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Phase Unwrapping Using Discontinuity Optimization

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Abstract -- In SAR interferometry, the periodicity of the phase must be removed using two-dimensional phase unwrapping. The goal of the procedure is to find a smooth surface in which large spatial phase differences, called discontinuities, are restricted to places where their presence is reasonable. The pioneering work of Goldstein et al. identified points of local unwrap inconsistency called residues, which must be connected by discontinuities. This paper presents an overview of recent work that treats phase unwrapping as a discrete optimization problem with the constraint that residues must be connected. Several algorithms use heuristic methods to reduce the total number of discontinuities. Constantini has introduced the weighted sum of discontinuity magnitudes as a criterion of unwrap error and shown how algorithms from optimization theory are used to minimize it. Pixels of low quality are given low weight to guide discontinuities away from smooth, high-quality regions. This method is generally robust, but if noise is severe it underestimates the steepness of slopes and the heights of peaks. This problem is mitigated by subtracting (modulo 2π) a smooth estimate of the unwrapped phase from the data, then unwrapping the resulting residual phase. The unwrapped residual is added to the smooth estimate to produce the final unwrapped phase. The estimate can be computed by lowpass filtering of an existing unwrapped phase; this makes possible an iterative algorithm in which the result of each iteration provides the estimate for the next. An example illustrates the results of optimal discontinuity placement and the improvement from unwrapping of the residual phase.

INTRODUCTION

In synthetic-aperture-radar interferometry (IFSAR), topography is measured as an image of phase angles [1]. This introduces an ambiguity interval, or cycle, of 2π radians of phase, that must be removed by adding an integer number of cycles to each pixel. This procedure is called two-dimensional phase unwrapping. It is assumed that the terrain is mostly smooth, with slopes small enough that phase differences of more than one-half cycle between adjacent pixels, called discontinuities or jumps, are rare. If noise is present, or if the topography contains large phase differences (such as layover in IFSAR data), then jumps cannot be avoided entirely. The

unwrapping algorithm should place jumps where the terrain is discontinuous and avoid them as much as possible elsewhere. This objective can be expressed by a *cost function* that penalizes each jump by an amount depending on its location and magnitude (in cycles, rounded to the nearest cycle).

Given the cost function, phase unwrapping is a problem of constrained optimization of the jumps. The constraints arise from "residues" or "discontinuity sources," as they are respectively called by Goldstein et al. [2] and Huntley [3]. In the unwrapped phase, a chain of jumps must originate at each residue, connecting it to a residue of opposite sign or to the boundary of the phase image. The algorithm must find the minimum-cost set of jumps that connects the residues correctly.

There are several algorithms that attempt to connect each residue with a nearby residue of opposite polarity, implicitly defining the cost to be the number of jumps. The algorithm of Goldstein et al. [2] searches the neighborhood of a residue for other residues, connecting them with "branch cuts" on which jumps are allowed, to form balanced sets of residues. Other algorithms [4,5,6] create a list of residue pairs, then modify it to reduce the sum of distances between paired residues. These algorithms produce good results in many cases, but may place jumps in smooth regions if the residues are dense. Better performance requires a more sophisticated cost function.

WEIGHTED OPTIMIZATION

The value of weighting in phase unwrapping is well known. Regions of noise or layover in an interferogram typically have a high concentration of residues and low complex correlation [7]. Jumps in these regions are less objectionable than in regions of high-quality data, and so their cost should be lower. The appropriate cost function is the weighted discontinuity sum, introduced by Constantini [8] and derived independently by Flynn [9]. Each possible jump location (pair of neighboring pixels) is given a weight based on the data quality in its vicinity. The contribution of each jump to the cost is its weight times its magnitude.

This sum can be described as the cost of a set of flows in a network: each positive residue is a source of one unit of "flow" that must be transported to a negative residue, or to the boundary of the image, by a chain of jumps. The cost of each chain is the sum of its weights. The problem is to compute a minimum-cost set of chains connecting the residues. This is a classical problem of optimization theory for which many

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efficient algorithms have been developed, as described in chapter 9 of [10]. In particular, the successive-shortest-paths algorithm (section 9.7) operates efficiently on phase unwrapping problems and is easy to implement.

SLOPE CORRECTION

Discontinuity optimization, like other phase unwrapping algorithms [11], tends to underestimate the steepness of slopes if the data are noisy. This is because the false jumps created by noise tend to have the same polarity as the mean slope. If such jumps are numerous enough, the algorithm removes them by introducing differences of the opposite polarity in the unwrapped phase. These errors often appear as contours, across which the unwrapped makes a step against the prevailing slope.

To reduce slope estimation errors, unwrapping can be done on a residual phase image with near-zero slope, rather than on the data directly. To create the residual requires a smooth estimate of the unwrapped phase whose slopes match those of the terrain at large scale. Subtracting the estimate from the data modulo 2π cancels large-scale slopes while preserving fine details. The unwrapped residual is added to the smooth estimate to produce the final result.

The estimate must, of course, be computed without knowledge of the true topography. A straightforward method is to unwrap the data directly, then apply a lowpass filter to the result. Although such a guess is affected by error contours, in many cases the contours cause fringes in the residual that are removed by unwrapping.

In principle, any unwrapped phase can be lowpass filtered to create a smooth guess. This suggests that an iterative algorithm, in which the unwrapped phase output of each iteration is used to form the estimate for the next, may enhance the quality of the final result. The iterative algorithm has this structure:

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input  $\phi$ , n_iter
 $z := \text{UW}(\phi)$ 
for  $i := 2 : n\_iter$ 
   $y := \text{LPF}(z)$ 
   $z := y + \text{UW}((\phi - y) \bmod 2\pi)$ 
end
output  $z$ 

```

The variable ϕ is the input phase, y is the smooth guess, and z is the unwrapped phase image. The operator LPF returns the result of applying a lowpass filter to its argument. The operator UW returns the result of unwrapping its argument. Any unwrapping algorithm with the property that $\text{UW}(\phi) = \phi \bmod 2\pi$ (phase congruency) can be used, but a noise-resistant algorithm will provide a better guess.

EXAMPLE

We now illustrate the performance of discontinuity optimization, with and without slope removal, on a phase image containing a steeply sloped region. An IFSAR image pair was taken by the Sandia radar testbed [12] over an area near Albuquerque, NM. The scene includes two elevated regions (mesas) and a gap between them. The phase data set prepared from these images is displayed in Figure 1. Closely spaced fringes appear at the edge of the mesa on the left. In Figure 2, the unwrapped phase with minimum weighted discontinuity is shown as a grayscale image. Jumps located within one pixel of a residue have weight 1; other jumps have weight 128. The bright contour near the edge of the mesa indicates where the unwrapping has failed to reconstruct the slope correctly. Figure 3 shows the lowpass phase computed by applying a 3×3 averaging box, five times in succession, to the unwrapped phase of Figure 2, then rewrapping. Noise and unwrap errors are largely smoothed out. Figure 4 displays the residual phase computed by subtracting the lowpass phase from the input phase, modulo 2π . It is near zero (medium gray) in most of the image, with a fringe visible at the edge of the mesa. The unwrapped residual phase is shown as a grayscale image in Figure 5. The bright region is about 2π radians higher than the rest, correcting the underestimated height of the mesa. The improved unwrapped phase is shown in Figure 6.

DISCUSSION

Phase unwrapping is a crucial step in the estimation of topography from interferometric measurements. The algorithm must control discontinuities, placing them where they are needed and minimizing their effect elsewhere. The formulation of cost functions based on the locations of discontinuities, and the use of algorithms from discrete optimization theory to find the minimum-cost solution, are transforming the practice of phase unwrapping. With these techniques, it is possible to compute solutions of good quality from phase data sets far larger, and with much greater degradation, than could be solved otherwise.

This, or any, phase unwrapping algorithm would benefit from improved methods of slope removal. An initial guess derived from an unwrapped phase may introduce errors that are propagated through later iterations. An unbiased estimator of slope, computed directly from the phase data without unwrapping, would avoid this shortcoming.

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Fig. 1: Phase data

Fig. 2: Unwrapped phase

Fig. 3: Lowpass phase

Fig. 4: Residual phase

Fig. 5: Unwrapped residual

Fig. 6: Unwrapped phase after iteration

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