismic wave is generated increases or is closer to the transducer when the electrodes are closer to each other and the strength of the electric field increases. If the spacing of the electrodes is fixed at different horizontal levels (Figure 6a), the electroseismic waves (Figure 6b) are almost the same. However, the arrival time and the amplitude vary with the depth of the electrodes. From Figure 6b we calculate the velocity of the electroseismic wave in a fluid-saturated porous medium (glued sand), which is the P-wave velocity.

The above experiments with the layered model confirm that the electroseismic wave is generated by the electric field between the electrodes and, as an acoustic wave, propagates in the medium.

6.5. PROPERTIES OF ELECTROSEISMIC CONVERSION

The above results show that the alternate electric field induces the relative motion between the pore fluid and the solid matrix in a fluid-saturated porous medium. Their interaction generates an acoustic wave known as an electroseismic wave. This phenomenon is essentially the conversion between electrical and mechanical energies.

To investigate the properties of the electroseismic conversion, we perform additional experiments with a Berea sandstone layered model (Figure 7). The spacing between the two centers of the electrodes is 8 cm. The acoustic receiver is a P-wave transducer. With this model we change the amplitude and the polarization of the exciting electric pulse, or the conductivity of the saturant, and record the electroseismic waves to investigate the relationship between the electroseismic wave and the exciting pulse or the conductivity of the pore fluid. Experiments also show the difference between the electroseismic conversion and the piezoelectric effect or electric spark.

To study the relationship between the electroseismic conversion and the electric conductivity of the porous medium, we cleaned the Berea sandstone block many times with pure water in a vacuum system. We can then change the conductivity of the water-saturated sandstone by changing that of the saturant water. Our experiments recorded the electroseismic waves generated by the same electric pulse in the layered model saturated by water with a different conductivity. Figure 8 shows the relationship between the normalized amplitude of the electroseismic wave and the conductivity of the saturant (water). When the conductivity increases, the amplitude of the electroseismic wave also increases. The opposite occurs in seismoelectric conversion (Zhu and Toksöz, 1996), where the amplitude of the seismoelectric signal generated by a seismic wave decreases when the conductivity of the saturant increases. This result also shows that the electroseismic conversion is closely related

to the pore fluid in the medium, because the interaction of the pore fluid flow on the solid matrix generates the electroseismic wave.

The principle of electroseismic conversion is completely different than that of the piezoelectric effect. There are many quartz grains in sandstone. When an electric field affects these grains, it generates an acoustic wave due to the piezoelectric effect of the quartz. The quartz grains are very small and arranged randomly, thus the generated acoustic wave is very weak or unmeasurable. We record the piezoelectric acoustic wave generated by an 800 V electric pulse in a dry sandstone sample shown in Figure 7, and compare it with the waves generated by the same pulse in the sandstone saturated with water of 400 μ S in conductivity (Figure 9). We see that the acoustic wave due to the piezoelectric effect is very weak compared with that of electroseismic conversion.

The principle of electroseismic conversion is also different than that of an electrical spark. When the voltage between two electrodes in an isolated medium increases to a certain value, the medium may be electrically punctured, and a strong temporal current passes through the medium and generates a spark and sound, resembling lightening and thunder in the sky. The spark, caused by the sudden change of the electric property of the medium, happens in an instance or does not vary continually with the voltage. Figure 10 shows the relationship between the amplitude of the electric pulse and the normalized maximum amplitude of the electroseismic wave when the pulse amplitude varies step by step from 100 V to 1000 V in the model shown in Figure 7. In the voltage range of our experiments, the amplitude of the generated electroseismic wave varies continually with that of the electric pulse. It is clear that the electroseismic wave can also be received at a lower voltage range (100–200 V). This means that the electroseismic conversion is not an electric spark. Electroseismic conversion does not change the chemical properties of the medium and can only generate an ion movement in the pore fluid. Electric signals with any amplitude can induce electroseismic conversion and the difference is only in the amplitude of the generated electroseismic waves.

6.6. CONCLUSIONS

The experimental results discussed here show that electric signals can generate electroseismic waves with different modes in water-saturated cylinders or layered models. The basic principle of electroseismic conversion is based on the motion of the ions in pore fluid under the forces of an electric field. This movement induces fluid flow relative to the solid matrix. The interaction between the fluid and the matrix generate seismic waves that propagate in the medium.

Electroseismic conversion relates not only to pore fluid flow but to the conductivity of the fluid as well. The more ions there are in the pore fluid, the larger the amplitude of the electroseismic wave generated by the same electric pulse. This phenomenon is opposite to that of seismoelectric conversion, where more ions would counteract the current of the seismoelectric conversion.

The results of the experiment show that the mechanism of the electroseismic conversion is completely different from that of the piezoelectric effect or electric spark. Comparing the electroseismic conversion, the piezoelectric effect is very weak due to the random arrangement of the quartz grains in rocks. The continual variation of the electroseismic conversion with the amplitude of the electric pulse shows that it is not an electrical spark. The measurement of the electroseismic conversion would be an effective method to explore pore fluid flow and some properties of porous formation.

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Figure 1: Diagram for measuring the electroseismic waveform generated by an electric pulse with ring electrodes (a), and the electroseismic waves (b) received by the P-wave transducer.

Figure 2: Diagram for measuring the electroseismic wave generated by an electric pulse with parallel electrodes (a), and the electroseismic waveforms (b) received by the S-wave transducer when its polarization is at #1, #2, and #3 positions.

Figure 3: Glued-sand layered model (a) and the electroseismic waveforms (b) recorded by fixing the electrodes on the top surface and moving the P-wave receiver with 2 cm/trace. The amplitude is normalized by the maximum in the plot.

Figure 4: Glued-sand layered model (a) and the acoustic waveforms (b) recorded by fixing the source transducer on the top surface and moving the P-wave receiver with 2 cm/trace. The amplitude is normalized by the maximum in the plot.

Figure 5: Glued-sand layered model (a) with horizontal electrodes inside the glued sand and the electroseismic waveforms (b) when the spacing of the electrodes varies from 14 cm to 4 cm with 4 cm/trace.

Figure 6: Glued-sand layered model (a) with horizontal electrodes inside the glued sand and the electroseismic waveforms (b) when the spacing of the electrodes varies with 1 cm/trace.

Figure 7: Sandstone layered model.

Figure 8: Relationship between the saturant (water) conductivity and the amplitude of the electroseismic waveforms in the sandstone layered model.

Figure 9: Waveforms recorded in the sandstone layered model when the sandstone is dry (1) or saturated (2) with water of $400 \mu\text{S}$ in conductivity.

Figure 10: Relationship between the voltage of the exciting electric pulse and the amplitude of the electroseismic waveforms in the sandstone layered model.

7. ELECTROSEISMIC INVESTIGATION OF THE SHALLOW SUBSURFACE: FIELD MEASUREMENTS AND NUMERICAL MODELING

7.1. SUMMARY

Electroseismic phenomena in porous media, first observed almost 60 years ago (Ivanov, 1939), were recently "rediscovered" due to their potential to detect zones of high fluid mobility and fluid chemistry contrasts in the subsurface (Thompson and Gist, 1993; Haartsen *et al.*, 1995). However, a limited number of field studies of these phenomena reported in the literature were not able to support the results with an explicit comparison to theoretical predictions. In this paper, we demonstrate that electroseismic phenomena in porous media can be observed in the field, explained, and modeled numerically, yielding a good agreement between the field and the synthetic data.

We first outline the design of our field experiment and describe the procedure used to reduce noise in the electroseismic data. Then, we present and interpret the field data, demonstrating how and where different electroseismic signals originated in the subsurface. Finally, we model our field experiment numerically and demonstrate that the numerical results correctly simulate arrival times, polarity, and the amplitude-versus-offset behavior of the electroseismic signals measured in the field.

7.2. INTRODUCTION

A seismic wave propagating in a medium can induce an electrical field or cause radiation of an electromagnetic wave. These phenomena can be caused by a number of different physical mechanisms, but are collectively referred to as electroseismic. An overview by Parkhomenko (1971) describes piezoelectric and triboelectric effects, as well as streaming currents, as the primary causes of electroseismic phenomena in rocks. In our study we focus on electroseismic phenomena in porous media generated by streaming currents.

In a fluid-saturated porous rock, adsorption of an electrical charge to the surface of solid grains creates an excess of mobile ions of the opposite sign in the pore fluid (Bockris and Reddy, 1970). When a seismic wave propagates in such a rock, it displaces the ion-carrying fluid with respect to the solid matrix, thus generating a streaming electrical current. The streaming current results in a macroscopic charge separation that induces an electrical field. The magnitude of this field depends on the electrochemical properties of the fluid-solid contact and the mobility of the pore fluid. Laboratory experiments (Parkhomenko and Tsze-San, 1964; Gaskarov and Parkhomenko, 1974; Parkhomenko *et al.*, 1975; Migunov and Kokorev, 1977; Mironov *et al.*, 1993) and theoretical studies (Frenkel, 1944; Neev and Yeatts, 1989; Pride, 1994) demonstrate that the magnitude of the induced electrical field depends on the type of pore fluid (air, water or hydrocarbons) and solid (siliciclastic or carbonate), as well as the mechanical properties and the structure of the medium (elastic moduli, porosity, permeability and saturation).

Our work investigates the electroseismic effects that occur when a seismic wave crosses or travels along an interface between two different porous media. The first effect studied in this paper is referred to as electroseismic conversion, and is shown in Figure 1. When a spherical P wave crosses an interface between two media, it creates a dipole charge separation due to the imbalance

of the streaming currents induced by the seismic wave on opposite sides of the interface. The electrical dipole radiates an electromagnetic wave, which can be detected by remote antennas. This phenomenon was first detected in the experiments of Martner and Sparks (1959), and later in the experiments of Thompson and Gist (1993) and Butler *et al.* (1994). The electroseismic conversion at an interface between two materials was also measured in a laboratory experiment by Zhu *et al.* (1994). A theoretical model of the electroseismic conversion was recently developed by Haartsen and Pride (1994). Numerical simulations (Haartsen, 1995) demonstrated that electroseismic conversion can take place at permeability or fluid chemistry contrasts. Therefore, electroseismic conversion has the potential to become a geophysical tool capable of detecting zones of high permeability such as fractured zones, and interfaces such as an oil-water contact (Haartsen *et al.*, 1995).

The other electroseismic effect studied in this paper is an electrical field generated by a seismic head wave (Figure 2). When a seismic head wave travels along an interface between two media, it creates a charge separation across the interface which induces an electrical field. This electrical field moves along the interface with the head wave and can be detected by antennas when the head wave passes under them. This phenomenon was previously identified in the experiments of Neishtadt and Osipov (1959) and Martner and Sparks (1959). It was also present in the experiments of Butler *et al.* (1994), although the authors did not identify it.

Historically, studies of electroseismic phenomena in porous media were limited to laboratory or theoretical investigations, primarily due to the difficulty in recording adequate signals in the field. There have been only a few reports of electroseismic field experiments published (Ivanov, 1939; Ivanov, 1940; Neishtadt and Osipov, 1959; Martner and Sparks, 1959; Zablocki and Keller, 1961; Broding *et al.*, 1963; Tome, 1975; Thompson and Gist, 1993; Butler *et al.*, 1994). Only a fraction of these reports explained the origin of the measured electroseismic signals (Neishtadt and Osipov, 1959; Martner and Sparks, 1959; Thompson and Gist, 1993; Butler *et al.*, 1994). None of these reports gave an explicit comparison of the field data with theoretical predictions.

The objective of our work is to demonstrate that electroseismic phenomena can be observed in the field, explained, and simulated numerically, yielding a good agreement between the field and synthetic data. In this paper, we first outline the design of our field experiment and describe the procedure used to reduce noise in electroseismic data. Next, we present and interpret the field data, demonstrating how and where different signals originated in the subsurface. In particular, we identify the electroseismic conversion and the head wave-generated electrical field. Finally, we model our field experiment numerically, and demonstrate that the numerical results correctly simulate the arrival times, polarities, and amplitude-versus-offset behavior of the electroseismic signals measured in the field.

7.3. FIELD DATA ACQUISITION AND NOISE REDUCTION PROCESSING

We conducted our field measurements at a site in Hamilton, Massachusetts. Figure 3 shows a diagram of the experimental layout and Figure 4 shows a generalized vertical cross-section of the subsurface at the site derived from a variety of previously collected geophysical data.

7.3.1. Field Experiments

During the field experiments, we recorded vertical ground motions and horizontal electrical fields generated by seismic waves propagating from a surface seismic source. The vertical ground motions were measured using an array of geophones. The horizontal electrical fields were measured with an array of dipole antennas. Each of the antennas consisted of a pair of grounded electrodes. A sledge hammer was used as the seismic source. For data recording, we used the DAS-1 data acquisition system manufactured by OYO Geospace Corporation. The system had a dynamic range of 132 dB and crosstalk between channels of less than -100 dB.

Trace A in Figure 5 is a typical example of an electrical signal recorded in the experiment. It is dominated by coherent signals of a nonelectroseismic nature which we will refer to as coherent noise. The major source of this coherent noise was electrical current induced in the ground by remote power lines. The other significant source of coherent noise was telluric current induced by the time variation of the Earth's magnetic field. In order to reduce the coherent noise induced by these currents, we stacked the data 100 times. To further reduce the coherent noise, we used the observation that the telluric and power line-induced currents did not change phase throughout the survey area. Therefore, the noise generated by them was effectively the same close and far away from the seismic source. During data acquisition we recorded the coherent noise on two remote, mutually perpendicular antennas (see Figure 3), and as part of the data processing we subtracted it from the electrical records. It was necessary to use two mutually perpendicular antennas to record both horizontal components of the coherent noise because the telluric and the power line-induced currents changed direction and amplitude (but not phase!) with location due to variation of ground resistivity. The details of noise reduction in the electrical data will be described in a later section.

In our field work we encountered a number of undesirable effects that interfered with the electroseismic measurements. The most significant of these effects are listed below.

1. When metal rods were used as electrodes, a chemical reaction between the electrodes and the ground material generated significant electrode noise. Therefore, we used stable Ag-AgCl electrodes to minimize this effect.
2. When an aluminum or steel base plate was used, a high frequency electromagnetic pulse was generated at the moment of sledge hammer impact with the plate. To eliminate this pulse we used a nonmetallic (Lucite) block as a base plate.
3. When a long trigger cable lay on wet ground, the electrical current flowing through the cable at the moment of the hammer impact induced a current in the ground. This cable-induced current was detected by the antennas. In some of the experiments it obscured the electroseismic signals. In order to eliminate this current, we cut the trigger cable as short as possible and isolated it from the ground. In the future, we suggest the use of an optical trigger cable for electroseismic field work.

7.3.2. Noise Reduction in Electrical Data

Prior to the interpretation of the field data, it was necessary to remove the power line and telluric noise from the electrical records. Figure 5 shows the results of our noise reduction procedure.

Trace A is an electrical signal recorded in the field. The signal-to-noise ratio in this trace is approximately 0.01. Trace F is the same trace after the noise was reduced and the signal-to-noise ratio was increased to about 10.0, i.e., by a factor of one thousand.

The first step in noise reduction was subtracting the coherent noise recorded on the remote antennas. We matched the coherent noise in each individual electrical trace with a linear combination of the two remote records of the noise. The coefficients of the linear combination were estimated to obtain the best match in the least-squares sense. After the coefficients were estimated, we subtracted the corresponding linear combination of the remote noise records from the electrical trace. In Figure 5, traces B and C are the remote noise records. Trace D is the result of subtraction of a combination of traces B and C from trace A, plotted to scale. Trace E is the same result magnified by a factor of one hundred. The signal-to-noise ratio in trace E is close to one. Therefore, subtraction of the remote noise records increased the signal-to-noise ratio in trace E by approximately a factor of one hundred.

The coherent noise remaining in the electrical traces after the remote records subtraction was due mostly to power line induced currents. Fourier analysis showed that the traces contained up to 30 harmonics (frequency range of 60 Hz to 1800 Hz) of the fundamental power line frequency, which in our experiments varied from 60 Hz by about 0.1 Hz. The electroseismic signals recorded in our experiments had the same frequency content as the seismic waves that generated them. The main energy of the electroseismic signals was contained in the frequency interval from 10 Hz to 150 Hz. To eliminate the power line harmonics with frequencies much higher than the frequencies of the electroseismic signals, we used low-pass filtering in the Fourier domain. We set the cutoff frequency of the low-pass filter to be 600 Hz, in order not to smear the first breaks in the electrical traces. To eliminate the remaining 10 harmonics of the fundamental power line frequency we first estimated the frequencies, amplitudes and phases of these 10 harmonics by a least squares fit in the time domain (Butler and Russell, 1993), and then subtracted the corresponding sinusoids from the electrical traces. Trace F in Figure 5 is the result of subtracting the power line harmonics from trace E. The signal-to-noise ratio in trace F is 10.0. Thus, it is increased by a factor of ten compared to trace E.

As a result of applying the procedure described above, we reduced the coherent noise in the electrical records by a total factor of one thousand. The highest signal-to-noise ratio in the records presented in this paper is about 50 and the lowest is about 5.0.

7.4. INTERPRETATION OF ELECTROSEISMIC DATA

Figures 6–9 show the electroseismic data collected at the site. These figures show electrical signals recorded on 4 ft (1.2 m), 8 ft (2.4 m), 8 ft (2.4 m), and 16 ft (4.8 m) antennas, respectively. The spacing between the antennas in Figures 6 and 7 is 2 ft (0.6 m), and in Figures 8 and 9 it is 4 ft (1.2 m). The antennas measured the potential of the electrode closer to the source with respect to the electrode further away from the source. To interpret the field data, we identified various electroseismic signals, and determined their origin by correlating their arrival times and moveout velocity with the known velocity structure and positions of interfaces in the subsurface.

Our knowledge of the subsurface was derived from seismic refraction, resistivity and hydrogeological data available at the experiment site. Figure 4 shows a generalized vertical cross-section

of the subsurface at the site. The top 2.5 ft (0.75 m) layer is organic soil. The P wave velocity of this layer increases gradually with depth, and on average is 650 ft/s (200 m/s). The resistivity of the soil is $2000 \Omega \cdot m$. Below the top soil is an 8 ft (2.4 m) layer of unsaturated glacial till. The P wave velocity of the unsaturated till is 2400 ft/s (730 m/s) and the resistivity is $2000 \Omega \cdot m$. The watertable is at a depth of 10.5 ft (3.2 m). Below the watertable is a 20 ft (6 m) layer of saturated glacial till with the P wave velocity of 4700 ft/s (1430 m/s) and the resistivity of $200 \Omega \cdot m$. Below the saturated glacial till is bedrock which appears to be fractured granite with the P wave velocity of 12200 ft/s (3700 m/s) and the resistivity of $5000 \Omega \cdot m$.

In our field data we identified the electroseismic conversion of the incident P wave at the top soil-glacial till interface and the electrical field generated by the head wave traversing this interface. We also detected signals that can be associated with the electroseismic conversion of the incident P wave at the watertable and at the glacial till-bedrock interface.

7.4.1. Electroseismic Conversion at the Top Soil-Glacial Till Interface

The records in Figures 6–9 show a negative electrical pulse arriving simultaneously at the antennas between 3 ms and 4 ms (event A-A). The pulse can be seen in the first ten traces in Figures 6 and 7 and in the first five traces in Figures 8 and 9. The amplitude of the pulse is always the strongest at the antenna closest to the source, and decreases further away from the source. Based on the following analysis of the data we concluded that this pulse is the electroseismic conversion of the incident P wave at the interface between the top soil and the glacial till. First, the arrival time of this pulse appears to be the same at all the antennas. Therefore, it traveled with an electromagnetic wave velocity in the medium. Then, the amplitude of the pulse is the strongest close to the source, which suggests that the pulse originated directly below the source. And finally, since the pulse arrived approximately at the time the P wave generated by the source reached the interface between the top soil and the glacial till, we concluded that this pulse was due to the electromagnetic wave radiated from the interface at the time when the incident P wave crossed it.

7.4.2. Electrical Fields Generated by the Head Wave Traversing the Top Soil-Glacial Till Interface

Traces 6 through 12 in Figures 8 and 9 show an electrical pulse travelling along the antenna array with a finite moveout velocity (event B-B). This pulse consists of a positive (shaded) peak, a negative double trough, and another positive peak. The horizontal velocity of the pulse is equal to the P wave velocity of the unsaturated glacial till. The pulse arrives at the antennas about 2 ms prior to the arrival of the P wave refracted from the interface between the top soil and the glacial till. Therefore, this pulse is due to the electrical field generated by the seismic head wave traveling along the interface between the top soil and the glacial till.

7.4.3. Electro seismic Conversion at the Watertable and the Glacial Till-Bedrock Interface

In our electro seismic records, we identified signals which may be attributed to the electro seismic conversion of the incident P wave at the watertable and the glacial till-bedrock interface. Traces 5 through 11 in Figure 7 and traces 6 through 8 in Figure 6 show a negative pulse arriving at approximately 7 ms (event C-C). We estimated the vertical P wave traveltime to the watertable to be 7.5 ms. Consequently, this pulse can be attributed to the electro seismic conversion at the watertable. However, this pulse is largely obscured by the electro seismic conversion at the top soil-glacial till interface. Further experiments with a higher frequency source are necessary to separate this pulse from the electro seismic conversion at the top soil-glacial till interface to confirm its origin.

Traces 6 through 9 in Figure 8 show a negative pulse arriving at approximately 14 ms (event D-D). We estimated the vertical P wave traveltime to the glacial till-bedrock interface to be approximately 11.5 ms. Therefore, this pulse may be attributed to the electro seismic conversion at that interface. The same pulse can be observed in traces 6, 7, and 8 in Figure 9. However, the amplitude of this pulse is very small, and further experiments with a stronger source are necessary to determine the nature of this signal.

7.5 NUMERICAL SIMULATION OF ELECTROSEISMIC PHENOMENA

We chose to simulate the electro seismic conversion and the head wave induced electrical fields at the top soil-glacial till interface because these phenomena were most clearly identified in the field data. We based the simulation on the coupled acoustic and electromagnetic equations for a fluid-saturated porous medium (Pride, 1994). Below we summarize these equations and describe the model of the subsurface used in numerical simulation.

The acoustic wave propagation in the medium is assumed to be governed by the Biot equations for a fluid-saturated porous medium. In the frequency (ω) domain:

$$\nabla \cdot \underline{\tau}_B = -\omega^2 [\rho_B \underline{u}_s + \rho_f \underline{w}]. \quad (1)$$

$$\underline{\tau}_B = [K_G \nabla \cdot \underline{u}_s + C \nabla \cdot \underline{w}] \underline{I} + G \left[\nabla \underline{u}_s + \nabla \underline{u}_s^T - \frac{2}{3} \nabla \cdot \underline{u}_s \underline{I} \right] \quad (2)$$

$$-P = C \nabla \cdot \underline{u}_s + M \nabla \cdot \underline{w} \quad (3)$$

Here, $\underline{\tau}_B$ is the bulk stress in the medium, P is the pressure in the pore fluid, $\underline{u}_s$ is the displacement in the solid, and $\underline{w}$ is the relative fluid-solid motion; ρ_B denotes the bulk density of the medium, and ρ_f denotes the fluid density; K_G , G , C , and M are the Biot moduli and incompressibilities of the medium (Biot, 1962; Pride *et al.*, 1992); and $\underline{I}$ is the identity tensor.

The electromagnetic effects in the medium are described by the Maxwell equations:

$$\nabla \times \underline{E} = i\omega \underline{B} \quad (4)$$

$$\nabla \times \underline{H} = -i\omega \underline{D} + \underline{J} \quad (5)$$

$$\underline{D} = \epsilon_0 \left[\frac{\phi}{\alpha_\infty} (\kappa_f - \kappa_s) + \kappa_s \right] \underline{E} \quad (6)$$

$$\underline{B} = \mu_0 \underline{H} \quad (7)$$

where $\underline{J}$ is the electrical current density, $\underline{E}$ is the electric field strength, $\underline{D}$ is the electric displacement, $\underline{H}$ is the magnetic field strength, $\underline{B}$ is the magnetic induction, ϵ_0 is the permittivity of free space, μ_0 is the permeability of free space, κ_f is the relative permittivity in the fluid, and κ_s is the relative permittivity in the solid. ϕ is the porosity of the medium, and α_∞ is the tortuosity.

The coupling of acoustic and electromagnetic fields is assumed to be due to streaming electrical currents and electroosmosis. These effects are included into transport equations for the medium:

$$\underline{J} = \sigma(\omega) \underline{E} + L(\omega) \left[-\nabla P + \omega^2 \rho_f \underline{u}_s \right] \quad (8)$$

$$-i\omega \underline{u} = L(\omega) \underline{E} + \frac{k(\omega)}{\eta} \left[-\nabla P + \omega^2 \rho_f \underline{u}_s \right]. \quad (9)$$

Here, $\sigma(\omega)$ is the frequency-dependent electrical conductivity of the medium, $k(\omega)$ is the permeability, η is the fluid viscosity, and $L(\omega)$ is the electrokinetic coupling coefficient. The expressions for $\sigma(\omega)$, $k(\omega)$, and $L(\omega)$, as well as the detailed derivation of equations 1–9, are given in Pride (1995). Pride and Haartsen (1995) obtained eigenvector solutions for these equations in the frequency-wavenumber domain. Haartsen and Pride (1994) developed a numerical algorithm for solving boundary value problems in layered media by integrating over the eigenvector solutions. We used this algorithm for modeling the results of the field experiments.

To simulate the electroseismic effects at the interface between the top soil and the glacial till, we used a two-layer model. We assumed both layers to consist of an elastic porous frame with solid grains coated with water and the pore space saturated with air. The media parameters used for modeling are given in Table 1. We did not have information about the exact values of most of these parameters for the top soil and the glacial till at the site, and instead relied on the experimental data available for similar types of materials. We assumed 35 percent porosity for the top soil, and 15 percent porosity for the glacial till. The value of permeability assumed for the top soil corresponds to the values given by Geli *et al.* (1987) for loose sediments. The value of permeability assumed for the glacial till corresponds to the values given by Freeze and Cherry (1979) for moderately permeable sands and glacial tills. We further assumed that the top soil had 2 percent of water saturation and the glacial till had 20 percent of water saturation. We determined the effective density and permittivity of the air-water mixture based on their volumetric fraction in the pore space. To determine the effective viscosity, we assumed that the air-water flow in the pores is annular, and obtained the values of the effective viscosity for such flows from Wallis (1969). We also assumed that the effective compressibility of the air-water mixture is determined by the compressibility of air.

Based on the above properties of the elastic frame and the pore fluid assumed for the top soil and the glacial till, we chose the frame moduli for both media such that the resulting P wave velocities matched the values measured in the field. The frame moduli chosen for the top soil and glacial till agreed with the values given by Geli *et al.* (1987) for loose sediments. We selected the

values for salinity of water to yield media resistivity matching the values measured at the site, assuming that the conductivity of the medium was controlled by water coating the solid grains.

Based on the model described above, we calculated electrical and seismic fields generated by a vertical point source in an axisymmetric geometry. In order to allow for a sufficient separation of different electroseismic events, we used a Ricker wavelet with a 300 Hz center frequency as the source in our calculations.

7.6. COMPARISON OF FIELD AND SYNTHETIC DATA

Figures 10 and 11 show the seismic and electrical traces calculated, respectively. The seismic records in Figure 10 contain the waves reflected and refracted at the interface (direct and surface waves were not simulated). The electrical records in Figure 11 contain the signals due to electroseismic conversion of the incident P wave and the head wave-generated electrical field at the top soil-glacial till interface. The electrical records also contain signals generated by the other seismic waves.

The electrical traces in Figure 11 show a negative pulse arriving simultaneously on the antennas at approximately 3.8 ms (event A-A). This negative signal is the electroseismic conversion of the incident P wave at the interface between the top soil and the glacial till. The arrival time and the polarity of this pulse are the same as in our field measurements.

Figure 11 also shows a pulse consisting of a positive peak and a negative trough traveling along the antenna array with the P wave velocity of the glacial till (event B-B). A comparison with the seismic data in Figure 10 shows that this pulse arrives 2 ms earlier than the P wave refracted from the interface. Therefore, this pulse is due to the head wave-generated electrical field. The arrival times and the polarity of the pulse calculated numerically are consistent with the field measurements.

To compare the amplitudes of the electroseismic signals calculated numerically with those measured in the field, it was necessary to match the strength of the source used in the numerical calculation with the strength of the sledge hammer blow in the field. In order to achieve this, we compared the numerical and field amplitudes of the P wave refracted at the interface and scaled the numerical source strength so that these amplitudes agreed. We then used the same scaling for the electrical signals.

Figure 12 presents the amplitudes of the electroseismic conversion of the incident P wave at the interface, plotted in logarithmic coordinates versus the distance from the source. The amplitudes of the signals calculated numerically have a qualitatively correct amplitude-versus-offset behavior. However, they are smaller than the observed amplitudes. Figure 13 presents the amplitudes of the electrical field generated by the head wave. Similar to the electroseismic conversion, the behavior of the amplitudes calculated numerically is qualitatively correct. However, their absolute values are smaller than the values measured in the field.

We therefore conclude that the numerical modeling correctly predicted the arrival times, polarity and qualitative amplitude-versus-offset behavior of the electroseismic conversion of the incident P wave at the top soil-glacial till interface and the electrical field generated by a head wave traversing this interface. It is also remarkable that despite the uncertainty of the lithology parameters and the modeling approximations, the numerical simulation was able to match the values of the amplitudes of the electroseismic signal within an order of magnitude.

7.7. CONCLUSIONS

We demonstrated that electroseismic phenomena can be observed in the field, explained, and simulated numerically, yielding a good agreement between the results of both field and numerical experiments.

We described the design of our field experiment and showed that it is possible to successfully eliminate noise and other undesirable effects to obtain clear electroseismic field records. In the field data we identified the electroseismic conversion of the incident P wave at the top soil-glacial till interface and the electrical field generated by the head wave traversing this interface. We also described signals that can be attributed to electroseismic conversion at the watertable and the glacial till-bedrock interface.

We numerically modeled the electroseismic conversion at the top soil-glacial till interface and the electrical field generated by the head wave traversing this interface. The numerical modeling correctly simulated the arrival times, polarities, and qualitative amplitude-versus-offset behavior of these electroseismic signals.

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Property	Top Soil	Glacial Till
porosity ϕ , [%]	35	15
dc permeability k , [m^2]	10^{-10}	10^{-12}
bulk modulus of the solid k_s , [Pa]	2.0×10^9	1.0×10^{10}
bulk modulus of the fluid k_f , [Pa]	1.4×10^5	1.4×10^5
bulk modulus of the frame k_{fr} , [Pa]	4.0×10^7	6.0×10^8
shear modulus of the frame g_{fr} , [Pa]	2.0×10^7	4.5×10^8
viscosity of the fluid η , [$Pa \cdot s$]	8.0×10^{-5}	6.0×10^{-4}
density of the solid ρ_s , [Kg/m^3]	2.2×10^3	2.4×10^3
density of the fluid ρ_f , [Kg/m^3]	20.0	200.0
salinity of the fluid C , [mol/l]	5.5×10^{-4}	1.2×10^{-3}
temperature T , [K]	298	298
permittivity of the solid κ_s	4	4
permittivity of the fluid κ_f	2.5	16
tortuosity α_∞	3	3

Table 1: The mechanical and electrical medium properties used for numerical modeling.

Figure 1: Diagram of the electroseismic conversion at an interface. When a spherical seismic wave crosses an interface, it creates a dipole charge separation across the interface. This electrical dipole radiates an electromagnetic wave which can be detected by remote antennas.

Figure 2: Diagram of the generation of an electrical field by a head wave traversing an interface. When a seismic head wave travels along an interface, it creates a charge separation across the interface which induces an electrical field. This electrical field can be detected by an antenna when the head wave travels below it.

Figure 3: Diagram of the experimental layout (top view). In different experiments we used antenna lengths of 4 ft (1.2 m), 8 ft (2.4 m) and 16 ft (4.8 m). The offset between the source and antennas (measured to the electrode closest to the source) ranged from 2 ft (0.6 m) to 48 ft (14.6 m). The remote antennas were placed approximately 70 ft (21 m) away from the seismic source.

Figure 4: Generalized vertical cross-section (not to scale) of the subsurface at the experimental site derived from seismic refraction, resistivity and hydrogeological data.

Figure 5: Noise reduction in the electrical data. Trace A is an electrical signal recorded in the field. The signal-to-noise ratio in trace A is about 0.01. Traces B and C are the remote noise records. Trace D is the result of subtracting a linear combination of traces B and C from trace A. Trace E is the same result magnified by a factor of one hundred. Trace F is the result of power line harmonics subtraction from trace E. The signal-to-noise ratio in trace F is about 10.0.

a. The original data

b. The interpretation

Figure 6: a) Electro seismic data collected at the site with 4 ft (1.2 m) antennas and 2 ft (0.6 m) spacing between the antennas. b) The first 60 ms of these data. Event A-A is the electro seismic conversion at the top soil-glacial till interface. Event C-C is possibly the electro seismic conversion at the watertable. Dashed lines mark the onset of these signals.

a. The original data

b. The interpretation

Figure 7: a) Electroseismic data collected at the site with 8 ft (2.4 m) antennas and 2 ft (0.6 m) spacing between the antennas. b) The first 60 ms of these data. Event A-A is the electroseismic conversion at the top soil-glacial till interface. Event C-C is possibly the electroseismic conversion at the watertable.

Figure 8: a) Electroseismic data collected at the site with 8 ft (2.4 m) antennas and 4 ft (1.2 m) spacing between the antennas. b) The first 60 ms of these data. Event A-A is the electroseismic conversion at the top soil-glacial till interface. Event B-B is the electrical field generated by the head wave traversing the same interface. Event D-D is possibly the electroseismic conversion at the glacial till-bedrock interface.

Figure 9: a) Electroseismic data collected at the site with 16 ft (4.8 m) antennas and 4 ft (1.2 m) spacing between the antennas. b) The first 60 ms of these data. Event A-A is the electroseismic conversion at the top soil-glacial till interface. Event B-B is the electrical field generated by the head wave traversing the same interface. Event D-D is possibly the electroseismic conversion at the glacial till-bedrock interface.

Figure 10: Synthetic seismic data. Event B-B is the refracted P wave. Two later events are the refracted *S* wave and the reflected P wave.

Figure 11: Synthetic electrical data. Event A-A is the electroseismic conversion at the interface. Event B-B is the electrical field generated by the head wave traveling along the interface. Two later events are due to electrical fields generated by the refracted S wave and the reflected P wave. In numerical calculations we used a noncausal Ricker wavelet. Therefore, we consider the time of a center peak as an arrival time, and the polarity of the center peak as the polarity of an electroseismic signal.

Figure 12: Comparison between the observed and the calculated amplitudes of the electroseismic conversion of the incident P wave at the top soil-glacial till interface.

Figure 13: Comparison between the observed and the calculated amplitudes of the electrical field generated by the head wave traveling along the top soil-glacial till interface.

8. USING BOREHOLE ELECTROSEISMIC MEASUREMENTS TO DETECT AND CHARACTERIZE FRACTURED (PERMEABLE) ZONES

8.1. SUMMARY

We present a new type of field measurement capable of detecting and characterizing fractured (permeable) zones intersecting a borehole. The method is based on measuring electrical fields generated by a borehole Stoneley wave. In this paper, we describe the measurement technique, present field data, and propose a theoretical model, which correctly predicts amplitudes and phases of the electrical fields measured in the borehole experiment.

The theoretical model and the field data demonstrate that the measurements of the Stoneley-wave-induced electrical fields can yield information about the interconnected porosity, and possibly about the permeability of the formation around the borehole. We derive an estimate of the interconnected porosity from the field data, and show that it correlates well with the density of fractures intersecting the borehole. Our results suggest that the borehole electroseismic method can be developed into a logging or a VSP tool, with possible applications in reservoir characterization.

8.2. INTRODUCTION

Electroseismic phenomena in fluid-saturated porous media provide geophysicists with a unique opportunity to detect a seismic-wave-generated flow of pore fluid with respect to the rock matrix. The term "electroseismic" describes phenomena in which a seismic wave induces an electrical field or causes radiation of an electromagnetic wave. Electroseismic phenomena take place in fluid-saturated porous rocks, because the pore fluid carries an excess electrical charge. When the charged pore fluid is forced to flow through the rock by pressure gradients within a seismic wave, a streaming electrical current is generated. This electrical current results in a charge separation, which induces an electrical field. Measuring this seismic-wave-induced electrical field allows detection of the fluid flow generated by the wave in the porous medium. In turn, detecting the fluid flow allows characterization of the fluid transport properties of the medium.

The potential ability of the electroseismic method to characterize the fluid transport properties of the subsurface can make it a useful tool for reservoir characterization. Theoretical studies (Haartsen and Pride, 1994; Haartsen, 1995) suggest that porosity and permeability contrasts in the subsurface can be detected by electroseismic measurements. Recent surface experiments (Thompson and Gist, 1993; Butler *et al.*, 1996; Mikhailov *et al.*, 1997) confirm that electroseismic signals from various interfaces in the subsurface can be detected. However, these studies also demonstrate that the penetration depth of the electroseismic method is limited by the exponential decay of the electrical field in a conductive earth. One way to overcome the penetration depth limitation is to place the detectors of the electrical field close to the formation of interest, i.e. to make electroseismic measurements in a borehole.

The idea of borehole electroseismic measurements has been around for quite a long time. However, only a few attempts of such measurements were made. Broding *et al.* (1963) made measurements with electrodes in a borehole and a seismic source away from a borehole (VSP-type measurements). Parkhomenko and Gaskarov (1971) placed electrodes in a borehole and excited

striking the well-head (i.e., the seismic wave source was effectively in a borehole, thus these were logging type measurements). Both studies reported that the amplitude of the recorded electrical signals varied with lithology (higher amplitudes in limestones than in shales). However, they did not comment on how the detected electrical fields were generated, or what information about the formation could be deduced from the borehole electroseismic measurements. Therefore, the main goal of our work is not only to perform borehole electroseismic measurements, but also to identify different electroseismic phenomena in the field data, to develop a theoretical model explaining the observations, and to determine which properties of the formation can be derived from such measurements.

In our borehole experiments we measured electrical fields generated by a Stoneley wave. When a borehole Stoneley wave travels past a fractured (permeable) zone, it induces a flow of the ion-carrying fluid within the fractures, thus creating a streaming electrical current from the borehole into the formation (Figure 1). The streaming electrical current results in a time-varying electrical charge separation, which locally induces an electrical field and also causes an electromagnetic wave radiation. Field data presented in this paper show both of these phenomena. Furthermore, the data demonstrate that a Stoneley wave induces stronger electrical fields in zones of higher fracture density, thus suggesting that the Stoneley-wave-induced electrical field can be used to detect and characterize fractured (permeable) zones around a borehole.

To determine what specific information about the fractured (permeable) zones can be derived from borehole electroseismic measurements, we developed a theoretical model and obtained an analytical solution for the amplitude and phase of the Stoneley-wave-induced electrical field. A quantitative comparison demonstrates that the analytical solution correctly predicts the amplitude and phase of the electrical fields observed in the experiment.

The analytical solution suggests that at a given frequency the amplitude of the Stoneley-wave-induced electrical field is proportional to the interconnected porosity of the formation around the borehole. We derive an estimate of the interconnected porosity from the field data, and demonstrate that it correlates with the density of fractures intersected by the borehole. Further analysis of the analytical solution suggests that if the borehole electroseismic measurements are performed over a wide frequency range, then the Biot critical frequency for the formation could possibly be determined, thus yielding information about the permeability. We suggest that the ability of the electroseismic method to determine the interconnected porosity, and possibly the permeability of a formation around the borehole, can make it a useful tool for reservoir characterization.

8.3. FIELD MEASUREMENT PROCEDURE AND NOISE REDUCTION PROCESSING

We conducted borehole electroseismic measurements in an open hole well in Hamilton, Massachusetts. Figure 2 shows a diagram of the experimental setup. During the experiments we generated a Stoneley wave at the top of the well, and recorded the pressure and electrical field generated by the wave as it traveled downward in the borehole. The Stoneley wave was generated by striking the wellhead with a sledge hammer. The pressure oscillations were recorded using a string of three hydrophones spaced one meter apart. The sensitivity of the hydrophones was $10\text{mV}/\text{Pa}$. The electrical fields were recorded as a potential difference between lead electrodes placed in an

array in the borehole. We used 8-electrode arrays with electrode spacings of 0.5m and 1.0m. To record signals measured by the hydrophones and the electrodes we used a data acquisition system with a dynamic range of 132dB and crosstalk between channels of less than -100dB.

In the experiments we made the pressure measurements and the electrical field measurements separately, to ensure that there was no crosstalk between channels. Since we were capable of only making either three hydrophone or four electrical measurements at a time, we had to consequently place the arrays at different depths in the borehole and repeat the source. Even though this was a disadvantage, we were able to demonstrate that the phenomena observed in the experiments were repeatable.

Nearby powerlines generated electrical noise in our data. We used remote referencing and powerline harmonic filters to reduce this noise. Details of the noise-reduction processing are given in Mikhailov *et al.* (1997). As a result of the processing, the signal-to-noise ratio in the electrical records was improved from 0.01 to about 5.0, thus allowing the data to be interpreted.

8.4. PRELIMINARY ANALYSIS OF THE FIELD DATA

The field experiments were conducted in an open-hole well drilled through fractured igneous rock. The top 140m of the well is in fractured granite, and below the depth of 140m is in fractured diorite (Figure 3). A 6m felsite dike intersects the well at a depth of 152m. Video and borehole televiewer logs show numerous fractures and fractured zones along the entire depth interval of investigation. The average fracture density in granite is 4 per meter. Figures 3 and 4 show caliper, P-wave slowness, and conductivity logs. Slowness and conductivity highs correspond to fractured zones and isolated fractures.

Figure 5 presents the hydrophone pressure measurements. The down-going wave is the borehole Stoneley wave (event A-A). Its velocity varies from 1380m/s to 1450m/s. Large fractures and fractured zones generate Stoneley wave reflections traveling upward. Figures 6 and 7 show the electrical measurements made with 0.5m and 1.0m electrode separation respectively. In these data the electrical field and the electromagnetic radiation induced by the Stoneley wave can be identified.

8.4.1. Electrical Field Induced by a Stoneley Wave

Figures 6 and 7 show an electrical signal, that arrives at the electrodes simultaneously with the Stoneley wave (event A-A). Figure 8 shows this signal enhanced by median-filtering. We suggest that the signal is the electrical field induced by the Stoneley wave forcing a flow of the ion-carrying pore fluid within the fractures (Figure 1).

Figure 9 shows the amplitude of the Stoneley-wave-induced electrical field normalized by the amplitude of the pressure oscillation in the wave. The amplitudes of the electrical field and the pressure oscillations are determined as a root-mean-squared value in a 15ms time window centered at the main pressure peak in the Stoneley wave. Figure 9 shows that the electrical fields' amplitudes measured with a 0.5m electrode separation, are practically the same as the amplitudes measured with a 1.0m electrode separation. Consistency of the electrical field amplitudes measured in different experiments demonstrates that the phenomena observed in the field are repeatable. The electrical field amplitude in Figure 9 is determined as the potential difference between the electrodes divided

by the distance between them. Thus, the 1.0m dipoles recorded a potential difference twice that recorded by the 0.5m dipoles. This suggests that the recorded signal is not due to the interaction of the Stoneley wave with the electrodes themselves.

Figure 9 also shows the phase shift between the Stoneley-wave-induced electrical fields and the pressure oscillations. The phases of both the pressure and the electrical field oscillations were determined by Fast Fourier Transformation in a cosine-tapered 50ms time window centered at the main pressure peak. The values plotted in Figure 9 are averages of phase shifts at different frequencies in an interval from 100Hz to 200Hz, where most of the Stoneley wave energy is concentrated. In granite around the depth of 110m and in diorite (below 140m) phase shifts estimated at different frequencies were not consistent with each other, due to a low signal-to-noise ratio. To indicate that at these depth the average value is not a reliable estimate of the phase shift, we plotted it using a dashed line.

Comparison of Figure 9 with Figures 3 and 4 shows that the amplitude of the electrical signal is high in fractured zones (zones of high P-wave slowness and conductivity in Figure 4) and low in unfractured zones. Therefore, the Stoneley-wave-induced electrical fields can be used to detect fractured zones intersecting a borehole.

8.4.2 Electromagnetic Radiation Caused by a Stoneley Wave

Electrical data in Figure 7 show an electrical signal which arrived simultaneously at all the dipoles at 46ms (event B-B). As was mentioned above, the electrical data were recorded 4 traces at a time. Event B-B was consistently recorded at the same time by electrodes placed at different depths in different experiments. Therefore, this signal cannot be attributed to noise (which changes from one group of 4 electrical traces to another).

Signal B-B originated at the time when the direct Stoneley wave encountered a fractured zone at the depth of 60m. Analysis of the amplitude of the signal shows that it decreases away from the depth of 60m. The simultaneous arrival time means that the signal traveled with the electromagnetic wave velocity. We suggest that this signal is an electromagnetic wave radiated by the Stoneley-wave-induced streaming current within the fractured zone at the depth of 60m.

8.5. THEORETICAL MODEL FOR THE ELECTRICAL FIELD INDUCED BY A STONELEY WAVE

The phenomenon detected most clearly and repeatedly in our experiment is the electrical field induced by the Stoneley wave. The amplitude of this electrical field is high in fractured zones and low in unfractured rock. To determine what specific characteristic of fractured zones can be deduced from the amplitude and the phase of the Stoneley-wave-induced electrical field, we developed the following theoretical model.

A Stoneley wave of frequency ω propagating with a phase velocity $c_s(\omega)$ downward in a borehole drilled through a homogeneous porous formation, induces oscillations of fluid pressure P_b in the

borehole:

$$P_b(r, z, t) = P_0 \frac{I_0\left(r \frac{\omega}{c_s}\right)}{I_0\left(R_b \frac{\omega}{c_s}\right)} \exp\left(-i\omega t + i \frac{\omega}{c_s} z\right). \quad (1)$$

Here r and z are the cylindrical coordinates (z -axis is pointing down), t is time, and R_b is the borehole radius. The Stoneley wave phase velocity $c_s(\omega)$ is in general a complex number (thus accommodating attenuation). In our experiment, the Stoneley wave's amplitude did not decrease significantly while the wave traveled along the borehole, except where part of the wave energy was reflected upwards by large fractures. Therefore, for the first-order analysis, we can neglect the attenuation, and consider c_s to be real-valued and equal to the apparent phase velocity of the Stoneley wave.

The pore pressure oscillation created in the formation by the Stoneley wave is (Tang *et al.*, 1991)

$$P_f(r, z, t) = P_0 \frac{K_0\left(r \sqrt{-\frac{i\omega}{D} + \left[\frac{\omega}{c_s}\right]^2}\right)}{K_0\left(R_b \sqrt{-\frac{i\omega}{D} + \left[\frac{\omega}{c_s}\right]^2}\right)} \exp\left(-i\omega t + i \frac{\omega}{c_s} z\right), \quad (2)$$

where D is the pore fluid dynamic diffusivity. In Equations 1 and 2, I_0 and K_0 are the zero-order modified Bessel functions.

The pore pressure gradient, induced in the formation by the Stoneley wave, causes a flow of an ion-carrying pore fluid, and results in a streaming electrical current. Thus, total electrical current density is the sum of the conductive current and streaming current densities:

$$\underline{j}_{total} = \underline{j}_{conductive} + \underline{j}_{streaming} = \sigma_r \underline{E} + L \nabla P_f. \quad (3)$$

Here $\underline{E}$ is the electrical field vector, σ_r is the formation conductivity, and L is the streaming current coupling coefficient, given by Pride (1994):

$$L = \frac{\phi_{ic}}{\alpha_\infty} \frac{\zeta \epsilon_f}{\mu} \left[1 - \frac{i\omega}{\omega_c}\right]^{-\frac{1}{2}}, \quad (4)$$

where ϕ_{ic} is the interconnected porosity, α_∞ is the pore space tortuosity, ζ is the zeta-potential (determined by the electrochemical interaction between the pore fluid and the rock), ϵ_f is the pore fluid permittivity, μ is the pore fluid viscosity, and ω_c is the critical frequency for the formation (fractured zone). The Biot theory determines ω_c as the frequency which separates the viscosity-dominated flow regime from the inertia-dominated flow regime. Equation 4 states that for the viscosity-dominated flow regime ($\omega \ll \omega_c$) the coupling coefficient does not change with frequency, and for the inertia-dominated flow regime ($\omega \gg \omega_c$) the coupling coefficient decreases as $\left[\frac{\omega}{\omega_c}\right]^{-\frac{1}{2}}$. The Biot critical frequency for the formation is (Johnson *et al.*, 1987)

$$\omega_c = \frac{\phi_{ic} \mu}{\alpha_\infty \rho_f k_0} \cdot \frac{2}{M}, \quad (5)$$

where k_0 is the formation static permeability, ρ_f is the pore fluid density, and the parameter M is close to 1 for most media.

The combination of Equations 2, 3 and 4 leads to an analytical solution for the vertical component of the electrical field in the borehole:

$$\frac{E_z}{P_b} = \frac{\frac{\phi_{ic}}{\alpha_\infty} \frac{\zeta \epsilon_f}{\mu} \left[-\frac{i\omega}{c_s} \right] \left[1 - \frac{i\omega}{\omega_c} \right]^{-\frac{1}{2}}}{\sigma_r + \sigma_f \frac{I_1\left(R_b \frac{\omega}{c_s}\right) K_0\left(R_b \frac{\omega}{c_s}\right)}{I_0\left(R_b \frac{\omega}{c_s}\right) K_1\left(R_b \frac{\omega}{c_s}\right)}}, \quad (6)$$

where σ_f is the pore fluid conductivity. Details of the derivation of Equation 6 are given in the appendix.

If the wavelength of the Stoneley wave is much greater than the borehole radius ($R_b \frac{\omega}{c_s} \ll 1.0$), then Equation 6 can be simplified as

$$\frac{E_z}{P_b} = \frac{\frac{\phi_{ic}}{\alpha_\infty} \frac{\zeta \epsilon_f}{\mu} \left[-i \frac{\omega}{c_s} \right] \left[1 - \frac{i\omega}{\omega_c} \right]^{-\frac{1}{2}}}{\sigma_r + \sigma_f \left[R_b \frac{\omega}{c_s} \right]^2 \frac{1}{2} \log \left(\frac{2c_s}{R_b \omega} \right)}. \quad (7)$$

Equation 7 gives the amplitude and phase of the electrical field induced by a Stoneley wave forcing a flow of the ion-carrying pore fluid within the permeable formation around the borehole. It is the central result of our theoretical analysis.

8.6. COMPARISON OF THE FIELD DATA AND THE THEORY

To test the validity of the theoretical model, we compare the amplitude and phase of the Stoneley-wave-induced electrical field observed in the field with the prediction of Equation 7. It is possible to make such a comparison for the granite section of the well (top 140m) because all the parameters in Equation 7 are known from independent field and laboratory measurements. In addition, the signal-to-noise ratio of the electrical signals is highest in the granite section.

Table 1 shows the values for the parameters in Equation 7 used for amplitude comparison. The Stoneley wave frequency ω and velocity c_s were determined from the hydrophone measurements (Figure 5). Borehole diameter R_b is taken from the caliper log (Figure 3). The value for the fluid conductivity σ_f corresponds to a salinity of 0.01mol/l, measured for a sample of the actual borehole fluid (water). The values of the fluid viscosity μ and permittivity ϵ_f are the values for water. The pore space tortuosity α_∞ was assumed to be close to 1.0 for fractures. For the borehole fluid properties in our experiments (salinity of 0.01mol/l and $pH = 7.5$), the value of the zeta-potential ζ for granite was taken from laboratory experiments (Ishido and Mizutani, 1981; Morgan *et al.*, 1989; Pride and Morgan, 1991). For a granite formation with a conductivity σ_r of 0.015S/m (an average value in granite in Figure 4), and an interconnected porosity ϕ_{ic} of 1% (a reasonable value for fractured granite), Equation 7 predicts the amplitude of the Stoneley-wave-induced electrical field to be $\left| \frac{E_z}{P_b} \right| = 12nV/(m \cdot Pa)$, assuming that $\omega \ll \omega_c$. This value of the amplitude is in agreement with the field measurements (Figure 9).

Equation 7 further predicts a (-90°) phase shift between the electrical field and the pressure oscillations. Figure 9 shows that in the granite section the phase shift in the field data is

around (-90°). Thus, the theory presented in this paper correctly predicts both the amplitude and the phase of the field data.

8.7. INTERPRETATION OF THE FIELD DATA BASED ON THE THEORETICAL MODEL

In this section, we analyze the field data and the analytical solution (Equation 7) for the amplitude and phase of the Stoneley-wave-induced electrical fields to determine what specific information about the formation can be deduced from borehole electroseismic measurements.

8.7.1. Estimating Interconnected Porosity From the Electrostatic Data

If the frequency of the Stoneley wave is less than the Biot critical frequency of the formation ($\omega < \omega_c$), then the interconnected porosity of the formation can be determined from Equation 7 as

$$\phi_{ic} = \left| \frac{E_z}{P_b} \right| \frac{\alpha_\infty \mu c_s}{\zeta \epsilon_f \omega} \left[\sigma_r + \sigma_f \left[R_b \frac{\omega}{c_s} \right]^2 \frac{1}{2} \log \left(\frac{2c_s}{R_b \omega} \right) \right]. \quad (8)$$

To derive an estimate of the interconnected porosity for the granite section (top 140m) using Equation 8, we use the values of the electrical field's amplitude, $\left| \frac{E_z}{P_b} \right|$, from Figure 9, and the formation conductivity σ_r from Figure 4. All the other parameters in Equation 8 are independent of depth and are given in Table 1. Figure 10 presents the porosity estimate for the granite section obtained using Equation 8 and a plot of the number of fractures per meter derived from a borehole video log. A visual comparison shows that the two curves correlate.

Electrostatic measurements suggest that, on the average, fracture porosity in granite is decreasing with depth. The borehole video log, however, shows that the number of visible fractures per meter on the average stays the same (Figure 10). To account for this discrepancy, the average fracture aperture has to decrease with depth. The borehole video log shows that the fractures intersecting the borehole are close to horizontal. It is possible that the overburden pressure forces deeper fractures to close more, resulting in a decrease of the fracture aperture with depth.

8.7.2. Correlation of Electrostatic Signals' Amplitude With Lithology

Figure 9 shows that the amplitude of the Stoneley-wave-induced electrical field is a factor of 5 higher in granite (top 140m) than in diorite (160m-290m). The simplest explanation of this observation is that there are less fractures in diorite than in granite. Smaller number of fractures results in lower porosity, and according to Equation 7, the Stoneley wave induces weaker electrical fields in diorite. The borehole video, P-wave slowness, and the conductivity logs also suggest that the fracture porosity in diorite is lower than in granite, thus supporting the above explanation.

However, there is another explanation to this observation. Analysis of the mineral composition of cuttings from the borehole shows that the diorite contains significantly less quartz than the granite. It is possible that due to the lower quartz content, the zeta-potential ζ is lower in diorite than in granite, and therefore the electrokinetic coupling is smaller (Equation 4). Further

experiments are necessary to determine zeta-potential values for different rock types in order to separate lithology effects from porosity effects in the field data.

8.7.3. Possibility to Determine Permeability From Electro seismic Measurements in a Wide Frequency Range

Figure 9 shows that the largest isolated fractures intersecting the borehole (depths 210, 255, and 290m) do not generate significant electro seismic signals, even though their "fracture porosity" is high. It is possible that the electrokinetic coupling in this fractures is low, because the frequency of the Stoneley wave in our experiment is higher than the Biot critical frequency for these fractures (a pressure wave traveling along a fracture is generated). Equation 4 states the electrokinetic coupling decreases above the Biot critical frequency ($\omega \gg \omega_c$). This variation of the electrokinetic coupling with frequency provides a possibility to determine the Biot critical frequency and permeability of the formation around the borehole.

Equation 7 shows that for low frequencies ($\omega \ll \omega_c$) the ratio of the electrical field amplitude to pressure, $\left| \frac{E_z}{P_b} \right|$, increases proportionally to the frequency ω and that the phase shift between the pressure and electrical field oscillations is (-90°). For high frequencies ($\omega \gg \omega_c$), $\left| \frac{E_z}{P_b} \right|$ decreases with frequency as $\omega^{-\frac{3}{2}}$ and the phase shift is (-45°). Therefore, if electro seismic measurements are made in a wide frequency range, then the Biot critical frequency ω_c could be determined as the frequency at which there is a change in frequency dependence and the phase shift of the electrical signals. Once the Biot critical frequency ω_c is determined, the permeability of the formation k_0 can be estimated from Equation 5 as

$$k_0 = \frac{2\phi_{ic}\mu}{\omega_c \alpha_\infty \rho_f M}. \quad (9)$$

To investigate whether it is feasible to determine permeability from the broad-band electro seismic measurements, we plan further experimental and theoretical work.

8.8 COMPARISON OF BOREHOLE ELECTROSEISMIC METHOD WITH STONELEY WAVE ATTENUATION METHODS

The electro seismic method may have advantages over Stoneley wave attenuation methods for detecting and characterizing fractured (permeable) zones. The first advantage is in the sensitivity of the electro seismic method. The analysis of the hydrophone data in Figure 5 shows that the amplitude of the pressure oscillation in the Stoneley wave decreased only by about 20% as the wave traveled from the top of the borehole to the depth of 290m. At the same time, the amplitude of the electrical field generated by the wave varied by 500% and was highest in the fractured zones. The second advantage of the electro seismic method is that it is not sensitive to borehole washouts, while the Stoneley wave attenuation methods require corrections for variations in borehole geometry. Thus, the sensitivity of the electro seismic borehole measurements to the presence of fractured zones, together with the simplicity of the method, could make it a useful tool for formation evaluation.

8.9. CONCLUSIONS

Downhole electroseismic measurements can be used to detect and characterize fractured (permeable) zones intersected by a borehole. The field data and the theoretical model presented in this paper indicate that at a given frequency the amplitude of the electroseismic signals is proportional to the interconnected porosity (provided that the frequency of the Stoneley wave is less than the Biot critical frequency for the fractures). Theoretical analysis also suggests that if electroseismic measurements are performed over a wide frequency range, then the critical frequency for the fractures could possibly be determined, thus providing information about the permeability. The results of this study suggest that the borehole electroseismic method could be developed into a logging or a VSP tool for detecting and characterizing permeable zones around a borehole.

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APPENDIX: Solution for the Amplitude and Phase of the Stoneley-Wave-Induced Electrical Field

We start our analysis with a full set of the Maxwell's equations. Comparison of the relative magnitude of the terms in the equations showed that at the frequencies and space-scales of the experiments, the terms containing differentiation with respect to time can be neglected. Thus, the full set of the Maxwell's equations reduces to the quasi-electrostatic equations for a conductive medium:

$$\nabla \cdot \underline{j} = 0 \quad (\text{A-1})$$

$$\nabla \times \underline{E} = 0. \quad (\text{A-2})$$

Our field data shows that at the sensitivity level of the measurements, electromagnetic waves were only detected in rare instances. At the same time, the electrical fields locally induced by a Stoneley wave were very clear in the field data. Therefore, the field data support the validity of neglecting "radiative" terms (i.e., the terms containing differentiation with respect to time) in the Maxwell's equations.

Equation A-2 allows introduction of the electrical potential:

$$\underline{E} = -\nabla\Phi. \quad (\text{A-3})$$

In the fluid filling the borehole, the electrical current density is determined by the fluid conductivity:

$$\underline{j} = \underline{j}_{\text{conductive}} = \sigma_f \underline{E}. \quad (\text{A-4})$$

In the formation around the borehole, the electrical current density is a sum of the conductive and streaming current densities:

$$\underline{j} = \underline{j}_{\text{conductive}} + \underline{j}_{\text{streaming}} = \sigma_r \underline{E} + L \nabla P_f. \quad (\text{A-5})$$

A combination of Equations A-3, A-4 and A-1 gives an equation for the electrical potential in the borehole:

$$\sigma_f \nabla^2 \Phi = 0. \quad (\text{A-6})$$

Similarly, a combination of Equations A-3, A-5 and A-1 gives an equation for the electrical potential in the formation:

$$\sigma_r \nabla^2 \Phi = L \nabla P_f. \quad (\text{A-7})$$

An axisymmetric solution for Equations A-6–A-7, that is finite at $r = 0$ and $r = \infty$ is

$$\Phi = A \cdot \frac{I_0\left(r \frac{\omega}{c_s}\right)}{I_0\left(R_b \frac{\omega}{c_s}\right)} \exp\left(-i\omega t + \frac{\omega}{c_s} z\right), \quad r < R_b \quad (\text{A-8})$$

$$\Phi = \frac{L}{\sigma_r} P_f + B \cdot \frac{K_0 \left(r \frac{\omega}{c_s} \right)}{K_0 \left(R_b \frac{\omega}{c_s} \right)} \exp \left(-i\omega t + \frac{\omega}{c_s} z \right), \quad r > R_b, \quad (\text{A-9})$$

where pressure P is given by Equation 2, and coefficients A and B are to be determined from boundary conditions.

At the borehole wall, the electrical potential and the normal electrical current have to be continuous. These boundary conditions provide two equations for coefficients A and B :

$$A = \frac{L}{\sigma_r} P_0 + B, \quad (\text{A-10})$$

$$-\sigma_f A \frac{\frac{\omega}{c_s} I_1 \left(R_b \frac{\omega}{c_s} \right)}{I_0 \left(R_b \frac{\omega}{c_s} \right)} = \sigma_r B \frac{\frac{\omega}{c_s} K_1 \left(R_b \frac{\omega}{c_s} \right)}{K_0 \left(R_b \frac{\omega}{c_s} \right)}. \quad (\text{A-11})$$

Solving Equations A-10 and A-11 results in:

$$A = \frac{LP_0}{\sigma_r + \sigma_f \frac{I_1 \left(R_b \frac{\omega}{c_s} \right) K_0 \left(R_b \frac{\omega}{c_s} \right)}{I_0 \left(R_b \frac{\omega}{c_s} \right) K_1 \left(R_b \frac{\omega}{c_s} \right)}}, \quad (\text{A-12})$$

$$B = -\frac{LP_0}{\sigma_r} + \frac{LP_0}{\sigma_r + \sigma_f \frac{I_1 \left(R_b \frac{\omega}{c_s} \right) K_0 \left(R_b \frac{\omega}{c_s} \right)}{I_0 \left(R_b \frac{\omega}{c_s} \right) K_1 \left(R_b \frac{\omega}{c_s} \right)}}. \quad (\text{A-13})$$

Equations A-8-A-9 and A-12-A-13 give the full solution for the electrical potential in and around the borehole. The vertical component of the electrical field can be obtained according to Equation A-3 by differentiating the expression for the electrical potential with respect to z :

$$E_z = \frac{LP_0 \left[-i \frac{\omega}{c_s} \right]}{\sigma_r + \sigma_f \frac{I_1 \left(R_b \frac{\omega}{c_s} \right) K_0 \left(R_b \frac{\omega}{c_s} \right)}{I_0 \left(R_b \frac{\omega}{c_s} \right) K_1 \left(R_b \frac{\omega}{c_s} \right)}} \frac{I_0 \left(r \frac{\omega}{c_s} \right)}{I_0 \left(R_b \frac{\omega}{c_s} \right)} \exp \left(-i\omega t + i \frac{\omega}{c_s} z \right). \quad (\text{A-14})$$

Finally, dividing Equation A-14 by Equation 1 gives the ratio of the electrical field to pressure in the borehole:

$$\frac{E_z}{P_b} = \frac{L \left[-i \frac{\omega}{c_s} \right]}{\sigma_r + \sigma_f \frac{I_1 \left(R_b \frac{\omega}{c_s} \right) K_0 \left(R_b \frac{\omega}{c_s} \right)}{I_0 \left(R_b \frac{\omega}{c_s} \right) K_1 \left(R_b \frac{\omega}{c_s} \right)}}. \quad (\text{A-15})$$

This equation is a solution for general L . If L is expressed in terms of the rock properties (e.g. Equation 4), then the solution for the amplitude and phase of the Stoneley-wave-induced electrical field is complete.

Parameter	Value
Stoneley wave frequency ω	$2\pi \cdot 150Hz$
Stoneley wave velocity c_s	$1400m/s$
borehole radius R_b	$0.15m$
fluid conductivity σ_f	$0.12S/m$
fluid viscosity μ	$10^{-3}Pa \cdot s$
fluid permittivity ϵ_f	$7.1 \cdot 10^{-10}coul^2/(N \cdot m^2)$
formation tortuosity α_∞	1
zeta potential ζ	$-40mV$

Table 1: Parameters of the experiment used for amplitude comparison and for estimating the interconnected porosity.

Figure 1: Diagram of a Stoneley wave inducing an electrical field. When a Stoneley wave travels past a fractured zone, it forces a flow of ion-carrying pore fluid within the fractures, thus generating a streaming electrical current. This current results in a time-varying charge separation, which induces an electrical field.

Figure 2: Diagram of the experimental setup.

Figure 3: Lithological description of the formation and the borehole caliper log.

Figure 4: P-wave slowness and conductivity logs for the formation.

Figure 5: Hydrophone measurements. Event A-A is the direct Stoneley wave.

Figure 6: Electrical measurements made with 0.5m electrode separation. Event A-A is the electrical field induced by the direct Stoneley wave.

Figure 7: Electrical measurements made with 1.0m electrode separation. Event A-A is the electrical field induced by the direct Stoneley wave. Event B-B is the electromagnetic wave radiated by the Stoneley-wave-induced electrical current within the fractured zone at the 60m depth.

Figure 8: Electrical data from Figure 6 median-filtered to enhance the electrical field induced by the direct Stoneley wave (Event A-A).

Figure 9: Amplitude of the Stoneley-wave-induced electrical field normalized by pressure, and the phase shift between the electrical field and pressure oscillations. Amplitude and phase of the Stoneley-wave-induced electrical field measured in the experiment agree with predictions of Equation 7.

Figure 10: Interconnected porosity estimate derived from the electroseismic field data, and a fracture density log derived from the borehole video.