The University of Alabama in Huntsville

Final Report

Submitted to Dr. David L. Hendrie and Dr. Dennis S. Kovar,
Division of Nuclear Physics, U.S. Department of Energy

PROGRAM ELEMENT: DE-FG-05-88ER40467

PROGRAM NAME:
Study of Isospin Correlation in High Energy S + Pb and Pb + Pb Interactions with a Magnetic-Interferometric-Emulsion-Chamber

PRINCIPAL INVESTIGATOR:
Dr. Yoshiyuki Takahashi / Research Professor
Department of Physics, The University of Alabama in Huntsville
Huntsville, ALABAMA 35899, TEL: (205) 890 - 6276 Ext 212

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

(Signature) Date: 12/12/97
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible electronic image products. Images are produced from the best available original document.
Summary of the Performed Research

The CERN EMU05 is the experiment to analyze all the charged particles emerging from \( S(200 \text{ GeV} / \text{Nucleon}) + \text{Pb} \) collisions using the emulsion chamber, MAGIC, (Magnetic-Interferometric Emulsion Chamber). Measurements on multiplicities, angular distributions, momenta, and charge-signs of the secondary charged particles produced in high multiplicity events (\( N_{ch} > 400 \)) from EMU05 and EMU09 have been performed to search for the possible phase transition of nuclear matter into Quark-Gluon Plasma (QGP) and Chiral Symmetry Restoration (CSR). In this report physics involved in heavy ion collisions is summarized and the methods to obtain the all-particle track data from the emulsion chambers are described. The results of the analysis with the new methods and algorithms (the canonical Fourier analysis and Periodogram) are reported for exploring the charge sign clusters. From these analyses of the limited number of the central collision samples we conclude that there is not a compelling evidence to require the DCC for heavy-ion collisions at the CERN energies.
CONTENTS

§ 1. Introduction

§ 2. QGP and DCC

§ 3. Theoretical and Experimental Review of High Density Nuclear Matter
   § 3 - 1 Possible Existence of Quark-Gluon Plasma in High Density Nuclear Matter
   § 3 - 2 Properties of Ordinary, Low-Density Nuclear Matter
   § 3 - 3 Theoretically Predicted Properties of Quark-Gluon Plasma
   § 3 - 4 Experimental Signature of Formation of QGP
   § 3 - 5 Experimental Signatures for the Chiral Symmetry Restoration

§ 4. Experimental Setup

§ 5. The Correlation Analyses

§ 6. Methodology

§ 7. Current Status of the Study of Heavy Ion Collisions with MAGIC Spectrometers.

§ 8. Publications

§ 9. Students Degrees resulted from this project and research at UAH.

§ 10. Conclusions

§ 11. References
Appendix A. Isospin Correlation Algorithms Developed by this Project

A1.1. Disoriented Chiral Condensate.

A.1.2. Past experimental study of the Isospin Clusters

Appendix A2. Isospin-cluster analysis for DCC.
- algorithms, and the nature of the low energy data (EMU05).

Appendix B1. Measurements on the charge-signs and momenta

Appendix B-2. Tracking Procedure

Appendix C. Fourier Analysis Data Compilation.
§ 1. INTRODUCTION.

This report describes the research results of the study of high energy heavy-ion interactions and multi-cluster correlations at the University of Alabama in Huntsville (UAH). This study has been performed as the CERN experiments, EMU05, EMU09 and EMU16, and a part of the RHIC PHENIX and its MVD Collaboration work. Physics objectives and methods are described in chapters 1, 2, 3 and Appendices A1 and A2. The experimental set-up, measurements, and the data analyses at UAH are described in chapters 4 through 10 and Appendices.

The UAH research was a quest for high density state of nuclear matter, in terms of finding analysis methods of multi-isospin correlations. The present work emphasized a study of the fluctuation of the particle density, discriminating the isospin for exploring the Disoriented Chiral Condensate (DCC). The analysis methods developed by us are:

1. Chi-square density test,
2. Run-test,
3. G-test,
4. Fourier analysis, and
5. Lomb’s Periodogram.

The application of these methods for central collision events in 2000 GeV/n S + Pb and 167 GeV/n Pb + Pb produced interesting DCC correlations for a few events. However, further investigation of fluctuations with Monte Carlo method guided us to understand various hidden degree of freedoms in such analyses. The results of the analysis of the experimental data in comparison with the Monte Carlo data did not support the DCC process as compelling.

The developed methods evolved for a plan to investigate the DCC in the PHENIX. The PHENIX will detect charged particle multiplicity at each segment of rapidity and azimuth angle with silicon micro-strip device, Multiplicity-Vertex-Detector (MVD). In a part of the covered rapidity-azimuth range, the PHENIX also installs pad-chamber and electromagnetic calorimeter (EMCAL) for recording gamma rays. By combining these two detector systems an investigation of the DCC could be allowed, in terms of the isospin density ratio, \( \pi^0/\pi^\pm \), via measurable \( \gamma/\text{charged} \) or \( \gamma/\pi^\pm \) ratio. Our study has obtained several mathematical analysis methods from the CERN EMU05/16 experiments for a possible use in RHIC experiments.
§ 2. QGP and DCC

One of the unique predictions of the Quantum Chromodynamics is that normal hadronic matter, where quarks are confined in individual hadrons, may melt into quark matter, where quarks and gluons behave as free particles in a volume larger than that of a hadron when the matter becomes very dense. Thus the behavior of high density nuclear matter, which can be created in high energy heavy ion collisions, has become one of the most prominent research subjects of contemporary nuclear physics. Active investigations have been performed both theoretically and experimentally, and our program has been one of these. The highest energy accelerator beams are used in order to create and study such an extreme condition in laboratory. Our program has been playing a unique role among these high energy experiments; having a capability of observing all charged particles in each event. The full acceptance particle detector (emulsion chamber) of ours is essential to examine the isospin correlations in the entire rapidity-azimuth space, while almost all the electronic counters had limited acceptance. Generally speaking, a large statistics of the data of all emitted particles with high accuracies is eventually needed to make quantitatively convincing analysis. Our study has been to explore what kind of physical analyses are possible and most powerful for examining the Disoriented Chiral Condensate (DCC) by using the all particle data.

The theoretical and experimental background is reviewed in chapter 3, the method of event analysis is presented in chapter 4, and finally, the significance of this work is stated in chapter 5 being followed by the summary of the current achievements.

§ 3. Theoretical and Experimental Review of High Density Nuclear Matter

§ 3 - 1 Possible Existence of Quark-Gluon Plasma in High Density Nuclear Matter

The standard quark model (SQM) describes a hadron as a complex of three quarks (baryons) or a quark and an antiquark (mesons) of three colors being bound by the color exchange force mediated by gluons to make color singlet states. The QCD (quantum chromodynamics) states that it takes an infinite amount of energy for a free quark or a free antiquark to exist. i.e.) Quarks are confined in a hadron (within the radius of \( \sim 1 \text{ fm} \)). On the other hand the attractive force binding quarks in a hadron becomes smaller as the inter quark distance becomes smaller (asymptotic freedom). The bag model explains this feature
very well. ref miy 1) The QCD assumes that the vacuum is the medium of which the magnetic permeability for the color field is infinite:

\[ \mu_{\text{vac}} = 1/\varepsilon_{\text{vac}} = \infty. \quad \text{(QCD vacuum)} \]

The bag model describes a hadron as a spherical shape perturbative vacuum space \((\mu = \varepsilon = 1)\) containing quarks and being filled with the color electric field which binds quarks together. (Fig. 1)

The energy of the bag is proportional to its volume if the mass of quarks are approximated to be zero.

**Fig 1. Bag Model:** \( E_{\text{bag}} = 4 B V \), where \( B \) is the energy density of the bag.

Since \( E \) is minimized by making the bag as small as possible, the energy density \( B \) acts as an effective pressure of the QCD vacuum onto the bag. The mechanisms of preventing the bag from collapsing are the kinetic energy of the quarks and the energy of the color electric field.

When the hadron density is very high so that the energy density becomes \( 4 B \), it is possible that the hadron bags will occupy the space fusing together to form a gaseous matter of free quarks and gluons where the identity of hadrons is no longer held. This state of matter is called quark-gluon plasma (QGP), which may represent the state of the early universe \((10^{-5} \sim 10^{-4} \text{ second after the big bang})\). Experimentally the QGP is expected to form in the relativistic heavy-ion collisions.

\[ \S 3 - 2 \text{ Properties of Ordinary, Low-Density Nuclear Matter} \]

To illustrate the difference of the QGP from the ordinary nuclear matter, here we review the nature of the latter.
A nucleus of the atomic number Z and the mass number A has been known to have the following properties.\textit{ref frm 1)}

i) The constituents (nucleons) are protons (p) and neutrons (n) being bound by the attractive force of exchanging mesons.

ii) The volume of a nucleus is approximately proportional to A (number of nucleons), which implies that there is a saturation of density. The rapid change of the density roughly from a constant to zero at a spherical surface suggests that the radius of the nucleus can be well approximated by the geometrical radius R. The relation of R and A obtained from the scattering and $\alpha$-decay, etc. is:

\[ R \simeq r_0 \times A^{1/3}, \quad r_0 \approx 1.5 \text{ fm} \quad \text{for } A > 20 \]

iii) The binding energy (B.E.) is defined as:

\[ \text{B.E.} = [ \Sigma M_{fn} - M(A, Z) ] c^2 \]

where \( M_{fn} \) = mass of a free nucleon and \( M(A, Z) \) = total mass

B.E. is approximately proportional to A, which implies that there is a saturation of binding energy at:

\[ \text{B.E.} \approx 8 \times A \text{ MeV} \quad \text{for } A > 20 \]

These saturation properties lead to an idea of the liquid drop model of nuclei from which a semi-empirical mass formula was established to fit the data (\( \Delta M/M \approx 1\% \)) considering the surface tension and the electrostatic repulsive forces among constituent protons. The mass formula thus derived is:

\[
M(A, Z) = [ 1.00898 A - 0.00085 Z - 0.01507 A + 0.014 A^{2/3} + 0.083 \ (A / 2 - Z)^2 / A \\
+ 0.000627 Z^2 / A^{1/3} + \delta(A, Z) ] \times 931 \text{ MeV} / c^2
\]

\[ \delta(A, Z) = 0.000277 A^{5/3} \frac{\text{MeV}}{c^2} \]

\[ \text{for } A > 20 \text{ and } Z > 2 \]
where the main corrections come from the binding energy (third term) and the surface tension (fourth term).

Although a detailed mechanism of nuclear forces is known to have shell structures, a short-range strong exchange force with a pion as a mediator, which depends on inter nucleon distance and parity-, spin-, and isospin-states, seems to well explain the saturation nature of nuclear force. Indeed, both the elastic and inelastic cross sections measured by the N-N scattering experiments show a free nucleon should have a finite volume with a radius $r_N$.

$$r_N \approx 0.45 \text{ fm}, \quad V_N \approx (4\pi/3) r_N^3$$

The effective radius of a nucleon for density saturation in a nucleus is given as:

$$r_{\text{eff}} \approx 1/2 \text{fm}$$

The repulsive nature of the core of nucleons is naively comprehensible when the fact that the nucleons are fermions and therefore the Pauli blocking is applicable in the form of repulsive force. Thus the ratio of the effective volume of nucleons ($V_{\text{Neff}} \approx (4\pi/3) r_{\text{eff}}^3$) in a nucleus is $1 \sim 2 \%$ to the volume of the nucleus ($V \approx (4\pi/3) R^3$). The success of the liquid drop model and the repulsive core of the nuclear force both support an idea that each individual nucleon is holding its identity in a nucleus.

§ 3 - 3 Theoretically Predicted Properties of Quark-Gluon Plasma

In order to discuss the possibility of the formation of QGP two extreme cases have to be considered. One is high temperature and high density case, and the other is low temperature and high density case.
[High Temperature Limit]  In the nuclear matter at high temperature nucleons would have large kinetic energy and collide to each other creating many pions (multiparticle production). At some critical temperature $T_C$ the heat energy would be mostly used to create pions and the temperature would no longer rise beyond $T_C$. $T_C$ is known as the Hagedorn limiting temperature estimated to be nearly equal to the pion mass.

$$T_C = 170 \text{ MeV} \text{ where the Boltzman constant } k_B \text{ is set to be unity.}$$

$$k_B = 8.616 \times 10^{-5} \text{ eV/deg} \implies 1$$

At $T_C$ the number density of hadrons becomes very high and the inter-quark distance between neighboring hadrons becomes very small ($\sim 1 \text{ fm}$), and the inter-quark interaction becomes negligibly small for quarks and antiquarks to behave like free particles. Thus a phase transition is expected from high density hadronic matter to the QGP phase where hadrons lose their identities and free quarks, antiquarks, and gluons occupy a large volume (compared to the size of a nucleon) at $T = T_C$.

[Zero Temperature Limit]  Since nucleons are Fermions, they fill up the lowest energy states up to the Fermi energy $\mu_F$ at $T = 0$.

$$\mu_F = (p_F^2 + m_N^2 c^4) - \text{ where } p_F \text{ is the corresponding Fermi momentum.}$$

The number density of nucleons $n_B$ can be expressed as:

$$n_B = 2 \frac{p_F^3}{3} \pi^2 h^3.$$
Thus the degenerate pressure of the nuclear matter increases as $n_B$ increases, and at some critical $n_{BC}$, $P_N$ becomes $(\text{QCD vacuum pressure } B) \times 4$. If $n_B$ goes beyond $n_{BC}$, then again the hadron bag would lose its identity and a phase transition to QGP would occur. This may be the situation at the inside of high density stars such as neutron stars. The possibility of the phase transition at the high temperature limit has been suggested to be in high energy heavy ion collisions. Theorists predict, from the results of the lattice QCD calculations, that the critical temperature would be:

$$T_C \sim 200 \text{ MeV}$$

Also in QGP phase, the Stephan-Boltzman law ($\varepsilon \propto T^4$) is expected to hold as we have free gas of quarks and gluons. The energy density required for the phase transition is estimated to be:

$$\varepsilon \sim \text{a few GeV/fm}^3.$$  

§ 3 - 4 Experimental Signature of Formation of QGP

[ Relativistic Heavy-Ion Collisions ]

In the center of mass frame both the target nucleus ($A_t$) and the projectile nucleus ($A_p$) look like pancakes in high energy collisions due to the Lorentz contraction. Nucleons in the overlapping region would interact and path through leaving a low baryon density region (central region) in between the two receding fragments of nuclei. This small hot cylinder would be the QGP if the accumulated energy is large enough.

J. D. Bjorken showed how to estimate the energy density to see whether the matter created in the central region is in the QGP phase or in the normal hadronic phase (mostly pion gas) by using the emitted pion distributions. The volume of this small cylinder is:  

$V = \frac{4}{3} \pi R L$
\[ V = \pi R_p^2 L \quad \text{where} \quad L = c \tau_0 \approx 1 \sim 2 \text{ fm.} \]

The energy (E) accumulated in this volume can be estimated by:

\[ E = \langle p_T^2 \rangle c^2 + m_{\pi}^2 c^4 \right) \text{d}N/\text{d}y \]

where \( \langle p_T^2 \rangle \) is average transverse momentum of central rapidity region, \( m_{\pi} \) is the rest mass of pion, and \( \text{d}N/\text{d}y \) is the multiplicity of +, −, and 0-charged particles in a unit rapidity (y). The physical variable y is the rapidity defined by:

\[
y = \frac{1}{2} \ln \frac{E/c - p_L}{E/c + p_L},
\]

Thus the energy density \( \varepsilon \) can be evaluated as:

\[
\varepsilon = \frac{E}{V} = \frac{1}{\pi R_p^2 L} \langle (p_T^2) c^2 + m_{\pi}^2 c^4 \rangle^{1/2} \frac{\text{d}N}{\text{d}y}
\]

which is called the Bjorken formula. \textbf{ref bj} 1)

If the thermal equilibrium is applicable to the system, then the temperature \( T \) is related to \( \langle p_T \rangle \). Therefore it can be judged by \( \langle p_T \rangle \) value if the temperature of the system is under the Hagedorn limiting temperature \( T_C \) (normal hadronic phase) or \( T \) is exceeding \( T_C \) satisfying the Stephan-Boltzman law of \( \varepsilon \propto T^4 \) (QGP phase).

JACEE (Japanese-American Cooperative Emulsion Experiment) reported that the cosmic ray data showed the increase of the \( \langle p_T \rangle \) as \( \varepsilon \) increased up to a limiting value of \( \sim 0.4 \text{ GeV}/c \). Also, above \( \sim 2 \text{ GeV}/\text{fm}^3 \), \( \langle p_T \rangle \) started to rise again with \( \varepsilon \). \textbf{ref jae} 1)

[Fig.5]
The method to estimate the space-time size of a thermal pion source (plasma ball) has been well established. Ref [zaj 1], Ref [nag 2]. The correlations of two identical particles, such as $\pi^-\pi^-$ or $p p$, can be calculated by HBT effect Ref [hbt 1] assuming that those particles can be treated as free particles. F. B. Yano and S. E. Koonin stated Ref [koo 1] that the plane wave approximation is very well applicable because the strong interaction and the Coulomb interaction between the particles require less than 0.1% modification in HBT effect. When two identical particles are emitted from two independent points, $(x_1, t_1)$ and $(x_2, t_2)$ in the same source and detected at $(x_{1'}, t_1')$ and $(x_{2'}, t_2')$ having the momentum and the energy of $(k_1, E_1)$ and $(k_2, E_2)$, the correlation function is given by Ref [nag 2]:

$$C_2 = \int P(X_1 T_1, X_2 T_2) \rho(x_1, t_1) \rho(x_2, t_2) dx_1 dx_2 dt_1 dt_2$$

where $P(X_1 T_1, X_2 T_2)$ is a symmetrized (for Bosons) or antisymmetrized (for Fermions) two-particle spectrum, which can be expressed as:

$$P(X_1 T_1, X_2 T_2) = \frac{1}{2} \left\{ \exp[i k_1 (X_1 - x_1) / h - i E_1 (T_1 - t_1) / h] \cdot \exp[i k_2 (X_2 - x_2) / h - i E_2 (T_2 - t_2) / h] \right\}^2$$

and $\rho(x, t)$ is the space-time structure of the source, which can be assumed to be Gaussian distribution function expressed as:

$$\rho(x, t) = \frac{1}{\pi^2 \tau^2} \exp(-x^2 / R^2) \exp(-t^2 / \tau^2).$$

After carrying out the integral $C_2$ becomes:
\[ C_2 = 1 \pm \lambda \exp[-|k_1 - k_2|^2 R^2 / 2h^2 - (E_1 - E_2)^2 \tau^2 / 2h^2] \]

where \( \lambda \) is a parameter introduced to indicate the correlation strength.

**Fig 6. Thermal Pion Source and Detectors**

The definition of the correlation function is the ratio of the probability of finding two particles having momenta \( k_1 \) and \( k_2 \); \( P(k_1, k_2) \) versus the probability of finding one particle having the momentum \( k_1 \) and the other with \( k_2 \); \( P(k_1) P(k_2) \), which suggests:

\[ C_2 = \frac{[dN(|k_1 - k_2| = q)]}{[dN/dk_1 \times dN/dk_2]}. \text{ ref emf 1} \]

Thus the parameters \( \lambda \) and \( R \) can be obtained by fitting the calculated \( C_2 \) function to the momentum spectra (neglecting the time dependence). The acceptance of EMU05 and EMU09 (almost \( 4\pi \) in the CM frame) is significantly large enough to determine the source radius. The results of EMU05 are shown in Fig. 7 below.
In summary the measurement of momenta of all particles is very important for the estimate of the energy density (i.e. temperature) and the source size.

§ 3 - 5 Experimental Signatures for the Chiral Symmetry Restoration

[Chiral Symmetry Restoration]

The QCD vacuum is understood to be a field of \( qq \) condensation which is isotropic and uniform. If we project quark's bispinor onto the left-handed and the right-handed states denoted by \( q \) and \( \bar{q} \) and ignore quark masses, the isospin invariance and the charge conservation requires the vacuum expectation value to be of the form:

\[
\begin{align*}
\langle u_L u_R \rangle &= \langle d_L d_R \rangle = \langle u_R u_L \rangle = \langle d_R d_L \rangle = \frac{1}{2} v
\end{align*}
\]

(1)

where \( v \) is a real constant.

The global chiral transformation is defined by
where $t$ is Pauli matrices for isospins.

The vacuum expectation value (1) changes the phase under this chiral transformation. Since the energy of the system is unchanged, our vacuum is infinitely degenerate (chiral condensate). But because it takes infinite amount of energy to change the phase, the quark's isospin points to one direction. Thus the chiral symmetry is spontaneously broken and the condition $\langle \bar{q}q \rangle \neq 0$ is the reason.

The lattice QCD calculation shows that the order parameter $\langle \bar{q}q \rangle \rightarrow 0$ at $T_c = 200$ MeV, which suggests the spontaneously broken chiral symmetry can be restored in high energy heavy ion collisions. However because $\langle \bar{q}q \rangle$ does not exactly vanish and quarks have finite masses, the vacuum would be disoriented from the direction favored in the ground state (disoriented chiral condensate, DCC). In the sigma model, which shall represent this system, the nonzero $\langle \bar{q}q \rangle$ stands for the nonzero $s$ component and zero $p$ components. i.e.) $\phi = (\sigma, \pi) = (\phi_\pi, 0)$: vector in internal space

Visible consequences of DCC are suggested as follows. J.Bjorken ref bj02) predicts the vacuum disorientation would lead to coherent pulses of semiclassical pion field which would give large fluctuations event to event in the ratio of neutral to charged pions produced. This could explain so called Centauro, where the neutral pions ratio is unusually small, or Anti-Centauro events. K. Rajagopal and F. Wilczek ref rwl 1) argues that in the region between two receding nuclei the temperature is well above $T_c$ and the system is in thermal equilibrium where $f$ points in $s$ direction. But as it expands, the plasma cools rapidly (quench), and the pion field in $f$ oscillates with a long wavelength resulting in creation of patches of coherent pions. M. Asakawa, Z. Huang, and X. Wang ref ahw 1) simulated both quench and annealing cases. They demonstrated that if the initial size of the region, where the chiral symmetry is restored, is 1 - 5 fm, the hedgehog structure of $p^+$ and $p^-$ is formed in the transverse plane to the beam axis.

The previous EMU05 analysis showed such clustering of charged particles in a few events. The "run test" and the "conjugate test" (G-test) were applied ref emf 1) to examine the degree of clustering vs. random coincidence. The run ($r$) is defined by the total number of sign-changes in a line of charges aligned in the order of azimuth angle (or

\[
q_L \rightarrow q_L' = e^{i\theta / 2} q_L \\
q_R \rightarrow q_R' = e^{-i\theta / 2} q_R
\]
rapidity) in one segment of rapidity-azimuth angle space. The normalized run \( (x) \), defined by 

\[
x = \frac{r - \langle r \rangle}{\sqrt{V}}
\]

should follow the normal distribution \((0, 1)\) if charge-signs are randomly distributed while clusters should show negative \( x \) excess. The G-test measures the degree of clustering defining \( G_i(\text{max}) \) for the i-th particle: 

\[
G_i(\text{max}) = \max G_{ij}(R),
\]

where \( G_{ij}(R) = \sum (C_k \times C_l) \) for \( k \) and \( l \) contained within \( R \). The \( C_i \) is +1 or -1 for positive or negative charge, respectively. The area \( R \) is measured from the i-th particle, \( 0 < R < R_{ij} \), where \( R_{ij} \) denotes the invariant mass of two particles. Fig. 8 shows an example of these tests for a cluster candidate. The run-test indicated a random coincidence dominance over any possible signals of clusters. The G-test, on the other hand, showed negative-charge clusters in the projectile region for some S + Pb events. Although both the run-test and the G-test are powerful for an observer to recognize the existence of clusters, the correlation of opposite sign clusters does not show clearly in a quantitative manner.

The canonical Fourier analysis as described below will incorporate the regularity of appearance of clusters for a quantitative characterization of the phase correlation. The proposed canonical Fourier analysis will be performed as follows.

A ring segment defined by rapidity is divided into \( n \) bins \((n = 2\pi / \Delta \phi)\) for the number distributions of + and - signs, \( N_j^\pm \). From the distribution of - sign a discrete Fourier cosine component with a phase parameter \( \delta \) is obtained as:

\[
a_k = 2 \sum_{j=0}^{n-1} N_j^- \cos\left(\frac{2\pi j}{n} + \delta\right), \quad k = 0, 1, 2, \ldots, \frac{n}{2}
\]

The phase parameter \( \delta \) is determined so that the highest peak of \( |a_k^-| \) is maximized. A discrete Fourier sine component is taken for + sign as:

\[
b_k^+ = 2 \sum_{j=0}^{n-1} N_j^+ \sin\left(\frac{2\pi j}{n} + \delta + \frac{\pi}{m}\right), \quad k = 1, 2, \ldots, \frac{n}{2}
\]

The parameter \( m \) is determined so that the highest peak of \( |b_k^+| \) is maximized. The parameter \( m \) characterizes the phase shift of + and - sign distributions for each ring segment. The width of the ring \((Dh)\) shall be determined to extract the cluster signal the most effectively. One possible way is to set the center of the ring \( h_i \) and \( Dh \) such that \( h_i \) is the i-th peak of the negative excess in the run-test and \( Dh \) is a constant or

\[
\Delta \eta = |\eta_i - \eta_{i+1}| / a \quad \text{where} \quad a \quad \text{is an integer}.
\]

Further requirement is the number of particles in
one bin \(|N_j|\) should not be below 2 on average. Thus all the parameter \(d, m\) and the spectra \(a_k, b_k\) will be uniquely determined without any human bias. The preliminary result for the same event shown in fig. 8 is presented in fig.8.5. The largest component of the wave number \(k\) is 9 and 11 for + and - charge distributions, respectively. The phase shift parameter \(m\) for this segment, defined by \(1.7 < h < 2.4\), is 1. The result implies, though tentatively, that the phase correlation of \(p^+\) and \(p^-\) seems to exist in the cluster candidates. The same analysis will be applied to events simulated with randomly distributed charges for the discussion of the probability of such a strong correlation to exist.
a) Apparent charge-sign clusters, (b) whose "Run-Test" and c) "Conjugate-test" for negative charge-sign clusters.

Fig 8.

§ 4. Experimental Setup

[EMU05] The EMU05 experiment let 200 GeV / Nucleon sulfur beam collide with lead target and detected almost all the emerging secondary charged particles with the
magnetic-Interferometric emulsion chamber: MAGIC. The MAGIC uses a 200 μm thick lead plate as a target, thin emulsion plates (50 μm thick emulsion on both sides of 300 μm thick polystyrene plates) as a particle detector, and low density Styrofoam plates (1000 ~ 5000 μm thick) as spacer between emulsion plates. Also a few thick emulsion plates (700 μm thick) were placed in the bottom the chamber so that the particle identification can be done according to the ionization loss in thick emulsion layers. All the plates (12 cm × 7.5 cm) were put in a box (~ 7 cm deep) and exposed vertically to the sulfur beam. A uniform 1.8 Tesla magnetic field was applied parallel to the plates so that the charge-sign and the momentum of each secondary particle can be obtained from the magnetic deflection of its track due to the Lorentz force. [Fig. 9, 10]

Fig 9. 3-dimensional view of the EMU05 emulsion chamber.

Fig 10. Vertical View of MAGIC
The CERN EMU09 experiment (spokesperson: Dr. Giorgio Romano Univ. of Salerno, Italy) was operated at the same time as EMU05. The EMU09 used the same sulfur beam and similar emulsion chambers as MAGIC (Fig. 11).

The chamber was constructed in a hollow black plastic box, and the emulsion plates (10 ~ 30 μm thick emulsion layers on both sides of 200 μm thick polystyrene plates) were bolted in slots (24.5 cm x 4.5 cm) with no mechanical supports in the middle.

Consequently the emulsion plates have warp which will require additional adjustment. The followings summarize a few features of EMU09 compared to EMU05.

〈advantages〉

i) The magnetic field applied to the emulsion chamber is stronger. (2.47 Tesla)
ii) The depth of the chamber is larger. (~ 10 cm)
iii) The amount of material in a chamber is less. (6 emulsion plates after the target and no spacer material)

〈disadvantages〉

iv) The emulsion plates are narrower. (4.5 cm x 2.4 cm)
v) The emulsion layer is thinner (~1/2), and the grain density is smaller.
vi) The number of available events is less. (~15 events)
vii) Most of the emulsion plates are slightly warping.
Since the momentum of the emitted particle is determined by the curvature of its track, it is necessary to trace the track downstream plate by plate. The expression for momentum in terms of the curvature is:

\[ p = e z B \rho / c \left[ \sin^2 \theta \cos^2 \phi + \cos^2 \theta \right]^{1/2} \]

where \( \rho \) is the curvature and \( \theta, \phi \) are the emission angles. [Fig. 12]

[Tracking] Each track was traced downstream plate by plate. Two-dimensional coordinates of each track with respect to the parent beam axis (z-axis) were measured with the microscope system: CUE-2. (Fig. 13) The emission angle (\( \theta, \phi \)) and the multiplicity are available after the first one or two plates are measured (\( z < 1000 \mu m \) downstream from the vertex.) In order to obtain charge-signs and momenta, individual tracing must be done throughout the emulsion telescope down to \( z = 7 \) cm (EMU05) and \( z = 10 \) cm (EMU09) according to the following procedures. (Detailed tracking procedure is in appendix B.)

[Fig 12 Definition of coordinate system]

![Fig 12 Definition of coordinate system]

[Fig 13. The Cue-2 Microscope System]
1. Event scanning was performed with the emulsion plate which was placed right below the target. Rough estimate of the event multiplicities (e.g., $N_{ch} \approx 50$) and the vertex heights (e.g., $z \pm 50 \mu m$) were obtained at this stage, and the relative location of scanned events were recorded for further measurements. For EMU09 the event selection was made according to the multiplicities ($N_{ch} \geq 350$ at least. Central collision events were selected by multiplicities and the residual target thickness. The contamination of electrons from the photon conversion were reduced to $1-2\%$ of charged tracks. Non-interacting beam tracks were selected ($N_{ref} \sim 50$: EMU05, $N_{ref} \sim 100$: EMU09) as reference beam tracks for two dimensional plate re-alignment (translational and rotational adjustments in the x-y plane) as shown in Fig. 14 (left).

![Diagram](image1)

Fig 15. EMU09 Reference Beams

2. On the emulsion plate which was placed above the target, the location of the parent beam is set to be the origin of the coordinate system, and the relative coordinates of the selected reference beams are measured.
3. The plate used for the scanning was set for tracking. After the reference beams were measured, each secondary track being displayed in the video monitor was converted into a single pixel and then analyzed for coordinates. On the bottom surface of the same plate those tracks were identified and measured by moving the stage using the target diagram pattern recognition [Fig. 16]. At this depth \(( z < 400 \, \mu m )\) and up to \( z = 15000 \, \mu m \), coordinates of each track were measured by the similar way using straight line prediction, and the recoordination (correction) for the depth coordinate was applied by a simple trigonometry.

4. Since the curvature of each track is defined at the end of plate \# 6 (EMU05) and plate \# 4 (EMU09), predictions by circular fit were used in the initial tracking at the immediate downstream plate. Each track was traced further downstream to obtain the curvature with better accuracy.

4. Since the curvature of each track is defined at the end of plate \# 6 (EMU05) and plate \# 4 (EMU09), predictions by circular fit were used in the initial tracking at the immediate downstream plate. Each track was traced further downstream to obtain the curvature with better accuracy.

The plate used for the scanning was set for tracking. After the reference beams were measured, each secondary track being displayed in the video monitor was converted into a single pixel and then analyzed for coordinates. On the bottom surface of the same plate those tracks were identified and measured by moving the stage using the target diagram pattern recognition [Fig. 16]. At this depth \(( z < 400 \, \mu m )\) and up to \( z = 15000 \, \mu m \), coordinates of each track were measured by the similar way using straight line prediction, and the recoordination (correction) for the depth coordinate was applied by a simple trigonometry.

4. Since the curvature of each track is defined at the end of plate \# 6 (EMU05) and plate \# 4 (EMU09), predictions by circular fit were used in the initial tracking at the immediate downstream plate. Each track was traced further downstream to obtain the curvature with better accuracy.

Almost all the tracks can be traced through the entire emulsion telescope except tracks going out of the chamber, largely scattering tracks, and decaying tracks. The present measurements have proven the efficiency \((\varepsilon)\) of the tracking for the determination of the momenta and charge-signs to be almost 100\% for both EMU05 (all particles) and for EMU09 (except for large emission angles, \( \theta_{LAB} > 15^\circ \): \( \varepsilon \approx 25\% \)).

The spatial resolution of the emulsion film is sub micron. The combination of the automated stage motion and the image analyzer was applied to the measurements. The
latter also has the resolution of sub micron (~0.5 μm). Although the former has mechanical backlash errors, a feed back from the encoder work independently from the stage driving motor, which minimizes the errors to be ±0.5 μm in a few successive stage motions of ±10000 μm in x and y directions. The individual reference beam tracks have the size of a few microns, but the centroid of many tracks allowed statistical improvement of reference reconstruction to better than 1 micron. For this purpose, we used a sufficiently large number of reference tracks (>60). Thus overall spatial resolution became ~±0.5 μm. [Fig. 17] Considering the spatial resolution mentioned above and the precision of the machine processed plastic base plates, the angular resolution in one emulsion plate was better than ±0.10 deg (EMU05) and ±0.14 deg (EMU09).

The accuracy of the momentum determination depends on the chamber design (as mainly due to multiple scattering) and the spatial resolution of curvature measurements. The overall analysis performed so far shows that the momentum error is:

\[ \Delta p_{LAB}/p_{LAB} = 5 + [p/(20 \text{ GeV}/c)] \% . \]  

[Fig. 18]
So far 17 events from EMU05 and 9 events from EMU09 (total of 26 events) have been measured. The fitting analysis was made by T. Virgili for the EMU09 events. The result of the momentum analysis and its determination accuracy is shown in Fig. 19.

1) The momentum error in the EMU09 data is:

\[ \Delta p_{LAB} / p_{LAB} = 3 + [p/(20 \text{ GeV/c})] \% \]
Thus the followings can be stated: EMU05 is suitable for all track analysis over almost all the emission angles, \(0 \leq \theta_{\text{LAB}} < 90^\circ\). EMU09 is suitable for the analysis of the tracks with better accuracy, especially in the forward region \( (y \geq 4) \).

Fig 19. Momentum Error (EMU09)

§ 5. The Correlation Analyses

1. High multiplicity events of 200 GeV / Nucleon S + Pb collisions were thoroughly investigated by the information of multiplicities, angular distributions, and momenta with charge-signs. The above information were obtained by the measurements of the three dimensional coordinates of all the charged secondary particles recorded in the EMU05 and EMU09 emulsion chambers using the microscope system CUE-2.

2. The number of central collision events being investigated were:
   
   EMU05  \( \Rightarrow \) new 17 events + 16 events that were reported in the literature.
   
   EMU09  \( \Rightarrow \) 16 events, \((16 \text{ is the maximum number of events available.})\)

3. The energy density of the nuclear matter created in 200 GeV / Nucleon S + Pb collisions were estimated using the Bjorken formula.

4. The transverse momentum spectra were obtained for positive and negative charged groups.

5. The size of the pion emitting source and the strength of the Bose-Einstein interference were obtained by fitting the correlation functions to measured two-body momentum-difference spectra.
6. The new canonical Fourier analysis were performed on each high multiplicity event of EMU05 and EMU09 to clarify whether a strong correlation of $p^+$ and $p^-$ clusters exists in the azimuth direction perpendicular to the beam axis.

§ 6. Methodology

i) Measurements

High multiplicity events of EMU-05 and EMU-09 were analyzed thoroughly using the CUE-2 microscope system for the data of multiplicities, angular distributions, momentum distributions with charge-signs.

ii) Monte Carlo Analysis

The obtained data were compared with the data of previous EMU05 and also with those of the other experiments for quantitative discussion. The probability of the existence of a strong correlation of clusters in the measured events were compared with those of the computer simulated events that assumed random charge distributions.

§ 7. Current Status of the Study of Heavy Ion Collisions with MAGIC Spectrometers.

The energy density obtained from Bjorken's formula for central collisions at 200 GeV / N (S + Pb) was:

$$\epsilon < 2 \text{ GeV} / \text{fm}^3 .$$

There were no second rise of $<p_T>$ that was reported in 1986 for very high energy cosmic ray events ($\epsilon > 2 \text{ GeV/fm}^3$, as seen in ref. jac 1). The $p_T$ spectra from EMU05 and EMU09 events are shown in fig. 20 and 21. ref tiz 1), emf 1)

The source radius obtained from $\pi^- \pi^-$ correlation was already reported by us:

$$R \approx 4.2 \pm 2.2 \text{ fm}$$

with the interference strength, $\lambda = 0.37 \pm 0.19$.  

24
\[
\frac{1}{P_T} \frac{dN}{dP_T} = e^{-P_T/P_0}
\]

\[P_0 = 227 \pm 3 \text{ MeV/c} \]

\[P_0 = 198 \pm 3 \text{ MeV/c} \]

Fig 20. Pt Spectra at \(2 < y < 4\)

§ 8. Publications

(The EMU05 Collaboration): Y. Takahashi, Nucl. Phys. A478, 675c (1988);

Y. Takahashi et al., Nucl. Phys. A498, 529c (1989);


Y. Takahashi et al., Nucl. Phys. A525, 365 (1991);

Y. Takahashi et al., Nucl. Phys. A525, 365 (1991);

A. Iyono et al., Nucl. Phys. A544, 455 (1992);

Y. Takahashi et al., 'High Energy Nuclear Collisions & Quark Gluon Plasma", pp. 276, Eds. M. Biyajima et al., World Scientific (1992);


T.H. Burnett et al., Phys. Rev. D35, 824 (1987);

Y. Takahashi and S. Dake, Nucl. Phys. A461, 263c (1987);


Y. Takahashi, P. B. Eby, T. A. Parnell, ibid. 6, 172 (1985), NASA Publ. 2376;

§ 9. Students Degrees resulted from this project and research at UAH.

Atsushi Iyono, Ph. D, 1989, (student from Okayama Univ. of Science).
Tiziano Virgil, Ph. D, 1994, (student from Univ. of Rome).
Kanaya Chevli, MS, 1996, (UAH).
Toshiyuki Shiina, Ph. D (qualified), 1997, (UAH).

§ 10. Conclusions

The current results on $P_T$-distributions, energy-density and particle correlation have not indicated any support for the formation of the QGP and DCC with the sulfur beam at this energy (200 GeV/n). However the apparent clustering in one of the EMU05 events suggested an interesting possibility of strong correlation of positive and negative charged clusters in these events. New analysis method for such clusters were explored as shown in the Appendix C, which might be useful for the further study of the DCC at the RHIC energies.

§ 11. References

  "How Disoriented Chiral Condensates Form: Quenching vs. Annealing"

  "Quark Matter and Nuclei"

  "Highly Relativistic Nucleus - Nucleus Collisions: The Central Rapidity Region"

  "Geometry of Multi-hadron Production"

  "Study of Correlation of Positive and Negative Charged Particles"

"Study of Particle Correlation in High Energy S + Pb and Pb + Pb Interactions
with a Magnetic - Interferometric - Emulsion - Chamber"

"Proposal for Emulsion - Hybrid Set-up for the Study of Sulfur - Nucleus
Collisions at 200 GeV/N"

frm 1) : J. Orear, Univ. of Chicago Press (1950)
"Nuclear Physics - A Course Given by E. Fermi at Univ. of Chicago"

inya 1) : A. Iyono, Ph. D Thesis (1991) Okayama Univ. of Science
"Study of Multi Particle Production in 200 GeV/AMU S + Pb interaction"

p.71
"A Cosmic-Ray Experiment on Very High Energy Nuclear Collisions"

"Determining Pion Source Parameters in Relativistic Heavy-Ion Collisions"

lan 1) : L. Landau, New York Consultants Bureau (1958)
"Lecture on Nuclear Theory"

"Quark Matter"

"Toward New Forms of Matter with Relativistic Heavy-Ion Collisions"

"Two-Particle Inclusive Correlations"

"The Chiral Phase Transition in QCD"

"Preliminary Results from EMU09 - Search for Correlation"

"Monte Carlo Calculation Methods for the Generation of Events with Bose-Einstein Correlations"
Appendix A. Isospin Correlation Algorithms Developed by this Project

A1.1. DISORIENTED CHIRAL CONDENSATE.

Recently, the most energetic secondaries have been the subject of active theoretical study in terms of what is now known as DCC.\textsuperscript{4 - 8} This theoretical development is based on the quantum chromodynamic (QCD) nature of the chiral axis orientation and also the QCD phase transition. The DCC is a result of the chiral oscillation or fluctuation of the chiral axis for isospin projection in nuclear collision, causing significant isospin asymmetries in interactions at very high energies.

This development may be capable of accounting for the famous “Centauro” phenomena in cosmic-ray interactions.\textsuperscript{24} Centauro indicated a substantial lack of $p^0$ mesons in the projectile fragmentation region where about 100 of hadrons were produced in cosmic ray interactions above 1,000 TeV/n, which is only slightly higher than the RHIC energy. Indeed, Bjorken was the first to embark on the consideration of the possible DCC, inspired by a puzzling nature of the Centauro phenomenon. If the DCC theory is correct, the photon multiplicity or inelasticity distribution will be much more enhanced at high $k_g$'s, which comes naturally from a simple phase space factor, $f(x)dx = 1/2\sqrt{x}dx$, where $x = p^0/(p^+ + p^-)$. Fig. A1-1 shows the significant difference between the ordinary binomial-type $k_g$ distribution (in terms of multiplicity ratio, $x_1$) from that ($x_2$) of the DCC. The enhancement factor of the effective inelasticity in cosmic ray experiments ($C_{kg}$) can be evaluated by the average of $x_1$ and $x_2$ values from Fig. A1-1 as $x = \langle x_2 \rangle / \langle x_1 \rangle = 0.7/0.38$. The DCC will increase the $C_{kg}$ value by a factor of about two ($x = 0.7/0.38 = 1.84$) in cosmic-ray experiments, and could potentially reduce the existing calorimeter data on cosmic ray intensity by a substantial factor of about $1.84^{b+1} (\approx 4.6 - 5.4)$. Such a significant correction to the calorimeter systematic is unlikely because the existing cosmic-ray spectra on protons and helium already indicate only a little deviation from extrapolations from the low energy data. Nevertheless, critical examination of this proposed phenomenon the heavy-ion interactions is yet to be made in cosmic ray physics,
and is seriously important for the study of Chiral Symmetry Phase and QGP by RHIC experiments.

\[ x = \frac{Np^-}{(Np^- + Np^+)} \]

Figure A1-1. The distributions of \( x = \frac{Np^-}{(Np^- + Np^+)} \) from superposition nucleus interaction models and the new theoretical concept of DCC \( (r = n^-_x, n = n^-_x + n^+_x) \). (a) for interactions with the monochromatic beam (accelerators), (b) for cosmic ray interactions, biased by cosmic ray energy spectrum, \( E^{-b-1}dE \).

Accelerator beams of heavy nuclei up to 200 GeV/n have been available at CERN. However, most CERN experiments have been unable to provide \( k_g \) data \( (k_g = \Sigma E\gamma/\Sigma E\gamma + \Sigma Ech) \) useful in cosmic ray experiments, mainly due to the difficulty in locating an instrument (calorimeter) in the proper angular region around and inside the forward beam pipe. This problem will become more acute for collider experiments (RHIC and LHC) at higher energies. Up to now the proton-antiproton colliders (CERN SppS and Fermilab Collider) have not measured \( k_g \) distributions or searched for DCC for protons.

Multiplicity ratio (gamma/charged) is getting measured by the MINMAX project at FNAL, led by Taylor and Bjorken. The MAGIC experiment (EMUOS at CERN),\(^{10}\) using 200 GeV/n ion beams, could not incorporate a calorimeter with the MAGIC, because the small
beam-size and high event rate prevented the separate observation of every photon cascades in the detector.

The effects of new states such as QGP and DCC, if experimentally confirmed, would need to be incorporated in future studies of the Air Shower energy spectrum in the energy region $10^{15} - 20$ eV.
A.1.2. Past experimental study of the Isospin Clusters

It was frequently recognized during the EMU05 data analysis since 1987, that some identical charge-sign data showed clustering or patches. Heavy-ion studies can be enriched by an investigation of the possible "charge-sign clustering," which may or may not be related to the "Disordered Chiral Condensate" and/or a "chiral quenching" phenomena. Fig. A1-2 shows a "target diagram" of produced particles in the x-y plane. Fig. A1-3 shows the "run" distribution along the rapidity axis. ("Run" and "G-test" are explained later in the Appendix 2). Figs. A2-3 and A2-4 display the G-value distribution in the pseudorapidity-azimuth plot, for which only tracks that have the "cluster indicator (G)" exceeding 20 are meaningful for a cluster analysis. (A cluster of 6 identical charge-sign tracks has $G = gC^2 = 15$ as described later.)

The observed phenomena shown in Fig. A1-2 ~ A2-4 exhibit a surprising nature. There are two independent theoretical predictions that encourage detailed investigation of the clustering phenomena of identical charges. According to the Lattice Monte Carlo QCD formulated by DeTar that incorporated light quarks, local charge conjugation may not necessarily hold in the Chiral Symmetry phase of matter. The significant consequence of this could be recognized as stated in DeTar's private communication in 1987, that the like-charge-signs (isospins) could form a cluster in the high density state of matter, due to the characteristics of the chiral condensate. More recently, (after 1992 - 93), Anselm & Ryskin, Bjorken, Blaizot et al., and
Wilczek & Rajagopalan have surmised that the isospin-patches in the hadronic collisions can appear as a result of the change of the chiral orientation axis due to the violent impact, correlated classical pion fields, and the chiral coherence during quench or annealing, respectively. This aspect of the Chiral coherence is what we have been exploring with the \(4\pi\) MAGIC detector since 1986, after the initial, but vague, hints recognized in the JACEE experiments in 1985.

Another source of charge-sign clustering is more conventional, multiple Bose-Einstein effect, as simulated by Zajc. The Bose-Einstein interference increases with the square of the rapidity density. Asymptotically, the isospin can be distinctively clustered but does not form side-by-side distribution of opposite-isospin clusters or any regularity of patches (Fig. 4).

The local charge-conjugation breakdown and the isospin patches may not monotonically increase with the (rapidity density)\(^2\) as above, but perhaps, with a "step-function" of the rapidity density, for the case of the "chiral quench". Experiments that have the capability to measure all the charged tracks with the identification of the charge-sign should explore this problem. Because the EMU05 experiment produces the only data at present, and probably for the foreseeable future, we developed by ourselves some algorithms of quantifying the charge-sign (isospin) clustering.\(^8\)

Other conventional multi-particle correlation studies are possible with the EMU05 all-particle data. The intermittency analysis for (positive) or (negative) particles were made, as in the rapidity spacing analysis. So far, fluctuation analysis by other experiments has been limited to "charged particles," which is subject to different fluctuations due to the proton, kaon, and pion contents. The factorial moment analysis, however, generally overlooks variations in the probability function as a function of the multiplicity and the rapidity, as well as binning effects. The power behavior at \(\Delta\eta > 0.1\) automatically reflects the trivial rapidity distribution changes for various multiplicities (van Hove, Annals of Physics, H. Feshbach anniversary issue, 1989; CERN preprint TH-5529/89, 1989). Similar small power behavior of the factorial moment is observed universally in the e+e\(^-\) and proton data, which may be because they are subject to trivial, single-particle rapidity distribution function, and the two-body Bose-Einstein correlation function (Satz, CERN TH-5589/89, 1989).
The EMU05 examined the rapidity fluctuations by means of the rapidity spacings, a technique developed in an analogous way to the "energy level spacings" in traditional nuclear physics. Details were shown in our published literature, in which we deduced a correlation for negative charged particles as consistent with a random ensemble at short-range ($\Delta y < 0.5$). However, a more "repulsive" nature at the long-range ($\Delta y > 0.5$) was clearly indicated. The isospin patches would show similar long-range "repulsion" of clusters, similar to the Gaussian Orthogonal Ensemble in the Level-Spacing physics.

In our current opinion, the so-called "G-Test" is more advanced to analyze the charge-sign cluster, as described below in Appendix A2.
A2. ISOSPIN-CLUSTER ANALYSIS FOR DCC.

- algorithms, and the nature of the low energy data (EMU05).

We have previously reported that, in S + Pb events, apparent clusters of identical charges were occasionally recognized. There are some non-trivial correlations of unlike-sign charges in two-particle correlation function in the event-by-event analysis. Recent theoretical work shed light on these observations. Local charge symmetry may not hold, and isospin clustering can happen, if long-wave oscillation of chiral isospin occurs, or if the chiral axis is not the constant. Below we will summarize the methods and algorithms to explore the DCC in terms of isospin cluster analysis.

\[ \chi^2 \text{-TEST OF BINOMIAL ISOSPIN DENSITIES} \]

Firstly, let's assume the trivial multi-particle ensemble (N) with two isospins (+, -) and their multiplicities, (n, N - n). For the probability of a particle to be the (+) isospin, we assume \( p = 1/2 \) in a large ensemble (\( N \to \infty \)). The combinatory counts of occurrence leads to the ordinary binomial distribution of having (n, N - n) ensemble,

\[ P(N|n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \]

The probability to have (n, N - n) in heavy-ion collisions can be a cross product of

\[ P = P_N \times P(N|n), \]

where \( P_N \) is the normalized multiplicity distribution function. If we consider only the very central collision events, \( P_N \) can be a Poissonian.

To evaluate the likelihood of observing a (n, N - n) isospin ensemble, the conventional \( \chi^2 \)-distribution can be used. If the entire rapidity-azimuthal space of the collision is segmented by some patches, and only a part of them (m-patches) are detectable, we will have the multiple ensemble of (\( n_i, N_i - n_i \)), \( i = 1, ..., m \). The individual patches are regarded independent, only if there are no correlations of any kind. Ignoring the Bose-Einstein correlation, the simplified random ensemble picture warrants that the products of \( \chi^2_i \) will rapidly lead to the asymptotic probability,

\[ P_{\text{total}}(\chi^2_1, \chi^2_2, \chi^2_3, ..., \chi^2_m) \to 1. \]

**Fig. A2-1** shows an example of the segmentation of the rapidity-azimuth (y, \( \phi \)) space. The event shown here is one of the high multiplicity event of \(^{16}\text{O} (200 \text{ GeV/n}) + \text{Pb} \) interactions. Each segment gives the \( \chi^2 \)/D.O.F.
Although the $\chi^2$-test is a simple and straightforward measure, it is very insensitive to any signals of correlation for high multiplicity ensemble, due to a very natural consequence of the Central Limiting Theorem, to which the principle of the testing itself is based upon. This was checked by the Monte Carlo for the events including the above example. When the multiplicity becomes very high, the ratio of the anomalous correlation to the random combinatory background becomes very small. For two particle correlation such as the gamma-gamma invariant mass observation, the very clear 2-body signals for the entire gamma rays, namely, the mass peak of $\pi^0 \rightarrow \gamma \gamma$ becomes invisible, due to a huge population (quadratic) of random phase space' combinatory background:

$$\frac{\text{Signal/Background}}{N(\pi^0)} \approx \frac{2}{N-1} \rightarrow 0, \text{ when } N \rightarrow \infty.$$ 

**RUN TEST**

To compliment the weak nature of the $\chi^2$, the statistical “run-test” can be envisaged, since the “run-test” is independent of the $\chi$-test. Similarly, the “Conjugate Sum” could be used as well, to enhance the non-statistical correlation.

Two statistical tests, the "run-test" and the "conjugate-test" (G-test), were previously performed by us for S + Pb events. The run (r) is defined by the total number of sign-changes in a chain of charges. The charge signs of tracks in a segment of the rapidity-azimuth angle space can be lined up as a series of (+) or (-) sign, following the order of the rapidity value or azimuth angle value. The change of the sign in this series is a "run". The normalized run (x), defined by $x = (r - <r>) / \sqrt{r}$, follows the normal distribution (0, 1) if charge-signs are randomly distributed. Clusters should appear as an excess at negative x. Fig. A2-2 illustrates a prominent "run-test" applied to a S + Pb event that had "apparent" appearance of the charge cluster.
The weakness of the “run-test” is its nature of projection in one-dimension. When a cluster exists, it is best recognized as the two-dimensional, non-trivial correlation. Therefore, we considered that the two-dimensional measure of a cluster is required for our study, and the G-test, described below, was introduced.

Furthermore, there could be correlations among “clusters”. Any regularity of clusters (such as periodicity) is independent of other 1- or 2-dimensional measure of counting the number of clusters. For this possibility, we also introduce the periodicity test, to account for potential regularity of cluster formation in a large ensemble. Canonical Fourier and/or Lomb’s Periodogram are adopted by us for the study of cluster analysis.

The RUN-TEST and $\chi^2$-TEST, and the periodogram-test (described later), are essentially independent to each other, so that the combined probability,

$$P = p(x|\alpha) \times p(\alpha) \times P(\text{Fourier/Lomb Periodogram}),$$
is the measure to describe the statistical significance of the cluster phenomena. The last factor is described later.

**CONJUGATE G-VALUE**

The G-test measures the degree of clustering. We define the degree of "clustering", $G_i(\text{max})$, for the $i$-th particle, as

$$G_i(\text{max}) = \text{Max} \ G_{ij}(R),$$

where $G_{ij}(R) = \sum (C_k \times C_l)$. The $k$-th and $l$-th particles are contained within the tested area ($R$). The $C_i$ is $+1$ or $-1$ for a positive or a negative charge, respectively. The area $R$ is measured from the $i$-th particle, $0 < R < R_{ij}$, where $R_{ij}$ denotes the invariant mass of two particles. Each particle is therefore having a specific G-value, reflecting in itself the all-particle correlation, because it is entirely dependent on all the other particles distribute around it. However, its sensitivity is limited to relatively short-range, because the G-value is defined in such a way that the contribution of the distant tracks (long-range correlations) to the G-max is increasingly reduced when $R_{ij}$-value increases, as the results of the quadratic increase of the random combinatory nature of other tracks in a large $R_{ij}$ circle (Central Limiting Theorem).

The G-value distribution of all the tracks in ($y$, $\phi$) space is the best method to discover the isospin clusters in the event-by-event analysis. **Figs. A2-3a-e** shows an example of these tests for cluster candidates. Four events from a total of 50 events were found by these tests as having candidates for isospin clusters. Notably, one event show a very large, negative-charge cluster in the projectile fragmentation region, which is an extremely rare occurrence in Monte Carlo simulations for high rapidity tracks ($P < 10^{-5}$).

The "run-test" and "G-test" are quick to signify candidates of charge-sign clusters. However, these tests require that the individual tracking data for all-particle, which would not be possible with the current MVD of the PHENIX. Moreover, the "run-test" has a weakness: random coincidence dominates any signals of clusters, if a test is statistically performed for random segmentation. It has to synchronize the ($y$, $\phi$) segmentation to the "cluster" distribution. The "G-test", on the other hand, can reveal the existence of anomalous charge-sign clusters, without having any segmentation or its synchronization. These tests can distinguish charge-sign clusters from random coincidence. However, none of them can incorporate more global correlations among clusters of different charge-signs.
These tests are not sensitive to the regularity (structural signals) among clusters, and therefore, the statistical tests are easily dominated by combinatory backgrounds, leading to negative but erroneous conclusions. In the use of “run-test” or “G-test”, the periodicity analysis must be coupled for deducing the probability of having the correlation of the isospin.

If one can successfully extract a nature of the regularity, and can incorporate inter-cluster correlations of coupled isospin, the candidates could more clearly emerge with stronger clustering signals. The Fourier spectrum can examine at the same time the degree of clustering of two different charges and the periodicity of isospin patches.

A cluster that has n-particles of a specific isospin state (+ or -) would have such a maximum sum, in the 2-particle cross product sum (around its center), as

$Gi = \Sigma_j \Sigma_k C_k \times C_j, (k \neq j)$

$+ \sum C_2 = n(n - 1)/2$, (for a pure cluster),

where charge-signs $C_k$ and $C_j$ are defined as (+ 1) for positive charges and (- 1) for negative charges, respectively.

Tracks in the rapidity-azimuth space are a mixture of different charge-signs. An isospin cluster, even if it exists, would be contaminated by other charges of different origin. To quantify the degree of clustering, we introduce a cluster indicator, $G_i$, for the i-th track. Each track (i) should have a specific $G$-value when it is defined by the maximum value of $G_i$ as a function of the invariant mass $m_i$. All the neighboring tracks are used (j, k): [(j, k ≠ i), j ≠ k]. To determine $(G_i)_{max} = \max G_i$ (m$_{ij}$, m$_{jk}$ < m$_i$) for the i-th particle, all the surrounding tracks (j and k; j ≠ k) within a specific invariant mass radius (m$_i$) are used for calculating $(G_i)_{max}$. By varying the m$_i$ from zero to infinity, the maximum $G_i$ value is determined. The $(G_i)_{max}$ is characteristic value for each track (i), and reflect how all other r tracks (isospins) are distributed around the i-th track.

A track that has a $G$ value greater than 20 would be equivalent to a tightly bound cluster with n > 6. Thus, we have a new physical quantity for each particle, as a characteristic measure of the multi-particle (isospin) correlation around the individual track.

The distribution of the $G$-values in a central collision event can be compared with that simulated by the Monte Carlo method with and without isospin clusters. We tentatively
use a cluster that has a multiplicity of 6 for the identical isospin. Events generated by a Monte Carlo simulation can have 6, 3, or 0 such clusters in average. Fig. A2-3 shows a comparison of the data with a Monte Carlo simulation for the event already presented in Figs. 11 and A1-17. In the given (y-ϕ) plot, each track is assigned with the symbol for positive or negative charge. The size of the symbol(r) is scaled to the G-value in terms of, \( r = c + aG \), where (c) is the constant (minimum size) for \( G = 0 \), and (a) is a dilation coefficient.

Several events exhibited some clustering of negative-charges in the highest rapidity region (projectile fragmentation region). Figs. A2-3 and A2-4 show such events. In the right portion of Fig. A2-3 and in the left portion of Fig. A2-4, the tracks with small G values (G < 20) have been eliminated to enhance the isospin patches. This figure is useful to analyze the correlation among clusters. In this three-dimensional G-y-ϕ display, one can quickly see whether there are isospin-patches in an event. Also, we can see how patches are distributed in the polar-azimuth angular space. Two events out of 50 EMU05 events suggest some clustering of the negative charges in the highest rapidity region in the central S + Pb collisions at 200 GeV/n: (4 ± 2 % of central collisions).

Fig. A2-3  [Right graph]: G-test of the event 4750X (S + Pb, only tracks that have G > 20 are plotted. [Left graph]: G-distribution of the event, data (points) and Monte Carlo simulations (curves) are compared. Random isospin fluctuations are shown by the lowest curves of the three curves; each three curves are for positive charge clusters (upper three) and negative-charge clusters (lower three curves). Simulations that produce 6 DCC clusters for the same the event are indicated by the uppermost curves, where each cluster is assumed to have the average multiplicity of 6 identical isospins and no unlike-isospins.
Fig. A2-4. Other events that show prominent charge-sign clusters (G > 20).
CANONICAL FOURIER METHOD

Charge-sign clustering, or patches of charge sign-groups, are further studied by improving the mathematical measures. In the Cartesian coordinates of the pion field, incoherent correlations of the $\pi_1$ and $\pi_2$ could not be transferred to that of $\pi^+$ and $\pi^-$. However, in case of a strong oscillatory coherence of the classical pion field ($\pi_1$ and $\pi_2$) in the DCC, the $\pi^+ = (\pi_1 + i \pi_2)/\sqrt{2}$ and $\pi^- = (\pi_1 - i \pi_2)/\sqrt{2}$ could also have strong amplitude of the Fourier components in $(y,\phi)$ space with a phase shift of $\sim\Delta\phi$ relative to that of $\pi_1$ and $\pi_2$ Fourier components, provided that the chiral oscillatory coherence is very strong.

The method to explore this oscillatory correlation is as follows: take the number distribution $(dN/d\phi)$, where $\phi$ denotes the azimuth angle of a track, with the bin size $\Delta\phi$, and get the maximum value of the $(dN/d\phi)$ for negative charge-sign ensemble. In doing so, allow an parameter of the initial azimuth off-set value of $\phi_0$. Take the maximum value of $(dN/d\phi)$ varying the off-set value from 0 to 60 degrees. The discreet Fourier cosine component with a phase parameter $d$ can be defined for the negative-charged particles: $a_k^- = (2/n) \sum N_j \cos(2\pi kj/n + d)$, $k = 0, 1, 2, .., n/2$, where $n$ is the number of the bins (n = $360^\circ/\Delta\phi$) and the $N_j$ is the normalization number of the particles in j-th bin. Parameters ($\phi_0, d$) are fixed so that they maximize the highest peak for $|a_k|^2$. Using the same parameters, one can consider that the positive-charged pions are the complimentary sine component of the same Fourier term. The correlated sine component for positive charged particles can well be the same Fourier sine coefficients, $b_k^+ = (2/n) \sum N_j \cos(2\pi kj/n + d + \pi/2)$, where the last argument is for a distortion of the correlation in the final state. If one can find the same maximum for $|b_k^+|^2$, it is a very good indication of two-dimensional correlated correlation, possibly due to chiral oscillation of classical pion field. One of the most notable candidates for such a pattern is the EMU05 AF4750X event. The target diagram and Fourier analyses of the event are shown in the following figures. Notable point was that the Fourier components had two peaks ($k = 4$ and 11 for positive charge, and $k = 3$ and 9 for negative charges.) The Monte Carlo studies of this method is also developed. This method can be applied to the study of chiral coherence in the actual experiments. Target Diagram of AF4750X event (S + Pb) is seen in Fig. A2-3.
To begin with, we divided the \((y, \phi)\) plane into rings. A \((y)\) ring is bordered by two rapidities \((y)\), and which is further divided into \(n\) azimuth-bins \((n = 2\pi / \Delta\phi)\) where the number densities of \((+\) and \((-\) signs are counted as \(\mathbb{N} \pm\). Using the \((-\) sign data, a discrete Fourier cosine component with a phase parameter \(\delta\) is defined by

\[
a_k^- = \frac{2}{n} \sum_{j=0}^{n-1} \mathbb{N}_j \cos\left(\frac{2\pi kj}{n} + \delta\right), \quad \text{where} \quad k = 0, 1, 2, \cdots, \frac{n}{2}.
\]

The phase parameter \(\delta\) is determined so that the highest peak of \(|a_k^-| (\delta)\) is maximized.

A discrete Fourier sine component is defined for \((+\) signs by

\[
b_k^+ = \frac{2}{n} \sum_{j=0}^{n-1} \mathbb{N}_j \sin\left(\frac{2\pi kj}{n} + \delta + \frac{\pi}{2}\right), \quad \text{where} \quad k = 0, 1, 2, \cdots, \frac{n}{2}.
\]

Similarly as above, the parameter \((m)\) is determined so that the highest peak of \(|b_k^+|\) is maximized. The parameter \((m)\) characterizes the phase shift of \((+\) and \((-\) sign distributions for each ring. The phase shift value \((m)\) for this ring was 1.

We applied this Fourier analysis to both a \(S + \text{Pb}\) event (shown in Fig. A2-5) and a \(\text{Pb} + \text{Pb}\) event (shown in Fig. A2-6).

**LOMB'S PERIODOGRAM**

Similarly, Lomb's Normalized Periodogram became as useful as the above described Canonical Fourier Analysis. The Lomb's Periodogram has been developed in Astrophysics cluster analysis of the stars and galaxies, and is defined for \(N\) data points of

\[
h_i = h(t_i), \quad i = 1, \ldots, N, \quad \text{with} \quad \bar{h} = \frac{1}{N} \sum_{i=1}^{N} h_i, \quad \text{and} \quad \sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (h_i - \bar{h})^2:
\]

\[
P_N(\omega) = \frac{1}{2\sigma^2} \left[ \frac{\sum_I (h_I - \bar{h}) \cos \omega(t_I - \tau)}{\sum_I \cos^2 \omega(t_I - \tau)} \right]^2 + \frac{\sum_I (h_I - \bar{h}) \sin \omega(t_I - \tau)}{\sum_I \sin^2 \omega(t_I - \tau)}
\]

where

\(\omega = 2\pi \nu > 0\)

and

\[\tan(2\omega \tau) = \frac{\sum_j \sin 2\omega t_j}{\sum_j \cos 2\omega t_j}.
\]
The 9-th and 11-th wave numbers in Fig. A2-5 had very high amplitude for (+) and (−) charges, respectively. Figs. A2-6 and A2-8 show highly significant periodicity for both positive and negative charge distributions in the central Pb + Pb event.

Two conclusions are due from these results: (1) There is a striking similarity of the two spectra, implying that the experimental data contain significant, alternating signals with similar cyclic regularity. (2) A strong amplitude in a high wave number implies that a phase correlation of \(35 - 40\) degrees is responsible for oscillatory correlation among (+) and (-) clusters.

Fig. A2-5. Fourier (S + Pb)  

Fig. A2-6. Fourier (Pb + Pb)
Fig. A2-7. Lomb (positive charge, upper curve; negative, lower curve).
ANISOTROPY MEASURE

Anisotropy in the transverse energy flow has been studied by using sphericity tensor on the transverse plane: $S_{ij} = \sum w_i(n) p_i(n) p_j(n)$, where the sum is over all the final state charged particles and the indices $i$ and $j$ denote the two transverse directions. The $w_i(n)$ is a weight (classical, $w = 1/2m$). J.Y. Olitraut (Phys. Rev. D48, 1132, 1993) suggests that the direction of the maximum of the momentum flow would give a collective effect. The distribution of the angle $q$ of the two sample directions can experimentally be measured. The ratio of the number of events with $\theta > 45^\circ$ to that with $\theta < 45^\circ$, $r = N(\theta > 45^\circ)/N( < 45^\circ)$, will be 1.0 if the events are isotropic, and tends asymptotically to zero for strong anisotropy. In two rapidity intervals, (2.5 - 3.5) and (3.6 - 6), the ratio was measured for 11 sample events. The $r$ value in this test sample was $r = A/B = 5/6$. This measure should shed light on a possible collective flow in the heavy-ion pionic ensemble of the scale RHIC experiments are anticipating.

<table>
<thead>
<tr>
<th>Event</th>
<th>$\theta$ (degrees)</th>
<th>$N(2.5 &lt; \eta &lt; 3.5)$</th>
<th>$N(\eta &gt; 3.6)$</th>
<th>$A: \theta &lt; 45^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.9</td>
<td>33</td>
<td>62</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>50.1</td>
<td>52</td>
<td>97</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>18.5</td>
<td>60</td>
<td>98</td>
<td>A</td>
</tr>
<tr>
<td>4</td>
<td>20.7</td>
<td>110</td>
<td>114</td>
<td>A</td>
</tr>
<tr>
<td>5</td>
<td>14.2</td>
<td>89</td>
<td>112</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>78.0</td>
<td>105</td>
<td>99</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>66.4</td>
<td>98</td>
<td>88</td>
<td>B</td>
</tr>
<tr>
<td>8</td>
<td>81.2</td>
<td>97</td>
<td>120</td>
<td>B</td>
</tr>
<tr>
<td>9</td>
<td>78.0</td>
<td>121</td>
<td>150</td>
<td>B</td>
</tr>
<tr>
<td>10</td>
<td>11.0</td>
<td>83</td>
<td>91</td>
<td>A</td>
</tr>
<tr>
<td>11</td>
<td>38.6</td>
<td>59</td>
<td>93</td>
<td>A</td>
</tr>
</tbody>
</table>
Appendix B

B.1. Measurements on the charge-signs and momenta

The Lorentz force acts on a charged particle moving in an electromagnetic field:

\[ F = q \left[ E + (v \times B) \right]/c. \]  (Gaussian system)

\( E \) and \( B \) field created by charged particles themselves is assumed to be negligibly small.

\[ E = 0 \]
\[ B = (0, -B, 0) \]
\[ v = (v_x, v_y, v_z) \]

\[ F = q/c \begin{vmatrix} x & y & z \\ v_x & v_y & v_z \\ 0 & -B & 0 \end{vmatrix} = -q \frac{B (v_x z - v_z x)}/c \]

And also ignoring the electromagnetic scattering the ionization loss, Bremsstrahlung, synchrotron radiation, etc., the orbit of the charged particle is circular in \( x-z \) plane and linear in \( y \) direction.

Equating the Lorentz force and the centrifugal force yields:

\[ m \frac{v_{xz}^2}{\rho} = q B \frac{v_{xz}}{c}, \quad \rho : \text{the radius of the curvature} \]

\[ p_{xz} = e B z \frac{\rho}{c}, \quad q = e z. \]

Expression of the magnitude \( xz \)-component of the momentum in the unit of \([eV/c]\) becomes:

\[ p_{xz} = 3.0 \times 10^{-2} \rho B z \]  \( \rho : [\text{cm}] \)

\( e : [\text{esu}] \)

\( B : [\text{Tesla}] \)

\( c : 3.0 \times 10^{10} [\text{cm sec}^{-1}] \)
The momentum $P$ can be expressed by the emission angle $(\theta, \phi)$ as follows:

$$P = p_x x + p_y y + p_z z$$

$$p_x = p \sin\theta \cos\phi$$

$$p_y = p \sin\theta \sin\phi$$

$$p_z = p \cos\theta$$

$$p_{xz} = p \left[ \sin^2\theta \cos^2\phi + \cos^2\theta \right]$$

$$p = p_{xz} / \left[ \sin^2\theta \cos^2\phi + \cos^2\theta \right], \quad p_\perp = \left[ p_x^2 + p_y^2 \right], \quad p_\parallel = p_z$$

Thus the momentum will be obtained by the radius of the curvature $\rho$ and the emission angle $(\theta, \phi)$ of individual track, and the charge-sign will be obtained from the direction of magnetic deflections: $\pm$ sign deflects in $\mp x$ direction.
Appendix B-2. Tracking Procedure

Since the tracking procedure is almost identical for both EMU05 and EMU09, the basic tracking procedure will be described for EMU05 in this appendix. Significant difference will be noted in [em9: ~] for EMU09.

- Plate Setting-

The emulsion plate is placed on a 500 μm thick lucite plate with a guiding lucite piece on each side. One of the guiding lucite piece is used to align the plate so that the plate is always set parallel to the stage motion in x-direction. The accuracy of the alignment has been proven to be:

\[ \tan \Delta \theta \leq 0.001. \]

Because emulsion plates are very sensitive to humidity and temperature change, the cover plate is placed above the emulsion plate and taped down with scotch tapes to insulate the emulsion plate and the whole set is placed on the microscope stage at least 30 minutes before the measurement so that the emulsion plate is in thermal equilibrium and stable under the microscope. This method is effective enough for manual tracking since no absolute focus is needed and also good for 10 minute automatic scanning (scanning area: \( \approx 250 \times 200 \) μm², 5 focus). Usually 5 - 7 events in one beam spot are measured in one plate setting. [em9: 5 - 7 events in the entire plate] The emulsion plate is removed from the lucite frame after the measurement and the next one is placed in the same slot.

- General Scan -

The event scanning is done with the plate placed right below the target by using low magnification objective lenses (10 × or 20 ×). Although only high multiplicity events are analyzed, the rough estimate of multiplicities, vertex heights, and the locations of almost all the events are recorded. [em9: Events of \( N_{ch} < 200 \) are ignored.] The distribution of beam tracks is shown in fig. 21. [em9: fig. 22]

The difference is not only the shape of the emulsion plates but also the way they were exposed. In EMU05 the chamber was fixed during one beam run and then shifted for another run. Thus the emulsion plate of EMU05 has two dense beam spots. Each spot has about ten events of \( N_{ch} > 400 \). On the other hand the emulsion chamber of EMU09 was
being moved across the beam line during one beam run. Therefore the beam distribution in the plate is less dense in the middle and rather dense on the edge. This reduced the number of analyzable events in EMU09 because the number of events are proportional to the beam intensity and the majority of events are located at the edge of the plate.

Fig B1. EMU05 Beam Spot.

![Fig B1. EMU05 Beam Spot.](image)

Fig B2. EMU09 Beams.

![Fig B2. EMU09 Beams.](image)

After the general scan the event selection was performed, the central collision events were chosen by multiplicity cut at 500 tracks. Furthermore, we asked the selected events to have a residual target thickness as small as 100 microns so as to warrant that the analysis of events are less contaminated by electrons and positrons due to photon conversions inside the residual target. The events thus selected were then measured for the coordinates of individual secondary tracks with respect to the parent beam axis.

- **Reference Beam Selection** -

Reference beam tracks were chosen for individual events, which were used for two dimensional chamber reconstruction (offset of event center and rotation). About twenty beam tracks were chosen near the event to locate the event center. [em9: ~ 40]
Appendix C. Fourier Analysis Data Compilation.

The Fourier analysis are summarized in the following figures. Each figure corresponds to the individual central collision event, whose event name is indicated inside the figure.
Method of the Canonical Fourier Analysis

The canonical Fourier analysis as described below will incorporate the regularity of appearance of clusters for a quantitative characterization of the phase correlation \( \text{ref tak 1} \). The proposed canonical Fourier analysis will be performed as follows.

A ring segment defined by rapidity is divided into \( n \) bins \( (n = \frac{2\pi}{\Delta \phi}) \) for the number distributions of + and − signs, \( N_j^{\pm} \). From the distribution of − sign a discrete Fourier cosine component with a phase parameter \( \delta \) is obtained as:

\[
a_k^- = \frac{2}{n} \sum_{j=0}^{n-1} N_j^- \cos\left(\frac{2\pi kj}{n} + \delta\right), \quad k = 0, 1, 2, \ldots, \frac{n}{2}
\]

The phase parameter \( \delta \) is determined so that the highest peak of \( |a_k^-| \) is maximized.

A discrete Fourier sine component is taken for + sign as:

\[
b_k^+ = \frac{2}{n} \sum_{j=0}^{n-1} N_j^+ \sin\left(\frac{2\pi kj}{n} + \delta + \frac{\pi}{mk}\right), \quad k = 1, 2, \ldots, \frac{n}{2}
\]

The parameter \( m \) is determined so that the highest peak of \( |b_k^+| \) is maximized. The parameter \( m \) characterizes the phase shift of + and − sign distributions for each ring segment. The width of the ring shall be determined to extract the cluster signal the most effectively. Thus all the parameter \( \delta, m \) and the spectra \( a_k, b_k \) will be uniquely determined without any bias.

In order to determine the pseudorapidity range and \( \delta \) that give the maximum signal, Lomb's Normalized Periodogram, as described below, is used. Lomb's Normalized Periodogram has been developed in astrophysics for the spectral analysis and is defined for \( N \) data points of

\[
h_i = h(t_i), i = 1, 2, \ldots, N \text{ with } \bar{h} = \frac{1}{N} \sum_i h_i \text{ and } \sigma^2 = \frac{1}{N-1} \sum_i (h_i - \bar{h})^2
\]

\[
P_N(\omega) = \frac{1}{2\sigma^2} \left\{ \frac{\sum_j (h_j - \bar{h}) \cos \omega(t_j - \tau))^2}{\sum_j \cos^2 \omega(t_j - \tau)} + \frac{\sum_j (h_j - \bar{h}) \sin \omega(t_j - \tau))^2}{\sum_j \sin^2 \omega(t_j - \tau)} \right\}
\]

where \( \omega = 2\pi \nu > 0 \)

and

\[
\tan(2\omega \tau) = \frac{\sum_j \sin 2\omega t_j}{\sum_j \cos 2\omega t_j}
\]

The statistical meaning of the periodogram \( \text{ref sea 1} \) is that the power at a given frequency \( \omega_0 \) is exponentially distributed. That is the probability distribution for

\[
Z = P_N(\omega_0)
\]

\[
p_Z(x)dx = Pr(x < Z < x + dx) = e^{-x} dx
\]

Hence the cumulative distribution function is
which gives the probability of observing $Z>x$ as

$$\Pr\{Z>x\} = e^{-x}.$$  

Thus as the observed power becomes larger, it becomes exponentially unlikely that such a power level (or greater) can be due to a noise fluctuation. So if we let the maximum value in the spectrum $Z=\max P_N(\omega)$, where the maximum is over the set of $N_f$ independent frequencies, then

$$\Pr\{Z>x\} = 1 - CDF_z(x) = 1 - [1 - e^{-x}]^{N_f}.$$  

This probability can be used as a threshold against the random noise coincidence. The noise probability threshold is tentatively set to 5%. The details of the parameter selection are as follows.

1. Define a ring segment by two pseudorapidity values, $\eta_{\text{high}}$ and $\eta_{\text{low}}$, satisfying the following requirements.
   i) The minimum number of either negative or positive tracks is 90.
   ii) The maximum number of tracks is 350.
   iii) The step size of the pseudorapidity is fixed to 0.15 regardless of how populated the selected ring segment is.

2. Take a number distribution for both charges, $N_j^-$ and $N_j^+$. The bin size is set to 10 degrees. ($j=1,2,\ldots,36$) (The origin of $\phi$ is shifted by 1.5 degree after each run of periodogram.)

3. Apply the periodogram to $N_j^-$ and $N_j^+$.

4. Repeat step 2 and 3 240 times shifting the origin of $\phi$ by 1.5 degree. Pick a parameter set, $\eta_{\text{high}}, \eta_{\text{low}}, \phi_{\text{shift}}$ that makes $(Pr_+ - Pr_-)/\sqrt{Pr_+ + Pr_-}$ minimum. If both $Pr_-$ and $Pr_+$ are less than 5%, store the parameter set as a signal candidate. $Pr_+/-$ is the probability that the maximum power is due to a random noise.

5. Repeat step 1, 2, 3, and 4 until the entire $\eta-\phi$ space is covered.

6. Select independent rings among candidates by imposing the following conditions.
   i) The ring segment that gives the minimum $Pr_\eta$ is the primary.
   ii) The overlap of two rings does not exceed 1.0 in $\eta$ regardless of the number of particles contained in the overlapped region.

7. Run the canonical Fourier analysis for each event with the selected set(s) of parameters.

The result of optimized periodogram for 21 events that passed 5% threshold is shown in Appendix C.

The above method will detect sinusoidal signals of charge distributions existing in $\eta-\phi$ space. However the question of how often such signal is detected still remains. Therefore the frequency of the occurrence of such events among randomly generated events has to be examined. The randomly generated events are simulated by fixing the $\eta_i$
i = 1, 2..., \( N_{\phi} \), and redistributing all particles in \( \phi \)-direction such that the probability of one particle falling into the jth bin \( N_j \), the number distribution in \( \phi \), is equal for all j’s. (1 ≤ j ≤ 36)

All currently available events were thus randomly regenerated and analyzed the same way. Table 5.1 shows the result.
<table>
<thead>
<tr>
<th>Event ID</th>
<th>$\eta$-high</th>
<th>$\eta$-low</th>
<th>$\phi$ shift</th>
<th>k-</th>
<th>Pr max - Nch - k+</th>
<th>Pr max + Nch + rando</th>
<th>Pr&lt;0.05</th>
<th>m run</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMU05 0385</td>
<td>5.739</td>
<td>2.502</td>
<td>286.5 4.3</td>
<td>0.033056</td>
<td>108 9.3</td>
<td>0.047214</td>
<td>117</td>
<td>300</td>
</tr>
<tr>
<td>0396</td>
<td>300</td>
<td>205</td>
<td>68%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0704</td>
<td>300</td>
<td>197</td>
<td>65%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0757</td>
<td>300</td>
<td>0</td>
<td>0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2764</td>
<td>244</td>
<td>178</td>
<td>73%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3141</td>
<td>4.027</td>
<td>1.867</td>
<td>48 5.9</td>
<td>0.006083</td>
<td>103 16</td>
<td>0.019771</td>
<td>143</td>
<td>300</td>
</tr>
<tr>
<td>3245</td>
<td>300</td>
<td>205</td>
<td>68%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3542</td>
<td>2.957</td>
<td>0.274</td>
<td>285 8.3</td>
<td>0.019895</td>
<td>104 16</td>
<td>0.017827</td>
<td>169</td>
<td>300</td>
</tr>
<tr>
<td>3745</td>
<td>5.321</td>
<td>1.089</td>
<td>267 16</td>
<td>0.039728</td>
<td>134 2.6</td>
<td>0.043465</td>
<td>164</td>
<td>300</td>
</tr>
<tr>
<td>4048</td>
<td>300</td>
<td>178</td>
<td>59%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4141</td>
<td>300</td>
<td>188</td>
<td>63%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4142</td>
<td>300</td>
<td>207</td>
<td>69%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4243</td>
<td>300</td>
<td>0</td>
<td>0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4251</td>
<td>3.918</td>
<td>1.272</td>
<td>127.5 12</td>
<td>0.023961</td>
<td>109 5.9</td>
<td>0.02041</td>
<td>139</td>
<td>300</td>
</tr>
<tr>
<td>4537</td>
<td>300</td>
<td>162</td>
<td>54%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4556</td>
<td>300</td>
<td>170</td>
<td>57%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4750</td>
<td>300</td>
<td>193</td>
<td>64%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5024</td>
<td>6.555</td>
<td>2.453</td>
<td>147 16</td>
<td>0.020243</td>
<td>150 5.1</td>
<td>0.023659</td>
<td>164</td>
<td>300</td>
</tr>
<tr>
<td>5882</td>
<td>300</td>
<td>203</td>
<td>68%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6106</td>
<td>5.885</td>
<td>2.477</td>
<td>58.5 13</td>
<td>0.046174</td>
<td>104 11</td>
<td>0.013434</td>
<td>126</td>
<td>300</td>
</tr>
<tr>
<td>6115</td>
<td>5.923</td>
<td>2.605</td>
<td>120 17</td>
<td>0.03108</td>
<td>109 15</td>
<td>0.028287</td>
<td>137</td>
<td>300</td>
</tr>
<tr>
<td>6116</td>
<td>6.256</td>
<td>2.248</td>
<td>232.5 2.8</td>
<td>0.032402</td>
<td>139 4.5</td>
<td>0.023346</td>
<td>173</td>
<td>300</td>
</tr>
<tr>
<td>6500</td>
<td>5.986</td>
<td>2.63</td>
<td>337.5 17</td>
<td>0.017446</td>
<td>119 15</td>
<td>0.012864</td>
<td>127</td>
<td>300</td>
</tr>
<tr>
<td>6621</td>
<td>3.925</td>
<td>2.088</td>
<td>82.5 13</td>
<td>0.013186</td>
<td>98 5.7</td>
<td>0.027685</td>
<td>110</td>
<td>300</td>
</tr>
<tr>
<td>6677</td>
<td>5.234</td>
<td>2.537</td>
<td>187.5 16</td>
<td>0.031698</td>
<td>107 15</td>
<td>0.049789</td>
<td>126</td>
<td>300</td>
</tr>
<tr>
<td>6690</td>
<td>5.858</td>
<td>2.623</td>
<td>199.5 12</td>
<td>0.023034</td>
<td>91 17</td>
<td>0.01769</td>
<td>130</td>
<td>300</td>
</tr>
<tr>
<td>6796</td>
<td>3.37</td>
<td>0.257</td>
<td>262.5 2.2</td>
<td>0.037792</td>
<td>137 6.3</td>
<td>0.03518</td>
<td>183</td>
<td>300</td>
</tr>
<tr>
<td>6807</td>
<td>300</td>
<td>211</td>
<td>70%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7188</td>
<td>300</td>
<td>164</td>
<td>55%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7697</td>
<td>3.542</td>
<td>1.41</td>
<td>247.5 15</td>
<td>0.008859</td>
<td>98 1.8</td>
<td>0.006426</td>
<td>120</td>
<td>300</td>
</tr>
<tr>
<td>EMU09 0324</td>
<td>4.491</td>
<td>2.238</td>
<td>304.5 5.5</td>
<td>0.020632</td>
<td>141 5.7</td>
<td>0.02268</td>
<td>121</td>
<td>300</td>
</tr>
<tr>
<td>4444</td>
<td>300</td>
<td>183</td>
<td>61%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4895</td>
<td>300</td>
<td>171</td>
<td>57%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5025</td>
<td>3.543</td>
<td>0.899</td>
<td>226.5 4.3</td>
<td>0.040164</td>
<td>98 16</td>
<td>0.005558</td>
<td>243</td>
<td>300</td>
</tr>
<tr>
<td>5040</td>
<td>300</td>
<td>173</td>
<td>58%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5336</td>
<td>6.625</td>
<td>1.875</td>
<td>159 2.6</td>
<td>0.04908</td>
<td>139 11</td>
<td>0.013958</td>
<td>184</td>
<td>300</td>
</tr>
<tr>
<td>5426</td>
<td>5.217</td>
<td>2.159</td>
<td>132 2.4</td>
<td>0.028175</td>
<td>96 6.1</td>
<td>0.015493</td>
<td>94</td>
<td>300</td>
</tr>
<tr>
<td>5538</td>
<td>300</td>
<td>147</td>
<td>49%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5684</td>
<td>300</td>
<td>171</td>
<td>57%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5986</td>
<td>300</td>
<td>0</td>
<td>0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9776</td>
<td>4.348</td>
<td>1.586</td>
<td>87 12</td>
<td>0.047301</td>
<td>96 18</td>
<td>0.041731</td>
<td>91</td>
<td>300</td>
</tr>
<tr>
<td>EMU16 160i</td>
<td>5.62</td>
<td>4.406</td>
<td>128 16</td>
<td>0.041791</td>
<td>100 6.7</td>
<td>0.043504</td>
<td>151</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 5.1
EMUS events

6116 6.256-2.248 232.5

6500 5.986-2.630 337.5

6621 3.925-2.068 82.5

6677 5.234-2.537 187.5

6690 5.858-2.623 199.5

6796 3.370-0.257 262.5

7697 3.542-1.410 247.5
EMU9 event

EMU9 event