THREE-DIMENSIONAL NATURAL CONVECTION OF A FLUID WITH TEMPERATURE-DEPENDENT VISCOSITY IN AN ENCLOSURE WITH LOCALIZED HEATING

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ABSTRACT
Three-dimensional natural convection of a fluid in an enclosure is examined. The geometry is motivated by a possible magma-energy extraction system, and the fluid is a magma simulant and has a highly temperature-dependent viscosity. Flow simulations are performed for enclosures with and without a cylinder, which represents the extractor, using the finite-element code FIDAP (Fluid Dynamics International). The presence of the cylinder completely alters the flow pattern. Flow-visualization and PIV experiments are in qualitative agreement with the simulations.

NOMENCLATURE

- c = specific heat
- g = gravitational acceleration (9.81 m/s²)
- k = thermal conductivity
- L = cylinder length, half-length of cubical enclosure edge
- m = mesh refinement parameter
- p = pressure
- P = pressure scale
- Pe = Peclet number
- Pr = Prandtl number
- Ra = Rayleigh number
- Re = Reynolds number
- T = temperature
- U = velocity scale
- u = velocity vector
- uₓ = x velocity component
- uᵧ = y velocity component
- uｚ = z velocity component
- x = coordinate vector
- x = horizontal coordinate normal to heater strip
- y = horizontal coordinate parallel to heater strip
- z = vertical coordinate
- β = coefficient of thermal expansion
- p = density
- μ = viscosity
- C₀c = at “cold” conditions
- C₀h = at “hot” conditions

INTRODUCTION
The three-dimensional natural convection of a fluid in an enclosure is examined. This study is motivated by a possible magma-energy extraction system discussed by Chu et al. (1990), in which a well drilled through the Earth’s crust penetrates the magma, and fluid circulation in the well keeps the rock adjacent to the penetration below the melting point (see Figure 1). Thus, the magma-energy extractor can be approximately described as a cold cylinder penetrating vertically downward into a hot fluid undergoing natural convection. An additional issue is that the viscosity of magma decreases strongly with increasing temperature (cf. Chu and Hickox, 1990; Hickox and Chu, 1991).

A model system is developed (see Figure 2) incorporating these features. The model geometry consists of a cubical enclosure (56 cm side), within which a closed cylinder (7.6 cm diameter, 28 cm height) representing the magma-energy extractor has been end-mounted to the center of the top side. The top side and cylinder are water-cooled, and a flush-mounted heater strip (56 cm length, 14 cm width) representing the magma heat source is centered on the bottom side of the box. All the other boundaries are insulated. The enclosure is filled with 42/43 corn syrup, which has been used by other investigators [e.g. Chu and Hickox (1990) and Hickox and Chu (1991)] as a magma simulant because of its highly temperature-dependent viscosity (see appendix).

¹This work was performed at Sandia National Laboratories, supported by the U.S. Department of Energy, under contract DE-AC04-94AL85000.
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The three-dimensional natural-convection flow that occurs in the above enclosure is examined numerically and experimentally. The computational fluid dynamics code FIDAP (Fluid Dynamics International, 1993) is used to simulate the flow for enclosures with and without the cylinder to ascertain its effect on the flow. FIDAP is a finite-element code capable of simulating a wide class of phenomena, including buoyancy and temperature-dependent material properties. Additionally, an experimental apparatus with the dimensions given above has been assembled. Although the heater strip, the cylinder, and the enclosure top are fabricated out of metal, the remainder of the enclosure is fabricated out of polycarbonate so that optical diagnostics can be applied. Some preliminary results of laser light sheet flow-visualization experiments and two-dimensional particle-image velocimetry (PIV) experiments are shown in Figures 3-4 (details of the PIV technique are presented in O'Hern et al. (1994)). Unfortunately, diurnally induced temperature drifts, refractive index gradients associated with the thermally induced density gradients, and material degradation (caramelization) after prolonged heating have precluded obtaining quantitative experimental results.

EQUATIONS AND NONDIMENSIONALIZATION

The equations governing the motion are taken to be the incompressible Navier-Stokes equations with the Boussinesq approximation and the energy-transport equation. Material properties are taken to have the temperature-dependences shown in the appendix: the specific heat and the thermal conductivity depend only weakly on temperature, but the viscosity decreases strongly with increasing temperature. The flow is further assumed to be steady, laminar, and symmetric about the geometric symmetry planes passing through the cylinder axis and parallel to the enclosure side walls (see Figure 2).

Prior to performing any calculations, the following scaling is employed to render the problem nondimensional, where L is the cylinder length, $T_C$ and $T_H$ are the cold (cylinder and enclosure top) and hot (heater strip) temperatures, respectively, and all quantities are evaluated at $T_C$ unless stated otherwise:

Table 1: Dimensionless quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prandtl number</td>
<td>$Pr = \mu c / k$</td>
</tr>
<tr>
<td>Rayleigh number</td>
<td>$Ra = \rho \beta c (T_H - T_C) L^3 / \mu k$</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>$Re = \rho \mu L / \mu = Ra^{1/2} / Pr$</td>
</tr>
<tr>
<td>Peclet number</td>
<td>$Pe = \rho c \mu L / k = Ra^{1/2}$</td>
</tr>
<tr>
<td>Position</td>
<td>$x / L$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$u / U, U = (k / \rho c L) Ra^{1/2}$</td>
</tr>
<tr>
<td>Pressure</td>
<td>$p / P, P = \rho g \beta (T_H - T_C) L$</td>
</tr>
<tr>
<td>Temperature</td>
<td>$(T - T_C) / (T_H - T_C)$</td>
</tr>
</tbody>
</table>

With this scaling, material properties (at cold conditions) take on the following values:

Table 2: Material and other properties at $T_C$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Scaled Value at $T_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$1 / Pr$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$1 / Ra^{1/2}$</td>
</tr>
<tr>
<td>Thermal Expansion Cf.</td>
<td>$Pr$</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>$Pr$</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$1 / Ra^{1/2}$</td>
</tr>
<tr>
<td>Gravitational Accel.</td>
<td>1</td>
</tr>
</tbody>
</table>

For the conditions under consideration, the Prandtl and Rayleigh numbers are on the order of $10^6$ and $10^5$-$10^6$, respectively, which yield Reynolds and Peclet numbers on the order of $10^4$-$10^5$ and $10^2$-$10^3$, respectively. Thus, the fluid motion is determined by a force balance between hydrostatic pressure, buoyancy, and viscous effects (inertial effects such as "jetting" are absent), but the thermal boundary layers driving the flow are thin. The combination of thin boundary layers and three-dimensionality makes this a computationally intense problem.

PREVIOUS WORK

Chu and Hickox (1990) and Hickox and Chu (1991) examined flow under similar conditions. They used 42/43 corn syrup as the working fluid but employed a thinner fluid layer in a shorter rectangular enclosure with a heater strip but without a cylinder. Thus, in their studies, they treated the flow as two-dimensional. For similar Prandtl numbers, Rayleigh numbers, and cold-to-hot viscosity ratios, they always observed both experimentally and computationally a single convection cell pair, with rolls parallel to the heater strip driven by fluid rising from the heater strip toward the top of the enclosure. Prior to performing any simulations or experiments on the enclosure with the cylinder, the flow pattern was expected to be similar to the previous studies except that the cylinder would probably weaken the motion.

SIMULATIONS

The computational fluid dynamics code FIDAP (Fluid Dynamics International, 1993) is used to perform simulations of the three-dimensional natural-convection flow in the enclosure described above with and without the cylinder. The penalty formulation with a penalty parameter of $10^{-9}$ is used to enforce the incompressibility constraint. Physical conditions of $T_C = 8.8^\circ C$ and $T_H = 31.7^\circ C$ are examined, which correspond to cold Prandtl and Rayleigh numbers of $6.7 \times 10^6$ and $2.2 \times 10^4$, respectively, a viscosity ratio of $\mu_c / \mu_H = 37$, and a velocity scale of $U = 62 \mu m / h$. To demonstrate that the results are insensitive to the mesh, simulations are performed on multiple grids having
appreciably different nodal density, where the nodal density in every coordinate direction is proportionate to $m$, the mesh-refinement parameter. For example, a grid with $m = 4$ has $(4/2)^3 = 8$ times as many nodes as the corresponding grid with $m = 2$. To address the possibility of nonunique flow solutions, a variety of different velocity and temperature fields have been used to initialize the simulations. In all cases examined, the converged flow solutions are found to be independent of the initialization, suggesting that the solutions are unique.

To examine the effect of the cylinder on the flow pattern, simulations are performed for enclosures both with and without the cylinder. Figure 5 shows the $m = 4$ mesh with the cylinder, and Figure 6 shows the variations of temperature and vertical velocity component with vertical position along the line collinear with the cylinder axis ($x = y = 0$) for three different meshes. Only small differences are observed, indicating that a reasonable degree of grid independence has been achieved. Figures 7-8 show temperature contours and velocity vectors for the $m = 4$ grid. Contrary to initial expectations, cold fluid is observed beneath the cylinder, where it is traveling downward toward the heater strip (rather than upward). Subsequently, this fluid warms as it travels outward along the strip toward the side wall. Ultimately, a plume of heated fluid above the end of the heater strip rises up along the side wall toward the top of the enclosure. Thus, this flow is composed of a convection cell pair, with rolls perpendicular to the heater strip (rather than parallel), entirely different from that observed by Chu and Hickox (1990) and Hickox and Chu (1991) in qualitative agreement with the flow-visualization and PIV experiments. Figure 9 shows the $m = 5$ mesh without the cylinder, and Figure 10 shows the variations of temperature and vertical velocity component with vertical position along the same line as in Figure 6 for four different meshes. As before, differences are small. Temperature contours and velocity vectors are shown in Figures 11-12. In this case, the flow pattern is essentially as seen by Chu and Hickox (1990) and Hickox and Chu (1991), a single convection cell pair driven by fluid rising from the heater strip, where the rolls lie parallel to the heater strip.

The cylinder influences the flow strongly, which in retrospect is not surprising since the cylinder surface area is comparable to the heater strip surface area. Because of this, the cylinder overwhelms the portion of the heater strip beneath it and cools the fluid in this region. The resulting buoyancy forces are downward beneath the cylinder but upward near the junction of the heater strip and the side wall. Convection cells resulting from these forces must have rolls perpendicular to the heater strip, as observed.

CONCLUSIONS

Simulations have been performed of the three-dimensional natural-convection flows that occur in enclosures with and without a cylindrical penetration. The cylinder is seen to alter the flow pattern completely, which is tentatively attributed to the fact that the cylinder and the heater strip have comparable areas. To test this idea, cylinders of significantly smaller diameters should be examined. Such cases would also correspond more closely to the magma-energy extraction system motivating this study.

ACKNOWLEDGMENTS

This work was performed at Sandia National Laboratories, supported by the U. S. Department of Energy under contract number DE-AC04-94AL85000, through a laboratory directed research and development (LDRD) contract.

REFERENCES


APPENDIX

This appendix contains the thermophysical properties of 42/43 corn syrup from Chu and Hickox (1990) and Hickox and Chu (1991), namely the viscosity $\mu$ in Poise, the thermal conductivity $k$ in W/m-K, the specific heat $c$ in J/g-K, the density $\rho$ in g/cm$^3$, and the coefficient of thermal expansion $\beta$ in K$^{-1}$, where the temperature $T$ is in °C.

Table 3: Material properties of 42/43 corn syrup

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$\mu = a_0 \exp \left[ a_1 \exp \left( -T/\alpha_2 \right) \right]$</td>
<td>$a_0 = 0.2412$, $a_1 = 12.5867$, $\alpha_2 = 55.7805$</td>
</tr>
<tr>
<td>$k$</td>
<td>$k = b_0 + b_1 T$</td>
<td>$b_0 = 0.3724$, $b_1 = 3.034 \times 10^{-4}$</td>
</tr>
<tr>
<td>$c$</td>
<td>$c = c_0 + c_1 T + c_2 T^2$</td>
<td>$c_0 = 2.2005$, $c_1 = 3.9532 \times 10^{-3}$, $c_2 = -6.7883 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\rho = \rho_0 \left( 1 - \beta \left( T - T_0 \right) \right)$</td>
<td>$\rho_0 = 1.4255$, $\beta = 4.1218 \times 10^{-4}$, $T_0 = 10$</td>
</tr>
</tbody>
</table>
Figure 1. Conceptual representation of a single well of a magma-energy extraction system during operation (cf. Chu et al., 1990).

Figure 2. Schematic diagrams of different views of the enclosure.
Figure 3. Laser-light-sheet flow visualization experiment: left, slice through cylinder along heater; right, slice through cylinder across heater.

Figure 4. Raw PIV velocity vectors at similar conditions: left, slice through cylinder along heater strip (plane $x = 0$); right, slice through cylinder across heater (plane $y = 0$). The flow pattern agrees with the flow visualization experiment. Flow is downward beneath cylinder.
Figure 5. $m = 4$ mesh with cylinder (tri-linear brick elements).

Figure 6. Grid-refinement study with cylinder.

Figure 7. Temperature contours with cylinder.

Figure 8. Velocity vectors with cylinder.
Figure 9. $m = 5$ mesh without cylinder (tri-linear brick elements).

Figure 10. Grid-refinement study without cylinder.

Figure 11. Temperature contours without cylinder.

Figure 12. Velocity vectors without cylinder.