Topics in Radiation at Accelerators: Radiation Physics for Personnel and Environmental Protection

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RADIATION PHYSICS FOR PERSONNEL AND ENVIRONMENTAL PROTECTION

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US PARTICLE ACCELERATOR SCHOOL
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These lecture notes are dedicated to my wife Claudia, and our children, Joe and Sally, who accommodated my long hours spent in the preparation of these notes with love, cheerfulness, and support. I acknowledge the opportunity provided by the Fermilab Director, John Peoples, to participate in this endeavor. I greatly appreciate the encouragement of Mel Month and A. Lincoln Read to participate in the USPAS. I have also greatly appreciated the support and advice of several members of the Fermilab Environment, Safety and Health Section during the preparation and revision of these materials: Alex Elwyn provided a great deal of helpful advice while these materials were initially developed. Nancy Grossman and Kamran Vaziri have offered many suggestions for improvement in these materials. I especially acknowledge Nancy's improvement of the problems. Many other helpful suggestions have been provided by Elaine Marshall, Vernon Cupps, and David Boehnlein. All of these individuals were not timid in calling to my attention my many minor errors and a few major ones which I have hopefully corrected in this revision.

J. Donald Cossairt
October 1996

PREFACE

The original version of this text was presented as part of a course taught at the session of the U. S. Particle Accelerator School held at Florida State University in January, 1993. Upon completion of the USPAS school, the notes were further refined and presented informally as a course at Fermilab in the Spring of 1993. Following this second presentation of the course, the materials were improved by taking into account the many suggestions of course participants. This third revision was prepared after the course was presented in depth at Fermilab in the autumn of 1994, at the U. S. Particle Accelerator School at Duke University in January 1995, and at the Health Physics Society Meeting in Boston in July 1995. This text represents a compilation of the work of many, many people and it is hoped that the reference citations leads the reader to those individuals who have developed this field of applied physics. The problems supplied with each chapter were developed with the goal of promoting better understanding of the text. Solutions to all of the problems are available by contacting the author.
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Chapter 1 Composition of Accelerator Radiation Fields

In this chapter, terminology, physical and radiological quantities, and units of measurement used to describe the properties of accelerator radiation fields are reviewed. The general considerations of primary radiation fields pertinent to accelerators are discussed. The primary radiation fields produced by electron beams are described qualitatively and quantitatively. In the same manner the primary radiation fields produced by proton and ion beams are described.

I. Review of Units and Terminology and Physical Constants and Properties

Radiological Units

In this section common units and terminology used in accelerator radiation protection are described or defined.

energy: The unit of energy in common use when dealing with energetic particles is the electron volt (eV). 1 eV = 1.602 X 10^-12 ergs or 1.602 X 10^-19 Joule; multiples in common use at accelerators are the keV (10^3 eV), MeV (10^6 eV), GeV(10^9 eV), and TeV (10^12 eV)

absorbed dose: The energy absorbed per unit mass of material. It is usually denoted by the symbol "D". The customary unit of absorbed dose is the rad while the Système Internationale (SI) unit of absorbed dose is the Gray:

1 rad = 100 ergs/gram = 6.24 X 10^13 eV/gm
1 Gray (Gy) = 1 J/kg = 100 rads = 6.24 X 10^15 eV/gm.

dose equivalent: This quantity has the same dimensions as absorbed dose. It is used to take into account the fact that different particle types have biological effects which are enhanced, per given absorbed dose, over those due to 200 keV photons (a "standard" reference particle). It is usually denoted by the symbol "H".

quality factor: This factor takes into account the relative enhancement in biological effects of various types of ionizing radiation. It is usually denoted by Q, and is used to obtain H from D through the following equation:

\[ H = QD. \]  

Q is dependent on both particle type and energy and, thus, for any radiation field its value is an average over all components. It is specifically defined to be equal to unity for 200 keV photons. Q ranges from unity for photons electrons, and high energy muons to a value as high as 20 for α-particles (4He nuclei) of a few MeV in energy. For neutrons, Q ranges from 2 to greater than 10, although recent guidance by the International Council on Radiation Protection (ICRP) and the U. S. National Council on Radiation Protection and Measurements (NCRP) recommends that Q be increased by a factor of 2 for neutrons. [Values have been proposed for photons and electrons that differ from unity!]

Q is presently defined to be a function of linear energy transfer (LET), L, which, crudely, is equivalent to stopping power, or rate of energy loss for charged particles (conventionally, in units of keV/micron). [All ionizing radiation ultimately manifests itself through charged particles so that LET is a "universal" parameterization of localized radiation damage.]
Chapter 1  Composition of Accelerator Radiation Fields

Quality Factor Relationship Graphs Taken from (Sw79)

Fig. 1.1  Quality Factor, Q, of charged particles as a function of collision stopping power (LET<sub>ww</sub>) in water as recommended by ICRP [Reproduced from (Pa73) and (IC73)].

Fig. 1.2  Quality factors of charged particles as a function of energy, as recommended by ICRP. [Reproduced from (Pa73) and (IC73)].
Chapter 1  Composition of Accelerator Radiation Fields

Fig. 1.3  Effective quality factor, \( Q \), for neutrons as a function of neutron kinetic energy: the maximum dose equivalent divided by the absorbed dose where the maximum dose equivalent occurs. The curve indicates values recommended by ICRP. [Reproduced from (Pa73) and (IC73).]

Most commonly, the "quality factor" actually used is an average over the spectrum of LET present:

\[
<Q> = \frac{\int_0^\infty Q(L)D(L)\,dL}{\int_0^\infty D(L)\,dL}.
\]  

(1.2)

Thus, \( H \) (rem) = \( QD \) when \( D \) is in rads (customary) and \( H \) (Sievert (Sv)) = \( QD \) when \( D \) is in Gy (SI). Figures 1.1, 1.2, and 1.3 from (Sw79) give the relationship between \( Q \) and LET and also between \( Q \) and particle energy for a variety of particles.

flux density - The number of particles that traverse a unit area in unit time, generally denoted by the symbol "\( \phi \),

\[
\phi = \frac{d^2n}{dAdt}
\]  

(1.3)

where \( d^2n \) is the differential number of particles traversing surface area element \( dA \) during time \( dt \). For radiation fields where the constituent particles move in a multitude of directions, \( \phi \) is determined from the number of transversals of a sphere of revolution of a small element of circular area \( dA \).
fluence, denoted by $\Phi$, is simply the integral over some time interval of the flux density,

$$\Phi = \int_{t_i}^{t_f} \phi(t) dt.$$  

(1.4)

The units of flux density are $\text{cm}^{-2}\text{s}^{-1}$ (customary) and $\text{m}^{-2}\text{s}^{-1}$ (SI) while the units of fluence are, of course, inverse area without the units of inverse time. Beware! Other units of time such as hours, minutes, days, years, etc $\text{cm}^{-2}$ and $\text{m}^{-2}$ are routinely used in the literature of radiation protection.

fluence or flux density-to-dose (or dose equivalent) conversion factors - Such factors have been derived through complex calculations supported in a limited way by measurements. These calculations also include effects due to the finite thicknesses of the material of reference (usually "tissue") and include secondary effects. Figures 1.4, 1.5, 1.6, and 1.7 give some graphs of typical fluence or flux density to dose equivalent conversion factors. Figures 1.4, 1.5, and 1.7 are taken from (Pa73) while Fig. 1.6 is taken from (Sw79).

For a radiation field containing a mixture of $n$ different components (e.g., different particle types), one determines the dose equivalent, $H$, from:

$$H = \sum_{i=1}^{n} \int_{E_{min}}^{E_{max}} P_i(E)\Phi_i(E)dE$$

(1.5)

where $\Phi_i(E)$ is the fluence of particles of type $i$ with energy between $E$ and $dE$ and $P_i(E)$ is a parameter that converts fluence to dose equivalent.

The cross section is an extremely important physical concept in describing particle interactions. The cross section represents the "size" of the atom or nucleus for some particular interaction. Consider a beam of particles of fluence $\Phi$ ($\text{particles/cm}^2$) incident on a thin slab of absorber of thickness $dx$. The absorbing medium will contain $N$ atoms/cm$^3$. The number of incident particles which interact, $\Phi dx$, will be given by:

$$-d\Phi = \sigma N \Phi dx$$

(1.6)

where $\sigma$ is the cross section in units of $\text{cm}^2$. But, $N = \rho N_A/A$, where $\rho$ is the density ($\text{g/cm}^3$), $N_A$ is Avagadro's number ($6.02 \times 10^{23}$ mol$^{-1}$) and $A$ is the atomic weight. Cross sections are often given in units of barns where 1 barn = $10^{-24}$ cm$^2$. If only one interaction is present with no other processes operative, this integrates, after some distance $x$ (e.g., in cm), to:

$$\Phi(x) = \Phi(0)e^{-N\sigma x}.$$  

(1.7)

Thus, the linear absorption coefficient, $\mu$, and its reciprocal, the attenuation length, $\lambda$, are given by:

$$\mu = N\sigma \text{ (cm$^{-1}$)} \quad \text{and} \quad \lambda = 1/N\sigma \text{ (cm)}.$$

(1.8)

Sometimes the mass attenuation length, $\lambda = \rho/N\sigma$ (g/cm$^2$) is used where $\rho$ is the density in g/cm$^3$. In the literature, the symbols are, unfortunately, often "confused" and one has to be careful to understand the particular definition of "$\lambda$" involved.
Chapter 1 Composition of Accelerator Radiation Fields

Flux Density to Dose Conversion Factors

Fig. 1.4 Conversion factors for flux density to dose-equivalent rate for electrons. [Reproduced from (Pa73) and references cited therein.]

Fig. 1.5 Conversion factors for flux density to dose-equivalent rate for photons. [Reproduced from (Pa73) and references cited therein.]
Chapter 1 Composition of Accelerator Radiation Fields

Flux Density to Dose Conversion Factors

Fig. 1.6 Conversion factors for neutrons as a function of incident neutron kinetic energy for unidirectional broad beam, normal incidence. The curves indicate the values recommended by ICRP. [Reproduced from (Sw79) and from (IC73).]

Fig. 1.7 Conversion factors for flux density to dose-equivalent rate for protons as a function of incident proton kinetic energy (protons cm\(^{-2}\) s\(^{-1}\) per mrem hr\(^{-1}\)) [Reproduced from (Pa73).]
Chapter 1  Composition of Accelerator Radiation Fields

Physical Constants and Atomic and Nuclear Properties

Tables 1.1 and 1.2 give physical constants and atomic and nuclear properties taken from (PR92). A number of these constants and properties will be used throughout the rest of this text, and in the solutions of the problems.

II. Summary of relativistic relationships

The rest energy, \( W_0 \), of a particle of rest mass \( m_0 \) is given by,

\[
W_0 = m_0c^2
\]

where \( c \) is the velocity of light. (1.9)

The total energy in free space, \( W \), is then given by

\[
W = mc^2 = m_0c^2(1-\beta^2)^{-1/2},
\]

where \( \beta = v/c \) and \( v \) is the velocity of the particle in a given frame of reference. (1.10)

The relativistic mass, \( m \), of a particle moving at \( \beta \) is another name for the total energy and is given by

\[
mc^2 = \frac{1}{\sqrt{1-\beta^2}}m_0c^2 = \gamma m_0c^2
\]

(1.11)

The kinetic energy, \( E \), is then;

\[
E = W - W_0 = (m - m_0)c^2 \quad \text{and}
\]

\[
\beta = \sqrt{1 - \left( \frac{W_0}{W} \right)^2}.
\]

(1.12) (1.13)

The momentum, \( p \), of a particle is

\[
p = mv = m\beta c = (1/c)(W^2 - W_0^2)^{1/2} = \frac{1}{c}\sqrt{E(E + 2W_0)},
\]

so that at high energies, \( p = E/c = W/c \).

It is usually most convenient to work in a system of units where energy is in units of eV, MeV, etc., velocities are expressed in units of the speed of light (\( \beta \)), momenta are expressed as energy divided by \( c \) (e.g., MeV/c, etc.), and masses are expressed as energy divided by \( c^2 \) (e.g., MeV/c^2, etc.).

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### Chapter 1 Composition of Accelerator Radiation Fields

**Table 1.1 Physical Constants [Reproduced from (PR92)]**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol, equation</th>
<th>Value</th>
<th>Uncert. (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light</td>
<td><em>c</em></td>
<td>299 792 458 m s⁻¹</td>
<td>(exact)</td>
</tr>
<tr>
<td>Planck constant</td>
<td><em>h</em></td>
<td>6.626 075 5(40) × 10⁻³⁴ J s</td>
<td>0.60</td>
</tr>
<tr>
<td>Planck constant, reduced</td>
<td><em>h/2π</em></td>
<td>1.064 377 66(63) × 10⁻³⁴ J s</td>
<td>0.60</td>
</tr>
<tr>
<td>electron charge magnitude</td>
<td><em>e</em></td>
<td>1.602 177 33(49) × 10⁻¹⁹ C</td>
<td>0.30</td>
</tr>
<tr>
<td>conversion constant</td>
<td><em>h/e</em></td>
<td>197.327 053(59) MeV fm</td>
<td>0.30</td>
</tr>
<tr>
<td>conversion constant</td>
<td>(hc)²</td>
<td>3.839 379 66(23) GeV² mbar</td>
<td>0.59</td>
</tr>
<tr>
<td>electron mass</td>
<td><em>me</em></td>
<td>0.510 909 06(15) MeV/c²</td>
<td>0.30</td>
</tr>
<tr>
<td>proton mass</td>
<td><em>mp</em></td>
<td>938.272 31(28) MeV/c²</td>
<td>0.30</td>
</tr>
<tr>
<td>deuteron mass</td>
<td><em>md</em></td>
<td>931.450 32(28) MeV/c²</td>
<td>0.30</td>
</tr>
<tr>
<td>permittivity of free space</td>
<td>ε₀</td>
<td>R_{\text{free}} = ε₀ \mu_0 = 1/c²</td>
<td>(exact)</td>
</tr>
<tr>
<td>permittivity of free space</td>
<td>μ₀</td>
<td>8.854 187 617 ... × 10⁻¹² F m⁻¹</td>
<td>(exact)</td>
</tr>
<tr>
<td>fine structure constant</td>
<td>α</td>
<td>1/137.035 969 (61)°</td>
<td>0.045</td>
</tr>
<tr>
<td>classical electron radius</td>
<td>r_e</td>
<td>2.817 940 92(38) × 10⁻¹⁰ m</td>
<td>0.13</td>
</tr>
<tr>
<td>electron Compton wavelength</td>
<td>λ_e = h/m_e c</td>
<td>3.861 593 23(35) × 10⁻¹³ m</td>
<td>0.099</td>
</tr>
<tr>
<td>Bohr radius</td>
<td>r_B = h/m_e c</td>
<td>0.529 177 249(24) × 10⁻¹⁰ m</td>
<td>0.045</td>
</tr>
<tr>
<td>Rydberg energy</td>
<td>h/ε_e</td>
<td>1.239 842 (37) × 10⁹ m</td>
<td>0.30</td>
</tr>
<tr>
<td>Thomson cross section</td>
<td>σ_T</td>
<td>6.656 246 (18) barn</td>
<td>0.27</td>
</tr>
<tr>
<td>Bohr magneton</td>
<td>μ_B = ε_0/2m_e</td>
<td>5.788 328 (52) × 10⁻¹¹ MeV T⁻¹</td>
<td>0.089</td>
</tr>
<tr>
<td>nuclear magneton</td>
<td>μ_N = ε_0/2m_p</td>
<td>3.152 451 (68) × 10⁻¹⁴ MeV T⁻¹</td>
<td>0.089</td>
</tr>
<tr>
<td>electron cyclotron freq. /field</td>
<td>ω_{e,c} / B = e/m_e</td>
<td>1.758 819 (63) × 10⁶ rad s⁻¹ T⁻¹</td>
<td>0.30</td>
</tr>
<tr>
<td>proton cyclotron freq. /field</td>
<td>ω_{p,c} / B = e/m_p</td>
<td>9.578 830 (99) × 10⁶ rad s⁻¹ T⁻¹</td>
<td>0.50</td>
</tr>
<tr>
<td>gravitational constant</td>
<td>G_N</td>
<td>6.672 58 (85) × 10⁻¹¹ m² kg⁻¹ s⁻²</td>
<td>128</td>
</tr>
<tr>
<td>standard grav. acc., sea level</td>
<td>g</td>
<td>9.806 55 m s⁻²</td>
<td>(exact)</td>
</tr>
<tr>
<td>Avogadro number</td>
<td>N_A</td>
<td>6.022 136 (76) × 10²³ mol⁻¹</td>
<td>0.59</td>
</tr>
<tr>
<td>Boltzmann constant</td>
<td>k = h/(2πN_A) c</td>
<td>1.380 548 (12) × 10⁻²³ J K⁻¹</td>
<td>8.5</td>
</tr>
<tr>
<td>Wien displacement law constant k = λ_{wien} T</td>
<td>= 6.671 385 (73) × 10⁻³ eV K⁻¹</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td>molar volume, ideal gas at STP</td>
<td>N_A(273.15 K)/(1 atmosphere)</td>
<td>22.414 10(19) × 10⁻³ m³ mol⁻¹</td>
<td>8.4</td>
</tr>
<tr>
<td>Stefan-Boltzmann constant</td>
<td>σ = 5.670 59(19) W m⁻² K⁻⁴</td>
<td>5.670 59(19) W m⁻² K⁻⁴</td>
<td>34</td>
</tr>
<tr>
<td>Fermi coupling constant</td>
<td>G_F/(h)c²</td>
<td>1.166 39(2) × 10⁻⁵ GeV⁻²</td>
<td>17</td>
</tr>
<tr>
<td>weak mixing angle</td>
<td>sin² θ_W (SB)</td>
<td>0.2325 ± 0.0008</td>
<td>3411</td>
</tr>
<tr>
<td>W boson mass</td>
<td>m_W</td>
<td>80.22 ± 0.26 GeV/c²</td>
<td>3241</td>
</tr>
<tr>
<td>Z² boson mass</td>
<td>m_Z</td>
<td>91.173 ± 0.020 GeV/c²</td>
<td>219</td>
</tr>
<tr>
<td>strong coupling constant</td>
<td>σ_{strong}</td>
<td>0.1134 ± 0.0035</td>
<td>3.1 ± 19⁴</td>
</tr>
</tbody>
</table>

**Notes:**
- *w = 3.141 592 653 589 793 238*
- *c = 2.718 281 828 459 044 235*
- *γ = 0.577 215 664 901 552 8621*
- *1 in = 0.0254 m*
- *1 barn = 10⁻²⁸ m²*
- *1 eV = 1.602 177 33(49) × 10⁻¹⁹ J*
- *1 g/cm³ = 10⁻¹⁰ kg cm⁻³*
- *1 atm = 1.013 25 × 10⁻⁵ N/m²*

* The meter is now defined to be the length of the path traveled by light in 1/299792458 seconds. See B.W. Pettew, Nature 303, 373 (1983).

* At Q² = m². At Q² = m², the value is approximately 1/128.
## Chapter 1 Composition of Accelerator Radiation Fields

### Table 1.2 Atomic and Nuclear Properties of Materials [Reproduced from (PR92)]

<table>
<thead>
<tr>
<th>Material</th>
<th>Z</th>
<th>A</th>
<th>Nuclear $^a$ total cross section</th>
<th>Nuclear $^a$ inelastic cross section</th>
<th>Nuclear $^a$ collision interaction</th>
<th>$dE/dx$ $^b$ in MeV/g•cm$^2$</th>
<th>Radiation length $^c$ ($\lambda_R$) in cm/g</th>
<th>Density $^d$ ($\rho$) in g/cm$^3$</th>
<th>Refractive index $^e$ $n$ for gas $^f$ in (n-1)×10$^5$ for gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>2</td>
<td>4</td>
<td>0.0387</td>
<td>0.033</td>
<td>43.3</td>
<td>12 6</td>
<td>61.28</td>
<td>865</td>
<td>0.0708(0.000)</td>
</tr>
<tr>
<td>O</td>
<td>8</td>
<td>16</td>
<td>0.0034</td>
<td>0.0011</td>
<td>45.7</td>
<td>2.07</td>
<td>122.5</td>
<td>143</td>
<td>0.102(0.177)</td>
</tr>
<tr>
<td>Ne</td>
<td>10</td>
<td>20.15</td>
<td>0.1024</td>
<td>0.029</td>
<td>36.1</td>
<td>1.73</td>
<td>28.96</td>
<td>40.0</td>
<td>1.14(1.43)</td>
</tr>
<tr>
<td>Ar</td>
<td>18</td>
<td>85.82</td>
<td>0.586</td>
<td>0.256</td>
<td>76.4</td>
<td>1.51</td>
<td>19.55</td>
<td>14.0</td>
<td>1.40(1.78)</td>
</tr>
<tr>
<td>Fe</td>
<td>26</td>
<td>55.85</td>
<td>1.20</td>
<td>0.703</td>
<td>131.9</td>
<td>1.48</td>
<td>13.84</td>
<td>1.76</td>
<td>7.97</td>
</tr>
<tr>
<td>Ca</td>
<td>20</td>
<td>63.35</td>
<td>0.92</td>
<td>0.722</td>
<td>134.9</td>
<td>1.44</td>
<td>12.86</td>
<td>1.43</td>
<td>8.96</td>
</tr>
<tr>
<td>Ge</td>
<td>32</td>
<td>72.59</td>
<td>1.365</td>
<td>0.858</td>
<td>140.5</td>
<td>1.40</td>
<td>12.25</td>
<td>2.30</td>
<td>5.33</td>
</tr>
<tr>
<td>Si</td>
<td>14</td>
<td>28.09</td>
<td>0.660</td>
<td>0.440</td>
<td>108.1</td>
<td>1.66</td>
<td>21.82</td>
<td>6.36</td>
<td>3.33</td>
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1. Atomic and Nuclear Properties of Materials [Reproduced from (PR92)]
2. Nuclear $^a$ total cross section $\sigma_T$ in barns ($\text{barn} = 10^{-24} \text{cm}^2$)
3. Nuclear $^a$ inelastic cross section $\sigma_{ei}$ in barns ($\text{barn} = 10^{-24} \text{cm}^2$)
4. Nuclear $^a$ collision interaction $\lambda_T$ in $\text{g/cm}^2$
5. Nuclear $^a$ interaction $\lambda_T$ in $\text{g/cm}^2$
6. $dE/dx$ in MeV/g•cm$^2$
7. Radiation length $^c$ ($\lambda_R$) in cm/g
8. Density $^d$ ($\rho$) in g/cm$^3$
9. Refractive index $^e$ $n$ for gas $^f$ in (n-1)×10$^5$ for gas

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Page 1-9
For moderately relativistic particles, the mean rate of energy loss (stopping power) is given approximately by (PR92):

\[
- \frac{dE}{dx} = 4 \pi N_A r_e^2 m_e c^2 z^2 Z \frac{1}{A \beta^2} \ln \left( \frac{2 m_e c^2 \gamma^2 \beta^2}{I} \right) - \beta^2 - \frac{\delta}{2},
\]

(1.15)

where \( N_A \) is Avogadro's number, \( Z \) and \( A \) are the atomic number and weight of the material transversed, \( z \) is the atomic number of the projectile, \( m_e \) and \( r_e \) are the mass and "classical radius" of the electron and \( I \) is the ionization constant. \( \delta \) is a small correction factor which approaches \( 2 \ln \gamma \). Substituting constants,

\[
- \frac{dE}{dx} = (0.3071) Z^2 \frac{1}{A \beta^2} \ln \left( \frac{2 m_e c^2 \gamma^2 \beta^2}{I} \right) - \beta^2 - \frac{\delta}{2} \quad \text{(MeV cm}^2\text{g}^{-1})
\]

(1.16)

where \( I = 16Z^{0.9} \) eV for \( Z > 1 \) and has the value of approximately 20 eV for diatomic hydrogen).

The decay length at a given velocity of a particle with a finite meanlife (at rest), \( \tau \), can be obtained from the product of the speed of light and the meanlife, \( ct \), which is often tabulated. The decay length is given by \( \gamma \beta c \tau \), where relativistic time dilation is taken into account. This length is to be distinguished from that called the decay path. The latter represents a distance in space in which a given particle is allowed to decay with no or minimal competition from other effects exemplified by scattering or absorption.
III. Primary Radiation Fields at Accelerators-General Considerations

The particle yield is a crucial parameter. It is typically a function of both angle and particle energy and is defined according to Fig. 1.8. Such particle yields are reported in terms of particle type, energy, fluence, and angular distributions. Scattered reaction products are found at a hypothetical "detector" or located at radius, r, and polar angle, θ, relative to the direction of the incident particle along the positive Z-axis.

Fig. 1.8 Conceptual interaction of incident beam with material which produces radiation at the location of a hypothetical detector located at polar coordinates (r, θ).
Chapter 1  Composition of Accelerator Radiation Fields

The rate of production of the desired reaction products and their energy spectra is, in general, a strong function of both θ and the incident particle energy E₀.

With a single exception, there is no dependence on the azimuthal angle, α, in this spherical coordinate scheme. With coordinates (r,θ,α) α is used, unconvendtionally, as the azimuthal angle to avoid confusion with φ, the flux density.

In principle, calculations of the particle yield could be taken directly from differential cross sections for given incident particle energy E (E usually denotes kinetic energy),

$$\frac{d\sigma(\theta,E)}{d\Omega}$$

where σ(Ω, E) is the cross section as a function of energy and Ω is the solid angle into which the secondary particles are produced. (The angular dependence is only on θ and not also on α due to the lack of azimuthal dependence.)

In general calculations of the radiation field which directly use the cross sections are not practical because targets hit by beam are not really thin (i.e., one cannot ignore energy loss or secondary interactions in the target) and there is incomplete knowledge of cross sections at all energies so one cannot integrate over θ and E to get the total yield.

For many applications, the details of the angular distributions of total secondary particle yield, dY(θ)/dΩ, and the angular dependence of the emitted particle energy spectrum, d²Y(θ,E)/dEdΩ, of the emitted particle spectra are very important.

Often, the particle fluence is needed at a particular location at coordinates (r,θ) from a known point source of beam loss while the angular distributions of dY/dΩ are generally expressed in units of particles/(steradian · incident particle).

To obtain the total fluence Φ(θ) [e.g., "particles"/(cm²·incident particle)], or differential fluence dΦ(E,θ)/dE [e.g., "particles"/(cm²·MeV·incident particle)] at a given distance r (cm) at a specified angle θ, one must simply multiply the plotted values by r⁻² (cm²):

$$\Phi(\theta) = \frac{1}{r^2} \frac{dY(\theta)}{d\Omega}$$ and $$\frac{d\Phi(E,\theta)}{dE} = \frac{1}{r^2} \frac{d^2Y(\theta,E)}{dEd\Omega}$$

(1.17)

¹The single exception is the case in which the spins of the target nuclei and/or the incident particle are oriented along some chosen direction in a "polarization" experiment.
Chapter 1  Composition of Accelerator Radiation Fields

IV. Radiation Production by Electron Accelerators (Most of this material is adapted from (Sw79), the work of the late William P. Swanson of SLAC and LBL)

At all energies photons produced by bremsstrahlung dominate the radiation field aside from the hazard of the direct beam. As the energy increases, neutrons become a significant problem. For $E_0 > 100$ MeV, the electromagnetic cascade must be considered (see Chapter 2)

An interesting rule of thumb is that electrons have a finite range in material proportional to the initial kinetic energy of the electron:

For $2 < E_0 < 10$ MeV, $R = 0.6E_0$ g cm$^{-2}$. (In air, $R = 5E_0$ meters with $E_0$ in MeV).  

Above approximately 10 MeV, radiative losses begin to dominate.

Direct Beam

Swanson (Sw79) has given what he expressed as an approximate, "conservative" rule of thumb for the energy domain of $1 < E_0 < 100$ MeV:

$$\frac{dH}{dt} = 1.6 \times 10^{-4} \phi \quad \text{where } \frac{dH}{dt} \text{ is the dose equivalent rate (rem h}^{-1} \text{ and } \phi \text{ is the flux density (electrons cm}^{-2} \text{s}^{-1} \text{)} \quad (1.19)$$

Others have calculated the conversion factor as a function of energy as in Fig. 1.9 taken from (Sw79). (The results in Fig. 1.9 should be regarded as more recent improvements to the results of Fig. 1.4.)

![Fig. 1.9 Conversion factor as a function of incident energy $E_0$ for a unidirectional broad beam of monoenergetic electrons at normal incidence. The curve indicates values represented by the ICRP. (Reproduced from (Sw79). See (Sw79) for references indicated on figure.)](image-url)
Bremsstrahlung

Bremsstrahlung is the radiative energy loss of electrons as they interact with materials. It appears in the form of photons. An important parameter when considering radiative energy loss of electrons in matter is the critical energy, \( E_c \). \( E_c \) is that energy above which the energy loss due to radiation exceeds that due to ionization for electrons. The value \( E_c \) is a smooth function of atomic number:

\[
E_c = \frac{800}{(Z + 1.2)} \text{(MeV)},
\]  

where \( Z \) is the atomic number of the material.

The transition from ionization to radiation is also a smooth one. The stopping power for electrons may be written as the sum of collisional and radiative components (Pa73):

\[
\left( \frac{dE}{dx} \right)_{\text{tot}} = \left( \frac{dE}{dx} \right)_{\text{coll}} + \left( \frac{dE}{dx} \right)_{\text{rad}}.
\]  

A parameter of significant importance for electrons is the radiation length, \( X_0 \), which (PR92) is the mean thickness of material over which a high energy electron loses all but \( 1/e \) of its energy by bremsstrahlung and is the approximate scale length for describing high-energy electromagnetic cascades. This parameter also plays a role in the "scaling" of multiple scattering for all charged particles. This parameter is approximated by:

\[
X_0 = \frac{716.4 \text{ g cm}^{-2} \text{A}}{Z(Z + 1)\ln(287/\sqrt{Z})}
\]  

where \( Z \) and \( A \) are the atomic number and weight of the material medium.

It turns out for high energy electrons that:

\[
\left( \frac{dE}{dx} \right)_{\text{rad}} = -\frac{E}{X_0},
\]  

so that under these conditions (where ionization can be neglected)

\[
E(x) = E_0 e^{-x/X_0}
\]

where the energy of the incident particle is \( E_0 \).

Figure 1.10 taken from (SW79) gives the percentage of \( E_0 \) which appears as radiation for various materials as a function of energy. External bremsstrahlung develops as a function of target thickness and is described by a "transition" curve. As the thickness increases, the radiation increases until reabsorption begins to take effect. Then, self-shielding begins to take over. One talks about the maximum as a "thick-target" bremsstrahlung spectrum. This can be used as a basis for conservative assumptions related to quantities of radiological concern. Figure 1.11 from (Sw79) shows the behavior for a high-Z target. This type of behavior has been developed into three "rules of thumb" by Swanson in (Sw79).
Chapter 1 Composition of Accelerator Radiation Fields

Fig. 1.11 Thick-target bremsstrahlung from a high-Z target. Absorbed dose rates at 1 m as a function of incident electron energy $E_0$. The dashed line at $0^\circ$ represents a reasonable extrapolation of the measured values. The dose rates measured in the sideward direction (smoothed for this figure) depend strongly on target and detector geometry and vary by more than a factor of two. The dashed line at $90^\circ$ represents the more penetrating radiation component to be considered in room shielding. [Reproduced from (Sw79).]

Fig. 1.12 Radiation yield (or bremsstrahlung efficiency) for electrons stopped in various materials. Fraction (in per cent) of kinetic energy as a function of incident energy $E_0$. The remainder is transferred to the medium by ionization. [Reproduced from (Sw79).]
These "Rules of Thumb" parameterize this behavior for the absorbed dose rates, $D$, at 1 meter and normalized to one kW of incident beam power:

Rule of Thumb 1:

$$D = [(Gy\cdot h^{-1})(kW\cdot m^{-2})^{-1}] = 20 E_0^2 \ \text{at } \theta = 0^\circ, E_0 < 15 \text{ MeV.} \quad (1.25)$$

Rule of Thumb 2:

$$D = [(Gy\cdot h^{-1})(kW\cdot m^{-2})^{-1}] = 300 E_0 \ \text{at } \theta = 0^\circ, E_0 > 15 \text{ MeV.} \quad (1.26)$$

Rule of Thumb 3:

$$D = [(Gy\cdot h^{-1})(kW\cdot m^{-2})^{-1}] = 50 \ \text{at } \theta = 90^\circ, E_0 > 100 \text{ MeV.} \quad (1.27)$$

One can scale to other distances by using the "inverse square" law. It should be noted that one can get higher dose rates at 90$^\circ$ in certain circumstances due to softer radiation components. The forward intensity is a slowly varying function of target material except at very low Z. The angular width, $\theta_{1/2}$, of the forward lobe (half-intensity) is approximately given by the relation:

$$E_0 \theta_{1/2} = 100 \ (\text{MeV degrees).} \quad (1.28)$$

This is displayed graphically in Fig. 1.12 taken from (Sw79).

---

**Fig. 1.12** Angular distribution of bremsstrahlung intensity from high-Z targets (relative units), plotted as a function of the variable $E_0 \theta$. [Reproduced from (Sw79).]
Figure 1.13 taken from Ref. 4 shows bremsstrahlung spectra at $\theta = 0^\circ$ for electrons incident on a high-Z material of intermediate thickness at a variety of energies.

Fig. 1.13  Bremsstrahlung spectra measured at $\theta = 0^\circ$ from intermediate-thickness (0.2 $X_0$) targets of high-Z material. The data points are measurements. [Reproduced from (Sw79).]
Spectra of bremsstrahlung photons emerging in various directions from thick tungsten targets irradiated by monoenergetic electron beams, normally incident. The target thickness is 2 or twice the mean electron range. The arrows indicate position annihilation radiation at 0.511 MeV. (a) Kinetic energy 30 MeV, target thickness = 24.8 cm. (b) Kinetic energy 60 MeV, target thickness = 33.8 cm.

Reproduced from (SW79).

Note: The prominent peak which corresponds to positron annihilation at 0.511 MeV electrons is also visible in the spectra at various angles.

Figure 1.4. From (SW79) shows typical spectra for 30 and 60 MeV electrons at various angles.
Chapter 1  Composition of Accelerator Radiation Fields

Synchrotron radiation

Reference (Sw90) presents a summary discussion of this important phenomenon. The movement of electrons in a circular orbit results in their centripetal acceleration. This gives rise to emission of photons and has been treated in much more detail and completeness by others.

At nonrelativistic energies, this radiation is largely isotropic. However, at relativistic energies, the photons emerge in a tight bundle along a tangent to any point on a circular orbit. Figure 1.15 taken from (Sw90) shows this bundle:

\[
\theta_c = \frac{1}{\gamma} = \sqrt{1 - \beta^2} \text{ radians.} \quad (1.29)
\]

The median energy of the power spectrum, \( \varepsilon_c \), is given in terms of the total energy, \( W \) (GeV) \([\gamma m_0 c^2]\), and bending radius, \( \rho \) (meters) by (Sw90):

\[
\varepsilon_c = 2.218 W^3/\rho \text{ (keV). [For protons, multiply by (me/mp)^3.]} \quad (1.30)
\]

From (Sw90), the radiated power, \( P \) (watts) for a circulating electron current, \( I \) (milliamperes) is

\[
P = 88.46 W^4 I/\rho. \text{ [For protons, multiply by (me/mp)^4.]} \quad (1.31)
\]

Figs. 1.16 and 1.17 taken from (Sw90) (and citations therein) give the universal radiation spectrum and calculations for high energies. These calculations were done in the course of the development of the LEP (Large Electron Positron) collider at CERN.
Chapter 1  Composition of Accelerator Radiation Fields

Fig. 1.16  Universal synchrotron radiation spectrum. The dimensionless quantity $G_2$ gives the relative power as a function of photon energy in units of characteristic energy, $e_0$. [Reproduced from (Sw83).]

Fig. 1.17  Primary synchrotron radiation spectrum at three high energies. [Reproduced from (Sw83).]
Chapter 1 Composition of Accelerator Radiation Fields

Neutrons

Several basic physical mechanisms have been described in (Sw79). The dominant one at electron machines, especially for kinetic energies $E_0 < 150$ MeV is that of photonuclear reactions; that is reactions in which a photon absorbed by a nucleus creates an excited nuclear state which subsequently decays by emitting a neutron. [A ($\gamma$, n) nuclear reaction as written in the scheme of notation in which the first symbol in the parentheses represents the incoming particle in a reaction while the second represents the outgoing particle.]

The total neutron yields and neutron energy spectra are typified by Figs. 1.18 and 1.19 taken from (Sw79). Note that saturation (normalized to beam power!) tends to occur at $E_0 = 100$ MeV. (Sw79) and (Sc90) contain more details about such scaling.

Because of the nature of the ($\gamma$,n) reaction, these neutron fields are nearly isotropic and the inverse square law may be used to estimate the flux density at any given distance, $r$. There is actually a slight enhancement at $\theta = 90^\circ$ of about a factor of 1.5. The production of these neutrons chiefly is influenced by giant resonances in the target nuclei. These resonances are nuclear excited states having very broad widths in energy. These states are excited by the photons and some finite time later decay by emitting neutrons. The yields of neutrons are approximately proportional to the beam power loss (and hence independent of energy) at high energies and isotropically distributed. Photoneutron energy spectra, $dN/dE_n$ fall rapidly as a function of neutron energy, typically as

$$\frac{dN}{dE_n} = E_n^{-\alpha} \text{ where, approximately, } 1.7 < \alpha < 3.6.$$  \hspace{1cm} (1.32)

The slope becomes steeper as $E_0$, the kinetic energy of the incident electron, is approached.

Table 1.3 taken from (Sc90) displays the following table of values for yields of giant resonance neutrons per watt of beam power ($s^{-1}W^{-1}$), the yield per GeV per sr ($Y_n$ GeV$^{-1}$ sr$^{-1}$) [measured and calculated], and a recommended dose equivalent source term (Sv cm$^2$ GeV$^{-1}$). The last column would be used in the following equation:

$$H = \frac{S}{r^2} E_0 I,$$  \hspace{1cm} (1.33)

where $H$ is the dose equivalent in Sieverts, $r$ is the radial distance from the target in cm, $E_0$ is in GeV, and $I$ is the total beam particles incident (e.g., in some time interval).

For $E_0 > 150$ MeV other, more complicated mechanisms come into play such as the quasi-deuteron effect (important in $30 < E_0 < 300$ MeV) and photopion reactions ($E_0 > 300$ MeV). The quasi-deuteron effect is so-named because for $E_0 > 30$ MeV the photon wavelength is in resonance with the average inter-nucleon distance so that the photon interactions tend to occur with "pairs" of nucleons. Only neutron-proton pairs have a nonzero electric dipole moment, which makes interactions of photons with such pairs (pseudo-deuterons) favorable.
Fig. 1.18 Neutron yields from infinitely thick targets per kW of electron beam power as a function of electron beam energy $E_0$, disregarding target self-shielding. [Reproduced from (SW79).]
Fig. 1.19  Photoneutron spectra produced at $\theta = 90^\circ$ by electrons of energy $E_0 = 150, 170, 182, 202, 235,$ and 266 MeV, incident on a thick lead target (4.3 $X_0$). The solid lines are predictions of a quasi-deuteron model. [Reproduced from (Sw79). See references cited therein.]
Table 1.3  Yields and source terms of giant resonance neutrons in an optimum target geometry. [Reproduced from (Sc90) as adapted from references cited therein.]

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<td>Yield per GeV, steradian and electron $\gamma_n$ GeV$^{-1}$ sr$^{-1}$</td>
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</table>

[Swa79b] All calculations at electron energies of 500 MeV or 1 GeV.
[Bat67b] Measurements at 6.3 GeV with indium in a moderator. In the case of copper the source term for neutrons up to 25 MeV is 2.8E-2 GeV$^{-1}$ sr$^{-1}$.
[DeS68] Measurement at 7 GeV with indium in a moderator.
[Han75] Electrons on tantalum and lead targets at 100 MeV.
[Als73] Calculation at 400 MeV.
[Ste83] Long-counter measurements: 2.7E-2 at 50 GeV, 3.2E-2 at 80 GeV and 3.6E-2 at 100 GeV.

*) In order to obtain source terms in $Sv cm^2 h^{-1} kW^{-1}$ the values have to be multiplied by 2.25E16.

**) The value for aluminium is recommended also for concrete.
Chapter 1 Composition of Accelerator Radiation Fields

Interactions in which the production of other elementary particles, perhaps best typified by pions, becomes energetically possible at still higher energies. These pions can then produce neutrons through secondary interactions as will be discussed in Chapter 3. The literature has very little on the yield values for such particles tailored to the needs of radiation dosimetry. H. DeStaebler of SLAC (De65) has parameterized the yield of high energy particles per GeV, steradian, and electron (taking experimental results into account):

\[ Y_n = \frac{7.5 \times 10^{-4}}{(1 - 0.75 \cos \theta)^2 A^{0.4}} \]  

(1.34)

where A is the atomic mass (g/mol) of the target material. It is reasonable to use a dose equivalent conversion factor of \( = 1 \times 10^{-13} \text{ Sv m}^2 \) for these neutrons.

Muons

With electron beams, muons become significant above an electron energy of approximately 211 MeV (the "di-muon" rest mass) by the pair production process in which a \( \mu^+, \mu^- \) pair results. They can, at much smaller fluxes, be produced by the decay of \( \pi^\pm \) and \( K^\pm \) which are, in turn, due to secondary production processes. Such decay muons will be discussed in more detail later. [The muon rest energy is 105.7 MeV, its meanlife \( \tau = 2.19 \times 10^{-6} \text{ s} \) and \( c\tau = 658.6 \text{ m} \).] These particles are highly forward peaked. Figures 1.20 and 1.21 taken from (Sw79) give the muon flux densities as a function of energy and at various energies and angles as well as the peak flux density at \( \theta = 0^\circ \). The reasonableness of scaling with energy to larger values of \( E_0 \) is well-demonstrated.

The flux density to dose equivalent conversion factor has been found by Stevenson [(St73), quoted in (Sw90)] to be 40 fSv m\(^2\) (25000 muons cm\(^{-2}\) per mrem) for 100 MeV < \( E_\mu \) < 200 GeV. [At lower energies range-out of muons in the body with consequential higher energy deposition gives a conversion factor of 260 fSv m\(^2\) (3850 muons cm\(^{-2}\) per mrem)].

A detailed theoretical treatment of muon production by incident electrons from a dosimetric perspective is given in (Ne68) and (Ne74).

Muons have very long mean ranges as shown in Fig. 1.22 taken from (Sw90). At high energies (> 100 GeV), range straggling becomes severe (Va87). Also, above a critical energy for muons of several hundred GeV (in, say, iron), radiative losses begin to dominate such that:

\[ -\frac{dE}{dx} = a(E) + b(E) E \]  

(1.35)

where \( a(E) \) is the collisional \( dE/dx \) and \( E \) is in GeV. Obviously, the range-energy relation of muons and considerations related to their energy loss mechanisms is relevant to shielding against muons regardless of the origin of the muons. The results presented here will thus be relevant to further discussion in this chapter and in Chapter 3.

\(^2\)The handwritten factor of \( 1 \times 10^5 \) is applied to the left-hand axis of Fig. 1.21 to correct a longstanding error that has been propagated through several publications. This correction was verified by a private communication between the author and W. R. Nelson.
Chapter 1  Composition of Accelerator Radiation Fields

For this equation, \( a(E) = 0.002 \text{ GeV/gm cm}^{-2} \) and \( b(E) \) is the radiative coefficient for \( E \) in GeV in Fig. 1.23 taken from (Sw90). The total \( dE/dx \) is also given in Fig. 1.24 taken from (PR92). The mean range is approximated by

\[
x_0 = \frac{1}{b} \ln(a + bE_0),
\]

where \( E_0 \) is the kinetic energy of the muon, not the incident electron.

![Graphs showing integral muon flux density](image)

Fig. 1.20  Integral muon flux density at 1 meter per unit electron beam power, versus fractional muon energy, \( E/E_0 \), for electron energies \( E_0 \) incident on a thick iron target. These data are normalized to 1 kW beam power, 1 meter from the target. [Reproduced from (Sw79), adapted from (Ne68).]
Fig. 1.21  Muon production at $\theta = 0^\circ$ from an unshielded thick iron target, as a function of electron energy, $E_o$. Left-hand scale: muon flux density at 1 meter per unit electron beam power (${(\text{cm}^2 \cdot \text{s}^{-1}) (\text{cm}^2 \cdot \text{s}^{-1})}$). Right-hand scale: unshielded dose-equivalent rate normalized to 1 meter per unit electron beam power (${(\text{rem} \cdot \text{h}^{-1}) (\text{kW} \cdot \text{m}^{-2})}$). [Reproduced from (Sw79), adapted from (Ne68).]

Fig. 1.22 Range-energy curves for muons in various materials. [Reproduced from (Sw83).]
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Fig. 1.23 Contributions to the fractional energy loss by muons in iron due to $e^+e^-$ pair production, bremsstrahlung, and photonuclear interactions. [Reproduced from (PR92), adapted from references cited therein.]

![Graph showing contributions to energy loss by muons in iron due to various processes.]

Fig. 1.24 The average energy loss of a muon in hydrogen, iron, and uranium as a function of muon energy. Contributions to $dE/dx$ in iron from ionization and the processes shown in Fig. 1.23 are also shown. [Reproduced from (PR92), adapted from references cited therein.]

![Graph showing the average energy loss of a muon in different materials.]

Muon range straggling (Va87) is chiefly due to the fact that, above 100 GeV, electron-positron pair production, bremsstrahlung, and deep inelastic nuclear reactions become the dominant energy loss mechanisms. The cross sections for the latter two mechanisms are such that only a few interactions can be expected. Although these processes have low probability, when they do occur they involve large energy losses and thus have quite significant effects.

Tables 1.4 and 1.5 below give fractional energy loss and comparisons of muon ranges at high energies, as taken from (Sc90) and derived from (Va87). The results of (Va87) illustrated in Fig. 1.25 taken from (Sc90) show this phenomena for muons incident on a soil shield having a density of 2.24 g cm$^{-3}$. At the higher energies the effect is very important since shielding calculations based upon using the mean range values can lead to significant underestimates of the number of muons which can penetrate the shield.
Table 1.4 Fractional energy loss of muons [(Sc90) adapted from (Va87)] in soil ($\rho = 2.0 \text{ g cm}^{-3}$). The fractions of the total energy loss due to the four dominant energy loss mechanisms are given.

<table>
<thead>
<tr>
<th>$E$ GeV</th>
<th>Ionization</th>
<th>Bremsstrahlung</th>
<th>Pair production</th>
<th>Deep inelastic nuclear interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.972</td>
<td>0.037</td>
<td>8.8E-04</td>
<td>9.7E-04</td>
</tr>
<tr>
<td>100</td>
<td>0.888</td>
<td>0.086</td>
<td>0.020</td>
<td>0.0093</td>
</tr>
<tr>
<td>1000</td>
<td>0.580</td>
<td>0.193</td>
<td>0.168</td>
<td>0.055</td>
</tr>
<tr>
<td>10000</td>
<td>0.167</td>
<td>0.335</td>
<td>0.388</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Table 1.5 Comparison of muon ranges (meters) in heavy soil ($\rho = 2.24 \text{ g cm}^{-3}$) [(Sc90) adapted from (Va87)]

<table>
<thead>
<tr>
<th>Energy</th>
<th>Calculations of Van Ginneken [Van 87]</th>
<th>Mean Ranges calculated from $dE/dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Range</td>
<td>standard deviation</td>
</tr>
<tr>
<td>10 GeV</td>
<td>22.8</td>
<td>1.6</td>
</tr>
<tr>
<td>30 GeV</td>
<td>63.0</td>
<td>5.6</td>
</tr>
<tr>
<td>100 GeV</td>
<td>188</td>
<td>23</td>
</tr>
<tr>
<td>300 GeV</td>
<td>481</td>
<td>78</td>
</tr>
<tr>
<td>1 TeV</td>
<td>1140</td>
<td>250</td>
</tr>
<tr>
<td>3 TeV</td>
<td>1970</td>
<td>550</td>
</tr>
<tr>
<td>10 TeV</td>
<td>3080</td>
<td>890</td>
</tr>
<tr>
<td>20 TeV</td>
<td>3730</td>
<td>1070</td>
</tr>
</tbody>
</table>
Fig. 1.25 Longitudinal distribution of monoenergetic muons stopping in heavy soil (2.24 g cm$^{-3}$). $M$ is the muon density (m$^{-1}$), $D$ is the depth of penetration (meters). [Reproduced (Sc90), adapted from (Va87).] a) $E_0 = 10$ GeV, b) $E_0 = 30$ GeV, c) $E_0 = 100$ GeV, d) $E_0 = 300$ GeV.
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Fig. 1.25 -Continued  e) $E_0 = 1$ TeV,  f) $E_0 = 3$ TeV,  g) $E_0 = 10$ TeV,  h) $E_0 = 20$ TeV.
Summary

In (Sw79), Swanson provided the content of Fig. 1.26 which illustrates the broad features of the radiation field due to the interactions of electrons with no shielding. This figure is useful for making crude estimates of the resultant radiation field. As one can see, at all angles, from the standpoint of dose equivalent, the unshielded field is always dominated by photons. At small angles, the field is dominated by photons with muons as the next most important ingredient at TeV energies.

![Graph showing dose-equivalent rates per unit primary beam power, produced by various types of "secondary" radiations from a high-Z target as a function of primary beam energy, if no shielding were present.](image)

**Fig. 1.26** Dose-equivalent rates per unit primary beam power, produced by various types of "secondary" radiations from a high-Z target as a function of primary beam energy, if no shielding were present (qualitative). The width of the bands suggests the degree of variation found, depending on such factors as target material and thickness. [Reproduced from (Sw79).]
V. Radiation Production by Proton Accelerators  *(Much of the material in this section is taken from (NC96) and the work referenced therein.)*

**The Direct Beam**

Direct beams at proton accelerators, from the dosimetric standpoint, nearly always dominate over any type of secondary phenomena since the beam current is generally concentrated into small dimensions. Figure 1.7 gives the fluence to dose equivalent conversion factor as a function of proton energy. The physical reason that the conversion factor shows such a prominent transition at about 200 MeV is that below that energy the proton range is less than the thickness of the human body. Hence as the energy is increased above 200 MeV, the energy largely escapes from the body so that it requires a far larger fluence of protons to deliver the same dose equivalent.

As the energy of a proton beam increases, the range of the protons increases to where the probability of the proton interacting before it has lost all of its energy due to ionization in a target gradually becomes significant. Klaus Tesch of HERA/DESY has illustrated this point in Fig. 1.27 taken from (Te85) for various materials and energies.
Neutrons (and other hadrons at high energies)

$E_0 < 10 \text{ MeV}$:

For nuclear reactions, the $Q$-value, $Q_v$, is defined in terms of the masses, $m_i$,

$$Q_v = [(m_1 + m_2) - (m_3 + m_4)]c^2$$ (1.37)

for nuclear reaction $m_1 + m_2 \rightarrow m_3 + m_4$. [In general such reactions are denoted $m_2(m_1,m_3)m_4$.] $Q_v > 0$ implies an exothermic nuclear reaction. Endothermic ($Q_v \leq 0$) reactions are characterized by a threshold energy, $E_{th}$, given by:

$$E_{th} = \frac{m_1 + m_2}{m_2} |Q_v|.$$ (1.38)

Fig. 1.28 Total neutron yield per proton for different target materials. [Reproduced from (Te85)].
Below 10 MeV, (p,n) reactions are important for some materials because these reactions commonly have very low thresholds (< 5 MeV). Many features are highly dependent upon the details of the structure of the target nuclei and are often highly dependent upon the target element, angle, and energy. For example, $^7$Li(p,n)$^7$Be has a threshold of 1.9 MeV and the total cross section, $\sigma$, quickly rises to a value of 300 mb.

For protons having kinetic energies, $E_0$, ranging from approximately 10 MeV up to the very highest energies, neutrons are usually the dominant feature of the radiation field that results from their interactions. At these energies, the yields are smoother functions of energy due to the lack of resonances, but are also more forward-peaked. Tesch (Te85) has summarized the total yields per incident proton for different materials as a function of energy in Fig. 1.28 taken from (Te85). In this figure these curves agree with the original primary data to within about a factor of two. An important feature is that for $50 < E_0 < 500$ MeV, $Y \propto E_0^2$ while for $E_0 > 1$ GeV, $Y \propto E_0$.

$10 < E_0 < 200$ MeV

In this region there are extensive angular distribution data as a result of nuclear physics research. The general features is that the distributions are forward-peaked. Representative examples are given in Figs. 1.29 and 1.30 taken from Nakamura (Na78) for 52 MeV protons and from Alsmiller (Al75) and Hagan (Ha88) for 200 MeV protons, respectively. The fluence above a 5 MeV threshold is plotted in Fig. 1.29 while yields are plotted in Fig. 1.30.
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Fig. 1.30 Calculated energy spectra of neutrons emitted by water, iron, and aluminum targets bombarded by 200 MeV protons for four ranges in $\theta$. The iron and water calculations are from (Ha81) while the aluminum results are from...
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200 MeV < $E_0$ < 1 GeV; ("intermediate" energy):

In this region, many more reaction channels become open and the number of protons emitted gradually becomes approximately equal to the number of neutrons. In fact, at the highest energies for such unshielded conditions, the radiation effects of protons and neutrons are essentially identical and both must be taken into account. Thus reliance on the Tesch yield curve in Fig. 1.28 could underestimate radiation effects by as much as a factor of two.

$E_0 > 1$ GeV ("high" energy region):

In this region, both the calculations and measurements become much more difficult. Often, "threshold" detectors are used to detect neutrons above some reaction threshold energy. Figures 1.31 (Gi68), 1.32 (Gi68), 1.33 (Ra72), and 1.34 (St85) show representative data at 14, 26, 22, and 225 GeV, respectively. In Figs. 1.31 and 1.32, the parameter $g(\theta)$ is the integral of $d^2Y/d\Omega dE$ above such a designated threshold energy. These should be regarded as thin target values. "Thin" target in this context means a target shorter than the removal mean free path for high energy protons. Table 1.6 summarizes common removal mean free paths.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>DENSITY (grams/cm³)</th>
<th>REMOVAL MEAN FREE PATH (grams/cm²)</th>
<th>REMOVAL MEAN FREE PATH (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hydrogen gas</td>
<td>9.00 X 10⁻⁵</td>
<td>43.3</td>
<td>4.81 X 10⁵</td>
</tr>
<tr>
<td>beryllium</td>
<td>1.85</td>
<td>55.5</td>
<td>30.03</td>
</tr>
<tr>
<td>carbon</td>
<td>2.27</td>
<td>60.2</td>
<td>26.58</td>
</tr>
<tr>
<td>aluminum</td>
<td>2.70</td>
<td>70.6</td>
<td>26.15</td>
</tr>
<tr>
<td>iron</td>
<td>7.87</td>
<td>82.8</td>
<td>10.52</td>
</tr>
<tr>
<td>copper</td>
<td>8.96</td>
<td>85.6</td>
<td>9.55</td>
</tr>
<tr>
<td>lead</td>
<td>11.35</td>
<td>116.2</td>
<td>10.24</td>
</tr>
<tr>
<td>uranium</td>
<td>18.95</td>
<td>117.0</td>
<td>6.17</td>
</tr>
<tr>
<td>air</td>
<td>1.29 X 10⁻³</td>
<td>62.0</td>
<td>4.81 X 10⁴</td>
</tr>
<tr>
<td>water</td>
<td>1.00</td>
<td>60.1</td>
<td>60.10</td>
</tr>
<tr>
<td>concrete (typical)</td>
<td>2.50</td>
<td>67.4</td>
<td>26.96</td>
</tr>
<tr>
<td>silicon dioxide</td>
<td>2.64</td>
<td>67.0</td>
<td>25.38</td>
</tr>
<tr>
<td>(quartz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>plastics</td>
<td>0.93</td>
<td>56.9</td>
<td>61.51</td>
</tr>
<tr>
<td>(polyethylene)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The angular distribution \( g(\theta) = \frac{d\Gamma}{d\Omega} \) of neutrons above 600 MeV produced by 14 and 26 GeV protons on a thin beryllium target.

Reproduced from (Gio88).

Fig. 1.32

Fig. 1.31

The angular distribution \( g(\theta) \) of neutrons above 20 MeV produced by 14 and 26 GeV protons on a thin beryllium target.

Reproduced from (Gio88).
Fig. 1.34 Comparison of the experimental (open symbols) and calculated (+) hadron fluences above different energy thresholds as a function of polar angle θ around a 15 cm long copper target bombarded by 225 GeV protons. The data have been multiplied by the indicated factors prior to plotting and represent hadrons/incident proton steradian. [Reproduced from (St85)].

Fig. 1.33 Comparison of calculated and measured angular distributions of hadron fluxes (particles cm⁻²) at 100 cm from a copper target bombarded by 22 GeV protons. Several choices of hadron energy thresholds are shown. [Reproduced from (Ra72)].
Anthony Sullivan of CERN (Su89) has developed a simple formula for the angular distribution of fluence, $\Phi(\theta)$ (cm$^{-2}$), of hadrons with $E_0 > 40$ MeV at one meter from a copper target struck by protons in the energy region $5 < E_0 < 500$ GeV per interacting proton:

$$\Phi(\theta) = \frac{1}{2\left[\frac{\theta}{35\sqrt{E_0}}\right]^2}$$

(1.39)

where $E_0$ is in GeV and $\theta$ is in degrees.

This formula also adequately accounts for the distributions of neutrons per incident proton produced by protons in the region of incident proton energy $0.025 < E_0 < 1$ GeV if it is multiplied by, approximately, a factor of two. This equation can be plotted as in Fig. 1.35, taken from the preprint of (Su89), in the "lateral" ($\theta = 90^\circ$) and "forward" ($\theta = 0^\circ$) directions.

Fig. 1.35  Flux of hadrons exceeding 40 MeV in energy, per interaction, at 1 meter from the target in both the forward ($\theta = 0^\circ$) and sideways ($\theta = 90^\circ$) direction as a function of the interacting proton. The proton is interacting in a copper target. [Reproduced from the preprint of (Su89).]
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Of course, the dose equivalent is often more important to know than is the "raw" fluence. In principal, the dose equivalent can be obtained by integrating thus:

\[ H = \int_0^{E_{\text{max}}} P(E) \Phi(E) dE, \]  

or by summation, taking into account the "coarseness" of available data and/or calculations:

\[ H = \sum_{j=1}^{m} P_j(E) \Phi_j(E)(\Delta E)_j. \]  

Tesch (Te85) has done this to obtain the dose equivalent at 1 meter from a copper target (\( \theta = 90^\circ \)) bombarded by protons of various energies. The result is plotted in Fig. 1.36 taken from (Te85).

![Graph showing dose equivalent per proton due to neutrons at \( \theta = 90^\circ \) with energies higher than 8 MeV at a distance of 1 meter from a copper target. Reproduced from (Te85).]
Fig. 1.37 Plots of absorbed dose per interacting proton at a distance of one meter from a 5 cm long copper target bombarded by 24 GeV/c (left) and 8 GeV/c (right) protons taken from Le72. The filled symbols are measurements while the curve is a theoretical prediction. Multiply the plotted values by 0.28 for the approximate results per incident proton.

The experimental results are normalized to the number of interacting protons incident on a Cu target, those are given in Fig. 1.37. The results are taken from Le72 (1972) and obtained experimental data on the angular distribution of absorbed dose for 8 GeV/c protons incident on a Cu target.

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Muons

Muons at proton accelerators arise from two principal mechanisms. Production by pion and kaon decay are outlined as follows where mass of the parent particles, the branching ratio (the percentage of time the parent particle decays by the reaction given), the meanlife, and the value of ct (PR92) are also given.

\[ \pi^\pm \rightarrow \mu^\pm + \nu_\mu \quad ; \quad m_\pi = 139.6 \text{ MeV}, \quad \tau = 2.6 \times 10^{-8} \text{ s}, \quad (99.99 \% \text{ branch}), \quad (c\tau = 7.804 \text{ m}) \]

\[ K^\pm \rightarrow \mu^\pm + \nu_\mu \quad ; \quad m_{K} = 493.6 \text{ MeV} \quad \tau = 1.2 \times 10^{-8} \text{ s}, \quad (63.51 \% \text{ branch}), \quad (c\tau = 3.709 \text{ m}) \]

The other important muon production mechanism associated with incident protons is the so-called "direct" muon production. These will discussed in more detail in Chapter 3.

At proton and ion accelerators, thus, the production of muons is usually dominated by a tertiary effect due to the decay of secondary particles. Muon fields are forward-peaked and, normally, dominated by those from pion decay (except, perhaps at the highest energies). Usually, Monte-Carlo techniques are needed to accurately estimate muon intensities. This is because of the need to:

A. calculate the production of pions from the proton interactions
B. follow the pions until they decay or interact
C. adequately account for the range-energy relation and range straggling
D. track the muons to the point of interest.

A full discussion of muon production and shielding must await Chapter 3.
VI. Primary Radiation Fields at Ion Accelerators

Because the ionization range for ions of a given kinetic energy decreases as a function of ion mass, targets become effectively "thicker" as the ion mass increases.

Light ions (ion mass number, $A < 5$)

For such ions there are "special case" exothermic reactions to be concerned with. Noteworthy examples (followed by their reaction Q-values, $Q_v$, in parentheses) are:

- $^3\text{H}(d,n)^4\text{He}$ ($Q_v = 17.586$ MeV)
- $^9\text{Be}(\alpha,n)^{12}\text{C}$ ($Q_v = 5.708$ MeV)
- $^2\text{D}(d,n)^3\text{He}$ ($Q_v = 3.266$ MeV)

In some cases monoenergetic beams of neutrons are possible using these or the following slightly endothermic reactions:

- $^1\text{H}(d,n)^{13}\text{N}$ ($Q_v = -0.281$ MeV)
- $^3\text{T}(p,n)^3\text{He}$ ($Q_v = -0.764$ MeV)
- $^7\text{Li}(p,n)^7\text{Be}$ ($Q_v = -1.646$ MeV).

The energies of such neutrons can range from 0 to 27 MeV for bombarding energies up to 10 MeV.

In general, deuteron stripping reactions [(d,n)] have the highest yields because the binding energy of the deuteron is only 2.225 MeV. (One gets an extra neutron "for free"!). This phenomena is especially pronounced at the lower energies. In the low energy region, and especially with light ions, one should carefully consider all possible reactions given the materials present in conjunction with the ions that are being accelerated.

Figure 1.38 taken from (Pa73) gives examples of typical light ion yield results.
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Fig. 1.38 Plots of total neutron yields for various light ions on a number of materials. One sec-μA (1 micro-Coulomb of electric charge) corresponds to 6.25 × 10^{12} incident protons, 3.125 × 10^{12} incident α^{++} ions, etc. [Reproduced in (Pa73), adapted from references cited therein.]
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**heavier ions (ions with \( A > 4 \))**

At higher energies and especially at higher masses, neutron yield and dose equivalent data and calculations are very sparse. The data often is normalized in terms of kinetic energy per atomic mass unit (specific energy, usually expressed in units of MeV/amu), or kinetic energy per nucleon because reaction parameters generally scale to that parameter. In the literature the technical distinction between energy/amu and energy/nucleon is often ignored. In the range up to 20 MeV/amu, this is illustrated by the Figs. 1.39 and 1.40 taken from (NC96) [adapted from (Hu60) & Oh80]) for both yield and dose equivalent for targets slightly thicker than the particle range:

[Diagram 1.39: Graph showing neutron yield vs. specific energy for different targets]

[Diagram 1.40: Graph showing dose equivalent vs. specific energy for different targets]

---

Fig. 1.39: Neutron yield vs. specific energy for different targets.

Fig. 1.40: Dose equivalent vs. specific energy for different targets.
Fig. 1.41 Comparison between measured and calculated yields per $10^{11}$ ions s$^{-1}$ at 86 MeV/amu $^{12}$C ions incident on an iron target for neutron energies below 20 MeV. Activation detectors with the following sensitive regions in neutron energy, $E_n$, were used: moderated In foils ($0.4 < E_n < 107$ eV), $^{33}$S(n,p)$^{32}$P ($E_n > 3$ MeV), $^{27}$Al(n,α)$^{24}$Na ($E_n > 7$ MeV). Measurements were made at the various radial distances. [Reproduced from (Tu84).]

Fig. 1.42 Comparison between measured and calculated yields per $10^{11}$ ions s$^{-1}$ at 86 MeV/amu $^{12}$C ions incident on an iron target for neutron energies above 20 MeV using the $^{12}$C(n,2n)$^{11}$C reaction. Measurements were made at the indicated radial distances [Reproduced from (Tu84).]
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Clapier and Zaidins (C183) have surveyed the existing data from 3 to 86 MeV/amu and have been able to parameterize it. They found that the following fits the angular distribution of flux density:

\[ \phi(\theta, \xi) = \frac{1}{4\pi} \left\{ \frac{1}{\log(1 + 1/\xi)} \right\} \left\{ \frac{1}{\xi + \sin^2(\theta/2)} \right\} \]

where \( \theta \) is in degrees and the fitting parameter \( \xi \) is determined by

\[ \xi = \frac{\phi(90^\circ)}{\phi(0^\circ) - \phi(90^\circ)} = \frac{1}{\phi(0^\circ)/\phi(90^\circ) - 1} \]

and where \( \phi(\theta) \) is the value of the fluence or dose equivalent at \( \theta \).

These same authors have found that the total yield, \( Y \) (neutrons/ion) can be approximately fit as a function of the target atomic number, \( Z \), and the specific energy, \( W \) (MeV/amu). [Again, note the lack of dependence on projectile atomic number!]

The expressions which result are:

\[ Y(W,Z) = C(Z)W^{\eta(Z)} \quad \text{with} \]

\[ \eta(Z) = 1.22\sqrt{Z} \quad \text{and} \]

\[ C(Z) = \frac{1.95 \times 10^{-4}}{Z^{2.75}}\exp[-0.475 (\ln Z)^2]. \]

These authors have tabulated the values of the parameters \( C(Z) \) and \( \eta(Z) \) in Table 1.7.

Table 1.7  Values of the parameters \( \eta(Z) \) and \( C(Z) \) as expressed in (C183).

<table>
<thead>
<tr>
<th>Atomic Number</th>
<th>(element)</th>
<th>( \eta(Z) )</th>
<th>( C(Z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(hydrogen)</td>
<td>1.5</td>
<td>1.7 \times 10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>(helium)</td>
<td>2.6</td>
<td>3.9 \times 10^{-6}</td>
</tr>
<tr>
<td>6</td>
<td>(carbon)</td>
<td>2.7</td>
<td>2.5 \times 10^{-6}</td>
</tr>
<tr>
<td>8</td>
<td>(oxygen)</td>
<td>3.6</td>
<td>3.6 \times 10^{-7}</td>
</tr>
<tr>
<td>10</td>
<td>(neon)</td>
<td>7.0</td>
<td>2.7 \times 10^{-10}</td>
</tr>
<tr>
<td>18</td>
<td>(argon)</td>
<td>7.0</td>
<td>5.1 \times 10^{-11}</td>
</tr>
<tr>
<td>36</td>
<td>(krypton)</td>
<td>7.9</td>
<td>6.0 \times 10^{-12}</td>
</tr>
<tr>
<td>82</td>
<td>(lead)</td>
<td>11.0</td>
<td>1.7 \times 10^{-13}</td>
</tr>
</tbody>
</table>
They also give a few examples of the parameter, \( \xi \), in the expression for fitting the angular distribution. They report values of 0.07 for uranium incident on uranium at 9 MeV/amu, 0.025 for neutrons of energy \(< 20\) MeV produced by 86 MeV/amu \(^{12}\)C incident on iron, and \(3 \times 10^{-4}\) for neutrons of energy \(> 20\) MeV produced by 86 MeV/amu \(^{12}\)C incident on iron. Fig. 1.43 gives the results found. One could use values given in Table 1.7 taken from (C183) or the direct calculation and obtain some idea of the uncertainties inherent in this fit to such a broad range of data.

![Diagram](image)

**Fig. 1.43** Total neutron yields as a function of specific energy for a variety of ions. The shaded region is representative of the uncertainties in the associated parametric fit to the available data. [Reproduced from (C183).]
McCaslin, et al (McC85) measured the angular distribution of yields of 670 MeV/amu Ne and Si ions and obtained the following results:

For incident 670 MeV/amu $^{20}$Ne ions including all neutrons above 6.5 MeV at a radius of 1 meter, McCaslin found:

$$\phi(\theta) = 372\;\theta^{-1}\;\text{neutrons m}^{-2}/\text{ion}$$
(for $2^\circ < \theta < 180^\circ$, $\theta$ in degrees)  

(1.47)

For incident 670 MeV/amu $^{20}$Ne ions including all neutrons above 20 MeV:

$$\phi(\theta) = 248\;e^{-0.29}\;\text{neutrons m}^{-2}/\text{ion}$$
(for $0^\circ < \theta < 20^\circ$, $\theta$ in degrees)  
and

$$\phi(\theta) = 10\;e^{-0.0389}\;\text{neutrons m}^{-2}/\text{ion}$$
(for $20^\circ < \theta < 120^\circ$, $\theta$ in degrees).  

(1.48) \hspace{1cm} (1.49)

The neutron yields at this high specific energy for heavy ions turn out to be quite large.
Chapter 1  Composition of Accelerator Radiation Fields

References


(Gi68) W. S. Gilbert, et. al, "1966 CERN-LRL-RHEL shielding experiment at the CERN Proton Synchrotron", UCRL-17941 (1968). [Much of this material is summarized in (PA73)].


(IC73) Data for protection against ionizing radiation from external sources, Supplement to ICRP 15, ICRP Report #21 (1973)


Chapter 1 Composition of Accelerator Radiation Fields


(Si83) G. R. Stevenson, "Dose and dose equivalent from muons", Rep. TIS-RP/099, CERN, Geneva Switzerland (1983). Important conclusions from this work are presented in (Sc90).


(Su89) A. H. Sullivan, "The intensity distribution of secondary particles produced in high energy proton interactions", Rad. Prot. Dos. 27 (1989) 189-192. [Fig 1.35 was obtained from the preprint to this paper, CERN Report TIS-RP/209/PP (1988).


Chapter 1  Composition of Accelerator Radiation Fields-Problems

1. a) To how many GeV/s does 1 kW of beam power correspond?
   
b) To how many singly charged particles per second does 1 ampere of beam current correspond?
   
c) To how many GeV/kg of energy deposition does an absorbed dose of 1 Gy correspond?
   
2. Which has the higher quality factor, a 10 MeV (kinetic energy) α-particle or a 1 MeV neutron? Write down the quality factors for each particle.
   
3. Calculate the number of $^{12}$C and $^{238}$U atoms per cm$^3$ of solid material.
   
4. Calculate the velocity and momenta of a 200 MeV electron, proton, iron ion, π$^+$, and μ$^+$. The 200 MeV is kinetic energy and the answers should be expressed in units of the speed of light (velocity) and MeV/c (momenta). Iron ions have an isotope-averaged mass of 52021 MeV (A = 55.847 X 931.5 MeV/amu). The π$^+$ mass is 140 MeV and the μ$^+$ mass = 106 MeV. Do the same calculation for 20 GeV protons, iron ions, and muons. It is suggested that these results be presented in tabular form. Make general comments on the velocity and momenta of the particles at the two energies. (The table may help you notice any algebraic errors that you have made.)
   
5. Calculate the mass stopping power of a 20 MeV electron (ionization only) and a 200 MeV proton in $^{28}$Si.
   
6. An electron accelerator has a beam profile in the form of a 2 mm diameter circle uniformly illuminated by the beam. Make a crude plot of the value of the dose equivalent rate in the beam as the energy increases from 1 MeV to 10 GeV. The average beam current is 1 microamp (1 μA). Assume the beam profile is unchanged during acceleration. Compare with Swanson’s simple formula (“conservative” value). Is his formula “conservative” above 100 MeV? (Hint: use Fig. 1.9)
   
7. Calculate the critical energy and length of material that corresponds to the radiation length for carbon and for lead. What does this say about the effectiveness of low-Z versus high-Z shielding materials for electrons?
Chapter 1  Composition of Accelerator Radiation Fields-Problems

8. A 100 MeV electron accelerator produces a 1.0 μA beam incident on a high-Z (thick) target. Estimate the bremsstrahlung absorbed dose rates at \( \theta = 0^\circ \) and \( 90^\circ \) at \( r = 2 \) m from the target using Swanson’s rules of thumb. Compare the \( 0^\circ \) result with the “in the beam dose equivalent rate” found in problem 6. How do the bremsstrahlung and in-beam dose rates compare?

9. Suppose the Tevatron enclosure at Fermilab is converted into an enclosure for an electron synchrotron. The radius of the synchrotron is 1000 m. If the circulated beam is \( 10^{12} \) electron, calculate the median energy of the synchrotron radiation photons for \( E_0 = 100 \) GeV. Also find \( \theta_c \) of the “lobe.”

10. For the accelerator of problem 8, calculate the neutron flux density at \( r = 2 \) m at large angles using the values in Table 1.3 for a high-Z (tungsten) target. Also use Table 1.3 to estimate the dose equivalent \( r = 2 \) m. Check this result by “guessing” the average neutron energy is 1-10 MeV and use the curve in Fig. 1.6. Compare this neutron dose with the Bremsstrahlung dose at large angles obtained in problem 8.

11. Fig. 1.21 gives both muon flux density and muon dose equivalent rate at one meter at \( \theta = 0^\circ \) as a function of electron beam energy. From the figure determine the fluence-to-dose equivalent factor used to obtain the dose equivalent rate from the primary calculation of muon flux density. Compare with Stevenson’s result.

12. Calculate the muon fluence necessary to produce a dose equivalent of 1 mrem assuming a quality factor = 1 and that tissue is equivalent to water for minimum ionizing muons. (Hint: use Table 1.2) Could this explain the slight discrepancy noted in this conversion factor found in problem 11? How?

13. For a 20 GeV electron accelerator, the electron beam strikes a beam stop made of aluminum or iron. How long (in Z) does the beam stop have to be to range out the muon having the average energy (for both Al & Fe)? (Hint: use curves from Fig. 1.20 to calculate the average energy by looking at flux versus energy.) What will the relative dose rates be at the immediate downstream ends of each material? Assume multiple scattering is not important. Compare the dose rates at the immediate downstream ends of each material. (Assume the production of muons from Fe is approximately equal to that from Al. Recall the inverse square law.)
Chapter 1 Composition of Accelerator Radiation Fields-Problems

14. One can use measurement results to check Sullivan’s formula for hadron fluence above 40 MeV for high-energy proton interactions. Check the agreement for the 22 and 225 GeV/c data in Figs. 1.33 and 1.34 for 3 representative angles at one meter. (Ignore the fact that the formula is for hadrons > 40 MeV while the only data provided is for hadrons >35 MeV and 50 MeV but do not ignore the difference between normalizing to incident versus interacting protons.) (It is valid to make the comparison on yield per interacting proton since the results in Fig. 1.34 is for targets approximately 1 interaction length long.) Comment on the quality of the agreement.

15. Calculations can also be used to check the Tesch curve for dose equivalent at $\theta = 90^\circ$ (Fig. 1.36). Use the 200 MeV calculations in Fig. 1.30 to do this by crudely numerically integrating the $60^\circ < \theta < 90^\circ$ yields to determine the average energy of the neutrons and the total fluence at $\theta = 90^\circ$ and at 1 meter. Use the results along with fluence-to-dose equivalent rate curves (Fig. 1.6) to obtain the dose equivalent per proton to compare with Tesch’s result. (Iron is considered equivalent to copper for this problem.)

16. Use Tesch’s curve in Fig. 1.36 to calculate the dose equivalent rate at 2 m and $\theta = 90^\circ$ from a target struck by 1 $\mu$A of 100 MeV protons. Compare with the neutron dose rate calculated in problem 10 for an electron accelerator having the same intensity and beam energy and discuss. (Scale the relevant result of problem 10 by the appropriate yield for copper vs. Tungsten.)

17. It is often necessary to work from fragmentary data to determine other quantities.

   a) Use McCaslin’s results and appropriate fluence to dose factors to calculate the dose equivalent rate at 1 meter and at $\theta = 30^\circ$ for a target struck by $10^8$ 670 MeV/amu $^{20}$Ne ions per sec. (Hint: use all available spectrum information.)

   b) Use McCaslin's results to obtain the total yield of neutrons per ion with $E_n > 6.5$ MeV. Assuming the target to be iron or copper, how does this yield correspond to that due to 700 MeV protons? Do this for both $E_n > 6.5$ MeV and $E_n > 20$ MeV to understand the overall composition. (Hint: Integrate over the unit sphere (double integral over spherical coordinates $\theta & \phi$)
Chapter 1  Composition of Accelerator Radiation Fields-Problems

The following indefinite integrals are needed:

$$\int \frac{\sin x}{x} \, dx = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!}$$

$$\int e^{ax} \sin (bx) \, dx = \frac{e^{ax} [a \sin (bx) - b \cos (bx)]}{a^2 + b^2}$$

The elemental area on the sphere of radius $R$ is $dA = r^2 \sin \theta \, d\theta d\phi$.

(See figure below.)
In this chapter the major features of the shielding of electrons and photons at accelerators are described. It includes extensive discussion of the electromagnetic cascade and a discussion of the shielding of photoneutrons and high energy particles that result from these interactions. The chapter concludes with a treatment of the generalized shielding problem with specific attention given to the Monte-Carlo method.

I. The Electromagnetic Cascade-Introduction

The "prime mover" in shielding design at electron accelerators is the electromagnetic cascade. This would also be true were a muon accelerator to be built.

One should recall the definitions of radiation length, $X_0$, and critical energy, $E_c$, from Chapter 1;

$$X_0 = \frac{716.4 \text{ A}}{Z(Z+1)\ln(287/Z)} \text{ (g cm}^{-2})$$ (2.1)

and

$$E_c = \frac{800}{(Z+1.2)} \text{ (MeV)}.$$ (2.2)

Another parameter of importance (PR92) for describing the electromagnetic cascade is the Molière radius, $X_m$:

$$X_m = X_0 E_s/E_c$$ (2.3)

where $E_s = (\sqrt{4\pi/\alpha})m_e c^2 = 21.2$ MeV. (2.4)

[\alpha is the fine structure constant (see Table 1.1) and $m_e$ is the mass of the electron.]

It turns out that $X_m$ is a good characteristic length for radial distributions in electromagnetic showers. Two more scaling dimensionless variables are commonly introduced to describe electromagnetic shower behavior;

$$t = x/X_0 \quad \text{(distance)}$$ (2.5)

and $$y = E/E_c \quad \text{(energy)}.$$ (2.6)

As an aside, for mixtures of n elements (PR92) states that these quantities and the stopping power dE/dx scale according to the elemental fractions by weight, $f_i$, as follows:

$$\frac{dE}{dx} = \sum_{i=1}^{n} f_i \left(\frac{dE}{dx}\right) \{\text{all dE/dx in energy/g cm}^{-2}\}$$ (2.7)

$$\frac{1}{X_o} = \sum_{i=1}^{n} \frac{f_i}{X_{oi}} \{\text{all X}_o \text{ in g cm}^{-2}\} \text{ and}$$ (2.8)

$$\frac{1}{X_m} = \frac{1}{E_s} \sum_{i=1}^{n} \frac{f_i E_c}{X_{oi}}.$$ (2.9)
Another term used is that of the so-called "Compton minimum" which, as the term is generally used, is the energy at which the total photon cross section is at a minimum. [The use of this term is an unfortunate occurrence of technical "slang" since at the higher energies the Compton scattering cross section monotonically decreases with energy!] This value always occurs at energies less than $E_C$ and is typically a few MeV. For high energy photons ($E_\gamma > 1$ GeV), the total $e^+e^-$ pair production cross section, $\sigma_{\text{pair}}$, is approximately given, for a single element, by

$$\sigma_{\text{pair}} = \frac{7}{9} \left( \frac{A}{X_\gamma N_A} \right) \text{ (cm}^2\text{)},$$

(2.10)

where $A$ is the atomic weight, $N_A$ is Avagadro's number, and $X_\gamma$ is the radiation length expressed in units of g/cm$^2$.

For energies larger than a few MeV, the pair production process dominates the total photon attenuation. The interaction length for pair production, $\lambda_{\text{pair}}$, is given by

$$\lambda_{\text{pair}} = \frac{\rho}{N_\sigma} \text{ (g/cm}^2\text{)} = \frac{\rho}{\rho N_A} \frac{\frac{A}{9} \frac{X_\gamma}{N_A}}{A} = \frac{9}{7} X_\gamma,$$

(2.11)

where the symbols all have the same meanings as used in Chapter 1 and thus far in this chapter.

This result, along with the facts about photon production by electrons interacting in matter, leads to the most important fact about the electromagnetic cascade:

**The electrons radiatively produce photons with almost the same characteristic length for which the photons produce more $e^+e^-$ pairs.**

This is so important because as a first order approximation it means that the "size" in physical space is independent of energy. (For hadronic cascades, the result is considerably different!)

Figure 2.1 taken from (PR92) illustrates the photon cross sections for the various physical processes responsible for photon attenuation.
Chapter 2 Shielding of Electrons and Photons at Accelerators

Fig. 2.1 Contributions to photon cross sections in carbon and lead. (Reproduced from Ref. 92 and references cited therein.)
The Electromagnetic Cascade Process

In the most simple terms, the electromagnetic cascade at an electron accelerator proceeds qualitatively according to the following steps:

1. A high energy electron produces a high energy photon by bremsstrahlung.
2. This photon produces an $e^+ e^-$ pair after traveling, on average, a distance of $X_0$ (each member of the pair will have half the energy of the photon).
3. After traveling an average distance of $X_0$, each member of the $e^+ e^-$ pair will produce yet another bremsstrahlung photon.
4. Each electron or positron may continue on to interact again and release yet more photons before its energy is totally absorbed.

{This chain could equally well be initiated by a high energy photon from a hadron accelerator.}

Eventually, after a number of generations, the individual energies of the electrons and positrons will be degraded to values below $E_0$ so that ionization processes then begin to dominate and terminate the shower. Likewise, the photon energies eventually are degraded so that Compton scattering and the photoelectric effect compete with the further production of $e^+ e^-$ pairs.

Figure 2.2 taken from (Sw79) shows, schematically, the electromagnetic cascade process. Of course, there are subtleties representing many different physical processes, such as the production of other particles, which must be taken into account. These are best handled by Monte-Carlo calculations. The most widely-used code incorporating the Monte-Carlo method as applied to electromagnetic cascades is that written by W. R. Nelson of SLAC called EGS (electron gamma shower, a current version is denoted EGS4) which has been described in (Ne90). Van Ginneken has also written a Monte-Carlo program which is very effective for calculating deep penetrations called AEGIS (Va78) [An appendix at the end of this discussion will briefly review the Monte-Carlo Method.] Analytical approximations have been developed and are summarized elsewhere [e.g., (Sw79)].

There are some published standard calculations from which estimates may be made which will now be introduced.

Longitudinal shower development

The dosimetric properties of the calculations of an electromagnetic cascade may be summarized in curves that give fluence, dose, or other quantities of interest as functions of shower depth or distance from the axis. Figure 2.3 taken from (Va75) shows the fraction of total energy deposited (integrated over all radii about the shower axis) versus depth from Van Ginneken and Awschalom (Va75). These authors found that a new scaling parameter, $\lambda_4$, given by

$$\lambda_4 = 325(\ln Z)^{-1.73}\ln(E_0) \text{ (g cm}^{-2}\text{)} \quad [E_0 \text{ is in MeV}] \quad (2.12)$$

When longitudinal lengths are expressed in units of $\lambda_4$, all curves merge approximately into this universal one.

---

1Monte-Carlo programs exist, in general, in a state of nearly continuous improvement. Thus the authors of such codes should be contacted to provide the current version.
Fig. 2.2 Development of an electromagnetic cascade in a semi-infinite medium at high energy (well above the critical energy). The dashed lines represent electrons or positrons and the wavy lines represent photons. An electron or positron of energy $E_0$ is incident at the left (a cascade can also be initiated by a photon). The spreading in the transverse direction is greatly exaggerated for clarity. Only bremsstrahlung (B) and pair production (P) events are shown, but Compton scattering also plays a role in the dispersal of energy. Energy is deposited in the medium along the dashed lines by ionization. Photonuclear reactions, as illustrated by the ($\gamma$,n) reaction at point N, may take place along any of the wavy lines if the energy of that photon is high enough. They occur much less frequently than might be inferred from this illustration. [Reproduced from (Sw79).]
Fig. 2.3 Fraction of total energy, $U$, deposited by an EM cascade shower versus depth, integrated over all radii about the shower axis. [Reproduced from (Va75).]
Rossi and Griesen (Ro41), in their development of analytical shower theory, have predicted (using their so-called "Approximation B") that the total number of electrons and positrons at the shower maximum, $N_{\text{show}}$, are proportional to the primary energy as follows:

$$N_{\text{show}} = \frac{0.31 E_o/E_c}{[\ln(E_o/E_c) - 0.37]^{1/2}}. \quad (2.13)$$

This makes sense intuitively; the result of the shower is to divide the energy at maximum among a number of particles with energies near $E_c$.

Also from this Approximation B, the location of the shower maximum $X_{\text{max}}$, (along the longitudinal axis usually represented by the Z coordinate) should be given by:

$$\frac{X_{\text{max}}}{X_o} = \ln\left(\frac{E_o}{E_c}\right) - C. \quad (\text{with } C = 1) \quad (2.14)$$

Experimentally, Bathow, Freytag, and Tesch (Ba67) found that $C = 0.77$ for Cu and 0.47 for Pb fit experimental data better. (Photon induced showers penetrate about 0.8 radiation lengths deeper. (Sc90) identifies slight differences between photon and lepton induced showers but these can normally be neglected.) The maximum energy deposited per radiation length is simply given by multiplying $N_{\text{show}}$ by the critical energy.

(Sc90) gives the mean squared longitudinal spread, $\tau^2$, (mean square distance lateral spread of the shower about $t = t_{\text{max}} = X_{\text{max}}/X_o$):

$$\tau^2 = 1.61 \ln\left(\frac{E_o}{E_c}\right) - 0.2 \quad \text{(electron-induced shower), and} \quad (2.15)$$

$$\tau^2 = 1.61 \ln\left(\frac{E_o}{E_c}\right) + 0.9 \quad (2.16)$$

(photon-induced shower, for which $E_o$ is the photon energy more conventionally denoted by $k_o$ by some authors).

For dosimetric purposes for a shower in a given material, EGS4 results tabulated in (Sc90) have been parameterized to determine "source terms" for longitudinal dose equivalents in materials over the energy region $1 \text{ GeV} < E_o < 1 \text{ TeV}$ for dose on the Z-axis (subscripts "a"), dose averaged over a 15 cm radius about the Z-axis, (subscripts "m") and for total energy deposited (subscripts "e"). Table 2.1 taken from (Sc90) gives the various parameters for calculating dose equivalent, $H_{\text{long}}$ (Sv per electron) at the end of a beam dump of length, $L$ (cm) of density, $\rho$ (g/cm$^3$) and gives fitted values of the various "attenuation lengths", $\lambda_i$ (g/cm$^2$). For absorbed dose calculations, the factor $C_i$, which is the ratio of dose equivalent in tissue (Sv) to absorbed dose in the material (not tissue) (Gy), should be set to unity. The following is the formula in which these parameters from Table 2.1 are to be used:

$$H_{\text{long}} = C_i e^{-\rho L/\lambda_i}. \quad (2.17)$$
Table 2.1 Source terms $S_a$, $S_m$, and $S_e$, and corresponding recommended attenuation lengths, $\lambda_a$, $\lambda_m$, and $\lambda_e$ for doses on the axis, averaged over a radius of 15 cm, and for total energy in the energy range from 1 GeV to 1 TeV in the forward direction for dumps and end-stops, respectively. Conversion factors $C$ from absorbed dose in shielding material to dose equivalent are given. $E_o$ is the beam kinetic energy in GeV. [Adapted from (Sc90).]

<table>
<thead>
<tr>
<th>Material</th>
<th>$C$</th>
<th>$S_a$</th>
<th>$\lambda_a$</th>
<th>$S_m$</th>
<th>$\lambda_m$</th>
<th>$S_e$</th>
<th>$\lambda_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Sv/Gy)</td>
<td>(Gy/electron)</td>
<td>(g/cm²)</td>
<td>(Gy/electron)</td>
<td>(g/cm²)</td>
<td>(Gy/electron)</td>
<td>(g/cm²)</td>
</tr>
<tr>
<td>Water</td>
<td>0.95</td>
<td>1.9X10^{-10}E_o^{2.0}</td>
<td>58</td>
<td>1.5X10^{-11}E_o^{2.0}</td>
<td>59.9</td>
<td>1.2X10^{-14}E_o^{1.7}</td>
<td>75</td>
</tr>
<tr>
<td>Concrete</td>
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<td>1.9X10^{-9}E_o^{1.8}</td>
<td>44</td>
<td>2.2X10^{-11}E_o^{1.8}</td>
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<td>9.0X10^{-14}E_o^{1.7}</td>
<td>52</td>
</tr>
<tr>
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<td>46</td>
<td>3.4X10^{-11}E_o^{1.7}</td>
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<td>1.0X10^{-13}E_o^{1.7}</td>
<td>55</td>
</tr>
<tr>
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<td>2.9X10^{-8}E_o^{1.7}</td>
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<td>1.1X10^{-12}E_o^{1.6}</td>
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</tr>
<tr>
<td>Lead</td>
<td>1.8</td>
<td>1.9X10^{-7}E_o^{1.4}</td>
<td>18</td>
<td>4.6X10^{-10}E_o^{1.4}</td>
<td>24.2</td>
<td>4.3X10^{-12}E_o^{1.2}</td>
<td>25</td>
</tr>
</tbody>
</table>

Lateral shower development

Figure 2.4 taken from (Ne68) shows the fraction $U/E_o$ of the incident energy which escapes laterally from an infinitely long cylinder as a function of cylinder radius for showers caused by electrons of various energies which bombard the front face of the cylinder. On this graph $R$ is in units of $X_m$. The curve has been parameterized as:

$$
\frac{U(r)}{E_o} = 0.8 \exp[-3.45 (r/X_m)] + 0.2\exp[-0.889 (r/X_m)]. \tag{2.18}
$$

In Fig. 2.4, the universality of electromagnetic cascade curves is clear. Similar results have been obtained using EGS4 (Sc90). For large radii, a material dependent phenomenon emerges in which the photons having the largest mean free paths determined by the photon cross section at the Compton minimum will dominate the slopes. These slopes, normalized to $X_m$, are also shown in this figure.²

²In several publications in which Fig.2.4 has appeared, including the original one, the decimal points in the "x-axis" coordinates have been nearly invisible!
Fig. 2.4  Fraction of total energy, $U$, deposited beyond a cylindrical radius, $R/X_m$, as a function of radius for showers caused by 0.1 - 20 GeV electrons incident on various materials (Reproduced from (Ne68) and references cited therein.)
Chapter 2 Shielding of Electrons and Photons at Accelerators

As was done for the longitudinal situation, EGS4 (Sc90) has been similarly used to give the maximum energy deposition (and by extension, the maximum absorbed dose and dose equivalent) as a function of radius R. Over the energy range 1 GeV < E_0 < 1 TeV, there is direct scaling with energy in the formula for dose equivalent:

$$H_{\text{lat}} = CE_0 S_1 e^{-\rho R \lambda_1},$$

(2.19)

where $H_{\text{lat}}$ is the maximum dose equivalent laterally (Sv per electron), C is the same as before, $E_0$ is the electron kinetic energy in GeV, $S_1$ is the source term from the EGS4 calculations (tabulated below), R is the lateral dimension of the shield (shield thickness) in cm, $\rho$ is the density (g cm$^{-3}$), $\lambda_1$ is the attenuation length (g/cm$^2$), and a is the distance from the axis, in cm, where the dose equivalent is desired.

Table 2.2 taken from from (Sc90) gives the parameters needed for the above formula and Fig. 2.5 also taken from (Sc90) shows the result of the EGS4 calculations.

<table>
<thead>
<tr>
<th>Material</th>
<th>C (Sv/Gy)</th>
<th>$S_1$ (Gy cm$^2$ GeV$^{-1}$ per electron)</th>
<th>$\lambda_1$ (g/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.95</td>
<td>2.5X10$^{-12}$</td>
<td>26</td>
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<td>Concrete</td>
<td>1.2</td>
<td>3.6X10$^{-12}$</td>
<td>27</td>
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<tr>
<td>Aluminum</td>
<td>1.2</td>
<td>3.4X10$^{-12}$</td>
<td>29</td>
</tr>
<tr>
<td>Iron</td>
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<td>4.7X10$^{-11}$</td>
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<tr>
<td>Lead</td>
<td>1.8</td>
<td>1.3X10$^{-10}$</td>
<td>26</td>
</tr>
</tbody>
</table>

[Adapted from (Sc90).]
Fig. 2.5 Maximum absorbed dose $D$ in a cylinder vs. radius $R$; curve 1: cylinder made of concrete of density 2.4 g/cm$^3$, curve 2: cylinder made of iron of density 7.2 g/cm$^3$. Curves calculated with EGS4 for electron beams of 1, 10, 100, and 1000 GeV and normalized per GeV become independent of energy. [Reproduced from (Sc90).]
Chapter 2 Shielding of Electrons and Photons at Accelerators

III. Hadron Production by the Electromagnetic Cascade.

As we have seen before, neutrons are produced by high energy electrons and photons. These neutrons must be taken into account to properly shield electron accelerators. The general issues concerning the shielding of neutrons are addressed in more detail in Chapter 3. K. Tesch has summarized shielding against these neutrons in his review article (Te88). He has summarized the feature of the neutron fields with simple analytical relations for cases where "thick" targets are struck by the electron beam.

For lateral concrete shielding, the dose equivalent per electron after shield thickness, \( d \) (g/cm\(^2\)), which begins at radius \( r \) (m) from an iron target struck by electrons having primary energy \( E_0 \) (GeV) is

\[
H(d, r) = \frac{4.0 \times 10^{-17}}{r^2} E_0 e^{-d/100} \text{ (Sv).} \tag{2.20}
\]

This is valid for \( E_0 > 0.4 \) GeV and \( d > 200 \) g cm\(^{-2}\).

The angular variations are not severe because of the nature of the mechanisms by which the neutrons are produced at electron accelerators (namely, photoneutron production). For other target materials one can scale this equation as follows.

The neutron production is proportional to the photoproduction cross section, the track length in cm, and the number of atoms cm\(^{-3}\). The interaction cross section is generally proportional to the atomic weight \( A \). The track length is proportional to \( X_0 \); the production becomes proportional to the radiation length in g cm\(^{-2}\). Thus one can, for rough estimates of dose equivalent in the environs of targets of materials other than iron, obtain results by scaling this value for iron by the factor \( f \);

\[
f = \frac{X_{0_{\text{material}}}}{X_{0_{\text{iron}}}}. \tag{2.21}
\]

For shields comprised of other materials, one can simply adjust the attenuation length (g cm\(^{-2}\)) in the exponent of the above to that appropriate to the material.

(Sc90) gives a somewhat more detailed treatment separately handling the giant resonance neutrons and high energy particle components of dose equivalent and deriving "source terms" and appropriate formulas. Fig. 2.6 taken from (Sc90) illustrates the geometry for using the formulas to be given below:

![Diagram of target and shielding geometry](image)

Fig. 2.6 Target and shielding geometry for the estimation of dose equivalent from bremsstrahlung, giant resonance neutrons, and high energy particles in the angular range with the beam \( \theta \) from 30 to 120 degrees; \( a \) is the lateral distance between the target and the shielding and \( d \) is the shielding thickness.

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The formulae given below are held to be valid for \(1 \text{ GeV} < E_0 < 1 \text{ TeV}\) and for \(30 < \theta < 120\) degrees.

For the giant resonance neutrons:

\[
H_n = \eta_n S_n E_0 \left( \frac{\sin \theta}{a + d} \right)^2 \exp \left[ - \frac{\rho d}{\lambda_n \sin \theta} \right]
\]

where \(E_0\) is the energy (GeV), \(E\) is the beam energy (GeV), \(\rho\) is the density (g/cm\(^3\)), \(a\) and \(d\) are as shown (cm) in Fig. 2.6, \(S_n\) is the source term from Chapter 1 (Sv cm\(^{-2}\) GeV\(^{-1}\)), and \(\lambda_n\) (g/cm\(^2\)) is the attenuation length recommended for giant resonance neutrons. Values of \(\lambda_n\) are as follows for representative materials:

<table>
<thead>
<tr>
<th>(\lambda_n) (g/cm(^2))</th>
<th>Water</th>
<th>Concrete</th>
<th>Iron</th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9</td>
<td>42</td>
<td>130</td>
<td>235</td>
</tr>
</tbody>
</table>

The factor \(\eta_n\) is a dimensionless factor (\(\eta_n \leq 1\)) which gives an estimate of the efficiency for the production of neutrons by the target. Figure 2.7 taken from (Sc90) can be used to estimate this quantity.

[Graph: Fig. 2.7 Relative yield \(Y\) of neutrons released by electron beams incident on a lead target at energies \(E_0 = 17, 34\) and 100 MeV vs. target thickness measured in radiation lengths \(X_0\). The curves are qualitatively similar for the other materials and energies but the initial rise will tend to be steeper, and saturation will be more quickly achieved, for higher \(E_0\) or greater \(Z\). [Reproduced from (Sc90) as adapted from references cited therein.]
Chapter 2  Shielding of Electrons and Photons at Accelerators

For the high energy particles:

In this case no correction for target thickness is generally employed. The formula for this is (starting with the De Staebler yield formula, Eq. (1.34)):

\[
H_h = \frac{7.5 \times 10^{-13}E_o}{(1 - 0.75 \cos \theta)^2} \left( \frac{\sin \theta}{a + d} \right)^2 \exp \left( -\frac{\rho d}{\lambda_h \sin \theta} \right). \tag{2.23}
\]

In this formula \( H_h \) is the dose equivalent due to these particles (Sv), \( E \) is the beam energy (GeV), \( A \) is the atomic weight of the target and \( \lambda_h \) (g/cm\(^2\)) is the attenuation length typical of these particles. Table 2.3 taken from (Sc90) gives values of \( \lambda_h \) for representative materials. (Sc90) goes further and describes a variety of special cases.

<table>
<thead>
<tr>
<th>Material</th>
<th>Energy Limit</th>
<th>Nuclear Interaction Length (g/cm(^2))</th>
<th>Recommended ( \lambda_h ) [Eq. (2.23)] (g/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>&gt; 14 MeV(^1) or &gt; 25 MeV(^2)</td>
<td>84.9</td>
<td>86</td>
</tr>
<tr>
<td>Aluminum</td>
<td>&gt; 100 MeV</td>
<td>106.4</td>
<td>128</td>
</tr>
<tr>
<td>Soil (sand)</td>
<td>&gt; 100 MeV</td>
<td>106.4</td>
<td>128</td>
</tr>
<tr>
<td>Concrete</td>
<td>&gt; 100 MeV</td>
<td>99.2</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>&gt; 25 MeV(^2)</td>
<td>96</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>&gt; 25 MeV(^2)</td>
<td>105</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>&gt; 25 MeV(^2)</td>
<td>120</td>
<td>117</td>
</tr>
<tr>
<td>Iron</td>
<td>&gt; 100 MeV</td>
<td>131.9</td>
<td>164</td>
</tr>
<tr>
<td>Lead</td>
<td>&gt; 100 MeV</td>
<td>194</td>
<td>253</td>
</tr>
</tbody>
</table>

\(^*\) Attenuation lengths for the indicated values are slightly dependent on angle with the higher value at \( \theta = 0^\circ \) and the smaller value in the backward direction for \( E > 15 \) MeV.

\(^+\) Same remark but for \( E > 25 \) MeV.
Chapter 2  Shielding of Electrons and Photons at Accelerators

IV.  Theory of Radiation Transport and the Monte Carlo Method

The theoretical material in this section is largely the work of Mr. Keran O'Brien of the University of Northern Arizona, (OB80). It is included to show clearly the mathematical basis of the contents of shielding codes, especially those which use the Monte-Carlo method.

General Considerations

Stray and direct radiations at any location are distributed in particle type, direction, and energy. To determine the amount of radiation present for radiation protection purposes we must assign a magnitude to this multidimensional quantity. This is done by forming a double integral over energy and direction of the product of the flux and an approximate flux-to-dose or flux-to-dose-equivalent conversion factor, summed over particle type;

\[
H(x,t) = \sum \int_{4\pi} d\Omega \int_{0}^{\infty} dE f_i(x,E,\Omega,t) H_i(E) 
\]

(2.24)

where \( \Omega \) is the direction vector of particle travel, \( x \) is the coordinate vector of the point in space where the dose or dose equivalent is to be calculated, \( E \) is the particle energy, \( t \) is time, and \( i \) is the particle type. (Here we adopt the conventional notation that bold-faced coordinates represent vector quantities.) \( H_i(E) \) is the flux-to-dose or flux-to-dose-equivalent conversion factor expressed as a function of energy and particle type. The inner integral is over all energies while the outer integral is over all spatial directions which contribute to the radiation field at the location specified by \( x \). The result of the integration is \( H \), the dose or dose-equivalent rate at location \( x \). Values of \( H_i \) are tabulated in (IC87). The angular flux, \( f_i(x,E,\Omega,t) \), the number of particles of type \( i \) per unit area, per unit energy, per unit solid angle, per unit time at location \( x \), with an energy \( E \), at a time \( t \) and traveling in a direction \( \Omega \) is related to the scalar flux, or flux density by integrating over direction,

\[
\phi_i(x,t) = \int_{4\pi} d\Omega \int_{0}^{\infty} dE f_i(x,E,\Omega,t),
\]

(2.25)

to the fluence by integrating over the intervening period of time,

\[
\phi_i(x) = \int_{4\pi} d\Omega \int_{0}^{\infty} dE \int dt f_i(x,E,\Omega,t),
\]

(2.26)

and to the energy spectrum at point \( x \) at time \( t \) by,

\[
\phi_i(x,t,E) = \int_{4\pi} d\Omega f_i(x,E,\Omega,t).
\]

(2.27)
To determine the proper dimensions and composition of a shield, the amount of radiation (expressed in terms of the dose or dose equivalent) which penetrates the shield and reaches locations of interest must be calculated. This quantity must be compared with the maximum permissible dose equivalent. If the calculated dose or dose equivalent is too large, either the conditions associated with the source of the radiation (e.g., the amount of beam loss allowed by the beam control instrumentation, the amount of residual gas in the vacuum system, or the amount of beam allowed to be accelerated) or the shield dimensions must be changed. It is difficult and expensive, especially in the case of the larger accelerators, to alter permanent shielding or operating conditions if the determination of shielding dimensions and composition has not been done correctly. The methods for determining these quantities have been investigated by numerous workers. The next section only summarizes the basics of this important work.

The Boltzmann Equation

The primary tool for determining the amount of radiation reaching a given location is the Boltzmann equation which, when solved, yields the angular flux: \( f_i(x,E,\Omega,t) \); the distribution in energy and angle for each particle type as a function of position and time. The angular flux is then converted to dose equivalent by means of Eq. (2.24).

This section describes the theory that yields the distribution of radiation in matter, and discusses some of the methods for extracting detailed numerical values for elements of this distribution such as particle flux, or related quantities, such as dose, activation or instrument response. The basis for this theory is the stationary form of the Boltzmann equation (henceforth, referred to simply as the Boltzmann equation) which is a statement of all the processes that the corpuscles of various types that comprise the radiation field can undergo.

The Boltzmann equation is an integrodifferential equation describing the behavior of a dilute assemblage of corpuscles. It was derived by Ludwig Boltzmann in 1872 to study the properties of gases but applies equally to the behavior of those "corpuscles" which comprise ionizing radiation.

Boltzmann's equation is a continuity equation of the angular flux, \( f_i(x,E,\Omega,t) \), in phase space which is made up of the three space coordinates of Euclidian geometry, the three corresponding direction cosines and the kinetic energy. The density of radiation in a volume of phase space may change in five ways:

1. Uniform translation; where the spatial coordinates change, but the energy-angle coordinates remain unchanged;
2. Collisions; as a result of which the energy-angle coordinates change, but the spatial coordinates remain unchanged, or the particle may be absorbed and disappear altogether;
3. Continuous slowing down; in which uniform translation is combined with continuous energy loss;
4. Decay; where particles are changed through radioactive transmutation into particles of another kind; and
5. Introduction; involving the direct emission of a particle from a source into the volume of phase space of interest: electrons or photons from radioactive materials, neutrons from an \( \alpha \)-n emitter, the "appearance of beam particles, or particles emitted from a collision at another (usually higher) energy.
Combining these five elements yields

\[ B_i f_i(x,E,\Omega,t) = Q_{ij} + Y_i \]  \hspace{1cm} (2.28)

where the mixed differential and integral operator, \( B_i \), is given by

\[ B_i = \Omega \cdot \text{grad} + \sigma_i + d_i - \left( \int \frac{dE}{E_dE} \right) S_i ; \]  \hspace{1cm} (2.29)

\[ Q_{ij} = \sum_j \int_{4\pi} d\Omega \int_0^{E_{\text{max}}} dE_B \sigma_{ij}(E_B \rightarrow E, \Omega' \rightarrow \Omega) f_i(x,e,\Omega',t) ; \]  \hspace{1cm} (2.30)

and

\[ d_i = \left[ \frac{\sqrt{1 - \beta_i^2}}{T_i c \beta_i} \right] . \]  \hspace{1cm} (2.31)

\( B_i \) is the Boltzmann operator for particles of type i;

\( Y_i \) is the number of particles of type i introduced by a source per unit area, time, energy, and solid angle;

\( \sigma_i \) is the absorption cross section for particles of type i. To be dimensionally correct, this is actually the macroscopic cross section or linear absorption coefficient \( \mu = N \sigma \) as defined in Chapter 1, Eq. (1.8);

\( d_i \) is the decay probability per unit flight path of radioactive particles (such as muons or pions) of type i;

\( S_i \) is the stopping power for charged particles of type i (assumed to be zero for uncharged particles);

\( Q_{ij} \) is the "scattering-down" integral, the production rate of particles of type i with a direction \( \Omega \), an energy \( E \) at a location \( x \), by collisions with nuclei or decay of j-type particles having a direction \( \Omega' \) at a higher energy \( E_B \);

\( \sigma_{ij} \) is the doubly-differential inclusive cross section for the production of type-i particles with energy \( E \) and a direction \( \Omega \) from nuclear collisions or decay of type-j particles with a direction \( E_B \) and a direction \( \Omega' \);

\( \beta_i \) is the velocity of a particle of type i divided by the speed of light c;

and \( T_i \) is the mean life of a radioactive particle of type i in the rest frame.

This equation is quite difficult to solve in general and special techniques have been devised to yield useful results. The Monte-Carlo method is the most common application to the field of radiation shielding.
Chapter 2 Shielding of Electrons and Photons at Accelerators

The Monte Carlo method—general principles

The Monte Carlo method is based on the use of random sampling to obtain the solution of the Boltzmann equation. It is one of the most useful methods for evaluating radiation hazards for realistic geometries which are generally quite complicated to model using analytic techniques. The calculation proceeds by constructing a series of trajectories, each segment of which is chosen at random from a distribution of applicable processes.

In the simplest and most widely used form of the Monte Carlo technique, a history is obtained by calculating travel distances between collisions, then sampling from distributions in energy and angle made up from the cross sections

\[ \sigma_{ij} (E_B \to E, \Omega' \to \Omega). \] (2.32)

The result of the interaction may be a number of particles of varying types, energies, and directions each of which will be followed in turn. The results of many histories will be processed, leading, typically, to some sort of mean and standard deviation.

If \( p(x)dx \) is the probability of an occurrence at \( x \pm \frac{1}{2} dx \) in the interval \([a,b]\), then

\[ P(x) = \int_a^x p(x')dx' \] (2.33)

is the probability that the event will occur in the interval \([a,x]\), and is monotonically increasing, satisfying \( P(a) = 0, P(b) = 1 \). If a random number \( R \) is chosen, uniform on the interval \([0, 1]\) from a computer routine, the equation

\[ R = P(x) \] (2.34)

amounts to a random choice of the value of \( x \), where the distribution function for the event \( P(x) \) can be inverted, as

\[ x = P^{-1}(R) \] (2.35)

As a simple illustration, to determine when an uncharged particle undergoes a reaction in a one dimensional system with no decays (\( d = 0 \)) or competing processes (\( S = 0 \)), we note from Eq. (1.6) and Eq (2.29) that the particle satisfies

\[ B \phi = \left( \Omega \cdot \text{grad} + \sigma_i \right) \phi \]

which in this simple situation reduces to the following (in view of the comment made above concerning the nature of \( \sigma_i \)), which is a continuity equation equivalent to Eq. (1.8):

\[ B \phi = \frac{d\phi}{dx} + N \sigma \phi = 0. \] (2.36)
The solution to this equation is the familiar

\[ \phi = \phi_0 \exp(-x/\lambda), \]  

(2.37)

where \( \lambda = 1/\text{No} \) as in Chapter 1. One can replace \( x/\lambda \) as \( r \), the number of mean-free-paths the particle travels in the medium. The differential probability per unit mean-free-path for an interaction is given by

\[ p(r) = e^{-r} \]  

(2.38)

with

\[ P(r) = \int_0^r \text{d}r' e^{-r'} = -e^{-r'} \bigg|_0^r = 1 - e^{-r} = R \]  

(2.39)

Selecting a random number, \( R \), then determines a depth \( r \) which has the proper physical distribution. By taking into account charged-particle slowing down during passage along \( r \), the correct energy-dependent cross section can be chosen. Of course, quite analogous methods apply to other exponential processes such as radioactive decay. In this simple case, it is clear that one can solve the above for \( r \) as a function of \( R \) and thus obtain individual values of \( r \) from random numbers. For some process, the inversion that is so simple in the above might not be possible analytically. In those situations, other techniques exemplified by successive approximation and "table look-ups" must be employed.

The next sampling process might select which of several physical processes would occur. Another sampling might choose, for instance, the scattering angle which would then provide a new energy.

The Monte Carlo result is the number of times the event of interest occurred for the random steps through the relevant processes. As a counting process it has a counting uncertainty and the variance will tend to decrease as the square root of the run time. Thus high probability processes can be more accurately estimated than low probability processes such as passage through an effective shield in which the radiation levels are attenuated over many orders of magnitude.

It is by no means clear that the distributions obtained using the Monte Carlo method will be normally distributed, so that a statistical test of the adequacy of the mean and standard deviation may be required.
Chapter 2 Shielding of Electrons and Photons at Accelerators

Monte-Carlo Example: A Sinusoidal Angular Distribution of Beam Particles

Suppose one has a distribution of beam particle particles such as exhibited in the following figure.

![Distribution of Beam Particles in $\theta$](image)

Say $p(\theta) = A \cos \theta$ for $0 < \theta < \pi/2$. Then, the fact that the integral of $p(\theta)$ over the relevant interval must be unity implies:

$$\int_0^{\pi/2} p(\theta) \, d\theta = 1; \quad \int_0^{\pi/2} A \cos \theta \, d\theta = A \sin \frac{\pi}{2} = A = 1.$$  

Then, $p(\theta) = \cos \theta$. The cumulative probability, $P(\theta)$, is given by:

$$P(\theta) = \int_0^\theta d\theta' p(\theta') = \int_0^\theta d\theta' \cos \theta' = \sin \theta \bigg|_0^\theta = \sin \theta.$$  

If $R$ is a random number, then $R = P(\theta)$ determines a unique value of $\theta$; hence:

$$\theta = \sin^{-1}(R)$$

One can perform a simple Monte Carlo using, for example, 50 random numbers. To do this one should set up at table such as that given below. One can set up a set of bins of successive ranges of $\theta$-values. The second column is a "tally sheet" for collecting "events" in which a random number $R$ results in a value of $\theta$ within the associated range of $\theta$-values. $\theta_{\text{mid}}$ is the midpoint of the bin (0.1, 0.3, ...). Column 4 is the normalized number in radians found from:

$$N = \frac{\text{Number Found in Bin in MC}}{\text{Total Number of MC} \times \text{(bin width)}} = \frac{\text{Number Found in Bin in MC}}{50 \times (0.2 \text{ radians})}$$
### Chapter 2 Shielding of Electrons and Photons at Accelerators

One can calculate exactly the mean value of $\theta$ for the exact distribution:

$$\langle \theta \rangle = \frac{\int_0^{\pi/2} \theta p(\theta) \, d\theta}{\int_0^{\pi} p(\theta) \, d\theta} = \frac{\int_0^{\pi/2} \theta \cos \theta \, d\theta}{1} = \left[ \cos \theta + \theta \sin \theta \right]_0^{\pi/2}$$

$$\langle \theta \rangle = \left[ 0 - 1 + \frac{\pi}{2} - 0 \right] = \frac{\pi}{2} - 1 = 0.57$$

<table>
<thead>
<tr>
<th>$\theta$ (radians)</th>
<th>R (random #)</th>
<th>Total R's in Bin</th>
<th>N (norm. #)</th>
<th>$\cos\theta_{\text{mid}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 0.199</td>
<td>+ + + + + + +</td>
<td>11</td>
<td>1.1</td>
<td>0.995</td>
</tr>
<tr>
<td>0.2 - 0.399</td>
<td>+ + + + + + +</td>
<td>13</td>
<td>1.3</td>
<td>0.955</td>
</tr>
<tr>
<td>0.4 - 0.599</td>
<td>+ + + + + + +</td>
<td>11</td>
<td>1.1</td>
<td>0.877</td>
</tr>
<tr>
<td>0.6 - 0.799</td>
<td>1111</td>
<td>4</td>
<td>0.4</td>
<td>0.765</td>
</tr>
<tr>
<td>0.8 - 0.999</td>
<td>+ + + + 11</td>
<td>7</td>
<td>0.7</td>
<td>0.621</td>
</tr>
<tr>
<td>1.0 - 1.199</td>
<td>1111</td>
<td>4</td>
<td>0.4</td>
<td>0.453</td>
</tr>
<tr>
<td>1.2 - 1.399</td>
<td></td>
<td></td>
<td></td>
<td>0.267</td>
</tr>
<tr>
<td>1.4 - 1.57</td>
<td></td>
<td></td>
<td></td>
<td>0.086</td>
</tr>
</tbody>
</table>

Multiplying the frequency of Monte-Carlo events for each eight angular bins from the table by the midpoint value of the bins, summing over the 8 bins and then dividing by the number of incident particles (50 in this example), one can determine the average value of $\theta$, $\langle \theta \rangle$ calculated by the Monte-Carlo technique:

$$\langle \theta \rangle_{\text{calc}} = \frac{(1)(0.1) + (13)(0.3) + (11)(0.5) + (4)(0.7) + (7)(0.9) + (4)(1.1)}{50} = 0.48.$$ 

It is easy to see from this simple example that the agreement is quite good in spite of the rather poor "statistics". This example also illustrates that the statistical errors are generally larger for the more rare events here represented by large values of $\theta$ (e.g., $\theta > 1$ radian).
Chapter 2 Shielding of Electrons and Photons at Accelerators

References


(Va78) A. Van Ginneken, "AEGIS, a program to calculate the average behavior of electromagnetic showers", Fermilab Report FN-309 (1978).
1. In the discussion of the longitudinal development of electromagnetic showers, there are two different formulations (Rossi-Griesen and Bathow, and Van Ginneken). Using Van Ginneken's scaling method, calculate the value of \( \lambda_4 \) (g/cm\(^2\)) for \( E_0 = 1000 \text{ MeV}, 10 \text{ GeV}, \) and \( 100 \text{ GeV} \) for copper and lead. Determine the number of radiation lengths to which \( \lambda_4 \), corresponds for each material at each energy.

2. Compare the results of Van Ginneken for the location of the longitudinal shower maximum with Bathow's result for copper and lead at the three energies given in problem 1. Is the agreement better or worse as the energy increases?

3. A hypothetical electron accelerator operates at either \( 100 \text{ MeV} \) or \( 10 \text{ GeV} \) and delivers a beam current of \( 1 \mu\text{A} \). Using the results of (Sc90) calculate the dose equivalent rates in both Sv/sec and rem/h at the end of a 300 cm long aluminum beam stop; averaged over a 15 cm radius. (The beam stop is a cylinder much larger than 15 cm in radius.) Then assume that, in order to save space, a high-Z beam stop is substituted. How long of a high-Z beam stop is needed to achieve the same dose rates? (Assume lead is a suitable high-Z material.) Why is the length of high-Z shield different for the 2 energies? [In this problem, assume the results of (Sc90) are valid for energies as low as 0.1 GeV.]

4. In the accelerator and beam stop of problem 3, if the radius of the beam stop is 30 cm, what is the maximum dose equivalent rate (Sv/s and rem/h) on the lateral surface (at contact at \( r = 30 \text{ cm} \)) of the beam stop for both energies, \( 100 \text{ MeV} \) and \( 10 \text{ GeV} \), and both materials? Again assume approximate validity at \( 100 \text{ MeV} \) of the (Sc90) results.

5. Calculate the dose equivalent rate outside a 1 meter thick concrete shield surrounding a radius tunnel (inner radius 1 meter) in which is located a copper target struck by \( 1 \mu\text{A} \) beam of \( 100 \text{ GeV} \) electrons. The geometry should be assumed to be optimized for producing giant resonance photoneutrons and the calculations should be performed at \( \theta = 30^\circ, 60^\circ \) and \( 90^\circ \) (Concrete has \( \rho = 2.5 \text{ g/cm}^3 \)). Express the result as Sv/sec and rem/h.

6. This problem gives two elementary examples of Monte Carlo techniques that are almost "trivial". In this problem, obtaining random numbers from a standard table or from a hand calculator should be helpful.
Chapter 2 Shielding of Electrons and Photons at Accelerators-
Problems

a) First, use a random number table or random number function on a calculator along with the
facts given about the cumulative probability distribution for exponential attenuation to
demonstrate that, even for a sample size as small as, say, 15, the mean value of paths
traveled is "within expectations" if random numbers are used to select those path lengths
from the cumulative distribution. Do this, for example, by calculating the mean and
standard deviation of your distribution.

b) An incident beam is subjected to a position measurement in the coordinate x. It is desirable
to "recreate" incident beam particles for a shielding study using Monte-Carlo. The x
distribution as measured is as follows:

<table>
<thead>
<tr>
<th>x</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Determine, crudely, p(x), P(x) and then use 50 random numbers to "create" particles
intended to represent this distribution. Then compare with the original one which was
measured in terms of the average value of X and its standard deviation. Do not take the
time to use interpolated values of x, simply round off to integer values of x for this
demonstration.
Chapter 3 Shielding of Proton and Ion Accelerators

In this chapter the major considerations pertinent to the shielding of proton and ion accelerators is addressed. Particular emphasis is placed on the shielding of neutrons in view of their general dominance of the radiation fields. The shielding of muons at such accelerators is also described. A short review of the various Monte-Carlo programs commonly used in shielding calculations at proton and ion accelerators is presented. The properties of various shielding materials commonly used at accelerators are reviewed. The chapter concludes with a discussion of various features of neutron energy spectra found at proton accelerators.

I. Hadron (Neutron) Shielding for Low Energy Incident Protons

For this discussion, the "low energy" region extends up to approximately $E_0 = 100$ MeV. The basic treatment follows that of (Pa73).

$E_0 < 15$ MeV:

This region is especially complex because it is the region of significant nuclear structure effects including:

- the resonances associated with the phenomena known as "compound nuclei" and
- the presence of a large number of nuclear excited states up to $20$ MeV excitation energy having a wide variety of nuclear structure quantum numbers and very narrow widths in energy.

The method most commonly used to calculate shielding thicknesses is that of removal cross section theory.

According to F. Clark (Cl71) there are three principles involved here:

A. "The shield must be sufficiently thick and the neutrons so distributed in energy that only a narrow band of the most penetrating source neutrons give any appreciable ultimate contribution to the dose outside the shield."

B. "There must be sufficient hydrogen in the shield, intimately mixed or in the final shield region, to assure a very short characteristic transport length from about 1 MeV to absorption at or near thermal energy."

C. "The source energy distribution and shield material (nonhydrogeneous) properties must be such as to assure a short transport distance for slowing down from the most penetrating energies to 1 MeV."

It has been found that the transmission of dose equivalent, $H(t)$, as a function of shield thickness, $t$, is approximately given for these neutrons by

$$H(t) = \Phi_0 P G e^{-\Sigma t},$$

where $\Phi_0$ is the fluence before the shielding (calculated from neutron yield information), $P$ is the fluence to dose equivalent conversion factor (where one may need to integrate over spectrum to get a true "average" value), $G$ is a "geometry factor", $t$ (cm) is the thickness. For parallel beams, $G = 1$ while for an isotropic point source, $G = 1/(4\pi t^2)$. $\Sigma$ is the macroscopic removal cross section:
where \( \sigma_r = \text{microscopic removal cross section in barns} \)
\[ \rho = \text{density (g/cm}^3) \]
\[ A = \text{mass number}. \]

For mixtures of \( n \) materials,
\[ \Sigma_r = \sum_{i=1}^{n} \left( \frac{\Sigma_{ri}}{\rho_i} \right) \rho_i \]  \hspace{1cm} (3.3)

where \( \rho_i \) is the partial density of the \( i^{th} \) material. (In this formulation the overall density is equal to the sum of the partial densities.)

For \( A > 8 \),
\[ \sigma_r = 0.21 A^{-0.58} \text{ barns} \]  \hspace{1cm} (3.4)

for neutrons of approximately 8 MeV. Figure 3.1 taken from (Pa73) gives measured values of \( \Sigma_r \) for various compounds (in units of \( \text{cm}^2/\text{g} \), with the density factor removed).

![Graph showing removal cross sections per unit atomic mass for fission neutrons as a function of mass number. Over the range 8 \( \leq A \leq 240 \), the values are well fit by Eq. (3.4). [Reproduced from (Pa73) and references cited therein].](image)

Table 3.1 taken from (Pa73) gives representative values for \( \sigma_r \) for some energies where this approach is applicable.
Table 3.1 Removal cross-section data, $\sigma_{\text{rem}}$ (barns) for low energy neutrons. The typical accuracy is quoted to be $\pm 5\%$. [Reproduced from (Pa73) and references cited therein.]

<table>
<thead>
<tr>
<th>Element</th>
<th>1 MeV spectrum</th>
<th>2.9 MeV</th>
<th>4 MeV</th>
<th>6.7 MeV</th>
<th>14.9 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>0.90</td>
<td>1.58</td>
<td>1.05</td>
<td>0.83</td>
<td>0.50</td>
</tr>
<tr>
<td>Aluminum</td>
<td>1.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron</td>
<td>1.1</td>
<td>1.96</td>
<td>1.98</td>
<td>2.26</td>
<td>1.60</td>
</tr>
<tr>
<td>Copper</td>
<td>2.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead</td>
<td>3.28</td>
<td>3.70</td>
<td>3.44</td>
<td>3.77</td>
<td>2.95</td>
</tr>
</tbody>
</table>

The removal cross sections work rather well despite the fact that as more shielding is penetrated, neutrons of lower energy tend to dominate the spectrum over those in the few MeV region.
II. Limiting Attenuation at High Energy

This is perhaps the most important feature of neutron shielding at accelerators. As a function of energy, the neutron inelastic cross sections increase rapidly until about 25 MeV where they level off and then fall rapidly with energy in the region $25 < E_n < 100$ MeV to a value which becomes flat with energy. This fact was first noticed by Lindelbaum (Li61).

This fact makes high energy neutron beams attenuate approximately exponentially with an attenuation length, $\lambda_{\text{atten}}$, which is not very sensitive to energy. Thus, in units of length,

$$\lambda_{\text{atten}} = \frac{1}{N\sigma_{\text{in}}}$$

(3.5)

where $\sigma_{\text{in}}$ is the inelastic cross section (roughly equivalent to the so-called "absorption cross section", often denoted $\sigma_a$). This cross section specifically does not include elastic scattering and so is always smaller than the total cross section. $N$ is the number of atoms of the absorbing material per unit volume determined as in Chapter 1 in association with Eq. (1.6). In a "simple-minded" approach, this cross section can be taken to be geometric and the nucleon radius is taken to be $1.2 \times 10^{-13}$ cm. It then follows that one can multiply by the density to get:

$$\rho\lambda_{\text{atten}} = 38A^{1/3} \quad (\text{g/cm}^2).$$

(3.6)

Fig. 3.2 taken from (Pa73) illustrates the neutron inelastic cross sections for several materials up to a kinetic energy of 1.4 GeV.

![Inelastic neutron cross sections as a function of energy in the range 0 to 1.4 GeV. [Reproduced from (Pa73) as adapted from (Li61).]]
Results stated in (Pa73) are well represented by
\[ \sigma_{\text{in}} = 43A^{0.69} \text{ (mb)}, \] (3.7)
which was initially verified by cosmic ray results.

The interaction length, \( \lambda_{\text{inel}} \) (g/cm\(^2\)), is thus given by:
\[ \lambda_{\text{inel}} = \frac{\rho}{N\sigma_{\text{in}}} = 38.5 A^{0.31} \text{ (g/cm}\(^2\)) \] (3.8)

The geometric approximation was thus not inaccurate!

Figure 3.3 taken from (Pa73) shows the results for absorption cross sections.

In general, the ratio of elastic cross section, \( \sigma_{\text{el}} \), to absorption (inelastic) cross section, \( \sigma_{\text{in}} \), \( (\sigma_{\text{el}}/\sigma_{\text{in}}) \) approximates a value of 0.57. Thus the attenuation length appropriate for the total cross section \( \sigma_{\text{tot}} \) can be obtained by dividing the values of \( \lambda_{\text{in}} \) by 1.57. Reference (Sc90) has extensive tabulations of the value of \( \sigma_{\text{in}} \) (mb) for a variety of particles, energies, and materials in the high energy region as function of particle momenta up to 10 TeV/c.

The saturation of attenuation length for concrete as function of neutron energy is especially important. Figures 3.4 and 3.5 taken from (Th88) gives the results for both neutrons (Fig. 3.4) and protons (Fig. 3.5). An important feature of these results is the equivalence of the attenuation lengths for protons and neutrons at high energies.
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Fig. 3.4  The variation of the attenuation length $\lambda$ for monoenergetic neutrons in concrete as a function of neutron energy. The solid line shows recommended values of $\lambda$ and the dashed line shows the high energy limiting value of 1170 kg m$^{-2}$. [Reproduced from (Th88). The symbols on the figure are the results of calculations referred to in (Th88).]

Fig. 3.5  Effective attenuation length $\lambda$ in concrete as a function of proton energy. [Reproduced from (Th88). The symbols on the figure are the results of calculations referred to in (Th88).]
Chapter 3 Shielding of Proton and Ion Accelerators

III. Intermediate and High Energy Shielding-the Hadronic Cascade

The hadronic cascade from a conceptual standpoint

The cascade is initiated at proton accelerators when the beam interacts with components to produce neutrons and other particles. It can also arise at electron accelerators since, as described in Chapters 1 and 2, high energy secondary hadrons are produced in such circumstances.

The collision of a high energy nucleon with a nucleus produces a large number of particles; pions, kaons, and other nucleons (production of "rare" particles other than these are seldom of importance with respect to shielding calculations) as well as fragments of the struck nucleus. Above 1 GeV and at forward angles, the pions, protons, and neutrons, can be nearly equal in number [see (Th88)]. The neutrons may be classified as either evaporation neutrons or cascade neutrons. Evaporation neutrons originate as decays from excited states of residual nuclei and average a few MeV in energy. These neutrons tend to be isotropically distributed. Cascade neutrons are emitted by direct impact and their spectrum extends in energy up to the incident energy with diminishing probability following a spectrum roughly characterized as "1/E".

As the proton kinetic energy increases, other particles, notably π± and K±, play roles in the cascade when their production becomes energetically possible. They are absorbed with absorption lengths comparable in magnitude to, but not identical with those of protons. These particles also decay into muons. Because of their long ionization ranges and lack of nuclear interactions, muons provide a pathway for energy to escape the cascade.

Nucleons with E_n > 150 MeV propagate the cascade. This is clear from the attenuation length information as summarized in Fig. 3.2. Nucleons in the range 20 < E_n < 150 MeV also transfer their energy predominantly by nuclear interactions but their energy gets distributed over many particles of all types energetically possible. The ones that are charged particles are ranged-out very quickly. The role played by the energy of approximately 150 MeV for hadronic cascades is similar in kind to that played by the critical energy for electromagnetic ones.

Neutral pions (π⁰) are produced when the kinetic energy of the incident proton significantly exceeds the pion rest energy. The π⁰ rest energy is 134.9 MeV, its mean lifetime τ = (8.4 ± 0.6) X 10⁻¹⁷ s and cτ = 25.2 nm. The principal decay (99 % branching ratio) is into two γ-rays. An energetic π⁰ thus "appears" as two forward-peaked photons each with half of the π⁰'s total energy. The decay photons from π⁰ decay readily initiate electromagnetic cascades along with the hadronic one. It is possible for the electromagnetic channel to feed back into the hadronic cascade because it too produces high energy hadrons. However this effect is generally of little importance and, for most shielding calculations, the electromagnetic component of a hadronic cascade can be ignored. The exceptions principally involve energy deposition calculations at forward angles (small values of θ). In fact, at hundreds of GeV, electromagnetic cascades dominate the energy deposition at forward angles. This feature can have important ramifications if one needs to consider radiation damage to equipment and the heat load on cryogenic systems.

In general, the neutrons are the principal drivers of the cascade because of the ionization energy loss for pions and for protons below 450 MeV where the ionization range becomes roughly equal to the interaction length. Also, any magnetic fields that are present which can deflect and disperse the charged particles present will not, of course, affect the neutrons. Furthermore, neutrons can be produced at large values of θ compared with the forward-peaked pions.
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These phenomena, in general, apply also to ions heavier than the proton with suitable corrections (especially at low energies) for nuclear structure effects. Scaling of proton results for heavier ions will, in general, roughly be according to the specific energy (MeV/amu).

Figure 3.6 taken from (Pa73) is a schematic flow chart of the hadronic cascade process.

Fig. 3.6 Schematic representation of the development of the nuclear cascade. [Reproduced from (Pa73).]
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A simple one-dimensional cascade model

Thomas and Stevenson (Th88) reviewed the simple one-dimensional model of a cascade by Lindenbaum (Li61) which gives instructive results and supplies some "intuition" into the nature of the hadronic cascade. Figure 3.7 taken from (Th88) defines the geometry.

![Diagram of cascade model]

**Fig. 3.7** a) Single collision geometry for Lindenbaum approximation. b) Two collision geometry for Lindenbaum approximation. [Reproduced from (Th88).]

Suppose one initially has $N_0$ incident high energy nucleons. After an individual collision, one of them continues in its original direction at a reduced energy but with the same attenuation length, $\lambda$, (approximately true at high energies due to the limiting effect discussed previously)

$$v_1 = N_0 \exp(-z/\lambda). \quad (3.9)$$

will generate one or more secondary particles also with the same $\lambda$, until it has undergone a number of collisions, $n$, to be degraded to $E = 150$ MeV (below which energy the inelastic cross sections greatly increase). At this point it is said to be removed from the cascade.

For simplicity, assume that $n$ is an integer (in reality it has a statistical distribution).

Thus, referring to the figure, the number, $v_1$ that reach $x = z$ having made $n0$ collisions is

$$v_1 = N_0 \exp(-z/\lambda).$$

Suppose that there is one collision between 0 and $z$. The number that reach $z$ is given by the product of the number that reach elemental coordinate $dr$ multiplied by the probability of subsequently reaching $z$, times the probability of interacting in $dr$ ($dr/\lambda$), times the multiplicity, $m_1$, in the first interaction. Integrating over dr:

$$\int_0^z [N_0 \exp(-r/\lambda)] [\exp(-(z-r)/\lambda)] [m_1 \frac{dr}{\lambda}] = (N_0 m_1 \frac{z}{\lambda}) \exp(-z/\lambda) = v_2. \quad (3.10)$$
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Now suppose there are two collisions. The number that reach $z$ is the product of those that reach $s$ having made one collision, multiplied by the probability of subsequently reaching $z$, times the multiplicity in the second interaction $m_2$, times the probability of interacting in $ds$:

$$\int_0^z \left[ N_0 m_1 \frac{8}{\lambda} \exp(-s/\lambda) \right] \left[ \exp(-(z-s)/\lambda) \right] \left[ m_2 \frac{ds}{\lambda} \right]$$

$$= (N_0 m_1 m_2 \frac{1}{\lambda^2}) \exp(-z/\lambda) \int_0^z sds = (N_0 m_1 m_2 \frac{2}{\lambda^2}) \exp(-z/\lambda) = v_3 \quad (3.11)$$

Therefore, with $n$ defined as above, one can write:

$$N_n(x) = N_0 \beta_n(z/\lambda) \exp(-z/\lambda) \quad (3.12)$$

where $\beta$ is a "buildup" factor,

for $n = 1 \quad N_1 = v_1 \quad \beta_1 = 1$

for $n = 2 \quad N_2 = v_1 + v_2 \quad \beta_2 = 1 + (m_1 z/\lambda)$

for $n = 3 \quad N_3 = v_1 + v_2 + v_3 \quad \beta_3 = 1 + (m_1 z/\lambda) + (m_1 m_2 z^2/2\lambda^2)$.

Thus this buildup factor is a monotonically increasing function of $z$.

If $m_1 = m_2 = \ldots = m$ (i.e., the assumption that the multiplicity stays the same for all interactions in this simple model) and $mz/\lambda >> n$, the absorption is approximately exponential with a mean free path, $\lambda_{cas}$, given by:

$$\lambda_{cas} = \lambda + \Delta \lambda, \quad \Delta \lambda = n \lambda/zm \quad (3.13)$$

This correction becomes small as $mz/\lambda$ increases, so that the true $\lambda_{cas}$ is not attained until large depths are reached. Figure 3.8 taken from (Th88) plots the number of particles after three generations as a function of $x/\lambda$ ($m = 2$ and $n = 3$). For this condition, the exponential region is not achieved until $z/\lambda = 10$. In concrete, this represents a depth of $\approx 1200$ g/cm$^2$. Figure 3.9, also taken from (Th88), compares a measurement and Monte-Carlo calculation which are in remarkably good agreement with a simple one-dimensional model similar to this one for a particular special case.
Fig. 3.8 Development of a one-dimensional cascade in the Lindenbaum approximation with \( n = 3 \) and \( m = 2 \). [Reproduced from (Th88).]

Fig. 3.9 The laterally integrated star density in nuclear emulsions produced by a 19.2 GeV/c proton beam incident on an iron slab as a function of depth in the slab. The Monte-Carlo calculations of Ranft are compared with measurements of Citron and an approximation of Passow. [Reproduced from (Th88 and references cited therein).]
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Such analytical approaches are constructive qualitatively but have severe limitations, among which are:

- restriction to one dimension
- neglect of ionization energy losses
- neglect of elastic and multiple Coulomb scattering
- assumption that all secondary particles go forward
- assumption that multiplicities are not dependent on energy and particle type
- assumption that $\lambda$ is a constant for all particles at all energies
- neglect of radiative effects
- neglect of escape of energy into the muon and electromagnetic cascade channels

THUS IT IS CLEAR THAT BETTER METHODS ARE NEEDED!

Semiempirical methods: The Moyer model for point source

References (Pa73), (IC78), (Sc90), (Ro76), (St82), (Th84), (McC87), (Te83), (Te85), (McC85), (Co82), and (Co85) all bear on the development of this model which is based, predominantly, on an exponential approximation with constants fitted to actual data spanning the range of proton beam energies from 7.4 to 800 GeV. The summary of this method here is largely taken from (Pa73) and (Sc90). It is called the “Moyer Model” in honor of the late Burton J. Moyer who developed it at LBL to solve particular shielding problems predating the development of large, fast computers and advanced Monte-Carlo techniques.

The starting point is Fig. 3.10 taken from (Pa73) which describes a "point" target:

![Diagram of target and shielding geometry](image)

Fig. 3.10  Target and shielding geometry which defines quantities used in the point source Moyer model.

The Moyer model for a point source is developed as follows. The number of neutrons, $dN/dE$ which are emitted into a given element of solid angle $d\Omega$ at angle $\theta$ relative to a target struck by $N_p$ protons/sec in an energy interval $E + dE$ is given by:
where \( B(E) \) is a "buildup factor" and the exponential attenuates the radiation field as it passes through the shield thickness, \( d \), at the "slant angle", \( \theta \). The role of the double differential of the yield is obvious. In the above, the flux density at distance \( r \) can be obtained by including the factor:

\[
\frac{d\Omega}{dA} = \frac{1}{r^2} = \frac{1}{(a + d)^2 \csc^2(\theta)}.
\]  

(3.15)

The integral flux density, \( \phi \), at the point where the ray emerges from the shield is given by

\[
\phi = \frac{1}{r^2} \int_{E_{\text{min}}}^{E_{\text{max}}} dE \ N_p \left( \frac{d^2Y}{dE d\Omega} \right) B(E) \exp \left[ -\frac{d \csc(\theta)}{\lambda(E)} \right].
\]  

(3.16)

Moyer introduced the following simplifying assumptions for this model. They are as follows:

A. \( \lambda(E) = \lambda = \text{constant} \) for \( E \geq 150 \text{ MeV} \) and \( \lambda(E) = 0 \) for \( E < 150 \text{ MeV} \). This is a simplified rendering of the leveling-off of the inelastic cross section at high energy. \[ \phi(E_n > 150 \text{ MeV}) = N_p \frac{1}{r^2} \exp \left[ -\frac{d \csc(\theta)}{\lambda} \right] \int_{150 \text{ MeV}}^{E_{\text{max}}} dE \ \left( \frac{d^2Y}{dE d\Omega} \right) B(E). \]  

(3.17)

B. The neutrons emitted at angle \( \theta \) can be represented by a simple function \( f(\theta) \) multiplied by a multiplicity factor \( M(E_{\text{max}}) \) that depends only on the incident energy, thus:

\[
\phi(E_n > 150 \text{ MeV}) = N_p \frac{1}{r^2} \exp \left[ -\frac{d \csc(\theta)}{\lambda} \right] M(E_{\text{max}}) f(\theta) = N_p \frac{1}{r^2} \exp \left[ -\frac{d \csc(\theta)}{\lambda} \right] g(E_{\text{max}}, \theta)
\]  

(3.18)

where \( g(E_{\text{max}}, \theta) \) is an angular distribution function constant for given \( E_{\text{max}} \) and given target.

C. The fluence to dose equivalent conversion factor for neutrons with energy \( > 150 \text{ MeV} \), \( P_{150} \), is not strongly dependent on energy (see Fig. 1.6). Thus the dose equivalent just outside of the shield due to neutrons \( > 150 \text{ MeV} \) is given by:

\[
H(E_n > 150 \text{ MeV}) = P_{150} \phi(E_n > 150 \text{ MeV}).
\]
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The total dose equivalent, $H$, then is given by

$$H = kH(E_n > 150 \text{ MeV}) \text{ where } k \geq 1. \quad (3.19)$$

This implicitly assumes that the low-energy neutrons are in equilibrium with those $> 150$ MeV so that the spectrum no longer changes with depth. This is a valid assumption for a shield more than a few mean free paths thick.

Thus, Moyer's assumptions lead to:

$$H = \frac{kP_{150}N_p g(E_{\text{max}}, \theta)}{(a + d)^2 \csc^2(\theta)} \exp \left[ -\frac{d \csc(\theta)}{\lambda} \right]. \quad (3.20)$$

(Sc90) has generalized the results for the geometry shown in Fig. 3.11 with multiple materials in the shield.

![Fig. 3.11 Sketch of the geometry for the empirical Moyer Model. The proton beam, $p$, impinges on the target, $T$. The shield materials $S_1$-$S_4$ could be, for example, iron, concrete, earth and air respectively. $R$ is the internal radius of the tunnel, assumed to offer no attenuation. The observer $O$ is situated at a radial thickness of $x$ equal to the sum of $x_1$ to $x_4$. [Reproduced from (Sc90).]]

The parameter $\zeta$ is introduced to take care of the multiple ($n$) shielding components:

$$\zeta = \sum_{i=1}^{n} \frac{x_i}{\lambda_i} \quad (3.21)$$

where the sum is over the $i$ layers of shielding.

Recent work, notably (St82) and (Th84), has determined that the data indicate that $f(\theta)$ is given by:

$$f(\theta) = \exp(-\beta \theta), \quad (3.22)$$

and that, in fact, $\beta = 2.3 \text{ rad}^{-1}$ (for $E_n > 150$ MeV).
Thus,

\[ H = \frac{H_0(E_p) \exp(-\beta \theta) \exp(-\zeta \csc(\theta))}{(r \csc(\theta))^2} \]  \hspace{1cm} (3.23)

in which \( r = R + \sum_{i=1}^{n} x_i \) and

where \( H_0(E_p) \exp(-\beta \theta) \) is determined from the yield data and empirical measurements.\(^1\) \( H_0(E_p) \) is best fit as a power law; \( H_0(E_p) = kE^n \). From such results (per incident proton):

\[ H_0(E_p) = [(2.84 \pm 0.14) \times 10^{-13}] E_p^{0.80 \pm 0.10} \text{ Sv m}^2. \]  \hspace{1cm} (3.24)

\[ = 2.84 \times 10^{-8} E_p^{0.8} \text{ mrem m}^2 = 2.8 \times 10^{-4} E_p^{0.8} \text{ mrem cm}^2 \]

with \( E_p \) in GeV (per proton). These results are derived for relatively "thick" targets (like accelerator magnets) in tunnel geometries. (Sc90), based on Monte-Carlo results gives values for "thin" targets of \( k = 2.0 \times 10^{-13} \) (Sv) and \( n = 0.5 \). A beam pipe would be an example of a "thin" target. The differences thus reflect buildup in the shower. For thick lateral shields close to the beam where the cascade immediately becomes fully developed, \( k = (6.9 \pm 0.1) \times 10^{-15} \) (Sv) independent of target material [(Sc90) and(St87)].

Similarly, recommended values of \( \lambda \) are;

- concrete: \( 1170 \pm 20 \text{ kg/m}^2 = 117 \text{ g/cm}^2 \)
- others: \( 428 \pm 11 \text{ kg/m}^2 = 42.8 \pm 11 \text{ g/cm}^2 \).

These values are 15-30% larger than the "nuclear interaction lengths" and are reflective of the shower phenomena discussed above.

If one sets the partial derivative \( \frac{\partial H}{\partial \theta} \) = zero, one can derive the following equation for determining the value of \( \theta = \theta' \) at which the maximum dose equivalent occurs. Generally this equation can be solved by successive approximation methods.

\[ \zeta \cos \theta' - \beta \sin^2(\theta') + 2\cos(\theta')\sin(\theta') = 0. \]  \hspace{1cm} (3.25)

One can substitute into the above equation to get the maximum dose equivalent at a given radial depth. According to (McC87), over a wide range of values of \( \zeta \), the following holds:

\[ H_{\text{max}} = 1.66 \times 10^{-14} E_p^{0.8} \exp(-\zeta) \frac{r^{0.245}}{r^2} \text{ Sv (r in m) (per proton)}. \]  \hspace{1cm} (3.26)

\(^1\) In this equation, the symbol \( r \) denotes the distance from the beam axis to the location of interest along a line perpendicular to the beam axis while in Fig. 3.10, the symbol \( r \) denotes the actual distance from the point of interaction to the location of concern.
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For values of $\zeta > 2$, the following is equally accurate:

$$H_{\text{max}} = 1.26 \times 10^{-14} E_p^{0.8} \exp \left( -1.023 \zeta \right) \frac{\text{Sv}}{r^2} \text{ (per proton).} \quad (3.27)$$

The Moyer Model for line sources

Assume a uniform source of one proton interacting per unit length. Then, the dose equivalent from the individual increments along the line source contribute to the total at any given point, P, external to the shield. Fig. 3.12 shows the integration variables.

![Diagram of Moyer model for line source](image)

Fig. 3.12  Variables of integration of Moyer point source result to obtain Moyer line source results.

One can integrate the elements $dl$ of a line source at given perpendicular distance $r$ as follows, making the change of variable of integration from the line integral the integral over angle $\theta$ ($dl = r \csc^2 \theta \, d\theta$);

$$H = H_o(E_p) \int_{-\infty}^{\infty} dl \frac{\exp(-\beta \theta) \exp(-\zeta \csc(\theta))}{r^2 \csc^2(\theta)}$$

$$H_o(E_p) \int_{0}^{\pi} d\theta \frac{\exp(-\beta \theta) \exp(-\zeta \csc(\theta))}{r \csc^2(\theta)} \quad =$$

$$\frac{H_o(E_p)}{r} \int_{0}^{\pi} d\theta \frac{\exp(-\beta \theta) \exp(-\zeta \csc(\theta))}{r \csc^2(\theta)} \quad =$$

$$\frac{H_o(E_p)}{r} \int_{0}^{\pi} d\theta \frac{\exp(-\beta \theta) \exp(-\zeta \csc(\theta))}{r} \quad = \frac{H_o(E_p)}{r} M(\beta, \zeta) \quad (3.28)$$

(per interacting proton per unit length).
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The integral in the above, \( M(\beta, \zeta) \), is the Moyer integral. The values of this integral have been tabulated in (Ro76).

In view of the above results, \( M(2.3, \zeta) \) has obvious special significance and is tabulated extensively in (Sc90). Tesch (Te83), has made an important contribution in that he determined an approximation to this integral which others have come to call the "Tesch approximation":

\[
M_T(2.3, \zeta) = 0.065 \exp(-1.09\zeta). \tag{3.29}
\]

For "intermediate" values of \( \zeta \), \( M_T(2.3, \zeta) \) can be used instead of \( M(2.3, \zeta) \) to simplify calculations. Table 3.2 adapted from (Sc90) gives the ratio \( M_T(2.3, \zeta)/M(2.3, \zeta) \) as a function of \( \zeta \).

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>( M_T(2.3, \zeta)/M(2.3, \zeta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.27</td>
</tr>
<tr>
<td>1.0</td>
<td>0.53</td>
</tr>
<tr>
<td>2.0</td>
<td>0.75</td>
</tr>
<tr>
<td>3.0</td>
<td>0.90</td>
</tr>
<tr>
<td>4.0</td>
<td>1.00</td>
</tr>
<tr>
<td>5.0</td>
<td>1.06</td>
</tr>
<tr>
<td>6.0</td>
<td>1.09</td>
</tr>
<tr>
<td>7.0</td>
<td>1.10</td>
</tr>
<tr>
<td>8.0</td>
<td>1.10</td>
</tr>
<tr>
<td>9.0</td>
<td>1.08</td>
</tr>
<tr>
<td>10.0</td>
<td>1.06</td>
</tr>
<tr>
<td>11.0</td>
<td>1.02</td>
</tr>
<tr>
<td>12.0</td>
<td>0.99</td>
</tr>
<tr>
<td>13.0</td>
<td>0.95</td>
</tr>
<tr>
<td>14.0</td>
<td>0.91</td>
</tr>
<tr>
<td>15.0</td>
<td>0.86</td>
</tr>
<tr>
<td>16.0</td>
<td>0.82</td>
</tr>
<tr>
<td>17.0</td>
<td>0.78</td>
</tr>
<tr>
<td>18.0</td>
<td>0.73</td>
</tr>
<tr>
<td>19.0</td>
<td>0.69</td>
</tr>
<tr>
<td>20.0</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Of course, few so-called "line sources" are actually infinite in length. Thus, the integration would need to be performed over only a finite angular range. Moreover, in practice only a limited angular range (and hence length) contributes significantly to the Moyer integral. Tables 3.3 and 3.4 taken from (Sc90) give angular integration limits (in degrees) corresponding to 90% of the \( M(2.3, \zeta) \) as a function of \( \zeta \) (Table 3.3) and the distances along the \( z \) axis corresponding to 90% of \( M(2.3, \zeta) \) as a function of the radial distance and \( \zeta \) (Table 3.4).
Chapter 3  Shielding of Proton and Ion Accelerators

Table 3.3 Angular integration limits in degrees which contain 90% of the Moyer Integral $M(2.3, \zeta)$. [Reproduced from (Sc90).]

<table>
<thead>
<tr>
<th>Depth($\zeta$)</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>31.52</td>
<td>106.58</td>
</tr>
<tr>
<td>3.0</td>
<td>34.35</td>
<td>107.15</td>
</tr>
<tr>
<td>3.5</td>
<td>36.19</td>
<td>107.47</td>
</tr>
<tr>
<td>4.0</td>
<td>39.00</td>
<td>107.64</td>
</tr>
<tr>
<td>4.5</td>
<td>40.91</td>
<td>107.72</td>
</tr>
<tr>
<td>5.0</td>
<td>42.67</td>
<td>107.73</td>
</tr>
<tr>
<td>5.5</td>
<td>44.10</td>
<td>107.71</td>
</tr>
<tr>
<td>6.0</td>
<td>45.77</td>
<td>107.66</td>
</tr>
<tr>
<td>6.5</td>
<td>47.22</td>
<td>107.57</td>
</tr>
<tr>
<td>7.0</td>
<td>48.51</td>
<td>107.48</td>
</tr>
<tr>
<td>7.5</td>
<td>49.58</td>
<td>107.38</td>
</tr>
<tr>
<td>8.0</td>
<td>50.68</td>
<td>107.28</td>
</tr>
<tr>
<td>8.5</td>
<td>51.86</td>
<td>107.17</td>
</tr>
<tr>
<td>9.0</td>
<td>52.70</td>
<td>107.04</td>
</tr>
<tr>
<td>9.5</td>
<td>53.51</td>
<td>106.92</td>
</tr>
<tr>
<td>10.0</td>
<td>54.34</td>
<td>106.79</td>
</tr>
<tr>
<td>10.5</td>
<td>55.21</td>
<td>106.67</td>
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<td>11.0</td>
<td>56.07</td>
<td>106.54</td>
</tr>
<tr>
<td>11.5</td>
<td>56.65</td>
<td>106.42</td>
</tr>
<tr>
<td>12.0</td>
<td>57.25</td>
<td>106.29</td>
</tr>
<tr>
<td>12.5</td>
<td>57.84</td>
<td>106.16</td>
</tr>
<tr>
<td>13.0</td>
<td>58.45</td>
<td>106.04</td>
</tr>
<tr>
<td>13.5</td>
<td>59.09</td>
<td>105.91</td>
</tr>
<tr>
<td>14.0</td>
<td>59.74</td>
<td>105.78</td>
</tr>
<tr>
<td>14.5</td>
<td>60.25</td>
<td>105.66</td>
</tr>
<tr>
<td>15.0</td>
<td>60.66</td>
<td>105.54</td>
</tr>
<tr>
<td>15.5</td>
<td>61.07</td>
<td>105.41</td>
</tr>
<tr>
<td>16.0</td>
<td>61.49</td>
<td>105.29</td>
</tr>
<tr>
<td>16.5</td>
<td>61.91</td>
<td>105.17</td>
</tr>
<tr>
<td>17.0</td>
<td>62.34</td>
<td>105.04</td>
</tr>
<tr>
<td>17.5</td>
<td>62.77</td>
<td>104.91</td>
</tr>
<tr>
<td>18.0</td>
<td>63.22</td>
<td>104.80</td>
</tr>
<tr>
<td>18.5</td>
<td>63.67</td>
<td>104.67</td>
</tr>
<tr>
<td>19.0</td>
<td>64.08</td>
<td>104.54</td>
</tr>
<tr>
<td>19.5</td>
<td>64.36</td>
<td>104.43</td>
</tr>
<tr>
<td>20.0</td>
<td>64.63</td>
<td>104.30</td>
</tr>
</tbody>
</table>

Table 3.4 Distances corresponding to 90% limits in Moyer Integrals. [Reproduced from (Sc90).]

<table>
<thead>
<tr>
<th>Radial distance m</th>
<th>Shield thickness m</th>
<th>$\zeta$</th>
<th>Upstream limit $z_1$ m</th>
<th>Downstream limit $z_2$ m</th>
<th>Total length $z_2 - z_1$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>1.0</td>
<td>-4.2</td>
<td>0.3</td>
<td>4.5</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>2.0</td>
<td>-3.7</td>
<td>0.6</td>
<td>4.3</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
<td>5.0</td>
<td>-3.8</td>
<td>1.1</td>
<td>4.9</td>
</tr>
<tr>
<td>6.0</td>
<td>5.0</td>
<td>10.0</td>
<td>-4.3</td>
<td>1.8</td>
<td>6.1</td>
</tr>
<tr>
<td>8.5</td>
<td>7.5</td>
<td>15.0</td>
<td>-4.8</td>
<td>2.4</td>
<td>7.2</td>
</tr>
<tr>
<td>11.0</td>
<td>10.0</td>
<td>20.0</td>
<td>-5.2</td>
<td>2.8</td>
<td>8.0</td>
</tr>
</tbody>
</table>
Chapter 3  Shielding of Proton and Ion Accelerators

The Moyer Model generally does not work at forward angles. For these situations, the Boltzmann equation must be solved. Monte-Carlo calculations are often the best approximation to such solutions. It should be pointed out that (McC85) demonstrates that the Moyer Model approach works for moderately energetic heavy ions. It has been found that the Moyer Model approach works well even into the intermediate energy region $200 < E_0 < 1000$ MeV. This may be interpreted as due to the relatively smooth dependence of neutron yield upon incident proton kinetic energy.

Review of Popular Monte-Carlo Codes [taken from (NC96) and (Sc90).]

HETC

This code, developed over many years under the leadership of R. G. Alsmiller by the Neutron Physics (now Engineering Physics) Division of the Oak Ridge National Laboratory, is the benchmark hadron shielding code. It has been upgraded many times and can, in suitably augmented versions, follow particles from the $20$ TeV region down to thermal energies. It is an extremely flexible code but has the important disadvantage that the events are written to tape. It is the responsibility of the user to write a program to analyze the results. In terms of CPU-time HETC is also relatively slow so that calculations to be done should be carefully selected. It is seen to be preferable to use selected HETC runs to "calibrate" some faster, but less accurate code. It is best described by Armstrong (Ref. 28) and Gabriel (29). It now uses the same event generator that FLUKA uses (see below). A simple example of an HETC calculation is given in Fig. 3.13 taken from (Al75) for the case of $200$ MeV protons incident on "thin" and "thick" aluminum targets. It plots $r^2H$ as a function of angle for several intervals of $\theta$ in a concrete shield.
Fig. 3.13 Calculations of $r^2N$ as a function of concrete shield thickness for several intervals of $\theta$ for 200 MeV protons incident on an aluminum target. [Reproduced from (A175).]
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FLUKA

FLUKA is a module program for computing hadronic and electromagnetic cascades written by J. Ranft of CERN and Leipzig. It can provide flexible multi-region multi-medium geometries and can use a variety of particle production models. It uses weighted Monte-Carlo techniques for special purposes. A recent version, FLUKA82, represents a full analog simulation of the cascade. The most up-to-date production mechanisms of any of the codes and, uncommon among this type of code, quantum number, momentum, and energy conservation are required as a constraint for all collisions. The production models used are scaled to experimental results. Internally, more than 100 hadron resonances are included in the models. Externally, nucleons, pions, kaons, some hyperons, and all their anti-particles are taken into account. Muons (and neutrinos) are not followed. Electromagnetic showers resulting from π⁰ production can be fed directly into EGS4. Particles are not followed below 50 MeV kinetic energy. However, this code can be coupled with the standard low-energy photon and neutron code MORSE (Em75) to follow neutrons down to thermal energies. A recent version is described in (Aa86). The geometries come in the form of calls to packages of "combinatorial geometries" which are quite similar to those originally developed at ORNL for MORSE.

CASIM

A. Van Ginneken at Fermilab has developed this program (Va75). It was designed to simulate the average behavior of hadrons in the region 10 to 1000 GeV but has recently been extended to 20 TeV (Va87). It uses inclusive production distributions directly in order to obtain the particles to follow. It uses the Hagedorn-Ranft thermodynamic model. Only one or two high energy particles are created in each collision and these carry a weight related to their probability of production and the energy carried with them. Path length stretching and particle splitting have been introduced. Electromagnetic showers resulting from π⁰ production are handled using AEGIS. Simple "standardized" geometries are available. However, the user generally writes a FORTRAN subroutine to set up the geometry of interest. This subroutine consists of "IF" statements used to deduce the location of the particle in space or in magnetic fields. The program readily allows magnetic-fields to be used. A muon version called CASIMU (now MUSIM) has been written (Va87). The accuracy of the hadron version has been verified for energies up to 800 GeV (Co82) and the muon version has been verified up to 800 GeV [production and transport in complicated shields, (Co89b)] and 500 GeV (transport in an earth shield (Co89a)). Normally, CASIM is not set up to follow particles with momenta less than 300 MeVc, which corresponds to a kinetic energy of 47 MeV for nucleons. All low energy phenomena, then, is obtained by matching energy spectra and fluence at this energy with results of codes capable of tracking lower energy particles (e.g., HETC).

MARS

This program was developed under the leadership of N. Mokhov at the Institute for High Energy Physics at Serpukhov, Russia (Ka84). The current version is denoted MARS10. It is somewhat similar to CASIM in a number of ways but uses a more modern production model; an additive quark model of hadron-nucleus collisions for events having higher momentum transfers while it uses a phenomenological model for the lower energy particle production. It, too, allows multimedia geometries. It now uses AEGIS (see Chapter 2) to follow the electromagnetic cascades. In its present version, it is "rated" to run at 30 TeV, and with crude estimation techniques, even at 10⁴ TeV for studies related to the deep underwater muon and neutrino detector (DUMAND) experiment.
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The three high energy codes (FLUKA, CASIM, AND MARS) have been compared in (Mo86). Star and energy deposition densities were calculated for several selected cases and demonstrated to be in generally good agreement with each other, and more importantly, with experiment. An interesting result of this comparison is that a determination of the Moyer Model energy scaling parameter, \( n \), (in \( E^n \)) over the range from \( 70 < E < 2 \times 10^4 \) GeV of \( n = 0.81 \) was made. Using the now obsolete CYBER-875 as a reference computer, MARS and CASIM are of comparable speeds with MARS being somewhat faster for comparable calculations. FLUKA is distinctively slower, but its more exact modeling is likely to be better for problems where studies of the statistical fluctuations are important.

General comments on Monte-Carlo star-to-dose conversions

All of the above codes, in general, calculate star densities. This quantity is more correctly called the density of inelastic interactions (stars/cm\(^3\)). The term "star" comes from historic cosmic ray work in which the high energy interaction events, with their large multiplicities, looked like "pointed stars". The conversion factor from star densities to dose equivalent is rather important and has recently been calculated by Stevenson [in (Sc90)]. While this conversion factor is somewhat dependent upon the position in the shield, after reasonable shield thicknesses (i.e., sufficient to establish "equilibrium" spectra), a constant value may be used. These values are given in Table 3.5 taken from (Sc90). This table also gives the star fluence obtained by multiplying by the nuclear interaction length. The star fluence roughly corresponds to the fluence of hadrons having energies above that where the cross section "levels off". For concrete a value of \( 4.9 \times 10^{-8} \) Sv cm\(^3\)/star is obtained.\(^2\) As one can see, the energy dependence is rather small.

Table 3.5  Coefficients to convert star densities \( S^* \) and star fluence \( \phi^* \) into dose equivalent. A star density is transformed into star fluence by the relation \( \phi^* = S^* \lambda' \) where \( \lambda' \) is the nuclear interaction length. [Reproduced from (Sc90).]

<table>
<thead>
<tr>
<th>Proton energy</th>
<th>Target material</th>
<th>Conversion coefficient ( S^* ) ( \times 10^{-8} )</th>
<th>( \lambda' ) cm</th>
<th>Conversion coefficient ( \phi^* ) ( \times 10^{-9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 GeV</td>
<td>Iron</td>
<td>2.04 ± 0.06</td>
<td>17.1</td>
<td>1.19 ± 0.04</td>
</tr>
<tr>
<td>100 GeV</td>
<td>Iron</td>
<td>2.15 ± 0.08</td>
<td>17.8</td>
<td>1.21 ± 0.05</td>
</tr>
<tr>
<td>1 TeV</td>
<td>Iron</td>
<td>2.12 ± 0.08</td>
<td>17.2</td>
<td>1.23 ± 0.05</td>
</tr>
<tr>
<td>Mean</td>
<td>Iron</td>
<td>2.10 ± 0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 GeV</td>
<td>Aluminium</td>
<td>4.62 ± 0.17</td>
<td>38.6</td>
<td>1.20 ± 0.04</td>
</tr>
<tr>
<td>100 GeV</td>
<td>Tungsten</td>
<td>1.19 ± 0.05</td>
<td>9.25</td>
<td>1.29 ± 0.05</td>
</tr>
<tr>
<td>Mean</td>
<td>All</td>
<td></td>
<td></td>
<td>1.22 ± 0.02</td>
</tr>
</tbody>
</table>

\(^2\)The version of this table appearing in (Sc90) contains an error in that the \( 10^{-9} \) multiplier applied to the values in the right-most column is incorrectly given in (Sc90) as \( 10^{-4} \). This was confirmed in a private communication with G. R. Stevenson.
Ref. (Sc90) contains a comprehensive set of Monte-Carlo results. The most popular way to display these results is to give contour plots of star density as function of longitudinal coordinate, Z, and radial coordinate, r, assuming cylindrical symmetry. Results for solid concrete and iron cylinders taken from (Sc90) are reproduced here, respectively, in Figs. 3.14 and 3.15. The dimensions used in the geometries are scaled in size with the proton energy. Table 3.6 adapted from (Sc90) lists the calculations in that reference which are given for Figs. 3.14 and 3.15.

Table 3.6 FLUKA Monte-Carlo calculations for solid concrete and iron beam dumps irradiated with high energy protons. [Adapted from (Sc90).]

<table>
<thead>
<tr>
<th>Material</th>
<th>Incident Proton Momentum (GeV/c)</th>
<th>Dimensions of Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Z (cm)</td>
</tr>
<tr>
<td>Concrete</td>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>10^2</td>
<td>1250</td>
</tr>
<tr>
<td></td>
<td>10^3</td>
<td>1250</td>
</tr>
<tr>
<td></td>
<td>10^4</td>
<td>1250</td>
</tr>
<tr>
<td>Iron</td>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>10^2</td>
<td>500</td>
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<td></td>
<td>10^3</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>10^4</td>
<td>500</td>
</tr>
</tbody>
</table>
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Fig. 3.14 Contours of equal star density in a cylindrical block of CONCRETE obtained from the Monte-Carlo program FLUKA. The beam enters along the axis of the cylinder (vertical arrow). The depth, Z, axis is vertical and the radial, R, axis is horizontal. All dimensions are given in cm. The numbers on the contours correspond to the star density (stars/cm$^3$) per incoming proton, a corresponds to a proton momentum of 1 GeV/c, b to 10 GeV/c, c to 100 GeV/c, d to 1 TeV/c, and e to 10 TeV/c. [Reproduced from (Sc90).]
Fig. 3.15 Contours of equal star density in a cylindrical block of IRON obtained from the Monte-Carlo program FLUKA. The beam enters along the axis of the cylinder (vertical arrow). The depth, $Z$, axis is vertical and the radial, $R$, axis is horizontal. All dimensions are given in cm. The numbers on the contours correspond to the star density (stars/cm$^3$) per incoming proton. a corresponds to a proton momentum of 1 GeV/c, b to 10 GeV/c, c to 100 GeV/c, d to 1 TeV/c, and e to 10 TeV/c. [Reproduced from (Sc90).]
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(Sc90) also gives results for the important calculations of equal star densities and dose equivalents due to protons incident on a target inside of a magnet located in a tunnel. (Cylindrical symmetry is used.) Table 3.7 taken from (Sc90) gives the dimensions used for the various momenta considered. Fig. 3.16 taken from (Sc90) shows the situation schematically. The magnet consists of iron and is centered in a concrete tunnel. Around the small target is an iron vacuum tube. The results of the FLUKA calculations (contour plots of equal star density) at 10 GeV/c, 100 GeV/c, and 10 TeV/c are presented in Fig 3.17 taken from (Sc90).

Table 3.7 Dimensions used in Fig. 3.16 for various momenta.
[Reproduced from (Sc90).]

<table>
<thead>
<tr>
<th>Momentum</th>
<th>10 GeV/c</th>
<th>100 GeV/c</th>
<th>10 TeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 ): Radius of target (beryllium) [cm]</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( R_1 ): Inner radius of vacuum pipe [cm]</td>
<td>4.9</td>
<td>4.9</td>
<td>4.9</td>
</tr>
<tr>
<td>( R_2 ): Outer radius of vacuum pipe = radius of magnet bore [cm]</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>( R_3 ): Outer radius of magnet (iron) [cm]</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>( R_4 ): Inner radius of tunnel [cm]</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>( R_s ): Outer radius of tunnel [cm]</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>( X_0 ): Start of tunnel [m]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( x_1 ): Upstream end of magnet [m]</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>( x_1 ): Target position [m]</td>
<td>11.0</td>
<td>11.0</td>
<td>101.0</td>
</tr>
<tr>
<td>( x_2 ): Target position [m]</td>
<td>11.1</td>
<td>11.1</td>
<td>101.1</td>
</tr>
<tr>
<td>( x_2 ): Downstream end of magnet [m]</td>
<td>20</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>( x_3 ): End of tunnel [m]</td>
<td>100</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td>Figure number</td>
<td>2.43a</td>
<td>2.43b</td>
<td>2.43c</td>
</tr>
</tbody>
</table>
Fig. 3.16 Geometry for the Monte-Carlo calculations of star densities in a concrete tunnel when a beryllium target in a magnet is hit by a proton beam. Note that in this figure, the coordinate labeled "x" is the longitudinal coordinate labeled "z" elsewhere. The resultant calculations are given in Fig. 3.17. [Reproduced from (Sc90).]

Fig. 3.17 Contours of equal star densities (star/cm$^3$) for an incoming proton. Z and R are longitudinal and radial coordinates, respectively. The particles interact first in the beryllium target (T, 4 cm diameter, 10 cm length) which sits inside a steel vacuum tube (V, 9.8 - 10 cm diameter), inside a steel cylinder representing a magnet (M, 10 m long, 10 - 50 cm diameter). This target/magnet configuration is inside a concrete tunnel of 3 m inside diameter. a proton momentum 10 GeV/c, b proton momentum 100 GeV/c, c proton momentum 10 TeV/c. [Reproduced from (Sc90).]
A comparison of FLUKA and CASIM in terms of the maximum dose equivalent per proton as a function of radius for different energies is given in Fig. 3.18 taken from (Sc90). The solid curve is essentially the Moyer Model.

Fig. 3.18 Variation of the dose equivalent per proton at the position of the longitudinal maximum multiplied by the square of the radius $H_r^2$ vs radius off-axis for proton induced cascades in iron of density 7.2 g cm$^{-3}$. Open circles are the FLUKA calculations; full circles indicate the CASIM calculations. The solid lines are derived from the empirical parametrizations. [Reproduced from (Sc90).]
Longitudinally, FLUKA and CASIM results in iron and concrete shields are compared in Figs. 3.19 and 3.20, respectively, taken from (Sc90).

Fig. 3.19 Variations of the dose equivalent per proton H on the longitudinal axis vs. depth Z in the shield for proton-induced cascades in IRON of density 7.2 g cm$^{-3}$. The solid symbols represent FLUKA calculations for incident proton momenta of 10 GeV/c, 100 GeV/c, 1 TeV/c, and 10 TeV/c. The open symbols correspond to CASIM calculations at the marked proton momenta. There are no CASIM results for 10 GeV/c. [Reproduced from (Sc90).]

Fig. 3.20 Variations of the dose equivalent per proton H on the longitudinal axis vs. depth Z in the shield for proton-induced cascades in CONCRETE of density 2.4 g cm$^{-3}$. The solid symbols represent FLUKA calculations for incident proton momenta of 10 GeV/c, 100 GeV/c, 1 TeV/c, and 10 TeV/c. The open symbols correspond to CASIM calculations at the marked proton momenta. There are no CASIM results for 10 GeV/c. [Reproduced from (Sc90).]
Chapter 3  Shielding of Proton and Ion Accelerators

Shielding Against Muons at Proton Accelerators

Muon production has been discussed previously in Chapter 1. At the higher energies, there are significant complications in that muon creation mechanisms in addition to pion and kaon production and subsequent decay are possible. However, the muons from pion and kaon decay generally, but not universally, represent the most important consideration in practical shielding calculations. In Monte-Carlo calculations, it is straightforward to "create" muons and follow them through the shielding medium. The previous discussion of Monte-Carlo programs discusses this topic.

Muon transport is well understood. Because of the lack of strong interactions, their absorption cross sections in shielding materials are negligible. The energy loss is dominated by ionization and excitation of atomic electrons. Coulomb scattering alters their paths. Because of their higher masses, radiative energy losses do not become important until their energies reach approximately 100 GeV. Other energy loss mechanisms also become important at the higher energies. The range-energy relations for muons were discussed in Chapter 1.

The effect of beam loss mechanisms on dose at proton and ion accelerators is, however, considerably different than in the electron situation. The particle energy downgrades quickly in hadronic showers so the most penetrating muons must originate in the first few generations of the process. These energetic muons are not "smeared out" in a large volume of phase space as are the neutrons. However, geometric effects or deflections by magnetic fields encountered near the point of production can affect the muon fluence at large distances. Thus, the presence of large "empty" spaces (vacuum or air) near the point of interaction provide opportunity for the pions or kaons to decay into muons before they can be removed by nuclear interactions in solid materials. This is particularly important for the typical situation of a target used to produce secondary beams followed (downstream) by an air or vacuum gap (the space for decay into muons) and then a beam dump. If magnetic fields are present, the muon fluence generally peaks in the bend plane.

As discussed before, muon calculations are extremely difficult except when using Monte-Carlo calculations because of the sensitivity to details of the geometry which determine the pion and kaon flight paths and influence the muon populations. (Sc90), however, contains useful information about the production of muons that one can use to make crude estimates. There are some features, discussed in (Sc90) and elsewhere, which are important.

Multiple scattering is an important effect in muon transport. There are several types of scattering that occur. The most important of these is due to elastic Coulomb scattering from the nuclei. An appropriate Gaussian approximation of such scattering for all charged particles carrying electronic charge $z$ ($z=1$ for muons) having mean width $\theta_0$ in space projected onto the plane of the initial direction of the particle is as follows:

$$\theta_0 = \frac{14.1 \, (\text{MeV/c}) \, z}{p \beta} \sqrt{L/X_0} \left[ 1 + \frac{1}{9}(L/X_0) \right] \text{ radians}$$

where $X_0$ is the radiation length defined as in Chapters 1 and 2, $p$ is momentum in MeV/c and $L$ is the shield thickness in the same units as the radiation length. The distribution is fit by the following function:

$$f(\theta)d\theta = \frac{d\theta}{\theta_0 \sqrt{2\pi}} \exp \left( -\frac{\theta^2}{2\theta_0^2} \right).$$

(3.31)
Chapter 3 Shielding of Proton and Ion Accelerators

Generally the most copious source of muons are those due to the decay of pions and kaons. There are several important facts about such muons which are summarized below.

A. The decay lengths (mean length for $\pi$ or $K$ to decay), $\Lambda$, are:

$$\Lambda_\pi = 55.9p \text{ (meters), where } p \text{ is the pion momentum in GeV/c},$$

$$\Lambda_K = 7.51p \text{ (meters), where } p \text{ is the kaon momentum in GeV/c}.$$  

The decay length can be used to estimate the total number of muons present. For example, a beam of $10^7$ pions at 20 GeV/c will decay in a distance of 50 meters into $10^7 \times [50 \text{ meters}] / [56 \times 20 \text{ meters decay length}] = 5 \times 10^5$ muons. [This uses the fact that the path length (50 meters) is small compared with the mean decay length of 1120 meters. If the path length, $x$, was comparable to the decay length, $\Lambda$, the intensity of $10^7$ would be multiplied by the exponential factor $\{1 - \exp(x/\Lambda)\}$.]

B. If $\beta = 1$, relativistic kinematics determines that the ratio, $k_i$, of the minimum momentum of the daughter muon ($p_{\text{min}}$) to the momentum of the parent pion or kaon ($p_i$) is given by:

$$k_i = p_{\text{min}} / p_{\text{parent}} = (m_\mu / m_{\text{parent}})^2.$$  \hspace{1cm} (3.32)

The result is that $k_i$ has a value of 0.57 for muons with pion parents and 0.046 for muons with kaon parents. Thus if, say, a beam transport system restricts the momentum of pions to some minimum value, then a minimum value given by the above is placed on the muon momentum at the time of decay.

C. Since in the center of mass frame of reference the decay is isotropic, and there is a one-to-one relationship between the muon momentum and the angle of emission, for muon momenta $\gg m_{\text{parent}}$ (in units where $c = 1$) the momentum spectrum of the muons can be expressed as $dN / dp = 1 / [p_{\text{parent}}(1 - k_i)]$. This means that the spectrum of daughter muons uniformly extends from the momentum of the parent down to the minimum established in Eq. (3.32).

D. Relativistic kinematics also gives the result that the maximum angle, in the laboratory frame of reference, between the momentum vector of the muon and that of the parent particle is given by:

$$\tan \theta_{\text{max}} = \frac{(m_{\text{parent}}^2 - m_\mu^2) / 2p_{\text{parent}}m_\mu}{(m_{\text{parent}}^2 - m_\mu^2) / 2p_{\text{parent}}m_\mu}.$$  \hspace{1cm} (3.33)

For muons originating from pion decay, $\theta_{\text{max}}$ is at most several milliradians. However, for muons originating from the decay of 5 GeV kaons, $\theta_{\text{max}}$ is a relatively large $12^\circ$. Thus $\pi \rightarrow \mu$ decays can be assumed to be collinear while $K \rightarrow \mu$ decays have significant divergence at the lower energies.
(Sc90) gives calculated values of angular distributions of muon spectra with an absolute normalization from pion and kaon decays for one meter decay paths. For other decay paths which are short compared with the decay length, one can simply scale by the length of the actual decay path. Results are given in Fig. 3.21.

Decays of other particles can be important sources of muons at higher energies. Especially notable are those from charm (D) and bottom (B) meson decays (Sc90). The muons from these sources are often called "direct" muons due to the short lifetimes and decay lengths involved. The masses of these parent particles and their meanlives, \( \tau \), are as follows:

\[
\begin{align*}
    m(D^\pm) &= 1869.3 \pm 0.5 \text{ MeV}, \quad \tau = (10.66 \pm 0.23) \times 10^{-13} \text{ s}, \quad ct = 320 \mu\text{m} \\
    m(B^\pm) &= 5278.6 \pm 2.0 \text{ MeV}, \quad \tau = (12.9 \pm 0.5) \times 10^{-13} \text{ s}, \quad ct = 387 \mu\text{m}.
\end{align*}
\]

Figures 3.22, and 3.23 taken from (Sc90) give results for muons originating from these decays. The results in Fig. 3.21 are for one meter decay paths. One must take care in reading Figs. 3.21 - 3.23. In these figures are presented cumulative spectra. That is, the ordinate is the yield of muons per unit solid angle having energy greater than the value of the abscissa. The abscissa is the ratio of muon energy to the proton energy.

An approximate method for calculating muon flux densities at proton accelerators has been developed by (Su92) and is based upon a semi-empirical fit to existing muon production data. The first result from Sullivan's work is an equation for the flux density of muons per meter of decay path as a function of shield thickness found along the proton beam axis (that is, on the straight-ahead maximum of the muons):

\[
\phi = 0.085 \frac{E x}{X^2} \exp \left\{ -\frac{\alpha t}{E} \right\}, \quad (3.34)
\]

where \( \phi \) is the flux density (muons/m²) per interacting proton, \( E \) is the proton beam energy (GeV), \( X \) is the distance of the point of concern to the point of interaction (meters), \( x \) is the average path length (i.e., the decay path) of the pions and kaons prior to interaction, and \( \alpha \) is an effective average energy loss rate (GeV/meter) for the muons in a shield of thickness \( t \) (meters). \( x \) can be taken to be the actual physical length of the decay path or, for a beam dump situation, according to Sullivan, it can reasonably be taken to have the value of 1.8 times the hadron nuclear interaction mean free path for the material comprising the beam dump. Values of \( \alpha \) have been tabulated by Sullivan and for typical shielding materials have the following values:

\[
\begin{align*}
    \alpha_{\text{concrete}} &= 9.0 \text{ GeV/meter} \quad (\text{for } \rho = 2.35 \text{ g cm}^{-3}) \\
    \alpha_{\text{water}} &= 4.0 \text{ GeV/meter} \quad (\text{for } \rho = 1.0 \text{ g cm}^{-3}) \\
    \alpha_{\text{iron}} &= 23.0 \text{ GeV/meter} \quad (\text{for } \rho = 7.4 \text{ g cm}^{-3}) \\
    \alpha_{\text{lead}} &= 29.0 \text{ GeV/meter} \quad (\text{for } \rho = 11.3 \text{ g cm}^{-3}).
\end{align*}
\]

The value for concrete can be used for earth if one adjusts it to the correct density. It is obvious that the argument of the exponential in Eq. (3.34) can be expanded as the sum over the materials comprising a composite shield. Sullivan has also given a prescription for calculating the full width at half maximum (FWHM) of the muon distribution at the boundary of such a shield. This is given by:

\[
\text{FWHM} = 4.6 \frac{X}{\sqrt{E \alpha t}} \quad (\text{meters}). \quad (3.35)
\]
Fig. 3.21 Muons from the decay of pions and kaons of both charges produced in proton-Fe collisions at various energies of the incident proton. The distance available for decay is assumed to be 1 meter. The abscissa, $E_\mu/E_p$, is the muon energy expressed as a fraction of the incident proton energy. The ordinate, $N$, is the number of muons per unit solid angle having an energy greater than $E_\mu$, expressed in $\text{sr}^{-1}$. The uppermost curve in each case is for production angle $\theta = 0$; the other curves represent the production at other angles in increments of $\Delta\theta$, listed below for the different proton energies $E_p$. 

- a $E_p = 10 \text{ GeV}; \Delta\theta = 30 \text{ milliradians}$
- b $E_p = 30 \text{ GeV}; \Delta\theta = 15 \text{ milliradians}$
- c $E_p = 100 \text{ GeV}; \Delta\theta = 15 \text{ milliradians}$
- d $E_p = 300 \text{ GeV}; \Delta\theta = 5 \text{ milliradians}$
- e $E_p = 1000 \text{ GeV}; \Delta\theta = 1.5 \text{ milliradians}$
- f $E_p = 3000 \text{ GeV}; \Delta\theta = 0.5 \text{ milliradians}$
- g $E_p = 10,000 \text{ GeV}; \Delta\theta = 0.15 \text{ milliradians}$

[Reproduced from (Sc90).]
Fig. 3.22 Muons from the decay of D-mesons produced in proton-proton collisions at various energies of the incident proton. The abscissa, $E_\mu/E_p$, is the muon energy expressed as a fraction of the incident proton energy. The ordinate, $N$, is the number of muons per unit solid angle having an energy greater than $E_\mu$, expressed in $\text{sr}^{-1}$. The uppermost curve in each case is for production at angles between $\theta = 0$ and $\theta = \Delta\theta$: the other curves represent the production in other angular bins in increments of $\Delta\theta$, listed below for the different proton energies $E_p$: a. $E_p = 100 \text{ GeV}; \Delta\theta = 10 \text{ milliradians}$, b. $E_p = 300 \text{ GeV}; \Delta\theta = 3 \text{ milliradians}$, c. $E_p = 1000 \text{ GeV}; \Delta\theta = 1 \text{ milliradians}$, d. $E_p = 3000 \text{ GeV}; \Delta\theta = 0.3 \text{ milliradians}$, e. $E_p = 10,000 \text{ GeV}; \Delta\theta = 0.1 \text{ milliradians}$. (Reproduced from (Sc90).)
Fig. 3.23 Muons from the decay of B-mesons produced in proton-proton collisions at various energies of the incident proton. The abscissa, $E_\mu/E_p$ is the muon energy expressed as a fraction of the incident proton energy. The ordinate, $N$, is the number of muons per unit solid angle having an energy greater than $E_\mu$, expressed in sr$^{-1}$. The uppermost curve in each case is for production at angles between $\theta = 0$ and $\theta = \Delta \theta$: the other curves represent the production in other angular bins in increments of $\Delta \theta$, listed below for the different proton energies $E_p$. a $E_p = 1000$ GeV; $\Delta \theta = 2$ milliradians, b $E_p = 3000$ GeV; $\Delta \theta = 0.6$ milliradians, c $E_p = 10,000$ GeV; $\Delta \theta = 0.2$ milliradians. [Reproduced from (Sc90).]
IV. Shielding Materials and Neutron Energy Spectra

Shielding Materials [largely taken from (NC96) and (Th88)]

Given the size of many modern accelerators, economic considerations often dominate shielding designs by requiring the use of relatively inexpensive, but less efficient shields. In all cases, good engineering practices concerning structural properties, appropriate floor loading allowances, and fire protection considerations must be appropriately taken into account to provide an acceptable degree of conventional safety.

In general, low atomic number materials are best used for targets, collimators, and beam stops at electron accelerators to reduce photon production, while high atomic number materials are preferred at proton and heavy ion accelerators for these components to reduce neutron production. However, at ion energies above 5 MeV neutrons are produced in most materials. Some materials have superior heat transfer characteristics which enhances reliability and thus can reduce personal exposures incurred in maintenance activities.

Earth

Earth has many admirable qualities as a shield material besides its economy. The water it contains enhances the effectiveness of the neutron attenuation, yet it is composed of sufficiently high atomic number elements to be effective against photons. Representative ranges of soil water content (per cent of dry weight) are: sand (0-10), sandy loam (5-20), loam (8-25), silty loam (10-30), dry loam (14-30), and clay (15-30). Dry earth has a typical elemental composition as given in Table 3.8 taken from (Ch84).

Earth is generally a "crackless" shield, not prone to neutron leakage by "streaming". The density of earth varies widely, from as low as 1.7 g-cm\(^{-3}\) to as much as 2.25 g-cm\(^{-3}\) depending upon soil type and water content. Given this variation, specific knowledge of soil characteristics at the accelerator site are needed to do effective shielding designs. Definitive measurements of the water content are also most useful if the shielding of neutrons is the intent and only small safety factors are being used.

Table 3.8 Elemental Composition, Dry-Weight Percent Basis, of Representative Soils. [Reproduced from (Ch84).]

<table>
<thead>
<tr>
<th>Element</th>
<th>Global Average (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>43.77</td>
</tr>
<tr>
<td>Si</td>
<td>28.1</td>
</tr>
<tr>
<td>Al</td>
<td>8.24</td>
</tr>
<tr>
<td>Fe</td>
<td>5.09</td>
</tr>
<tr>
<td>Mn</td>
<td>0.07 ± 0.06</td>
</tr>
<tr>
<td>Ti</td>
<td>0.45 ± 0.43</td>
</tr>
<tr>
<td>Ca</td>
<td>3.65</td>
</tr>
<tr>
<td>Mg</td>
<td>2.11</td>
</tr>
<tr>
<td>K</td>
<td>2.64</td>
</tr>
<tr>
<td>Na</td>
<td>2.84</td>
</tr>
</tbody>
</table>
Concrete has obvious advantages in that it can either be poured in place permanently or be cast into modular blocks. Sometimes concrete is used to shield targets, beam stops, etc. in a manner that allows their ready access if the need for maintenance arises. The use of concrete blocks generally requires the overlapping of the blocks to avoid streaming through the cracks.

It is sometimes efficient to use a heavy material as part of the aggregate in the concrete recipe. This can increase the density of the material as well as its average atomic number. The latter, of course, increases the effectiveness against photons. Table 3.9 adapted from (Ch84) gives some partial densities of various concretes used in shielding. When shielding neutrons, the water content is quite important because it incorporates almost all of the hydrogen. Under "extreme" low-humidity conditions, the water content of concrete can decrease with time, to as little as 50% of the initial value over a 20 year period.

Table 3.9 Partial Densities of Representative Concretes After Curing. [Adapted from (Ch894).]

<table>
<thead>
<tr>
<th>Type: Additive</th>
<th>Ordinary</th>
<th>Magnetite (FeO, Fe₂O₃)</th>
<th>Barytes BaSO₄</th>
<th>Magnetite &amp; Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm³)</td>
<td>2.34</td>
<td>3.53</td>
<td>3.35</td>
<td>4.64</td>
</tr>
<tr>
<td>H</td>
<td>0.013</td>
<td>0.011</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
<td>O</td>
<td>1.165</td>
<td>1.168</td>
<td>1.043</td>
<td>0.638</td>
</tr>
<tr>
<td>Si</td>
<td>0.737</td>
<td>0.091</td>
<td>0.035</td>
<td>0.073</td>
</tr>
<tr>
<td>Ca</td>
<td>0.194</td>
<td>0.251</td>
<td>0.168</td>
<td>0.258</td>
</tr>
<tr>
<td>Na</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg</td>
<td>0.006</td>
<td>0.033</td>
<td>0.004</td>
<td>0.017</td>
</tr>
<tr>
<td>Al</td>
<td>0.107</td>
<td>0.083</td>
<td>0.014</td>
<td>0.048</td>
</tr>
<tr>
<td>S</td>
<td>0.003</td>
<td>0.005</td>
<td>0.361</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.045</td>
<td></td>
<td>0.159</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>0.029</td>
<td>1.676</td>
<td></td>
<td>3.512</td>
</tr>
<tr>
<td>Ti</td>
<td></td>
<td>0.192</td>
<td></td>
<td>0.074</td>
</tr>
<tr>
<td>Cr</td>
<td></td>
<td>0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td></td>
<td>0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td>0.011</td>
<td></td>
<td>0.003</td>
</tr>
<tr>
<td>Ba</td>
<td></td>
<td></td>
<td></td>
<td>1.551</td>
</tr>
</tbody>
</table>

[Local densities of "ordinary" concrete vary considerably, up to 2.5 g/cm³.]

Other hydrogenous materials

Polyethylene and other materials subject to boration: (CH₂)ₙ is a very effective neutron shield because of its hydrogen content (14% by weight) and its density (0.92 g/cm³). The addition of boron can reduce the buildup of 2.2 MeV photons released in the thermal neutron capture by hydrogen by instead capturing the thermal neutrons in the boron, where the decay reaction produces an easily attenuated α-particle plus a more readily attenuated 0.48 MeV photon.

Commercially, polyethylene is available including additives of boron (up to 32%), lithium (up to 10%) and lead (up to 80%) in various forms such as planer sheets, spheres, and cylinders.
Chapter 3  Shielding of Proton and Ion Accelerators

These materials can be useful, if it is necessary, to economize on space and also to accomplish shielding of photons and neutrons simultaneously.

Pure polyethylene is flammable, but some of the commercial products available contain self-extinguishing additives. Some of these materials are available in powder form, for molding into a desired shape by the user. Besides polyethylene, boron has been added to other materials to form effective thermal neutron shields. These include other plastics, putties, clays, and glasses to accomplish specific shielding objectives.

The three materials water, wood, and paraffin are superficially attractive neutron shields because of their very high hydrogen contents.

Water, of course, tends to rust out its containers and there is the omnipresent question as to whether the shield has gone "down the drain". Exposed to thermal neutrons, it also emits the 2.2 MeV capture γ-ray from hydrogen. Boration is more difficult because of the relative insolubility of boron salts in water.

Wood was found in the early years of operation at the Bevatron to be as effective as concrete for shielding intermediate energy neutrons per unit length. Thus it is essential that the neutron energy spectrum to be attenuated is known. In the past, wood has been discouraged as a shielding material because of its flammability. Recently, chemically treated wood that is nearly completely fireproof has become available, but it is not clear that the flammability problem has been solved with complete satisfaction. For example, questions have been raised by reports of a reduction in structural strength of such treated wood products.

Paraffin historically has been used for neutron shielding but has been spurned in recent years because of the fire hazard. Under some conditions it can be used if it is packaged in metal containers. Recently, paraffin treated with fire retardant additives has become available. It is still subject to "plastic" flow problems.

Iron

A relatively high density in conjunction with its low cost make iron an attractive shielding material. Caution is required because the density can vary widely from a low of 7.0 for low grade cast iron to a high value of 7.8 g·cm⁻³ for some steels. (The "textbook" value of 7.9 g·cm⁻³ is almost never attained in the bulk quantities necessary for radiation shielding).

Because of its nonmagnetic properties, stainless steel is often used as part of accelerator components. Because of concerns about radioactivation, a knowledge of the elemental composition of various alloys can sometimes be of interest.

Iron has a very important deficiency as a neutron shield; this will be discussed a bit later.

High atomic number materials

The materials in this category are valuable because of their high atomic number, especially where the shielding of photons is important. The most obvious material in this category is lead. It has high density (11.3 g·cm⁻³) and is resistant to corrosion. Pure lead, as is well-known, has major drawbacks because of its poor structural characteristics and low melting point (327.4 °C). It is usually best used when it can be laminated to some other, more structurally stable, material. Some alloys represent improvements on the structural properties.
Chapter 3  Shielding of Proton and Ion Accelerators

It is often available as an additive to other materials in order to improve their capacity for shielding photons. Fabric blankets containing shredded lead can be effectively used to shield radioactivated components to minimize exposures associated with accelerator maintenance activities. The high chemical toxicity of lead requires care in its fabrication and handling to properly protect personnel.

Tungsten is an excellent, but relatively expensive, shielding material. Its high density (19.3 g·cm⁻³) and high melting temperature (3410 °C) make it extremely useful as a component in photon shields and in beam collimators.

Uranium is a somewhat attractive shielding material, most often in its "depleted" form in which 235U is removed from the dominant 238U down to some residual fraction (usually 0.2 %) much lower than the natural value of 0.72 %. Its high density (19.0 g·cm⁻³) and relatively high melting point (1133 °C) are positive attributes, especially in places where space efficiency is a concern. It is obviously not a good choice of material in environments having a high neutron flux density. In the depleted form, it is relatively safe, but if combined with hydrogenous materials, criticality should be considered for the specific material and geometric arrangement to be employed. Even in the absence of hydrogen, thermal neutrons under certain conditions can result in the possibility of criticality.

Major drawbacks are the material properties. It has a large anisotropic thermal expansion coefficient and also readily oxidizes when exposed to air (especially humid air). The oxide is readily removable and presents a significant internal exposure hazard. Prevention of oxidation by sealing it with epoxy or paint meets with only limited success due eventual embrittlement and chipping accelerated by radiation damage. Sealed containers filled with dry air seems to be the best storage solution to limit oxide formation. Small chips of this element are also pyrophoric, complicating machining-type processes by posing yet another safety hazard.

-beryllium, aluminum, and zirconium

These three materials find considerable usage as accelerator components because of various properties. Beryllium is often used as a target material in intense beams because of its resistance to thermal effects (especially when in the form of the oxide, BeO). It has been used at high energy accelerators in relatively large quantities as a "filter" to enrich one particle type at the expense of another. A serious concern is the extreme chemical toxicity of the metal and its compounds, which makes it difficult to fabricate. Aluminum is used as an accelerator component because of its nonmagnetic properties and its resistance to corrosion. It is not an effective shield against neutrons. Zirconium has a very small thermal neutron capture cross section and very good thermal properties. It is therefore not a good neutron absorber but has been found to be useful in beam-handling component material in some situations.

Measured Neutron Energy Spectra Outside of Shields

In the most simple approximation, outside of thick shields of soil or concrete that contain some hydrogen content (usually in the form of H₂O), accelerator neutron shields can most generally said to be a "1/E" spectrum with the energies extending from those of thermal neutrons (\(<E_n> = 0.025 \text{ eV}\)) up to the energy of the incident protons. In this approximation, the spectrum is given as:
where \( k \) is a normalizing constant.

N. Rohrig (Ro83) observed from this that it is more convenient to plot such spectra as flux per logarithmic energy interval by simply plotting \( E\phi(E) \);

\[
\frac{d\phi(E)}{d\ln(E)} = E\phi(E). 
\] (3.37)

In the terminology of textbooks on "neutron physics" (i.e., a terminology that is somewhat obscure to particle and nuclear physicists), this is also called a "lethargy" plot. This, effectively, suppresses the \( 1/E \) dependence seen in typical neutron energy spectra.

Detailed features of the geometry involved can produce peaks in the neutron energy spectrum. Some of these features have been discussed in (Pa73), (Nc96), (Th88), (El86), and (Co88). These peaks are typically encountered in the few MeV region. Figures 3.24, 3.25, and 3.26 are plots of neutron spectra and sketches of the shielding geometry involved taken from (Co88). These are typical of the spectra found at high energy proton accelerators. These results were obtained using the Bonner sphere technique discussed in more detail in Chapter 6. Spectrum C is particularly interesting because its shape was demonstrated to be essentially independent of proton energy over the range of 150 to 900 GeV (McC88).
Chapter 3  Shielding of Proton and Ion Accelerators

Fig. 3.24  Shielding geometries (left) and corresponding unfolded neutron energy spectra (right) for situations A, B, and C described in (Co88). The ordinate is a linear scale in arbitrary units of fluence per logarithmic energy interval. Spectrum A arose from 8 GeV protons being targeted on a magnet in the Fermilab Debuncher storage ring normally used to store antiprotons for the Fermilab colliding beams physics program. The Bonner spheres were located external to a 671 g cm$^{-2}$ shield of earth and concrete. Spectrum B resulted from 8 GeV protons being targeted on a magnet in a different part of the same Debuncher storage ring. There spheres were located external to a 402 g cm$^{-2}$ shield of earth and concrete plus some iron just below the spheres. Spectrum C was obtained inside the Tevatron tunnel with the spheres located on the wall opposite the accelerator elements. Data were collected using an array of 8 detectors operated primarily by personnel from the Lawrence Berkeley Laboratory. Neutrons were produced from 800 GeV protons interacting with residual gas in the Tevatron vacuum chamber during circulating beam conditions. [Reproduced from (Co88) and references cited therein.]
Chapter 3  Shielding of Proton and Ion Accelerators

Fig. 3.25  Shielding geometries (left) and corresponding unfolded neutron energy spectra (right) for situations D, E, and F described in (Co88). The ordinate is a linear scale in arbitrary units of fluence per logarithmic energy interval.  Spectrum D was obtained relatively far downstream of a large target and beam dump system struck by 800 GeV protons and shielded by iron and concrete.  Spectra E and F were obtained laterally to a large electromagnet which contained a beam dump within its gap.  This beam dump was struck by 800 GeV protons.  For E, the spheres viewed bare iron return yoke while for F the return yoke viewed by the spheres was covered by additional concrete shielding, at least 91.4 cm thick, as shown.  [Reproduced from (Co88) and references cited therein.]
Shielding geometries (left) and corresponding unfolded neutron energy spectra (right) for situations G, H, and I described in (Co88). The ordinate is a linear scale in arbitrary units of fluence per logarithmic energy interval. Spectrum G was obtained on top of the downstream end of a beam dump and target assembly involving 800 GeV protons incident on a target followed by bending magnets and a beam dump. The entire assembly was shielded by an inner layer of iron and an outer layer of concrete in the form of large (= 91.4 cm thick) blocks. Spectrum H was obtained in a beam enclosure in which 800 GeV protons struck a target in an iron cave downstream from the spheres. The detectors thus view "backscatter" from this target. Spectrum I was obtained in the second "leg" of labyrinth (see Chapter 4). The neutrons were produced by 400 GeV protons striking an aluminum target inside a large pipe beneath the floor of the main enclosure. [Reproduced from (Co88) and references cited therein.]
One of these peaks which commonly appear in such spectra is particularly important. As discovered by Alsmiller and Barish (Al73), iron has a major deficiency as a shield for fast neutrons. Containing no hydrogen, the primary attenuation mechanism for fast neutrons is by inelastic scattering from the iron nuclei. At energies below the first excited state of any nucleus, inelastic scattering becomes impossible and elastic scattering becomes the only removal process. As will be seen in Chapter 6, elastic scattering is a very inefficient means of energy removal for neutrons scattering off the much more massive iron nucleus. Billiard balls scattering off of bowling balls comes to mind as an analogy. It is intuitive that billiard balls scattering off other billiard balls of equal mass provides for much more efficient energy transfer. Likewise, neutrons scattered by the "free" protons in hydrogenous materials is much more efficient in terms of energy transfer than is the elastic scattering of neutrons from iron nuclei.

The first excited state of $^{56}$Fe [the dominant (92%) isotope in natural iron] is at 847 keV. This has the consequence that the neutrons build up below this energy because of the inefficiency of the transfer of energy by means of elastic scattering. Thus neutrons above 847 keV in a given spectrum will be slowed by inelastic scattering only to build up in this region. Amplifying this effect when one considers the dose equivalent external to such shields is the fact that the quality factor for neutrons as a function of energy also has its maximum value at about 700 keV. Thus, pure iron shields are rather ineffective in attenuating neutrons in this energy region.

Spectra E and F in Fig. 3.25 illustrate this phenomena concerning iron shielding. They were both measured at about $\theta = 90^\circ$ from a beam dump struck by secondary particles due to 800 GeV proton interactions far upstream of the beam dump (Co88). One of the spectra is for a bare iron shield while the other is after the iron was covered by a 91.4 cm thick layer of concrete. For the bare iron the dose equivalent rate external to the shield was over 40 times that measured after the concrete was installed. This factor is far in excess of the approximate factor of 10 expected from simple attenuation of the equilibrium cascade neutron spectrum. The concrete also reduced the average quality factor from 5.4 to 2.8. In general, an iron shield "capped" or "backed" by such a concrete shield will be an efficient use of space. It has been determined that 60 cm of concrete is the most efficient thickness to use for this purpose ([Yu83] and [Za87]).

Shielding properties of other elements near iron (chiefly copper and nickel) in the periodic table are comparable.

Finally, one must be concerned with the per cent of fluence and per cent of dose equivalent in specific energy bins. This can affect the potential to produce radioactivity and also guides the designer of shielding. Tables 3.10 and 3.11 taken from (Co88) give these properties for the nine spectra displayed in Figs. 3.24, 3.25, and 3.26. Fig. 3.27 from (Va75) is a plot of cumulative values of the same quantities for 1000 GeV protons incident on the face of a thick cylindrical concrete shield. As determined by Van Ginneken and Awschalom (Va75), the dependence upon incident proton energy of the distributions of fluence and dose equivalent is slight.

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Table 3.10 Percent Fluence in Specific Energy Bins for Unfolded Neutron Spectra [Reproduced from (Co88).]

<table>
<thead>
<tr>
<th>Spectrum-&gt;Energy</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1.5 eV</td>
<td>31.5</td>
<td>42</td>
<td>19.5</td>
<td>29</td>
<td>28</td>
<td>55</td>
<td>33.5</td>
<td>42</td>
<td>71</td>
</tr>
<tr>
<td>0.0015-100 keV</td>
<td>12.5</td>
<td>4.5</td>
<td>36</td>
<td>39</td>
<td>46</td>
<td>43</td>
<td>62.1</td>
<td>36.2</td>
<td>24</td>
</tr>
<tr>
<td>0.1-2 MeV</td>
<td>8.5</td>
<td>1</td>
<td>36</td>
<td>19.5</td>
<td>17.5</td>
<td>2</td>
<td>0</td>
<td>19.4</td>
<td>2</td>
</tr>
<tr>
<td>2-25 MeV</td>
<td>40.5</td>
<td>2.5</td>
<td>7</td>
<td>10</td>
<td>4.5</td>
<td>0.1</td>
<td>2.1</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>&gt; 25 MeV</td>
<td>7</td>
<td>50</td>
<td>1.5</td>
<td>2.5</td>
<td>4</td>
<td>0</td>
<td>2.3</td>
<td>0.9</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 3.11 Percent of Dose Equivalent in Specific Energy Bins for Unfolded Neutron Spectra Along with Average Quality Factor [Reproduced from (Co88).]

<table>
<thead>
<tr>
<th>Spectrum-&gt;Energy</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1.5 eV</td>
<td>1.5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>41.5</td>
<td>12.5</td>
<td>9</td>
<td>32</td>
</tr>
<tr>
<td>0.0015-100 keV</td>
<td>0.5</td>
<td>0.2</td>
<td>6</td>
<td>6</td>
<td>11.5</td>
<td>37</td>
<td>22.3</td>
<td>11.9</td>
<td>16</td>
</tr>
<tr>
<td>0.1-2 MeV</td>
<td>9</td>
<td>0.4</td>
<td>58.5</td>
<td>41</td>
<td>35</td>
<td>17</td>
<td>0.1</td>
<td>59.8</td>
<td>9</td>
</tr>
<tr>
<td>2-25 MeV</td>
<td>75</td>
<td>4</td>
<td>26</td>
<td>38</td>
<td>24</td>
<td>3.5</td>
<td>28</td>
<td>11.5</td>
<td>13</td>
</tr>
<tr>
<td>&gt; 25 MeV</td>
<td>14</td>
<td>93.5</td>
<td>7.5</td>
<td>12</td>
<td>25</td>
<td>1</td>
<td>37.1</td>
<td>7.9</td>
<td>30</td>
</tr>
<tr>
<td>Average Q.F.</td>
<td>5.8</td>
<td>4.2</td>
<td>6.9</td>
<td>6.2</td>
<td>5.4</td>
<td>2.5</td>
<td>3.4</td>
<td>5.7</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Fig. 3.27 Fraction of the omnidirectional flux, entrance absorbed dose, and maximum dose equivalent below hadron kinetic energy on abscissa (in MeV) for the region between zero and 450 cm depth and between 300 cm and 750 cm radius for 1000 GeV/c protons incident on the face of a solid concrete cylinder. [Reproduced from (Va75).]
Chapter 3 Shielding of Proton and Ion Accelerators

References


(Al73) R. G. Alsniiller, Jr. and J. Barish, "Shielding against the neutrons produced when 400 Mev electrons are incident on a thick copper target", Particle Accelerators 5 (1973) 155.


(Co85) J. D. Cossairt, S. W. Butala, and M. A. Gerardi, "Absorbed dose measurements at an 800 GeV proton accelerator, comparison with Monte-Carlo calculations, Nucl. Instr. and Meth., A238 (1985) 504.


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(Su92) A. H. Sullivan, *A guide to radiation and radioactivity levels near high energy particle accelerators* (Nuclear Technology Publishing, Ashford, Kent, United Kingdom), 1992


## Chapter 3 Shielding of Proton and Ion Accelerators

<table>
<thead>
<tr>
<th>Ref</th>
<th>Author(s)</th>
<th>Title</th>
<th>Publication Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Va75)</td>
<td>A. Van Ginneken and M. Awschalom</td>
<td>&quot;High energy particle interactions in large targets: Volume I, hadronic cascades, shielding, and energy deposition&quot;</td>
<td>Fermilab (1975).</td>
</tr>
<tr>
<td>(Yu83)</td>
<td>P. M. Yurista and J. D. Cossairt</td>
<td>&quot;Concrete shielding exterior to iron,&quot;</td>
<td>Fermilab Report TM-1204, Fermi National Accelerator Laboratory (1983).</td>
</tr>
</tbody>
</table>
Chapter 3 Shielding of Proton and Ion Accelerators—Problems

1. It is asserted that if the assumption is made that the limiting attenuation is simply geometric, with the nucleon radius equal to $1.2 \times 10^{-13}$ cm, then $\rho \lambda_{\text{atten}} = 38A^{1/3}$ (g/cm²). Show this to be the case using the volume of a nucleus and nucleons along with the cross section.

2. a) Use the Moyer Model to calculate the dose equivalent rate (mrem/hr) lateral ($\theta = 90^\circ$) to a magnet centered in a 1.5 m radius tunnel. The magnet is struck by $10^{12}$ protons at 100 GeV (per sec). The tunnel walls consist of 1/3 m concrete followed by soil having the same composition [$\rho(\text{concrete}) = 2.5$ g/cm³, $\rho(\text{soil}) = 2.0$ g/cm³]. Perform the same calculation for several thicknesses of soil out to 6 meters of soil radially. Do this for increments of 1 meter from 1 meter to 6 meters of soil.

b) Calculate the result if the same beam loss occurs over a 100 meter length of tunnel at the same soil thicknesses as in 2) (use the Tesch approximation). Approximately how many meters of beam loss does it take to cause 90% of the calculated dose equivalent rate at 6 m of lateral soil shield?

c) For the point loss in part a), at what value of $\theta$ does the maximum dose equivalent rate occur and what is its magnitude outside of 6 meters of soil shield? (Use successive approximations to solve.)

3. For the accelerator and beam delivery conditions of problem 2a, use the results of FLUKA calculations in Fig. 3.16 to determine the approximate dose equivalent rate outside of 1 meter of concrete shielding and compare with a result using the Moyer equation for point loss on a "magnet". Both calculations should be at the location of the dose equivalent maximum. Assume $\rho(\text{concrete}) = 2.5$ g/cm³. Why might there be an explainable disagreement between the two results?

4. Using the results of Monte-Carlo hadron calculations (FLUKA/CASIM) calculate, for solid shields of iron (cylinders), what longitudinal thickness of iron is needed to achieve the same hadron dose equivalent per proton on the beam axis as found at $R = 50$ cm at 10 GeV/c, 100 GeV/c, 1000 GeV/c and 10 TeV/c. Use the maximum value of $H (r = 50$ cm). One may need to extrapolate calculations shown in Fig. 3.18.
5. In the Chapter 1, Fig. 1.30 we have calculations of neutron energy spectra for 200 MeV protons incident on various targets, including aluminum. In Fig. 3.12 of this chapter, calculations of dose equivalent values for concrete shielding surrounding aluminum targets @ $E_p = 200$ MeV are given. At shielding thicknesses approaching zero and at forward angles, are the two results in "sensible" (that is, approximate, agreement)? (Hint: "Integrate" crudely over the forward spectrum to obtain the fluence/proton and convert this fluence to dose equivalent.)

a) Make the comparison for zero shield thickness and in the angular range $0 < \theta < 30^0$.

b) Now use the shielding calculations to obtain the dose equivalent rate (rem/h) due to a 1 $\mu$A beam incident at 200 MeV on such a thick target at a distance of 4 m from the target with 0, 1, 2, & 3 m of intervening concrete shielding ($\rho = 2.5$g/cm$^3$) for $\theta = 15^0$ and $\theta = 75^0$. (Hint: Use the center of the angular bins.)

6. Assume that a target is struck by 100 GeV protons and that a 10 m long decay space exists for $\pi$ and $K$ decay. Use the curves in Fig. 3.21 to crudely estimate the muon flux density and dose equivalent rates (mrem/h) at $\theta = 0^0$ if $10^{12}$ p/s are targeted in this manner at 1 km away if:

a) there is no shielding present (neglect air scattering and in-scattering from the ground). (Hint: The muon yield for this decay space will scale with the length of the decay space.)

b) there is 100 meters of intervening shielding of earth ($\rho = 2$ g/cm$^3$) (Hint: use Fig. 1.22 range-energy curves to determine the mean energy of muons which will penetrate this much shielding). Neglect multiple scattering and range-straggling.

c) If the beam operates for 4000 h/yr, is 100 mrem/yr exceeded? Will multiple scattering increase or decrease this dose equivalent? (Answer both questions for the soil shielded case only.)

d) Repeat Part b) of the same calculation using Sullivan's semi-empirical approach. If the disagreement between the results obtained using the two methods is large, please suggest an explanation of a possible cause of the difference.
Chapter 4  Low Energy Prompt Radiation Phenomena

In this chapter two special considerations pertinent to nearly all accelerators are discussed. These are the phenomena of the transmission of photons and neutrons by penetrations and of neutron skyshine. It will be shown that these phenomena are nearly independent of incident particle type and energy. Methods for taking proper account of these two phenomena in the design of accelerator shielding are discussed in detail.

I. Introduction

Accelerators of sufficient energy to produce neutrons copiously (either ion or electron accelerators) exhibit two phenomena involving low energy neutrons which must be addressed. These are the transmission of neutrons by penetrations and the control/prevention of "skyshine". In both cases, the neutrons involved are generally of low energy (compared to the beam particles) and the phenomena are also rather independent of the beam particle energy.

All accelerators evidence the need to control the transmission of neutrons by penetrations since all accelerators have access-ways to permit entry of personnel and equipment as well as cable penetrations. Concerning skyshine, while it is "preventable" through the application of sufficient roof shielding, all major accelerators constructed to date (at least the large ones!) have encountered this problem at some point due to "oversights" or the need to accommodate modifications in the accelerator design or the associated experimental program.

II. Transmission of photons and neutrons by penetrations  [This material largely follows (NC77), (Sc90), and (Th88).]

Personnel access penetrations will typically have cross-sectional dimensions of 1 meter by 2 meters (door-sized) while utility ducts will generally be much smaller, typically no larger than 0.2 by 0.2 m. Often the utility penetrations are partially filled with cables and other items, and even cooling water in pipes.

Two general rules are advised for such penetrations:

It is an unwise practice to arrange any penetration so that a particle or photon beam is aimed directly toward it. Therefore, one is always primarily transporting scattered radiation through the penetrations.

The sum of the wall thickness between the source and the "outside" should be equivalent to that which would be required if the labyrinth were not present.

Before describing the details of penetration design as such, one should review some simple parameterizations of the reflections of photons and neutrons. Photons are being considered here simultaneously because their simple treatment can usefully be done with the same methods, with appropriate albedo parameters. These coefficients have applications more general than merely the design of penetrations. They represent the ability of solid materials to "reflect" particles.

Figs. 4.1 and 4.2 adapted from the results of Refs. (NC77), (Ch63), (Ch64), (Ch65a), and (Ch65b) give the reflection coefficients $\alpha_\gamma$ and $\alpha_n$ for monoenergetic photons and neutrons (respectively) incident on flat surfaces of infinite dimensions of concrete, iron, and lead plotted as functions of energy for various conditions of incidence. As is obvious from these curves, the albedo of neutrons is typically larger and somewhat less strongly dependent on energy than is that of photons.
Chapter 4  Low Energy Prompt Radiation Phenomena

Fig. 4.1 Reflection coefficients, $\alpha_x$, for monoenergetic photons incident on ordinary concrete, iron, and lead as a function of incident photon energy, for several angles of reflection assuming normal incidence and for equal angles of incidence and reflection. Values are given for ordinary concrete and iron, based on existing available information, both theoretical and experimental. For photon energies higher than 10 MeV, the use of the 10 MeV values of $\alpha_x$ is expected to be conservative. Values of $\alpha_x$ for photons incident on lead are not as readily calculable but a conservative upper limit is $5 \times 10^{-3}$ for any energy and scattering angle. [Adapted from (NC77) and references cited therein.]
Fig. 4.2 Reflection coefficients, $\alpha_n$, for monoenergetic neutrons incident on ordinary concrete, iron, and lead as a function of incident neutron energy, for several angles of reflection assuming normal incidence and for equal angles of incidence and reflection. Values of $\alpha_n$ are given for ordinary concrete and iron, based on existing available information, both theoretical and experimental. Values of $\alpha_n$ for neutrons incident on lead are approximately an order of magnitude larger than those given above. [Adapted from (NC77) and references cited therein.]
Photons

A particular application of these coefficients to the design of labyrinths is given here to illustrate the method using these albedo curves. Figure 4.3 taken from (NC77) shows an example of a labyrinth providing access to a collimated photon source of some known dose equivalent (or dose equivalent rate), $H_0$, at one meter. In general, some knowledge of the photon energy spectrum at this location is also needed. Such a photon "beam" is relevant to the subject of this text because, for example, it could arise from the target of a beam from an electron accelerator.

Fig. 4.3 Generalized labyrinth design illustrating successive reflections of photons from a collimated source through the maze. The source could just as well originate from an electron beam incident from the right side of the figure on a target located at the point in space labeled "collimated x-ray source". The various path lengths can be approximated by a sequence of centerline distances, as shown in the diagram. [Reproduced from (NC77).]
Chapter 4 Low Energy Prompt Radiation Phenomena

With the reflection coefficients $\alpha_x$, one can use the following formula to obtain a conservative estimate of the dose equivalent (or dose equivalent rate), $H_{rj}$, after $j$ sections (not counting the initial path length to the wall, $d_i$) of the maze:

$$H_{rj} = \frac{H_0 \alpha_x \prod_{k=1}^{j} \alpha_k A_k}{d_i^2 \prod_{k=1}^{j} d_k^2} \quad j \geq 1. \tag{4.1}$$

In this formula, the coefficient $\alpha_i$ is selected to be representative of the photon energy while $A_i$ estimates the cross-sectional area of the wall struck by the initial photons evaluated by projecting the beam cross-sectional area to the wall. For successive legs, taking the value of $\alpha$ appropriate for 0.5 MeV photons is often considered to be a conservative approach. This is substantiated physically because, if $E_0$ is the initial photon energy in MeV, the photon energy, $E_{\text{scatt}}$, following Compton scattering is given by:

$$E_{\text{scatt}} = \frac{E_0}{1 + (E_0/0.511)(1-\cos \theta)} \quad \text{(MeV)}. \tag{4.2}$$

Thus, $E_{\text{scatt}}$ has a maximum value after a scatter of 90$^\circ$ of 0.511 MeV for $E_0/0.511 >> 1$.

$A_k$, then, is the cross-sectional area of the $k^{\text{th}}$ leg of the maze. If the maze is uniform with cross section $A$, and has $j$ legs, then the product in the numerator is simply raised to the $j^\text{th}$ power; $(\alpha A)^j$. Otherwise individual factors for each leg must be used in place of the quantity taken to the $j^\text{th}$ power. In the denominator, the distances are just those defined in the figure above and, of course, represent the inverse-square law dependence. This formula is more conservative for photon energies exceeding 10 MeV, but at the higher energies the uncertainties are larger. The above formula is probably most accurate if the ratios $d_{ik}/(A_k)^{1/2}$ lie between 2 and 6.

**Neutrons**

Unfortunately, complications in the transport of neutrons discourage the use of a similar formula. The radiation source (or potential radiation source for situations of concern from the standpoint of accidental beam losses) should be evaluated according to the methods described previously. The calculation of the attenuation of penetrations is a difficult one to do in great theoretical detail.

Typical methods employed involve the use of the results of calculations made by sophisticated Monte-Carlo codes. These can be used for both curved and rectilinear labyrinths with the primary practical experience being with the latter type. In this section, the results of such work will be presented in order to give the reader useful information in the evaluation of such penetrations.

An overwhelming conclusion of the body of existing data is that the bombarding particle energy has very little effect upon the attenuation of a labyrinth viewing a source of beam loss other than the increased total yield of "source" neutrons as a function of incident energy and ion type. One can thus estimate the dose, dose equivalent, or neutron fluence at the exit of a labyrinth by using attenuation estimates in conjunction with an estimate of the neutron fluence or dose equivalent at the entrance of the penetration into the beam enclosure. This "factorization" approximation
allows attenuation measurements and calculations obtained at proton accelerators to be of rather general utility.

Another general "rule of thumb" is that labyrinth attenuations of neutrons scale with a unit length equal to the square root of its cross-sectional area provided that the height to width ratio does not vary greatly outside of the range 0.5 to 2.0 (Th88).

For penetrations exposed to targets struck by hadrons, we first consider straight penetrations viewing a point of beam loss at 90° to the incident beam. Figures 4.4 and 4.5 taken from (Gi69) show measurements of the relative transmission of an exceptionally long straight tunnel of dimensions 2.8 m high by 1.8 m wide and 100 m long. 14 GeV protons were incident on a target providing a good "point source" 3.2 m from the tunnel entrance. Various activation detectors having different energy thresholds made it possible to obtain some information about the neutron energy spectrum. The experimental conditions did not allow an absolute normalization to beam loss.

Table 4.1 taken from (Th88) gives the thresholds of various reactions used in the measurement summarized in Figs. 4.4 and 4.5 where the number of events above the reaction threshold is used as an indicator of the energy spectrum:

<table>
<thead>
<tr>
<th>Detector</th>
<th>Reaction</th>
<th>Energy range (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare gold foils</td>
<td>$^{197}$Au(n,γ)$^{198}$Au</td>
<td>thermal</td>
</tr>
<tr>
<td>Large bare indium foils</td>
<td>$^{115}$In(n,γ)$^{116}$In$^m$</td>
<td>thermal</td>
</tr>
<tr>
<td>Moderated gold foils</td>
<td>$^{197}$Au(n,γ)$^{198}$Au</td>
<td>0.02 to 20</td>
</tr>
<tr>
<td>Kodak Type-B neutron films</td>
<td>Proton recoil</td>
<td>0.5 to 25</td>
</tr>
<tr>
<td>Sulphur</td>
<td>$^{32}$S(n,p)$^{32}$p</td>
<td>3 to 25</td>
</tr>
<tr>
<td>Aluminium</td>
<td>$^{27}$Al(n,α)$^{24}$Na</td>
<td>6 to 25</td>
</tr>
<tr>
<td>Plastic scintillator</td>
<td>$^{12}$C(n,2n)$^{11}$C</td>
<td>&gt;20</td>
</tr>
<tr>
<td>Beta/gamma film and LiF TLD</td>
<td>Gamma and charged particles</td>
<td></td>
</tr>
</tbody>
</table>

[Reproduced from (Th88).]
Fig. 4.4 The relative transmission of neutron flux density and gamma dose rate along a large straight tunnel. [Reproduced from (Gi69).]

Fig. 4.5 Approximate exponential absorption of neutron flux density and gamma dose rate in a large straight tunnel for a variety of detectors. [Reproduced from (Gi69).]
It is clear that the low energy part of the spectrum attenuates more rapidly by air and wall scattering than do the higher energy neutrons. Also, for short tunnels (< 20 m long) the "attenuation" of the fast neutrons is almost entirely accomplished by inverse-square law considerations. (In these figures, the coordinate "d" or "Z" was used to represent distance along the tunnel.) It is also clear that the transmission, when inverse-square effects are excluded, is approximately an exponential with effective mean free paths, $\lambda(E)$, corresponding to effective, energy-dependent, removal cross sections. These also were determined in (Th88) and op. cit. and are given in Table 4.2.

Table 4.2 Mean free paths and removal cross sections for tunnel transmission as exhibited by the measurements summarized in Figs. 4.4 and 4.5. [Reproduced from (Th88).]

<table>
<thead>
<tr>
<th>Detector</th>
<th>Mean free path (m)</th>
<th>Removal cross-section (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic scintillator</td>
<td>100</td>
<td>1.9</td>
</tr>
<tr>
<td>Aluminium</td>
<td>60</td>
<td>3.2</td>
</tr>
<tr>
<td>Film and TLD</td>
<td>55</td>
<td>3.3</td>
</tr>
<tr>
<td>Bare gold</td>
<td>30</td>
<td>6.2</td>
</tr>
</tbody>
</table>

It is of some interest that the effective removal cross sections determined by this measurement are about a factor of 1.5 to 2 "too small" compared to those that would be inferred from the known cross sections for the constituents of air. "In-scattering" of the concrete walls may well provide some explanation since more neutrons were observed at the larger distances into the tunnel.
Of course, details of the source geometry are very important in such a straight penetration. K. Goebel [(Go75) and summarized in (Sc90)] has calculated "universal"curves for "first" legs of labyrinths (i.e., equivalent to such a "straight" penetration). Goebel used and compared results from the the codes SAM-CE (Co73), AMC (Ma67), and ZEUS (D'H68). Gollon and Awschalom (Go71) have generated similar curves using the ZEUS code for a variety of geometries. The three conditions of point source, line source, and plane or point source off axis for a straight tunnel displayed as universal dose attenuation curves as a function of the distance into the tunnel in units of the square root of the cross-sectional area as calculated by Goebel are given in Fig. 4.6 taken from (Go75) [also in (Sc90)]. In these results it is obvious that extended, or "off axis", sources are more readily attenuated because the tunnel aperture provides less "acceptance". As a matter of terminology, the "mouth" of a given leg is the innermost opening of the leg under consideration.

**Fig. 4.6** Universal transmission curves for the first leg of a labyrinth. [Reproduced from (Go75).]
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Figure 4.7, taken from (St73), shows a two-legged penetration at the NIMROD synchrotron. This penetration was of cross section 2.3 X 2.3 m² and the walls were made of concrete. The target at the mouth of the labyrinth was bombarded by 7 GeV protons.

![Diagram of experimental layout used to study the transmission of neutrons around right angle bends.](image)

**Fig. 4.7** Experimental layout used to study the transmission of neutrons around right angle bends. [Reproduced from (St73).]

Figure 4.8, taken from (St73), is a plot of the transmission of particle flux density along this tunnel for four different nuclear reactions, again used because of their thresholds. One can see that (proceeding from the target outward in the passageway) beyond the abrupt jump which arises because the corner hides the target from view, the fast neutron components are attenuated greatly below the thermal one. Stevenson and Squier (St73) reported changes in the spectrum at a right-handed bend in a labyrinth and a change in attenuation in the second leg of such a labyrinth. [This phenomena was also verified in the results of (Co85a).] The results are illustrated in Fig. 4.8 in the plot of neutron transmission.
Fig. 4.8 Relative transmission of particle flux density along the tunnel layout shown in Fig. 4.7. [Reproduced from (St73).]

Second and successive legs of such "rectilinear" penetrations thus change the situation dramatically, principally by modifying the spectrum of the transmitted neutrons. Fig. 4.9 taken from (Go75) [also in (Sc90)] is a universal curve for second and succeeding legs which is a companion to that given for the first leg in Fig. 4.6. [In Fig. 4.9, the "distance from tunnel mouth" is the distance from the mouth of a given leg, not from the initial entrance into the passageway from the beam enclosure.] The results using the three codes mentioned previously disagree somewhat. The upper (dot-dashed) curve was obtained using SAM-CE, the middle (solid) curve resulted from the use of AMC, while the bottom (dashed) curve was obtained using ZEUS.
Fig. 4.9 Universal transmission curves for the second and subsequent legs of labyrinths. [Reproduced from (Go75).]
Tesch (Te82) has developed an entirely analytic approach to the problem of dose equivalent rate attenuation by multi-legged labyrinths at proton accelerators. For the first leg (the one directly viewing the point of beam loss) the expression is essentially an inverse-square law dependence with a simple "in-scattering" factor of two included. The expression for succeeding legs is in the form of the sum of two exponentials:

\[ H_i(r_i) = 2H_0(r_0) \frac{r_i^{2-2}}{r_0^{2-2}} \quad 1^{st} \text{leg} \]

\[ H_i(r_i) = \left\{ \frac{\exp(-r_i/0.45) + 0.022A_{i}^{1.3}\exp(-r_i/2.35)}{1 + 0.022A_{i}^{1.3}} \right\} H_{i-1} \quad i^{th} \text{leg} \quad (i > 1) \]

In these formulae, the mouth of the labyrinth is \( r_0 \) meters from the source, the coordinate \( r_1 \) (meters) is the distance from the source into the first leg and \( H_0(r_0) \) is the dose equivalent at the mouth (from a point source). In the formula for the \( i^{th} \) leg, \( H_{i-1} \) is the dose equivalent at the entrance to it, \( r_i \) is the distance into it (in meters), and \( A_i \) is the cross sectional area of the enclosure (meters\(^2\)). Thus the second formula is used "recursively" to determine the dose equivalent at the exit. This formula is easily used on a small calculator, but does not contain the expected scaling with the square root of the tunnel aperture. It is best used for personnel (i.e., "door-sized") labyrinths having cross sectional areas of approximately 2 m\(^2\).

Figure 4.10 taken from (Co85a) shows a four-legged labyrinth providing entrance to a tunnel above a target stuck by 400 GeV protons from the Tevatron at Fermilab.
Fig. 4.10 Plan and elevation views of the access labyrinth studied in (Co85a). Coordinates are defined in the figure. Locations of Bonner Sphere (S) neutron spectra and recombination chamber (R) measurements of quality factor are indicated. [Reproduced from (Co85a).]
Figure 4.11 taken from (Sc90) compares experimental results obtained in the Fermilab labyrinth (Co85a) with several methods of calculation. As one can see, all three methods of calculating the attenuation are approximately valid even for this four-legged labyrinth. [As a coincidence, the "transmission factor scale" on the ordinate is also the absolute scale of the Fermilab absorbed dose measurements as indicated.] The assumption that succeeding legs can be considered the same as the second leg is approximately verified.

---

Fig. 4.11  Measurements and predictions of transmission in a tunnel at Fermilab. [Reproduced from (Sc90).]
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For this labyrinth, a recombination chamber technique (see Chapter 6) was used to measure the neutron quality factor \( Q \) at two locations, one at the end of the first leg and one in the middle of the short second leg (locations denoted "R" in Fig. 4.10). The results were \( Q = 5.5 \pm 0.6 \) and \( Q = 3.4 \pm 0.1 \), respectively. This indicates a softening of the neutron energy spectrum in the second leg which was further verified by a measurement of the neutron energy spectrum using a multisphere technique (see Chapter 6) which resulted in \( Q = 3.1 \pm 0.7 \). The measured second-leg spectrum exhibited domination by thermal, or near-thermal neutrons. It is clear that several approaches to the design of labyrinths are equally effective for practical radiation protection work.

Curved tunnels are principally used to provide access for large equipment items that cannot negotiate right-angle bends. These have not been treated in nearly the same detail as have the rectilinear passageways. It appears that the attenuation is effectively an exponential with an attenuation length, \( \lambda \), a function of only the radius \( R \) of the tunnel. Patterson and Thomas (Pa73) determined

\[
\lambda = 0.7 (R)^{1/2},
\]

where \( R \) is in meters and \( 4 < R < 40 \) meters. This appears to fit the extremely sparse available data.

A final piece of information that is needed in practical labyrinth calculations is the answer to the following question: "What happens to the neutrons beyond the "exit" to the passageway?"

Direct observational evidence is that beyond the exit, the neutrons "disappear" rather rapidly. This phenomenon is probably due to the fact that the neutron energy spectrum is heavily dominated by thermal and near-thermal neutrons in all "legs" after the first [(St73) and (Co85a)]. Such neutrons, therefore, having suffered many scatters would not be "collimated in any particular direction". Elwyn (El91) has quantified this phenomena as follows:

Assume that the exit of the labyrinth is a circular disk of area \( A \) (equivalent in area to that of the exit opening) and that the neutrons emerge from this disk at all random directions with source strength (neutrons/unit area) \( S_A \). Fig. 4.12 illustrates the geometry.

![Diagram of labyrinth exit neutron calculation.](image-url)
Further, assume that there is only emission into the $2\pi$ steradian hemisphere outside the exit. Then the differential flux density at $P$ on the axis of the disk is:

$$d\phi = \frac{S_A \cos \alpha \, dA}{2\pi \rho^2}$$  \hspace{1cm} (4.6)$$

where $dA = r \, dr \, d\theta$, $p^2 = h^2 + r^2$, and angle $\alpha$ is defined in Fig. 4.12 ($\cos \alpha = h/\rho$). The $\cos \alpha$ factor is present to take into account the area of the source elemental area projected in the direction of point $P$. Thus,

$$d\phi = \frac{S_A h \, r \, dr \, d\theta}{2\pi \rho^3}.$$  \hspace{1cm} (4.7)$$

Integrating,

$$\phi(h) = \frac{S_A h}{2\pi} \int_0^{\pi} d\theta \int_0^R dr \frac{r}{(r^2 + h^2)^{3/2}} = \frac{S_A h (2\pi)}{2\pi} \int_0^R dr \frac{r}{(r^2 + h^2)^{3/2}}$$

$$= hS_A \left[ \frac{-1}{\sqrt{h^2 + r^2}} \right]_0^R = S_A \left[ 1 - \frac{1}{\sqrt{1 + \left( \frac{R}{h} \right)^2}} \right]$$  \hspace{1cm} (4.8)$$

where attenuation by the air is neglected. Thus one can use this by approximating the area of the exit opening by the area of a disk have an equivalent area. The rapidity of the decrease of fluence is illustrated by the tabulation of a few values in Table 4.3.

**Table 4.3 Estimates of relative neutron flux or dose equivalent as a function of scaled distance from the exit of a labyrinth.**

<table>
<thead>
<tr>
<th>$h/R$</th>
<th>$\phi(h)/S_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>1.0</td>
<td>0.29</td>
</tr>
<tr>
<td>2.0</td>
<td>0.11</td>
</tr>
<tr>
<td>4.0</td>
<td>0.03</td>
</tr>
<tr>
<td>10.0</td>
<td>0.005</td>
</tr>
</tbody>
</table>
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At large distances, one can apply a "point source" approximation due to the fact that:

\[
\phi(h) = S_A \left[ 1 - \left(1 + \frac{R^2}{h^2}\right)^{-1/2} \right] = \frac{S_A \left[ \left(\frac{R}{h}\right)^2 \right]}{2 \left(\frac{R}{h}\right)^2} = \frac{S_A R^2}{2 h^2} \quad \text{for} \quad h \gg R. \quad (4.9)
\]

For \( h = 0 \), \( \phi(0) = S_A \) as expected.

To summarize the results of this section so far; one can use a calculation or measurement of the neutron flux density or dose equivalent at the entrance (beam enclosure side) of the labyrinth in conjunction with one of the above methods of calculating the attenuation of the neutrons by the passageway to get an estimate of the dose equivalent or fluence at the exit of the passageway.
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Generally, the dose at the entrance can be obtained using Monte-Carlo techniques. Approximations that use Moyer Model parameters discussed in Chapter 3 are likely to overestimate the dose equivalent at the entrance. This is due to the fact that the Moyer parameter implicitly assumes development of the shower (a "buildup" mechanism, as seen in Chapter 3) in the enclosure shielding which is, in effect, short-circuited by the passageway.

For high energy proton accelerators, a rule of thumb for the source term which has been found to be very successful for the degree of accuracy generally required for personnel protection purposes has been developed at Fermilab by Ruffin and Moore (Ru76), and recently improved by inclusion of the Moyer energy scaling by Rameika (Ra91). In this model, it is observed that about one fast neutron/GeV of proton beam energy is produced with an isotropic distribution in addition to the much higher multiplicity in the forward direction. The neutrons which will dominate the spectrum and determine the dose equivalent at the entrance to the labyrinth are those around 1 to 10 MeV of kinetic energy. From the fluence to dose equivalent information, 8.3 neutrons/(cm² s) corresponds to 1 mrem/hour. Hence, $3 \times 10^7$ neutrons/cm² approximately corresponds to 1 rem.

Thus, at distance $R$ (cm) from the source, Rameika obtains

$$H \text{ (rem)} = \frac{E^0 N_p}{4\pi R^2 (3 \times 10^7)} = \left(2.65 \times 10^{-9}\right) \frac{E^0 N_p}{R^2}. \quad (4.10)$$

where $R$ is in cm, $E_0$ is in GeV, and $N_p$ is the number of incident protons (This could be generalized to applying to the number of incident hadrons). The constant, $2.65 \times 10^{-9}$ (rem cm²), turns out to be approximately 1/3 the value obtained by using the Moyer source (see Chapter 3) parameter along with the Moyer angular factor at $\theta = \pi/2$;

$$(2.8 \times 10^{-7} \text{ rem cm}^2) \exp(-2.3\pi/2) = 7.6 \times 10^{-9} \text{ rem cm}^2. \quad (4.11)$$
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III. Skyshine

Thin roof shielding represents a "curse" that has plagued a number of accelerators such as the Cosmotron (BNL), the Bevatron (LBL), and the Fermilab experimental areas. This "curse" actually represents situations in which the roof of some portion of the accelerator or an associated experimental facility is shielded more thinly than are the sides of the same enclosure which "directly viewed" the radiation source. The first attempt to calculate the so-called "skyshine" contribution was made by Lindenbaum (Li61). Reference (Sc90) gives a rather complete description of the phenomena. References (Pa73), (Ri75), (St84), and (Co85b) give some specific results.

Figure 4.13 taken from (Sc90) shows a summary of existing measurements of the neutron fluence times the square of the distance from the source, $r^2 \phi(r)$. As one can see in this figure, in general, the value of $r^2 \phi(r)$ is characterized by a buildup followed by an exponential falloff. As exhibited by the typical skyshine data, $\lambda$, the effective attenuation length, has been found to vary between a minimum value of about 200 meters and much larger values which approach one kilometer.

(Pa73) gives a satisfactory formula that can empirically describe such behavior for $r > 50$ meters.

$$\phi(r) = \frac{aQ}{4\pi r^2} (1-e^{-r/\mu})e^{-r/\lambda}. \quad (4.12)$$

In this equation, the isotropy of the distribution is obvious, $a = 2.8$ and represents an empirical "buildup" factor, while $\mu$ is the effective "buildup" relaxation length and $\lambda$ is an effective interaction length. From the existing data, typically $\mu = 56$ meters. $Q$ is the source strength which dimensionally must be consistent with $\phi(r)$. Thus, for the standard meaning of $\phi(r)$ as the flux density, $Q$ is in units of neutrons emitted by the source per second.

Values of $\lambda > 830$ meters are possible if very high energy neutrons ($E > 150$ MeV) are present. [Such "apparent" large values can also result from the presence of extended or multiple sources.] Concerning "high energy sources", for example, it was known that high energy neutrons were present in one of the situations examined in (St73), where the shielding was extremely thin. For that situation a value of $\lambda = 1200$ meters was obtained. After the addition of more shielding (presumably bringing the energy spectrum into "equilibrium" such that a large number of lower energy neutrons were present), a value of $\lambda = 340$ meters was obtained. A value of 830 m (100 g/cm$^2$ of air at STP) corresponds to the interaction of the $\approx 100$ MeV neutrons likely to control the propagation of hadronic cascades in air (see Chapter 3). Thus, $\lambda$ is determined by the neutron energy spectrum present at the thinly shielded location.

The procedure, then, for using the above equation is to:

A. Estimate the total emission rate of neutrons from the source. This can be done by using a neutron spectrum information to choose an "average" energy. The flux density to dose equivalent rate factor at that energy can then be used in conjunction with a dose equivalent rate survey over the thinly shielded region to determine the total neutron emission rate ($Q$) by numerically integrating over the area of the top of the shield.
Fig. 4.13 Neutron flux measurements performed around different accelerators. The abscissa is the distance $r$ from the accelerators, the ordinate is the product of the measured neutron flux density $\phi$ and the square of the distance. In these coordinates, a $1/r^2$ variation is represented by a horizontal line. 

a) Measurements of the fast neutron flux density performed at the CERN 28 GeV Proton Synchrotron.  
b) Measurements of the fast neutron flux density performed at the Dubna 10 GeV Proton Synchrophasotron.  
c) Measurements of the dose equivalent rate performed at the Brookhaven 30 GeV proton AGS.  
d) Measurements of the fast neutron flux density performed at the CERN 600 MeV Proton Synchrocyclotron.  
e) Measurements of the fast neutron flux density performed at the DESY 7.5 GeV Electron Synchrotron.  
f)Measurements of the fast neutron flux density performed at the Rutherford Laboratory Proton Linear Accelerator (solid dots for 30 MeV protons, open dots for 50 MeV protons).  
g) Measurements made at the 12 GeV Proton Synchrotron at KEK. [Reproduced from (Sc90) as adapted from references cited therein.]
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B.  Estimate the value of \( \lambda \) from the spectral information.

C.  Apply the above equation to determine the radial dependence.

A somewhat more rigorous treatment has been reported by Stevenson and Thomas ([St84] summarized also in [Sc90]). Stevenson and Thomas used as a starting point the separate work of Nakamura and Kosako (Na81) and of Alsmiller, Barish, and Childs (Al81). Stevenson and Thomas parameterized the phenomena by the following equation:

\[
\phi(r) = \frac{Q'}{4\pi r^2} e^{-r/\lambda}. \tag{4.13}
\]

In this equation, the buildup exponential has been suppressed so the formula is valid only at large distances (i.e., \( r \gg 56 \) meters). In addition, the quantity denoted by \( Q' \) ignores the buildup factor of 2.8.

The Nakamura and Alsmiller groups have separately performed extensive Monte-Carlo calculations of the neutrons emitted into cones of small vertex angle. Nakamura and Kosako used the Monte-Carlo code MORSE while Alsmiller, Barish, and Childs used the Discrete Ordinates Transport Code DOT. [Despite the title of Ref. (Al81), the calculations were performed in the context of the design of the 400 GeV accelerator ISABELLE at Brookhaven National Laboratory.] For certain selected distances from the skyshine source, these workers have calculated the dose equivalent as a function of both the source neutron energy and the emission cone's semivertical angle (that is, the half-angle the rotation of which defines the cone). The authors define this quantity, the "neutron importance", as the dose equivalent per neutron. The results are given in Fig. 4.14 taken from (Al81) [also in (Sc90)]. The Alsmiller calculations are for a semivertical angle of 37\(^\circ\).

Stevenson and Thomas (St84) were able to derive an alternative "recipe" for skyshine neutron calculations by making two assumptions:

A.  The neutron spectrum has the \( 1/E \) form up to the proton energy and zero at higher energies. This likely overestimates the contribution of the higher energy neutrons.

B.  The neutrons are emitted into a cone whose semivertical angle is about 70-80\(^\circ\). This may overestimate the doses by up to a factor of three for sources of very small semivertical angles.

Further, they used the results of Fig. 4.14 to estimate the value of \( \lambda \) based upon the upper energy of the \( 1/E \) spectrum. Fig. 4.15 taken from (St84) [also in (Sc90)] uses the values for several choices of upper energies at the three distances given in Alsmiller's calculation in a plot in which the \( 1/r^2 \) dependence is suppressed. (The curve labeled "BNL" is the result of a measurement at 28 GeV at Brookhaven National Laboratory.) The slopes were, then, used to obtain values of \( \lambda \) as a function of "upper energy" which are plotted in Fig. 4.16.
Fig. 4.14 Neutron skyshine importance functions for small semivertical cone angles. The histogram gives the results of the calculations of (Al81), the crosses, open circles, and closed circles are from (Na81). [Reproduced from (Sc90).]

Fig. 4.15 Variation of dose equivalent with distance \( r \) for \( 1/E \) neutron spectra with different upper energies. Ordinate is dose equivalent \( H \) times distance squared. The curve labeled "BNL" is a measurement. [Reproduced from (Sc90).]
Slopes determined in this way were successfully applied to data taken at 30 and 50 MeV at the Rutherford Laboratory Proton Linear Accelerator (thus demonstrating the lack of sensitivity to proton energy) as shown in Fig. 4.17 taken from (St84) [also in (Sc90)].

Fig. 4.17 Comparison of the effective absorption lengths for 30 and 50 MeV data obtained at the Rutherford Laboratory Proton Linear Accelerator. The abscissa is the distance $r$ from the source while the ordinate is the flux density times the distance squared in arbitrary units. [Reproduced from (Sc90) as adapted from references cited therein.]
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To determine the all important source term, the straight lines in Fig. 4.15 (on the semi-logarithmic plot) were extrapolated to zero and used to determine intercepts at \( r = 0 \) ranging from \( 1.5 \times 10^{-15} \) to \( 3 \times 10^{-15} \) Sv m\(^{-2}\)/neutron (\( 1.5 \times 10^{-13} \) to \( 3 \times 10^{-13} \) rem m\(^{-2}\)/neutron). Note that the BNL data also are in agreement with such a result. Hence, conservatively, Stevenson and Thomas found that

\[
H(r) = \frac{3 \times 10^{-13}}{r^2} e^{-r/\lambda} \text{ (rem/neutron, r in meters)} .
\]  

(4.14)

Again, one has to determine the total number of neutrons emitted. This can be done as before by measuring the integral of dose equivalent times area over the thinly shielded location and using the reciprocal of the fluence-to-dose equivalent conversion factor to get the total number of neutrons emitted. The use of the above formula will lead to an overestimate of neutrons for values of \( r \) less than approximately 100 meters because the extrapolation ignores the observed exponential buildup of the skyshine.

(St84) gives a convenient table of dose-equivalent-to-fluence conversion factors derived from data in ICRP Publication 21 (IC73) which is reproduced here as Table 4.4

<table>
<thead>
<tr>
<th>Upper Energy (MeV)</th>
<th>Spectrum Averaged Dose Equivalent Conversion ( (10^{-9}\text{rem}/(\text{n cm}^{-2})) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>3.9</td>
</tr>
<tr>
<td>2.5</td>
<td>4.8</td>
</tr>
<tr>
<td>4.0</td>
<td>5.6</td>
</tr>
<tr>
<td>6.3</td>
<td>6.4</td>
</tr>
<tr>
<td>10</td>
<td>7.2</td>
</tr>
<tr>
<td>16</td>
<td>7.9</td>
</tr>
<tr>
<td>25</td>
<td>8.6</td>
</tr>
<tr>
<td>40</td>
<td>9.4</td>
</tr>
<tr>
<td>63</td>
<td>10.1</td>
</tr>
<tr>
<td>100</td>
<td>10.9</td>
</tr>
<tr>
<td>160</td>
<td>11.7</td>
</tr>
<tr>
<td>250</td>
<td>12.5</td>
</tr>
<tr>
<td>400</td>
<td>13.4</td>
</tr>
<tr>
<td>630</td>
<td>14.6</td>
</tr>
<tr>
<td>1000</td>
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</tr>
<tr>
<td>4000</td>
<td>25.0</td>
</tr>
<tr>
<td>6300</td>
<td>30.0</td>
</tr>
<tr>
<td>10000</td>
<td>36.5</td>
</tr>
</tbody>
</table>

Table 4.4 Dose equivalent per neutron/cm\(^2\) for 1/E Neutron Spectra of Different Upper Energies [Reproduced from (Ref. 30).]
Measurements at Fermilab (Co85b) have confirmed this general method for a "source" involving the targetry of 400 GeV protons. Figure 4.18 taken from (Co85b) shows two measured and fitted radial distributions. These fits were made using the formula explicitly showing the buildup factor and employing the source term Q rather than Q'.

![Graph showing radial distributions for two surveys](image)

**Fig. 4.18** Skyshine data from two different surveys plotted as \( r^2 \phi \) as a function of distance from the source \( r \). The solid curves are from the least squares fit of Eq. (4.12) to the data points while the dashed curve is the fit if \( \lambda \) is constrained to have a value of 830 m. Error bars represent 1 standard deviation counting statistics. [Reproduced from (Co85b).]

In Fig. 4.18, "Survey 2" corresponds to a shielding configuration where the neutron energy spectrum was inferred to be of very high energy while "Survey 4" was likely to involve a softer spectrum. Survey 4 was made for the same beam and target after the concrete shield thickness was greatly increased compared with the shield present when "Survey 2" was obtained. (The normalization to "counts-m^2/hr" can, for purposes of this discussion, be ignored.) The instrument calibration of "counts/hr" made possible an estimate of dose rates at \( r = 204 \) meters for the two surveys. If one makes an educated guess that the conditions of "Survey 2" correspond to an upper energy of 1 GeV and that the conditions of "Survey 4" correspond to an upper energy of 100 MeV (again, by making an "educated guess"), one can then apply the conversion factors. Experimentally, the value of Q for the Survey 2 conditions was determined to be \( 2.5 \times 10^5 \) mrem m^2/hr (obtained from the measured absorbed dose surface integral of \( 5 \times 10^4 \) mrad m^2/hr and assuming a quality factor of 5). For the Survey 4 conditions Q was found to be \( 4.0 \times 10^4 \) mrem m^2/hr (again, obtained from the measured absorbed dose surface integral of \( 8.1 \times 10^3 \) mrad m^2/hr and assuming a quality factor of 5).
Table 4.5 makes a comparison with the prescription of (St84). In the table, H is the dose equivalent in one hour at 200 meters. The prescription of Stevenson and Thomas (St84) is used to calculate the dose equivalent in one hour at 200 meters.

**Table 4.5 Comparisons of Fermilab skyshine data with results of parameterizations. The quantities are all for a one hour time period.**

<table>
<thead>
<tr>
<th>Survey</th>
<th>( \lambda ) (meters)</th>
<th>( E_{\text{max}} ) (MeV)</th>
<th>Fluence-Dose Conversion ( \text{mrem/n cm}^{-2} )</th>
<th>Q-measured ( \text{n m}^{-2}/\text{mrem} )</th>
<th>H (200 m) ( \text{calculated experiment} ) (mrem)</th>
<th>H (200 m) ( \text{inferred} ) (St84) (mrem)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey 2</td>
<td>1200</td>
<td>1000</td>
<td>16.2 ( \times 10^{-6} )</td>
<td>6.17 ( \times 10^{8} )</td>
<td>2.5 ( \times 10^{5} )</td>
<td>1.0</td>
</tr>
<tr>
<td>Survey 4</td>
<td>340</td>
<td>100</td>
<td>10.9 ( \times 10^{-6} )</td>
<td>9.17 ( \times 10^{8} )</td>
<td>4.0 ( \times 10^{4} )</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The agreement is well within all uncertainties involved.

A final illustration comes from (El86) where the thin iron shield was involved. Fig. 4.19 taken from (El86) shows the radial dependence of neutron flux. In this measurement, \( \lambda \) was determined to be 184.4 m by fitting this radial dependence using the formula involving \( Q \) rather than \( Q' \). This value is consistent with a neutron spectrum otherwise found to be dominated by neutrons of a few MeV (see above). In fact, the spectrum is dominated by neutrons of energies near 847 keV insofar as skyshine is concerned (i.e., a "thick" iron spectrum, see Chapter 3 and (El86)). One can use the parametrization of (El86) to reproduce the measured fluence (normalized to \( 10^{12} \) incident protons) at, say, \( r = 200 \) meters (where \( Q = 1.75 \times 10^{10} \) was determined by fitting the skyshine data):

\[
\phi(r) = \frac{aQ}{4\pi r^2}(1-e^{-\eta r})e^{-\eta \lambda}; \quad \phi(200) = \frac{2.8(1.75 \times 10^{10})}{4\pi(200)^2}(1-\exp(-200/56))\exp(-200/184.4) = 3.20 \times 10^{14} \text{ n m}^{-2}.
\]  

(Pa73), at 847 keV gives a flux density to dose equivalent rate conversion factor of 10.2 (n cm\(^{-2}\))/(mrem hr\(^{-1}\)) which is equivalent to 3.67 \( \times 10^{8} \) neutron m\(^2\) mrem\(^{-1}\). Thus, taking the measured neutron flux at \( r = 200 \) meters and applying this factor gives a dose equivalent per \( 10^{12} \) incident protons of 8.7 \( \times 10^{-5} \) mrem at \( r = 200 \) meters.

(El86) also estimated the total neutron emission of the source to be \((3.4 \pm 2.0) \times 10^{10}\) per \( 10^{12} \) incident protons. Applying the prescription of (St84):

\[
H(r) = \frac{3 \times 10^{-13}}{r^2}e^{-\eta r} \text{ (rem/neutron, r in meters)}; \\
H(200) = \frac{3 \times 10^{-13}(3.4 \pm 2.0) \times 10^{10}}{200^2}(200)^{-2} \exp(-200/184.4) \text{ (rem)} = (8.6 \pm 5.1) \times 10^{-5} \text{ mrem at this same location}.
\]  

Thus the methodology is verified.
The product of $r^2$ and the neutron fluence $\phi(r)$ per $10^{12}$ protons incident on a target as a function of the distance from the source $r$. The smooth curve is a fit to Eq. (4.12) with parameters $\lambda = 184.4$ m and $Q = 1.74 \times 10^{10}$ neutrons per $10^{12}$ protons. [Reproduced from (El86).]
Chapter 4  Low Energy Prompt Radiation Phenomena

References


(Ch65a) A. B. Chilton, "Backscattering of gamma rays from point sources by an infinite-plane concrete surface", Nucl. Sci. Eng. 21 (1965) 194-200.


(Gi69) W. S. Gilbert, "Shielding measurements at the CERN 25 GeV proton synchrotron", Rept. CONF-691101, USAEC 323 (1969)


(Go75) K. Goebel, G. R. Stevenson, J. T. Routti, and H. G. Vogt, "Evaluating dose rates due to neutron leakage through the access tunnels of the SPS", CERN Report. LABII-RA/Note/75-10 (1975).


Chapter 4  Low Energy Prompt Radiation Phenomena

(Ma67)  R. E. Maerker and V. R. Cain, "AMC, a Monte-Carlo code utilizing the albedo approach for calculating neutron and capture gamma-ray distributions in rectangular concrete ducts", Rep. ORNL-3964, Oak Ridge National Laboratory, Oak Ridge, Tennessee (1967).


(St73)  G. R. Stevenson and D. M. Squier, "An experimental study of the attenuation of radiation in tunnels penetrating-the shield of an extracted beam of the 7 GeV proton synchrotron NIMROD, Health Physics 24 (1973) 87.


(Te82)  K. Tesch, "The attenuation of the neutron dose equivalent in a labyrinth through an accelerator shield", Particle Accelerators 12 (1982) 169-175.

Chapter 4  Low Energy Prompt Radiation Phenomena-Problems

1. A 1 μA 100 MeV electron beam is incident on an "optimized bremsstrahlung" target in a shielding configuration and labyrinth like that in Fig. 4.3. Using the facts given in Chapter 1 (Swanson's Rules of Thumb, etc.) about bremsstrahlung, calculate the dose equivalent rate at the exit of a labyrinth having 2 legs. Set all distances, $d_1$, $d_2$, and $d_2 = 3$ meters. If the goal is to get the dose equivalent rate at the exit < 1 mrem/hr, is this a sensible design? The 2 legs are 1 X 2 meter$^2$ in cross section and, since no other information is available, use $\alpha = 10^{-2}$ as a "conservative" value. [Hint: One needs to calculate the projected diameter of the beam at the wall where the first scatter occurs. This can be done using Eq. (1.28).]

2. A 500 GeV proton beam ($10^{11}$ protons/second) strikes a magnet 2 m from the mouth of a 3-legged labyrinth. Each of the 3 legs is 4 meters long and 1 X 2 m$^2$ in cross section. Using Goebel's "universal" curves ("point source" for first leg and AMC calculation for legs after the first) and Rameika's source term, what is the dose equivalent rate at the exit expressed in rem/hr. Based on the results of Elwyn, how far away from the exit does the value of $dH/dt$ fall to 10 mrem/hr. (Assume point source and "on-axis" conditions.)

3. A high energy accelerator has a section of beamline which was poorly designed. Beam losses and skimpy shielding have resulted in a region of roof 10 meters wide and 50 meters long where a neutron dose equivalent rate averaging 100 mrem/hour (averaged over the surface of the weak shield) is found. A spectrum measurement indicates the spectrum to be approximately $1/E$ with an upper end point of $\sim 500$ MeV. Calculate the dose equivalent rate due to skyshine at distance $r = 50, 100, 200, 500, \& 1000$ m using both formulae presented.
Chapter 5 Induced Radioactivity at Accelerators

In this chapter the production of induced radioactivity at accelerators is described. This discussion begins with a review of the basic principles of the production of radioactivity. It proceeds with a discussion of the activation of accelerator components including some generalizations that may be used for practical health physics applications. The chapter also considers the production of airborne radioactivity from both the standpoints of occupational and environmental radiological protection. Finally, the chapter concludes with a discussion of soil and groundwater activation pertinent to environmental protection concerns.

I. Fundamental Principles of Induced Radioactivity at Accelerators

In principle, induced radioactivity can be produced at all accelerators capable of liberating neutrons and other hadrons. When the accelerated beam strikes a nucleus, it converts it into a different nuclide which may be radioactive. In these discussions, the activity of a given radionuclide refers to the number of atoms that decay per unit time.

The customary unit of activity is the Curie (and its submultiples) which was originally defined to be the activity of 1 gram of natural radium but now is defined to be $3.7 \times 10^{10}$ decays per second.

The SI unit of activity is the Becquerel (and its multiples) which is defined to be 1 decay per second.

A related quantity of considerable importance is the specific activity which is defined to be the activity per unit volume or, alternatively, the activity per unit mass.

Radioactive decay is a random process characterized by a mean-life (time) denoted by $\tau$ (sec), and its reciprocal, the decay constant $\lambda$ [$\lambda = 1/\tau$ (sec$^{-1}$)]. If a total of $N_{tot}(t)$ atoms of a radionuclide are present at time $t$, the total activity $A_{tot}(t)$ is determined by the random nature of radioactive decay to be

$$A_{tot}(t) = \frac{dN_{tot}(t)}{dt} = \frac{N_{tot}(t)}{\tau} = \lambda N_{tot}(t).$$

(5.1)

If, at time $t = 0$, $N_{tot}(0)$ atoms are present, then this simple differential equation has the solution:

$$A_{tot}(t) = \lambda N_{tot}(0) \exp(-\lambda t) = A_{tot}(0) \exp(-\lambda t).$$

(5.2)

Often, the time required to decay to half of the original activity is tabulated. This half-life, denoted as $t_{1/2}$, is related to the mean-life by the following:

$$\tau = \frac{1}{\ln 2}t_{1/2} = \frac{1}{0.693}t_{1/2} = 1.442t_{1/2}.$$  

(5.3)

The most simple activation situation at accelerators is illustrated by the constant irradiation of some material by a constant spatially uniform flux density of neutrons (or other high energy hadrons at the higher energies) that begins at some time $t = 0$, continues for an irradiation period that ends at $t = t_i$, followed by a decay period called the "cooling time" and denoted $t_c$. $t_c$ is a period which begins at $t = t_i$ and ends at $t = t_i + t_c$. For this simplest situation, self-absorption of the hadrons by the target is ignored. Also ignored is the fact that a whole spectrum
of neutrons might be present. Thus the process of producing the radioactivity is characterized by a single cross section, $\sigma$ [cm$^2$ or barns (1 barn = $10^{-24}$ cm$^2$)] which, in the more complicated generalized situations could be an appropriate average cross section.

The number of atoms of the radionuclide of interest per unit volume will thus be governed by the following differential equation during the period of the irradiation:

$$\frac{dn(t)}{dt} = -\lambda n(t) + N\sigma\phi,$$  \hspace{1cm} (5.4)

where $n(t)$ is the atoms of the radionuclide per cm$^3$ at time $t$. $N$ is the number of "target" atoms per cm$^3$, $\sigma$ is in units of cm$^2$, and $\phi$ is the flux density (cm$^{-2}$ sec$^{-1}$) of incident particles. $N$ is defined in chapter 1 immediately following Eq. (1.6). On the right hand side of the above equation, the first term represents the loss of radionuclides through decay during the irradiation while the second term represents the gain of radionuclides through the production reaction under consideration.

The equation has the following solution for $0 < t < t_i$:

$$n(t) = \frac{N\sigma\phi}{\lambda}(1 - e^{-\lambda t}).$$  \hspace{1cm} (5.5)

Thus the specific activity (Bq/cm$^3$) induced in the material as a function of time during the irradiation is given by $a(t) = \lambda n(t)$, hence

$$a(t) = N\sigma\phi(1 - e^{-\lambda t}) \quad \text{for} \quad 0 < t < t_i.$$  \hspace{1cm} (5.6)

[To obtain specific activity in Curies/cm$^3$, one must simply divide by the constant $3.7 \times 10^{10}$ Curies/Bq.]

At the instant of completion of the irradiation ($t = t_i$), the specific activity will thus be:

$$a(t_i) = N\sigma\phi[1 - \exp(-\lambda t_i)],$$  \hspace{1cm} (5.7)

so that we see that the specific activity as a function of time is characterized by a buildup from zero to the saturation value of $N\sigma\phi$ for infinitely long irradiations.

After the irradiation has ceased ($t > t_i$), the specific activity as a function of the "cooling time", $t_c$, will obviously decay exponentially and be given by:

$$a(t_c) = N\sigma\phi[1 - \exp(-\lambda t_i)]\exp{-\lambda t_c}$$  \hspace{1cm} (5.8)

where $t_c$ is the cooling time; $t_c = t - t_i$.  \hspace{1cm} (5.9)

For total activities in situations where uniform flux densities of particles of constant energy are incident on a homogeneous "target", one can simply multiply by the volume of the "target"; or in more complex cases involving nonuniform flux densities, integrate the above over the volume of the target.
Chapter 5  Induced Radioactivity at Accelerators

For $\gamma$-ray emitters, the absorbed dose rate, $\frac{dD}{dt}$ (rad/h), at a distance $r$ (meters) from a "point" source of typical activation $\gamma$-rays (those in the range from about 100 keV to 10 MeV) is given in terms of the source strength, $S$, (Ci), and the photon energy, $E_{\gamma}$ (MeV) by:

$$\frac{dD}{dt} = 0.4 \sum E_{\gamma_i} \left( \frac{S}{r^2} \right),$$

(5.10)

where the summation is over all $\gamma$-rays present, including appropriate branching fractions if more than one photon is emitted per decay. If $dD/dt$ is desired as an approximate absorbed dose rate in Gy/h at a distance, $r$ (meters), from a source strength $S$ in GBq (GBq, $10^9$ Bq, is a more practical unit than is Bq), the factor 0.4 becomes $1.08 \times 10^{-4}$. Thus one can use the above to determine the absorbed dose rate from a given activated object if it is a point source. For non-point sources, an appropriate spatial integration must be performed.
II. Activation of Accelerator Components [Parts of this discussion follow that of (NC96)]

Proton accelerators whose energy exceeds about 10 MeV will produce radioactivity. This will also occur for accelerators of other ions above a specific energy of about 10 MeV/amu. In some special cases radioactivity can be produced at much lower energies because of exothermic nuclear reactions which either produce radionuclides directly or emit neutrons capable of inducing radioactivity through their secondary interactions. If a given accelerator is properly designed with respect to the shielding against prompt radiation and has proper access controls to avoid direct beam-on exposure to people, the induced radioactivity is very likely to be the dominant source of exposure to people. In fact, the experience at most accelerators bears this out in that the vast majority of the radiation exposure incurred by the workers is due to maintenance activities on radioactivated components, handling and moving of activated items, radiation surveys, and radioactive waste handling. An understanding of the production of radionuclides can help reduce personnel exposures through the selection of more appropriate machine component materials and the optimization of decay ("cool-down") times recommended after the beam has been turned off.

The primary focus of this section is on proton accelerators because the activation is much more severe at such machines. Marcel Barbier (Ba69) has rather adequately handled activation by photons and electrons and also considered special problems associated with heavy ions.

For the lower incident energies (< 30 MeV), one is first concerned with production of radionuclides by such processes as \((p,\gamma)\) and single- and multi-nucleon transfer reactions. While the details of the total cross sections for such reactions continue to form an interesting subfield of nuclear physics, the systematics and approximate energy dependencies are globally well understood. In general, one is dealing with endothermic nuclear reactions which have a threshold, \(E_{th}\), below which the process is forbidden by conservation of energy. For nuclear reactions induced by ions, \(E_{th}\) is related through masses of the projectile, \(m\), and the target, \(M\), to the energy released in the reaction, (the reaction "Q value", see discussion in Chapter 1) \(Q_v\), by:

\[
E_{th} = \frac{m + M}{M} |Q_v|, \tag{5.11}
\]

where the \(Q_v\) is negative in an endothermic reaction having a positive value of \(E_{th}\). In this equation, \(m\) is the mass of the incident projectile while \(M\) is the mass of the target atom, assumed to stationary. Thick target yields of radionuclides for targets having a range of atomic numbers have been systematically plotted for numerous reaction processes by B. L. Cohen (Co78). Representative plots for the more significant reactions are given in Fig. 5.1 taken from (Co78). It is assumed that the target thickness comfortably exceeds the range of the incident ion and that the irradiation period greatly exceeds the half-life of the radionuclide of interest. If shorter bombarding periods are used, one can correct by multiplying the plotted value by the factor \([1 - \exp(-\lambda t)]\). The values of \(\mu\text{Ci}/\mu\text{A}\) (microCuries per targeted microamperes of beam current) should be accurate to within a factor of about three.

It should be emphasized that these are plots of radionuclide yield as a function of the energy above threshold, \(E - E_{th}\). As one can see, a general feature is that the yield rises as the threshold energy is exceeded by the bombarding energy by a few MeV. At higher energies, the rate of increase of yield with energy rises more slowly and, in some cases, appears to level off to a "saturation" value. Over the energy range of these curves, the importance of activation by secondary particles is small compared to that encountered at higher energies.
Fig. 5.1 Thick target yields from charged particle reactions. Curves give microcuries per microamperes for elements of given atomic number Z as a function of particle kinetic energy E minus the threshold energy $E_{th}$ for the reaction. Nuclides of the same Z have about the same approximate yield (order of magnitude) for the same value of $E - E_{th}$. [Reproduced from (Co78).]
Chapter 5  Induced Radioactivity at Accelerators
For particle accelerators of higher energy, the neglect of secondary reactions and the restriction to few- and multi-nucleon transfer reactions can become a serious deficiency in the accuracy of estimation of induced radioactivity because of the rise in importance of such processes as spallation. Fig. 5.2 taken from (Pa73) illustrates how the number of radionuclides produced increases with increased bombarding energy for the case of protons incident on bismuth. At 40 MeV, only few-nucleon transfer reactions are available while at 3 GeV, essentially the entire periodic table of nuclides having mass numbers less than that of the target material becomes available. The "bipolar" peak obtained at 480 MeV clearly represents the optimization of the fission process. The variety of radionuclides that can be produced increases as one increases the bombarding energy because more thresholds are exceeded. As a general rule, at high energies $(E_0 = 1 \text{ GeV or greater})$, one must consider that all radionuclides in the periodic table which have mass numbers less than that of the material exposed to the flux of hadrons may be produced. Of course, many of these are of little significance due to short lifetimes and small production cross sections.

![Diagram](image)

Fig. 5.2 Mass yield curves for the proton bombardment of bismuth for various energies. [Reproduced from (Pa73) as adapted from references cited therein.]
Table 5.1 taken from (NC96) gives a list of radionuclides typically encountered in accelerator installations and their half-lives. In this table only nuclides with half-lives between 10 minutes and 5 years are listed. Also, all "pure" (that is, with no γ-ray emitted) β± emitters are ignored.

Table 5.1 Summary of radionuclides commonly identified in materials irradiated around accelerators.

<table>
<thead>
<tr>
<th>Target material</th>
<th>Radionuclides</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastics and oils</td>
<td>⁷Be</td>
<td>53.6 days</td>
</tr>
<tr>
<td></td>
<td>¹¹C</td>
<td>20.4 minutes</td>
</tr>
<tr>
<td>Duralumin</td>
<td>As above, plus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>²²Na</td>
<td>2.60 years</td>
</tr>
<tr>
<td></td>
<td>²⁴Na</td>
<td>15.0 hours</td>
</tr>
<tr>
<td>Steel</td>
<td>As above, plus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>⁴²K</td>
<td>12.47 hours</td>
</tr>
<tr>
<td></td>
<td>⁴³K</td>
<td>22.4 hours</td>
</tr>
<tr>
<td></td>
<td>⁴⁴Sc</td>
<td>3.92 hours</td>
</tr>
<tr>
<td></td>
<td>⁴⁴mSc</td>
<td>2.44 days</td>
</tr>
<tr>
<td></td>
<td>⁴⁶Sc</td>
<td>84 days</td>
</tr>
<tr>
<td></td>
<td>⁴⁷Sc</td>
<td>3.43 days</td>
</tr>
<tr>
<td></td>
<td>⁴⁸Sc</td>
<td>1.83 days</td>
</tr>
<tr>
<td></td>
<td>⁴⁸V</td>
<td>16.0 days</td>
</tr>
<tr>
<td></td>
<td>⁵¹Cr</td>
<td>27.8 days</td>
</tr>
<tr>
<td></td>
<td>⁵²Mn</td>
<td>5.55 days</td>
</tr>
<tr>
<td></td>
<td>⁵²mMn</td>
<td>21.3 minutes</td>
</tr>
<tr>
<td></td>
<td>⁵⁴Mn</td>
<td>300 days</td>
</tr>
<tr>
<td></td>
<td>⁵⁶Co</td>
<td>77 days</td>
</tr>
<tr>
<td></td>
<td>⁵⁷Co</td>
<td>270 days</td>
</tr>
<tr>
<td></td>
<td>⁵⁸Co</td>
<td>72 days</td>
</tr>
<tr>
<td></td>
<td>⁵⁵Fe</td>
<td>2.94 years</td>
</tr>
<tr>
<td></td>
<td>⁵⁹Fe</td>
<td>45.1 days</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>As above, plus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>⁶⁰Co</td>
<td>5.27 years</td>
</tr>
<tr>
<td></td>
<td>⁵⁷Ni</td>
<td>37 hours</td>
</tr>
<tr>
<td></td>
<td>⁶⁰Cu</td>
<td>24 minutes</td>
</tr>
<tr>
<td>Copper</td>
<td>As above, plus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>⁶⁵Ni</td>
<td>2.56 hours</td>
</tr>
<tr>
<td></td>
<td>⁶¹Cu</td>
<td>3.33 hours</td>
</tr>
<tr>
<td></td>
<td>⁶²Cu</td>
<td>9.80 minutes</td>
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<td></td>
<td>⁶⁴Cu</td>
<td>12.82 hours</td>
</tr>
<tr>
<td></td>
<td>⁶³Zn</td>
<td>38.3 minutes</td>
</tr>
<tr>
<td></td>
<td>⁶⁵Zn</td>
<td>245 days</td>
</tr>
</tbody>
</table>
Chapter 5  Induced Radioactivity at Accelerators

An extensive treatise on induced activity, dealing with the multitude of complications has been written by Barbier (Ba69) in which methods for systematizing the large body of nuclear physics data are described. The reader is encouraged to refer to this reference for an extensive discussion of the mechanisms, including many excitation functions for many nuclides of interest.

Concerning the activation of accelerator components, one must have a systematic way of handling the great multiplicity of radionuclides produced, as illustrated above, since it is simply not practical to handle them all separately. Global properties of the distribution of radionuclides must be used. Sullivan and Overton (Su65) have treated this problem in an elegant manner which will now be restated here. The initial starting point is an equation of the form derived previously for describing activity, but now related to dose rate (which is, of course, proportional to activity):

$$\delta(t, t_c) = G\phi[1 - \exp(-\lambda t)]\exp\{-\lambda(t_c)\}, \quad (5.12)$$

where $\delta(t, t_c)$ is the absorbed dose rate, $\phi$ is the flux density, and $G$ is a collection of many contributing factors from among the following:

- production cross section
- energy of the beam
- types of secondaries produced
- isotopic composition of the irradiated component
- geometry
- energy of the $\gamma$-rays produced
- attenuation coefficients for the $\gamma$-rays produced.

If the number of radionuclides produced by the irradiation which have decay constants in the interval between $\lambda$ and $\lambda + d\lambda$ is represented by the differential $dm$, then the corresponding increment in absorbed dose rate, $d\delta(t, t_c)$, is given by:

$$d\delta(t, t_c) = G\phi[1 - \exp(-\lambda t)]\exp\{-\lambda(t_c)\} dm. \quad (5.13)$$

If it is assumed that the value of $G$ is independent of $\lambda$, or its dependence on $\lambda$ is small compared to other factors, then one can integrate:

$$\delta(t, t_c) = G\phi \int_{\lambda_0}^{\infty} d\lambda \frac{dm}{d\lambda}[1 - \exp(-\lambda t)]\exp\{-\lambda(t_c)\}. \quad (5.14)$$

Here, $\lambda_0$ is the shortest decay constant (longest mean-life) to be considered. Barbier (Ba69) has followed up on the work of Sullivan and Overton and has plotted the distributions of isotopes with respect to their half-lives below a given atomic mass as shown in Fig. 5.3. (This, then, corresponds to the distribution of isotopes that could be produced in a target of mass number $A$ irradiated by high energy hadrons.) Figures 5.4 and 5.5 taken from (Ba69) show the integrals of these distributions up to a given value of half-life.

---

1This implicitly makes the assumption that, on average, the cross sections that produce the radionuclides of concern are independent of both the half-lives and the particle energies. Somewhat remarkably, this approximation is sufficiently accurate.
Fig. 5.3 Distribution of radioactive isotopes below a given atomic number with respect to their half lives. [Reproduced from (Ba69).]
Fig. 5.4 Total number of isotopes up to a given half-life as a function of half-life (cases $A < 60$ and $A < 25$). [Reproduced from (Ba69).]

Fig. 5.5 Total number of isotopes up to a given half-life as a function of half-life (cases $A < 209$, $A < 101$, and $A < 60$). [Reproduced from (Ba69).]
Sullivan and Overton (Su65), also as discussed in (Ba69), found that these cumulative distributions are well-described for values of half-life between $10^{-3}$ and $10^3$ days by the following form:

$$N(t_{1/2}) = a + b \ln(t_{1/2}),$$

(5.15)

where $N(t_{1/2})$ is the number of isotopes with half-lives less than the value of $t_{1/2}$ and $a$ and $b$ are fitting parameters. Because of the one-to-one correspondence between values of $t_{1/2}$, $\tau$, and $\lambda$, one can just as well write

$$m(\lambda) = a + b \ln \lambda,$$

(5.16)

where $m(\lambda)$ is the number of radionuclides with decay constants greater than $\lambda$ for the material of concern. Thus,

$$\frac{dm(\lambda)}{d\lambda} = \frac{b}{\lambda}.$$

(5.17)

Substituting into Eq. (5.14), one gets:

$$\delta(t_i, t_c) = Gb\int_{\lambda_0}^{\lambda_0} \frac{d\lambda}{\lambda} [1 - \exp(-\lambda t_i)] \exp(-\lambda t_c).$$

$$= Gb\left\{ \int_{\lambda_0}^{\lambda_0} \frac{d\lambda}{\lambda} \exp(-\lambda t_c) - \int_{\lambda_0}^{\lambda_0} \frac{d\lambda}{\lambda} \exp(-\lambda (t_i + t_c)) \right\}.\quad (5.18)$$

The changes of variables $\alpha = \lambda t_c$ (first term) and $\alpha' = \lambda(t_i + t_c)$ are helpful;

$$\delta(t_i, t_c) = Gb\left[ \int_{\lambda_0 t_c}^{\lambda_0 t_c} d\alpha \frac{e^{-\alpha}}{\alpha} - \int_{\lambda_0 (t_i + t_c)}^{\lambda_0 (t_i + t_c)} d\alpha' \frac{e^{-\alpha'}}{\alpha'} \right].\quad (5.19)$$

Recognizing that the integrands are identical and simplifying by rearranging the limits of integration, we have

$$\delta(t_i, t_c) = Gb\int_{\lambda_0 t_c}^{\lambda_0 t_c} d\alpha \frac{e^{-\alpha}}{\alpha}.\quad (5.20)$$

The integral is of a form that integrates to a series expansion found in standard tables of integrals;

$$\int_{x_1}^{x_2} \frac{e^{ax}dx}{x} = \ln x + \frac{ax}{1!} + \frac{a^2x^2}{2(2!)} + \frac{a^3x^3}{3(3!)} + \ldots \bigg|_{x_1}^{x_2}.\quad (5.21)$$
Substituting,
\[
\int_{\lambda_0 t_c}^{\lambda_0(t_i + t_c)} \frac{e^{-\alpha d\alpha}}{\alpha} = \ln \alpha - \frac{\alpha^2}{4} + \frac{\alpha^3}{18} + \ldots \int_{\lambda_0 t_c}^{\lambda_0(t_i + t_c)} \lambda_0 \, dt_c.
\]
(5.22)

Evaluating, one obtains
\[
\delta(t_i, t_c) = G \phi \left[ \ln \left( \frac{t_i + t_c}{t_c} \right) - \lambda_0 t_i + \ldots \right].
\]
(5.23)

Since \( \lambda_0 \) approaches zero (corresponding to large lifetimes), the following is obtained:
\[
\delta(t_i, t_c) = B \phi \ln \left( \frac{t_i + t_c}{t_c} \right),
\]
(5.24)

where several constants are merged in parameter \( B \).

Peter Gollon (Go76) has further elaborated on these principles and determined some very useful "rules of thumb" for high energy hadron accelerators at which the extranuclear hadron cascade process produces the major fraction of the induced activity. Four rules are extremely useful for approximate radioactivity estimates:

**Rule 1**: (Repeated here for convenience) The absorbed dose rate, \( dD/dt \) (rad/h), at a distance \( r \) (meters) from a "point" source of typical activation gamma-rays is given in terms of the source strength (Ci) and the photon energy, \( E_\gamma \) (MeV) by:
\[
\frac{dD}{dt} = 0.4 \sum_i E_\gamma \left( \frac{S}{r^2} \right),
\]
(5.25)

where the summation is over all \( \gamma \)-rays present, including appropriate branching fractions if more than one photon is emitted per decay. If \( dD/dt \) is desired as an approximate absorbed dose rate in Gy/h at a distance \( r \) (meters) from a source strength \( S \) in GBq, the factor 0.4 becomes \( 1.08 \times 10^{-4} \).

**Rule 2**: In many common materials, about 50% of the nuclear interactions produce a nuclide with a half-life longer than a few minutes. Further, about 50% of these have a half-life longer than one day. Thus approximately 25% of the nuclear interactions (e.g., the "stars" discussed in Chapter 3) produce a radionuclide having a half-life exceeding approximately one day.

**Rule 3**: For most common shielding materials, the approximate dose rate \( dD/dt \) due to a constant irradiation is given as above:
\[
\frac{dD(t_c)}{dt_c} = B \phi \ln \left( \frac{t_i + t_c}{t_c} \right).
\]
(5.26)
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In the above, the geometry and material dependent factor $B$ can often be determined empirically, or by using rule 2, while $\phi$ is the incident flux density. This expression appears to be valid also for heavy ion beams at 86 MeV/nucleon according to Tuyn (Tu84).

**Rule 4:**

In a hadronic cascade, a proton produces about four inelastic interactions for each GeV of energy.

These rules can be illustrated by examples. In a short target, $1/10$ of an interaction length long, approximately $10\%$ of an incident beam of $10^{11}$ protons s$^{-1}$ will interact. Assume this has been occurring for several months (long enough to reach saturation production for many radionuclides). Using Rule 2 in conjunction with the above rate, one determines that the decay rate after one day of the shutdown is $2.5 \times 10^9$ Bq (68 mCi). If each of these decays produces a 1 MeV $\gamma$-ray, then Rule 1 will indicate an absorbed dose rate of $27$ mrad/h ($= 0.27$ mGy/h of absorbed dose rate) at one meter away.

Rule 3 can be used along with such a calculation to predict the absorbed dose rate from a point source at some future time after beam shutdown. Furthermore, this rule is not restricted to "point" sources but can be used for more massive ones, with suitable adjustments to the geometry factors. Sometimes one can estimate the product $B\phi$ or use a measurement of the exposure or absorbed dose rate to determine it empirically for the purpose of using the formula to predict the "cooldown". In this way, Rule 3 is also useful for extended shields irradiated by secondary particles from a well-developed cascade. Rule 4 can be used to crudely estimate the activation of a beam dump by incident high energy particles when it is coupled with Rule 2.

Rule 4 can be used thus: A beam of $10^{12} 400$ GeV p/s ($= 0.16$ $\mu$A or 64 kW) produces a total of $4 \times 400 \times 10^{12}$ stars/s in a beam dump. If $25\%$ of these produce a radionuclide with a half-life $> 1$ day (rule 2), then the total amount of the moderately long-lived radioactivity (at saturation) is:

\[
\frac{(0.25 \text{ atoms/star})(1.6 \times 10^{15} \text{ stars/sec})}{3.7 \times 10^{10} \text{ s}^{-1} \text{Ci}^{-1}} = 10.8 \text{ kCi}.
\]

At sufficiently large distance (say 10 meters), rule 1 could be used to calculate an absorbed dose rate assuming all decays are 1 MeV $\gamma$-rays:

\[
\frac{dD}{dt} = 0.4(1 \text{ MeV}) \left(\frac{1.08 \times 10^4 \text{Curies}}{10^2 \text{ meter}^2}\right) = 43 \text{ rads/hour}.
\]

A valuable parameter used to quantify the absorbed dose rate, $dD/dt$, at the surface of a thick target is the danger parameter, $D$, as developed by Barbier (Ba69) for a thick object irradiated by beam with a uniform flux density $\phi$. If this source of radioactivity subtends solid angle $\Omega$ at the point of concern, then

\[
\frac{dD}{dt} = \frac{\Omega}{4\pi} \phi D.
\]
Fig. 5.6 Values of the Barbier Danger Parameter. [Reproduced from (Ba69).]
Fig. 5.6 Values of the Barbier Danger Parameter. [Reproduced from (Ba69).]
Fig. 5.6 Values of the Barbier Danger Parameter. [Reproduced from (Ba69).]
Fig. 5.6 Values of the Barbier Danger Parameter. [Reproduced from (Ba69).]
For contact with a semi-infinite slab of uniformly irradiated material, the fractional solid angle factor \( \Omega/4\pi \) has the intuitively obvious value of 1/2. The danger parameter \( D \) has the physical interpretation as the absorbed dose rate found inside a cavity of arbitrary form embedded in an infinite volume of a material which has been uniformly irradiated by a unit flux density (one particle per second per square centimeter). Figures 5.6 taken from [Ba69] give representative examples of plots of \( D \) for several elements and a few compounds. These curves thus can be used to predict cooling of various components around accelerators.

Gollon [Go76] provided "cooling curves" for iron struck by high energy protons. These are given in Fig. 5.7 taken from [Go76] and include both calculations by Armstrong and Alsmiller [Ar73] and empirical measurements at the Brookhaven National Laboratory AGS, the Fermilab Main Ring Accelerator, and the Fermilab Neutrino target "train".

Of course, one is often concerned with situations where the determination of \( \phi \) in the danger parameter equation is not at all simple. For example, one can have activation in a large object where the hadronic cascade is contributing numerous hadrons at a variety of energies from a multitude of directions. Fortunately, important features of activation phenomena have little or no correlation with energy. The chief of these is evidenced by the excitation functions of various reactions. In general, the cross sections rise just above the threshold and then, somewhere in the region of 10's of MeV, a leveling-off occurs. Furthermore, in general the cross sections for production of radionuclides by neutrons and protons (and even other ions and particles) do not differ from each other at the higher energies. Results from Barbier's book [Ba69] illustrate this and are given as Fig. 5.8.
Fig. 5.8 Curves of excitation functions for nuclear reactions. [Reproduced from (Ba69).]
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Fig. 5.8 Curves of excitation functions for nuclear reactions. [Reproduced from (Ba69).]
Fig. 5.8 Curves of excitation functions for nuclear reactions. [Reproduced from (Ba69).]
Chapter 5  Induced Radioactivity at Accelerators

The "leveling-off" of the cross section has some very important implications the most important is the fact that for estimating activation, one can perform approximate calculations without performing an integration over energy if one has some reasonable estimate of the flux above the reaction threshold of interest. An average "effective cross section" can then be used. [The "leveling off" also renders reasonable the use of "threshold" detectors in instrumentation as discussed in Chapter 6.] Another feature of these excitation functions is the fact that the leveling off occurs in the region from a few 10's to a few 100's of MeV precisely where relatively fast Monte-Carlo hadron shielding calculations are available from several different codes (e.g., CASIM, FLUKA, HETC, and MARS).

It is often possible to relate the flux density of high energy hadrons (i.e., those with energies above the "leveling off") to the star density, $S$, calculated from such Monte-Carlo calculations through the relationship,

$$\phi(\vec{r}) \left( \text{cm}^{-2}\text{sec}^{-1} \right) = \frac{\lambda \left( \text{g/cm}^2 \right) dS(\vec{r})}{\rho \left( \text{g/cm}^3 \right) dt} \left[ \text{stars cm}^{-3} \text{sec} \right]$$

(5.30)

where $\phi(\vec{r})$, the flux density at position vector $\vec{r}$, is related to the rate of star density production $\frac{dS(\vec{r})}{dt}$ (stars cm$^{-3}$ s$^{-1}$) at the same location. The density is denoted by $\rho$ and the interaction length is denoted by $\lambda$.

In the context of this discussion, care must be taken not to confuse interaction length with activity constant since they are customarily denoted by the same symbol, $\lambda$. The value of $\phi(\vec{r})$ so determined could, in principle, be substituted into the equation given above for calculating absorbed dose rate due to residual activity using the Barbier danger parameter, $D$, if one were to make suitable adjustments in the solid angle. However, the limitation of this approach is the fact that the Monte-Carlo cutoffs may introduce an energy (or momentum) cutoff (e.g., typically 300 MeV/c in CASIM) which is not necessarily matched to the reaction threshold. In order to calculate dose equivalent rates, Gollon (G076) made detailed calculations and obtained the following formula:

$$\frac{dD(\vec{r})}{dt} = \frac{\Omega}{4\pi} \frac{dS(\vec{r})}{dt} \omega(t_i, t_c)$$

(5.31)

where $\omega(t_i, t_c)$ is related to the Barbier danger parameter. For iron, this parameter has the following values for two useful situations:

$$\omega(\infty, 0) = 9 \times 10^{-6} \text{ rad h}^{-1}/(\text{star cm}^{-3} \text{ s}^{-1})$$

(infinite irradiation, zero cooling time) and

$$\omega(30 \text{ d}, 1 \text{ d}) = 2.5 \times 10^{-6} \text{ rad h}^{-1}/(\text{star cm}^{-3} \text{ s}^{-1})$$

(30 days irradiation, 1 day cooling time).

For materials other than iron, estimates of the $\omega$-values can be made by scaling the values of $D$ (Ba69) for the same values of $t_i$ and $t_c$. 
Finally, Gollon derived a simple relationship between dose rates involving cooling times different from "standard" ones for which values of $D$ and $\omega$ are available. As stated previously, the dose rate after irradiation time $t_i$ and cooldown time $t_c$ is

$$\delta(t_i,t_c) = \sum_{\mu} A_{\mu} [1 - \exp(-\lambda_{\mu} t_i)] \exp(-\lambda_{\mu} t_c)$$

(5.33)

where the summation over index $\mu$ includes all relevant radionuclides and the product of flux density and geometry factors are absorbed (and allowed to vary with radionuclide) in the quantity $A_{\mu}$.

Rearranging, Gollon obtained:

$$\delta(t_i,t_c) = \sum_{\mu} A_{\mu} \left[ \exp \left\{ -\lambda_{\mu} (t_c) \right\} - \exp \left\{ -\lambda_{\mu} (t_i + t_c) \right\} \right]$$

$$= \delta(\infty, t_c) - \delta(\infty, t_i + t_c).$$

(5.34)

Thus, the infinite irradiation curve can be used to determine any other combination of the times $t_i$ and $t_c$. In fact, this formula is exact, it is "model independent" and may be used also with empirical results such as, for example, radiation survey data.

A final method for connecting the production of "stars" in material (e.g., as calculated by a Monte-Carlo code) to the production of atoms of some radionuclide is by the ratios of cross sections. Thus, at some point in space, $\vec{r}$, the rate of production of atoms per cm$^3$, $n(\vec{r})$, of some radionuclide is approximately given by:

$$\frac{dn(\vec{r})}{dt} = \frac{\sigma_r}{\sigma_{in}} \frac{dS(\vec{r})}{dt} = \frac{\Sigma_r}{\Sigma_{in}} \frac{dS(\vec{r})}{dt}$$

(5.35)

where one essentially scales the star density production rate [e.g., stars/(cm$^3$-s)] by the ratio of the production (reaction) cross section for the nuclide of interest, $\sigma_r$, to the total inelastic cross section $\sigma_{in}$ or, alternatively, by the macroscopic cross section ratio ($\Sigma_r/\Sigma_{in}$). At saturation, this will be the rate of decay as well. The phenomena will obey the usual activation equation.

The reason this is approximate is due to the standard concerns about constancy of cross sections, lack of perfect "matching" of thresholds, etc.

Somewhat special considerations may apply to the concrete shielding surrounding accelerators. As was seen before, ordinary concrete typically contains a partial density of 0.04 g/cm$^3$ of Na. This "typical" value varies a great deal due to the variety of minerals which might be present in local concrete. The significance of this seemingly small additive is that the naturally dominant isotope present is $^{23}$Na. This nucleus has the relatively large thermal neutron capture cross section of 535 mb.

Patterson (Pa58) determined that average thermal flux density, $\phi_{th}$, in a concrete room is approximately given as follows:

$$\phi_{th} = \frac{1.25 Q}{S}$$

(5.36)
where $Q$ is the number of fast neutrons produced per second in the enclosure and $S$ is the inside surface area of the enclosure ($\text{cm}^2$). Thus, a substantial flux density of thermal neutrons can be present in an accelerator room and this flux can produce significant amount of $^{24}\text{Na}$ with its 15 hour half-life. The relatively high energy photon emitted in its decay (2.75 MeV) also can enhance the radiation hazard.

Furthermore, while the dose due to activated components falls off radially with distance, if absorption by the air is not significant, the absorbed dose rate due to residual activation in an empty cylindrical room uniformly irradiated by such thermal neutrons is a constant and the dose equivalent rate anywhere inside the enclosure will be equal to the dose equivalent at the wall.

This has been explicitly demonstrated for cylinders by Armstrong and Barish (Ar69) and is also true for the interior of all mathematically well-behaved closed surfaces. This fact can readily be demonstrated by analogy to the Gauss Law in electrostatics as follows by examining the situation in Fig. 5.9. Consider a simple, closed surface which emits an omnidirectional flux density of some particle $\phi_0$ (e.g., particles cm$^{-2}$s$^{-1}$) that is constant over the surface. One wants to calculate the flux density at some point in space $P$ within the surface. $P$ is located at radius vector $\vec{r}$. Consider further the contributions of the particle emitted by some elemental area $d\vec{A}$ at $P$ where $d\vec{A}$ is perpendicular to the surface at coordinate vector $\vec{r}$.

![Fig. 5.9 Geometry for deriving relationship between a surface of uniform emission and the flux density at any point within it.](image)

The solid angle subtended at $P$ by $d\vec{A}$ is;

$$d\Omega = \frac{d\vec{A} \cdot \hat{n}}{|\vec{r}' - \vec{r}|^2}$$  \hspace{1cm} (5.37)
where the unit vector $\hat{n}$ is given by

$$\hat{n} = \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|}$$ (5.38)

and is along the direction of $\vec{r}' - \vec{r}$.

But the increment of flux at point $P$ due to elemental area $dA$ is given by:

$$d\phi = \frac{\phi_o \, dA \cdot \hat{n}}{4\pi |\vec{r}' - \vec{r}|^2}$$

thus

$$d\phi = \frac{\phi_o}{4\pi} d\Omega$$

and

$$\phi = \int_{4\pi} \frac{\phi_o}{4\pi} d\Omega = \frac{\phi_o}{4\pi} \int_{4\pi} d\Omega = \phi_o.$$ (5.39)

In some cases it has been important to minimize the amount of sodium in the concrete ingredients in order to reduce exposures to maintenance personnel. In fact, the phenomena described above has been noticed at accelerators and sometimes leads to "disappointment" in how little gamma-ray exposure rates are reduced when "hot" components are removed from enclosures with equally "hot" walls. For example, Armstrong and Barish (Ar69) have calculated residual dose rates inside a cylindrical accelerator tunnel due to both the magnets and the concrete walls for 3 GeV protons incident on iron. [These authors have also included some other reactions which are capable of also producing $^{24}$Na (spallation) which also must be included.] The results are shown in Fig. 5.10 taken from (Pa73) adapted from (Ar69) for the surface at the tunnel wall.

![Graph](image)

**Fig. 5.10** Photon dose rate at surface of tunnel wall after infinite irradiation time for concrete containing one percent sodium by weight. [Reproduced from (Pa73) as adapted from (Ar69).]
III. Production and Propagation of Airborne Radioactivity

Production of Airborne Radioactivity

Thomas and Stevenson have presented a very useful synopsis of the production of activity in the atmosphere in (Th88) which is largely followed here. Some of this same discussion appears in (Sw90). The principal source of radioactivity in air at accelerators is due to the interaction of primary and secondary particles directly with the constituent target nuclei in the air in accelerator enclosures. Activated dust and gaseous emission from activated liquids are of secondary importance. One must be reminded of the isotopic composition of the atmosphere and this is given in Table 5.2 taken from (Th88).

Table 5.2 Most Abundant Isotopes in the Atmosphere

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Percentage by volume in the atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>14N</td>
<td>78.1</td>
</tr>
<tr>
<td>16O</td>
<td>21.2</td>
</tr>
<tr>
<td>40Ar</td>
<td>0.46</td>
</tr>
<tr>
<td>15N</td>
<td>0.28</td>
</tr>
<tr>
<td>18O</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Since low energy accelerators contain their beams in continuous vacuum pipes, the activation of air at these machines is greatly minimized. At high energy accelerators, it is quite common to have air gaps at certain "interface points" and where complicated "gadgets" associated with beam targetry or beamline instrumentation render continuous vacuum impractical. (These "air gaps" are only characteristic of extracted beam lines, the machines themselves are, in general, contained in continuous vacuum.) In addition, the large multiplicity of secondary particles produced as a part of cascade (both electromagnetic and hadronic) processes can produce airborne radioactivity even where the beams themselves are contained in vacuum.

Table 5.3 taken from (Sw90) gives the radionuclides that can be produced from the principle constituents in air along with the reaction mechanisms associated with their production and an estimate of the average production cross section. The large cross section for neutron induced (n, γ) and (n, p) reactions are for captures of thermal, E_n = 0.025 eV, neutrons while the remaining cross sections can be anticipated to be the saturation cross sections found in the 10s of MeV region and above. The γ-induced reactions are present at virtually all accelerators and most energies. In this table "spallation" reactions refer to the intermediate energy process by which the target nucleus is effectively shattered into a number of its constituents.

If the accelerator enclosures were completely sealed, there would be no releases to the outside world and the hazard of these airborne radionuclides would be entirely restricted to those who might have to enter the enclosures. This would, however, allow the longer-lived radionuclides to build up in accord with the activation formula. Also, ventilation is generally needed to provide cooling of components and fresh breathing air for workers. Typically, the "residence time" of air in accelerator enclosures is 30 minutes to one hour and not much longer.
Thus, the typical half-lives of the accelerator environment "in equilibrium" will have half-lives only up to the order of one hour. The residence time of the air in conjunction with the cross sections determines the radionuclides of importance.

### Table 5.3 Radionuclides with half-life > 1 minute that can be produced in air at accelerators.

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>Half-life</th>
<th>Emission</th>
<th>Parent element</th>
<th>Production reaction</th>
<th>Cross section (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$H</td>
<td>12.3 yr</td>
<td>$\beta^-$</td>
<td>N</td>
<td>Spallation</td>
<td>30</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>53.3 days</td>
<td>$\gamma$, EC</td>
<td>N</td>
<td>Spallation</td>
<td>10</td>
</tr>
<tr>
<td>$^{11}$C</td>
<td>20.4 min</td>
<td>$\beta^+$</td>
<td>N</td>
<td>Spallation</td>
<td>10</td>
</tr>
<tr>
<td>$^{14}$C</td>
<td>5730 yr</td>
<td>$\beta^-$</td>
<td>N</td>
<td>(n, p)</td>
<td>1640</td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>9.96 min</td>
<td>$\beta^+$</td>
<td>N</td>
<td>Spallation</td>
<td>10</td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>70.6 s</td>
<td>$\beta^+$, $\gamma$</td>
<td>O</td>
<td>Spallation</td>
<td>1</td>
</tr>
<tr>
<td>$^{15}$O</td>
<td>2.03 min</td>
<td>$\beta^+$</td>
<td>O</td>
<td>Spallation</td>
<td>40</td>
</tr>
<tr>
<td>$^{18}$F</td>
<td>1.83 h</td>
<td>$\beta^+$, EC</td>
<td>Ar</td>
<td>Spallation</td>
<td>6</td>
</tr>
<tr>
<td>$^{24}$Ne</td>
<td>3.4 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>10</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>2.6 yr</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>7</td>
</tr>
<tr>
<td>$^{24}$Na</td>
<td>15.0 h</td>
<td>$\beta^-$</td>
<td>Ar</td>
<td>Spallation</td>
<td>2.5</td>
</tr>
<tr>
<td>$^{27}$Mg</td>
<td>9.46 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>0.4</td>
</tr>
<tr>
<td>$^{28}$Mg</td>
<td>20.9 h</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>13</td>
</tr>
<tr>
<td>$^{28}$Al</td>
<td>2.25 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>4</td>
</tr>
<tr>
<td>$^{29}$Al</td>
<td>6.6 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>6</td>
</tr>
<tr>
<td>$^{31}$Si</td>
<td>2.62 h</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>4.4</td>
</tr>
<tr>
<td>$^{30}$P</td>
<td>2.50 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>25</td>
</tr>
<tr>
<td>$^{33}$P</td>
<td>14.3 d</td>
<td>$\beta^-$</td>
<td>Ar</td>
<td>Spallation</td>
<td>9</td>
</tr>
<tr>
<td>$^{35}$S</td>
<td>87.5 d</td>
<td>$\beta^-$</td>
<td>Ar</td>
<td>Spallation</td>
<td>23</td>
</tr>
<tr>
<td>$^{34}$Cl</td>
<td>32.0 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>Spallation</td>
<td>0.6</td>
</tr>
<tr>
<td>$^{35}$Cl</td>
<td>37.2 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>(\gamma, pn)</td>
<td>4</td>
</tr>
<tr>
<td>$^{39}$Cl</td>
<td>55 min</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>(\gamma, p)</td>
<td>7</td>
</tr>
<tr>
<td>$^{41}$Ar</td>
<td>1.8 h</td>
<td>$\beta^+$, $\gamma$</td>
<td>Ar</td>
<td>(n, \gamma)</td>
<td>660</td>
</tr>
</tbody>
</table>

* After Rindi (1972b).
Chapter 5  Induced Radioactivity at Accelerators

In general, the positron emitters $^{11}$C, $^{13}$N, $^{15}$O along with $^{41}$Ar (produced by thermal capture) are the nuclides most frequently seen. Recent work at Fermilab described in (Bu89), (Va93), and (Va94) has also confirmed these identifications and, additionally, detected $^{39}$Cl. At electron accelerators, the copious presence of photons will enhance the photon-induced production processes and hence the production of $^{38}$Cl and $^{39}$Cl. It should be pointed out that distinguishing between the positron emitters must principally be done by analysis fits to decay curves because their $\gamma$-ray spectra are all comprised of 0.511 MeV photons from positron annihilation. Such decay curves have been analyzed (by fitting with sums of exponentials representing the half-lives possible) and used to determine proportions of the various radionuclides in (Th88), (Sw90), (Bu89), (Va93), and (Va94).

It appears, especially from the results of (Bu89), that the geometry of target stations can significantly affect the composition. For example, high intensity targets immediately surrounded with large volumes of iron and shielded directly by contact with concrete without allowing the secondary (cascade) particles emerging from the iron to interact with the air, had much less $^{41}$Ar than did those where the bulk iron shield was located in a "open" room. Presumably, the open space provided opportunity for the large flux of 0.85 MeV neutrons expected external to a pure iron shield (see Chapter 3) to "thermalize" and thus enhance the production of $^{41}$Ar in that air space. The large thermal capture ($n,\gamma$) cross section ($\sigma_{th} = 660$ mb) for $^{40}$Ar also may have provided the photons necessary to enhance the ($\gamma$, $p$) and ($\gamma$, $pn$) reactions required to produce significant quantities of $^{39}$Cl and $^{38}$Cl, respectively. Some typical percentages of the various radionuclides (by activity concentration) are given in Table 5.4.

### Table 5.4 Radionuclide composition of typical airborne releases from accelerators

<table>
<thead>
<tr>
<th>Situation</th>
<th>Radionuclides (Activity Per Cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN (Th88) 28 GeV</td>
<td>$^{11}$C 31.0 $^{13}$N 47.0 $^{15}$O 8.0 $^{38}$Cl 14.0</td>
</tr>
<tr>
<td>Fermilab (Bu89) 800 GeV</td>
<td></td>
</tr>
<tr>
<td>(no gap between iron and concrete)</td>
<td>$^{11}$C 46.0 $^{13}$N 19.0 $^{15}$O 35.0</td>
</tr>
<tr>
<td>(gap between iron and concrete)</td>
<td>$^{11}$C 30.0 $^{13}$N 10.0 $^{15}$O 0.0 $^{38}$Cl 10.0 $^{39}$Cl 30.0 $^{41}$Ar 30.0</td>
</tr>
<tr>
<td>Fermilab (Va93) 120 GeV</td>
<td>$^{11}$C 58.5 $^{13}$N 37.9 $^{15}$O 1.0 $^{38}$Cl 1.1 $^{39}$Cl 1.5</td>
</tr>
<tr>
<td>Fermilab (Va94) 120 GeV</td>
<td>$^{11}$C 64.6 $^{13}$N 30.5 $^{15}$O 1.0 $^{38}$Cl 1.1 $^{39}$Cl 1.5</td>
</tr>
</tbody>
</table>

Patterson and Thomas (Pa73), have used the expanded general activation equation to derive the total specific activity, $S$ (typically Bq/cm$^3$) of an enclosed volume of radioactive air;

$$S = C \sum_i \left[ \sum_j \phi_i \sigma_{ij} + \sum_j \phi_{th,i} \sigma_{ij,th} + \sum_j \phi_{HE,i} \sigma_{ij,HE} \right] (1 - e^{-\lambda_i T}) e^{-\lambda_i t} \quad (5.40)$$

where $\phi_i$, $\phi_{th,i}$, and $\phi_{HE,i}$ represent the average photon, thermal neutron and high energy flux densities. For clarity, in this equation $T$ is the irradiation time while $t$ represents the decay time. The $\sigma_{ij}$ values are the corresponding cross sections averaged with the energy-dependent flux density over energy.
Chapter 5  Induced Radioactivity at Accelerators

\[ \sigma_{ijk} \phi_k = \int_{E_{\text{min}}}^{E_{\text{max}}} dE \sigma_{ijk}(E) \phi_k(E) \]  

(5.41)

where the limits of integration correspond to the three ranges in the summation. The constant, \( C \), is the conversion to specific activity and is equal to unity for activity in Becquerels/cm\(^3\). The outer sum over index 'i' is over the possible radionuclides and the sums over the index j represent the sums over the parent atoms of atomic density \( N_j \) atoms/cm\(^3\) in air. The flux densities should, without further information, be the average for the enclosure.

Adjustments for the presence of ventilation can be quite conveniently made by substituting effective decay constants, \( \lambda' \),

\[ \lambda' = \lambda + \frac{D}{V} \]  

(5.42)

where \( D \) is the ventilation rate and \( V \) is the volume of the enclosure. That this is so can be shown as follows: Consider,

\[ \lambda' = \lambda + \frac{D}{V} = \lambda + r \]

where \( D \) is the vent rate, \( V \) is the volume and thus \( r \) is the air changes per unit time.

The differential equation with ventilation included is, then:

\[ \frac{dn'}{dt} = -\lambda' n'(t) + N\sigma = -\lambda n'(t) - r n'(t) + N\sigma \]  

(5.43)

The solution is:

\[ n'(t) = \frac{N\sigma}{\lambda + r} \left[ 1 - \exp\left[ -(\lambda + r) t \right] \right] \]  

(5.44)

And the specific activity is:

\[ a'(t) = \frac{\lambda N\sigma}{\lambda + r} \left[ 1 - \exp\left[ -(\lambda + r) t \right] \right] \]  

(5.45)

But \( N\sigma \) is just the saturation concentration, \( a_{\text{sat}} \), without mixing. Hence, with mixing the saturation concentration \( a' \) is:

\[ a'_{\text{sat}} = \frac{\lambda a_{\text{sat}}}{\lambda + r} \]  

(5.46)

The airborne radioactivity is of primary concern to workers who might enter the enclosure to perform maintenance activities. Since the principal radionuclides are of relative short half-life, the hazard is largely due to the "immersion" in a source of external dose rather than a gaseous ingestion hazard such as might be found in operations involving the processing of long-lived radioactive materials. Nevertheless, regulatory authorities (guided by ICRP and NCRP recommendations) have established quantities called "Derived Air Concentrations" (DAC) for radiation workers. DACs are based upon the receipt of 5000 mrem of dose equivalent if the entire working year (\( = 2000 \) hours) is spent working in a concentration corresponding to "1 DAC". A one DAC concentration is generally a quite large concentration that is rarely encountered in accelerator radiation environments. Similarly, for members of the general
public, values of "Derived Concentration Guides" (DCGs) have been tabulated that would result in the receipt of 100 mrem of dose equivalent by an individual who spent all of the time in one year breathing such air. Table 5.5 gives representative values of these quantities based upon present U. S. Department of Energy Orders (DOE90) and Regulations (CFR93) and along with some values determined for accelerator-produced radionuclides not included in the DOE documents calculated by M. Höfert of CERN (Ho69). For some radionuclides commonly found at accelerators (CFR93) gives two values of DAC, one for air inhaled into the lungs and the other for immersion in an infinite cloud of γ-emitting radionuclides. The latter condition is more likely to be the dominant exposure mechanism due to activated air at accelerators. Not all radionuclides of concern in the air at accelerators are included in the U. S. Department of Energy tabulations and thus must be determined independently. Hence, the Höfert calculations are very important because they provide values for these accelerator-produced radionuclides that are missing from the Department of Energy tables or are only included there as immersion in an "infinite" cloud. Also, Höfert recognized that such "immersion dose" is highly sensitive to the size of the cloud and that clouds of infinite extent are rare inside buildings at accelerators. He then calculated the equivalent of DACs for clouds of various sizes; Table 5.5 gives those for clouds of 4 meters radius. For the general population, Höfert postulated an infinite cloud, since such exposure would presumably occur outdoors.

Table 5.5 DACs and DCGs (Air) for radiation workers and the general population. (μCi/m³)

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>DAC-Radiation Worker</th>
<th>DCG-General Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 rem/yr</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(40 hrs/week)</td>
<td>(168 hrs/week)</td>
</tr>
<tr>
<td>(CFR93)</td>
<td>(CFR93)</td>
<td>(Hö69)</td>
</tr>
<tr>
<td>(Hö69)</td>
<td>(DOE90)</td>
<td>(Hö69)</td>
</tr>
<tr>
<td>3H</td>
<td>20</td>
<td>0.1</td>
</tr>
<tr>
<td>7Be</td>
<td>9</td>
<td>0.04</td>
</tr>
<tr>
<td>11C</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>13N</td>
<td>4</td>
<td>41</td>
</tr>
<tr>
<td>15O</td>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>41Ar</td>
<td>3</td>
<td>47</td>
</tr>
<tr>
<td>22Na</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>54Mn</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>60Co</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>238U</td>
<td>3.0 x 10⁻⁴</td>
<td></td>
</tr>
</tbody>
</table>

Propagation of Airborne Radioactivity-Tall Stacks

The other consideration concerning airborne radioactivity is that associated with the dose to members of the general public. The U. S. Environmental Protection Agency (EPA) has placed a 10 mrem/year limit on dose equivalent to members of the general public due to the operations of DOE facilities and has also placed stringent regulations on how such releases are to be measured (CFR90). The regulations prescribe the specific computer codes that must be used to calculate the dose to the public due from a given release point using a Gaussian plume model. Such computer modeling will not be described in detail here. Examples of such plume models are given in standard text books and the results depend on details of the meteorological conditions. Such concentrations can be estimated analytically using the so-called "Sutton's equation" [Eq. (5.47)]. A good description that applies to rather tall (> 25 m) release points has been given by H. Cember (Ce69). The dispersion is mainly characterized by dilution of the radionuclides and
their eventual return to ground level breathing zones. The meteorological conditions are of
major importance and are illustrated in Fig. 5.11 taken from (Ce69). Especially important are the
stability classes:

*stable*: No heat is gained or lost by a parcel of air that rises and expands adiabatically
with falling temperature. The adiabatic cooling with rise normally

\[
\Delta T = \frac{\Delta z}{5.4} \quad \text{for dry air}
\]

\[
\Delta T = \frac{\Delta z}{3.5} \quad \text{for moist air}
\]

If the temperature gradient is less than adiabatic, but still negative, stability is
achieved because a rising parcel cools faster than its surroundings and then
tends to sink. A sinking parcel is warmer than its surroundings and thus
is less dense and tends to rise. This restricts the width of the plume and
consequently decreases dilution.

*inversion*: If the temperature gradient is such that the temperature increases with height,
then an inversion occurs. Rising effluent from a "stack" becomes much
denser than its surroundings and thus sinks. The effluent is thus more limited
in its ascent and this, too, serves to limit dilution.

*superadiabatic*: If the rate of decrease is greater than that in adiabatic conditions, an unstable
condition results which promotes the vertical dispersion, and hence dilution.
A rising parcel does not cool fast enough due to its expansion and therefore
remains warmer and continues to rise. Likewise, a falling parcel continues to
continue.

Table 5.6 gives certain parameters to be used in Sutton's equation as expressed by (Ce69) for tall
stacks. In this table, the "chimney height" is the effective chimney height as calculated
according to Eq. (5.48).

<table>
<thead>
<tr>
<th>Lapse rate</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superadiabatic</td>
<td>0.20</td>
<td>0.043</td>
<td>0.030</td>
<td>0.024</td>
</tr>
<tr>
<td>Stable</td>
<td>0.25</td>
<td>0.014</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td>Moderate inversion</td>
<td>0.33</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>Large inversion</td>
<td>0.50</td>
<td>0.004</td>
<td>0.003</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 5.6 Diffusion (C²) and Stability (n) parameters for Sutton's
Equation (Eq. 5.47). [Reproduced from (Ce69).]
Chapter 5 Induced Radioactivity at Accelerators

Fig. 5.11  Effect of atmospheric temperature gradient-or lapse rate- on a displaced volume of air.  a  Unstable lapse rate;  b  Stable lapse rate;  c  Neutral lapse rate [Reproduced from (Ce69), originally taken from (Si68).]

Sutton's equation, as adapted here for consideration of short-lived radionuclides, is:

\[
\tilde{c}(x,y) = \frac{2Q \exp\left[-\frac{\lambda}{u}\sqrt{x^2 + y^2}\right]}{\pi C^2 u x^{-n}} \exp\left[-\left(\frac{1}{C^2 x^{2-n}}\right)\left(h^2 + y^2\right)\right] \quad (5.47)
\]

where the exponential involving decay constant \(\lambda\) conservatively allows for radioactive decay in transit for a particular radionuclide and;


\[ \bar{c}(x,y) \text{ is the average concentration (activity per m}^3\text{)} \]

\[ Q \text{ is the emission rate of activity per sec} \]

\((x,y)\) are coordinates to the point of measurement from the foot of the stack (meters)

\((x)\) is on the centerline of the plume as determined by the wind direction (downwind) or average thereof.

\(u\) is the mean wind speed, meters per second

\(C\) is the virtual diffusion constant in lateral and vertical directions (see Table 5.6)

\(n\) is a dimensionless parameter related to the atmospheric conditions (see Table 5.6)

\(h\) is the effective chimney height (if the gas has significant emission velocity) determined as follows from the actual chimney height \(h_a\):

\[
\begin{align*}
    h &= h_a + d(y)^{1.4}(1 + \frac{\Delta T}{T}) . \\
\end{align*}
\]

(5.48)

In the above, \(h_a\) is the actual height in meters, \(d\) is the outlet diameter in meters, \(v\) is the exit velocity of the gas (meters/sec) and \(\Delta T\) is the difference between the temperature of the gas and the ambient outdoor temperature divided by the absolute temperature of the gas.

**Propagation of Airborne Radioactivity-Short Stacks**

The above representation of Sutton's equation is a useful one where tall stacks are involved. However, at typical accelerator facilities it is uncommon for stacks to be as tall as 25 meters. (SI68) is a complete treatise on the subject that describes atmospheric releases of contaminants. For purposes of this discussion, only steady state conditions continuous in time are treated here. In this treatment, the concentration as a function of coordinates \((x,y,z)\), defined as for the tall stacks, is given by a somewhat different formulation of Sutton's equation;

\[
\bar{c}(x,y,z) = \frac{Q \exp\left(-\lambda \frac{1}{u} \sqrt{x^2 + y^2}\right)}{2\pi \sigma_y \sigma_z u} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left\{ \exp\left[-\frac{(z - h)^2}{2\sigma_z^2}\right] + \exp\left[-\frac{(z + h)^2}{2\sigma_z^2}\right] \right\} . \\
\]

(5.49)

For the common situation of interest where the receptor location of concern is at ground level \((z = 0)\), this simplifies to

\[
\bar{c}(x,y,0) = \frac{Q \exp\left(-\lambda \frac{(x^2 + y^2)^{1/2}}{u}\right)}{\pi \sigma_y \sigma_z u} \left\{ \exp\left[-\frac{y^2}{2\sigma_y^2} + \frac{h^2}{2\sigma_z^2}\right] \right\} , \\
\]

(5.50)

where the presence of the ground as a "barrier" to the flux is taken into account. In these equations, the quantity \(h\) is the elevation of the stack top above the ground in meters and the \(\sigma_y\) and \(\sigma_z\) are the dispersion coefficients and have units of length (meters). All other quantities are the same as given above for tall stacks. In the above equations, \(\sigma_y\) and \(\sigma_z\) are implicitly functions of the coordinate \(x\). These variables are, of course, determined from the meteorological conditions.
Table 5.7 taken from (S168) gives a scheme for classifying these conditions. The meteorological condition classification may then be used with the curves in Figs. 5.12 and 5.13 taken from (S168) to determine the values of \( \sigma_y \) and \( \sigma_z \) as a function of the coordinate \( x \).

Airborne radioactivity emissions can be minimized by:

- limiting the ventilation rates during operations when people are not present in the enclosure.
- delaying the actual emissions by requiring long pathways to the ventilation "stacks".
- minimizing air gaps in the beam.

**Table 5.7 Relation of turbulence types to weather conditions.** [Reproduced from (S168).]

<table>
<thead>
<tr>
<th>Surface wind speed, m/sec</th>
<th>Daytime insolation</th>
<th>Nighttime conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strong</td>
<td>Moderate</td>
</tr>
<tr>
<td>&lt;2</td>
<td>A</td>
<td>A – B</td>
</tr>
<tr>
<td>2</td>
<td>A – B</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>B – C</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>C – D</td>
</tr>
<tr>
<td>&gt;6</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

*Applicable to heavy overcast, day or night.
†The degree of cloudiness is defined as that fraction of the sky above the local apparent horizon which is covered by clouds.
Fig. 5.12  Lateral diffusion, $\sigma_y$, as a function of downwind distance from source for Pasquill's turbulence types as defined in Table 5.7. [Reproduced from (S168).]
Fig. 5.13  Vertical diffusion, $\sigma_z$, as a function of downwind distance from source for Pasquill's turbulence types as defined in Table 5.7. [Reproduced from (S168).]
IV. Soil and Groundwater Activation

The protection of groundwater resources has become a significant public concern. This concern includes the need to assure protection of groundwater resources from contamination with radionuclides. In principal, activity can be produced in both the earth itself and in the water content it holds but in practice it is not so simple (or important!) to separate these two sources. One could initiate calculations of groundwater activation at accelerators by starting from "first principles" and by using the activation formula. However, in practice such calculations have been done more frequently by analyzing results obtained using irradiated samples. The work of Borak, et al, (Bo72) is of singular importance in this regard.

In this paper, radioactivity induced in soil by high energy hadrons was measured by radiochemical analysis of soil samples irradiated near high energy synchrotrons (the 12 GeV Argonne ZGS and the 28 GeV Brookhaven AGS). The radionuclides $^3$H, $^7$Be, $^{22}$Na, $^{45}$Ca, $^{46}$Sc, $^{48}$V, $^{51}$Cr, $^{54}$Mn, $^{55}$Fe, $^{59}$Fe, and $^{60}$Co were identified. Laboratory experiments were then performed to determine which radionuclides, and what fractions of them could be leached by water. This study determined macroscopic production cross sections and ion velocities relative to ground water flow in soil. Of these nuclides, only $^3$H, $^{22}$Na, $^{45}$Ca, and $^{54}$Mn were observed in leach waters. The $^3$H was assumed to be all leachable and was measured by driving it out of the sample by baking. Radionuclides with half-lives exceeding 15 days were the only ones considered. The activities at saturation, $A_j$, are given (in Bq) by:

$$A_j = \phi \sum_i n_i \sigma_{ij}$$

(5.51)

where $\phi$ is the flux density, $n_i$ is the number of target nuclei of the $i^{th}$ nuclide per gram of the soil sample, and $\sigma_{ij}$ is effective cross section for the transformation from target nucleus $i$ to radionuclide $j$. The sum is over the soil constituents. Borak, et al were able to measure the summations, $\sum n_i \sigma_{ij}$, to determine the total macroscopic cross sections for each radionuclide of interest. Table 5.8 taken from (Bo72) gives the results of the measurements.

Some comments should be made with respect to each of the four nuclides identified as leachable in this work.

$^3$H- The leaching process was able to collect all the tritium measured by the bake-out process. The average value of the macroscopic cross section in soil was found to be $5.1 \times 10^{-3}$ cm$^2$/g of water. An important conclusion is that the tritium will migrate with the same velocity as any other water in the soil.

$^{22}$Na- Typically 10-20 % of this nuclide was leachable. On average, it appeared that the migration velocity of this nuclide is approximately 40% of that of water through the soil due to ion exchange processes.

$^{45}$Ca- At most 5 % of this nuclide was leached from the soil. The migration velocity was determined to be extremely small.

$^{54}$Mn- At most 2 % of this nuclide was leached from the soil. It was determined that this nuclide will not migrate significant distances.
Table 5.8  Induced activity and macroscopic cross section for soil normalized to unit flux of particles with kinetic energies greater than 30 MeV. [Reproduced from (Bo72).]

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Sample soil depth</th>
<th>A-1 Glacial till</th>
<th>B-1 Gray sandy clay</th>
<th>B-2 Red sandy clay</th>
<th>B-3 Gray clay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Activity (pCi/g)</td>
<td>$\Sigma n\sigma_{HI}$ (cm$^2$/g)</td>
<td>Activity (pCi/g)</td>
<td>$\Sigma n\sigma_{HI}$ (cm$^2$/g)</td>
<td>Activity (pCi/g)</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>$7.9 \times 10^{-3}$</td>
<td>$2.9 \times 10^{-4}$</td>
<td>$9.9 \times 10^{-3}$</td>
<td>$3.7 \times 10^{-4}$</td>
<td>$8.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{54}$Cr</td>
<td>$4.7 \times 10^{-4}$</td>
<td>$1.7 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$3.7 \times 10^{-5}$</td>
<td>$7.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{58}$Na</td>
<td>$5.6 \times 10^{-3}$</td>
<td>$2.1 \times 10^{-4}$</td>
<td>$6.1 \times 10^{-3}$</td>
<td>$2.3 \times 10^{-4}$</td>
<td>$5.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{64}$Mn</td>
<td>$1.6 \times 10^{-3}$</td>
<td>$5.9 \times 10^{-5}$</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-5}$</td>
<td>$9.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{65}$Sc</td>
<td>$8.2 \times 10^{-4}$</td>
<td>$3.0 \times 10^{-5}$</td>
<td>$3.6 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-5}$</td>
<td>$2.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{68}$V</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-4}$</td>
<td>$2.9 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-5}$</td>
<td>$1.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{66}$Fe</td>
<td>$2.5 \times 10^{-3}$</td>
<td>$9.3 \times 10^{-5}$</td>
<td>$3.2 \times 10^{-3}$</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{68}$Fe</td>
<td>$8.7 \times 10^{-5}$</td>
<td>$3.2 \times 10^{-5}$</td>
<td>$4.6 \times 10^{-5}$</td>
<td>$1.7 \times 10^{-5}$</td>
<td>$3.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>$8.9 \times 10^{-4}$</td>
<td>$3.3 \times 10^{-5}$</td>
<td>$3.9 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-5}$</td>
<td>$3.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{48}$Ca</td>
<td>$4.4 \times 10^{-3}$</td>
<td>$1.6 \times 10^{-4}$</td>
<td>$5.4 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-5}$</td>
<td>$8.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{46}$Ca</td>
<td>$2.3 \times 10^{-3}$</td>
<td>$8.2 \times 10^{-4}$</td>
<td>$2.9 \times 10^{-3}$</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$9.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{8}$H</td>
<td>$1.6 \times 10^{-1}$</td>
<td>$5.9 \times 10^{-2}$</td>
<td>$1.6 \times 10^{-1}$</td>
<td>$5.9 \times 10^{-3}$</td>
<td>$1.1 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

* Activity and cross sections per gram of water in the soil.
One can thus calculate the quantities of radionuclides that might pose a risk to groundwater in the environs of an accelerator. This can be done, as demonstrated by Gollon (Go78), by performing, for example, Monte-Carlo calculations in which the total stars (or nonelastic interactions above some threshold) produced in some volume of earth shielding are determined. The total number of atoms, $K_i$, of the $i^{th}$ nuclide that can be produced per star in that same volume would then be given by

$$K_i = \frac{\Sigma_i}{\Sigma_{ne}} \quad (5.52)$$

where $\Sigma_i$ is the macroscopic cross section (cm$^2$/gram) for the $i^{th}$ radionuclide and $\Sigma_{ne}$ is the total macroscopic nonelastic cross section (cm$^2$/gram) for soil. Gollon quotes a value of $\Sigma_{ne} = 1.1 \times 10^{-2}$ cm$^2$/gm for soil. Thus, a calculation of total stars in some soil volume per unit time can be taken directly from the Monte-Carlo calculations. Gollon used the following values for $^3$H and $^{22}$Na as selected from Borak's paper for soils peculiar to Fermilab (glacial till):

$$K_3 = \frac{8.2 \times 10^{-4}}{1.1 \times 10^{-2}} = 0.075 \quad (5.53a)$$

$$K_{22} = \frac{2.1 \times 10^{-4}}{1.1 \times 10^{-2}} = 0.02 \quad (5.53b)$$

One can then calculate the total number of atoms of radionuclides produced during some time interval in some volume by simply multiplying these factors by the number of stars (or nonelastic interactions) in the same volume. The number of atoms then can be converted to activity using the decay constant.

The quantity of ultimate concern, of course, is the resultant concentration in some drinking water supply. The United States Environmental Protection Agency (CFR76) limits such concentrations to those that would produce a dose equivalent of 4 mrem/year and specifically gives a limit of 20 pCi/ml for tritium as a legal limit. (An explicit limit for $^{22}$Na is not specified by EPA.) The U.S. Department of Energy (DOE90) specifies limits using a more up-to-date methodology which results in a limit of 80 pCi/ml for $^3$H and 0.4 pCi/ml for $^{22}$Na. At any rate, the concentration in the water must satisfy the following inequality:

$$\sum_i \frac{C_i}{C_{max, i}} \leq 1. \quad (5.54)$$

The numerator in the summation is the concentration of some particular nuclide $i$ while the denominator is the allowed limit. One needs to ultimately determine the concentration of the various radionuclides in the groundwater. The methods for calculating these concentrations will vary with the regulatory authority and the "conservatism" of the institution. The most conservative assumption is to assume that saturation values of production are reached (tantamount to assuming no movement of the radionuclides) and that the dilution in water is solely by the water contained in the soil in the immediate vicinity of the accelerator. This is almost absurdly conservative given the fact that there are no known methods for increasing the specific activity once the water departs the vicinity of the accelerator. This means that massive shielding is needed inside accelerator enclosures to reduce the neutron flux densities outside of the enclosures to very small values. Though certain types of soils (particularly clays) allow only very modest water movement (= a few centimeters or meters per year, dependent upon the details
of the soil type), the "nonmovement" requirement is especially serious for $^3$H in that it implies the accumulation of the saturation value of the specific activity must be sufficiently diluted to meet the above criteria.

At Fermilab, a standard model allowing some movement and further dilution of water has been employed for many years (Go78). In this model, the vertical migration of water of about 2.2 meters per year is taken for water. The tritium vertical velocities are taken to be this value while the results obtained in (Bo72) are used to obtain a reduced value of about 1 m/year for $^{22}$Na (the leachable fraction of the $^{22}$Na according to (Bo72) is the only portion of that particular radionuclide included). The procedure that has been used at Fermilab is to allow decay during the downward migration of the radionuclides produced in one year to the highest aquifer (all Fermilab targets and beam dumps are above this level). At that point, it is assumed that the radionuclides are rapidly transported to a shallow well where it is assumed that the flow of water collecting the radionuclides is entirely used by a single, miserly user who consumes a very low value of 150 liters per day. Thus the annual production, as transported vertically, is diluted into the $5.5 \times 10^7$ cm$^3$/year that this represents. This simple model is generally conservative but does, in fact, neglect that fact that the water movement may not be uniform from year-to-year.

It is clear that better methods may be needed and a new model has been developed for use at Fermilab (Ma93). There currently is much research and development effort in this general area of hydrogeology given the need to carefully design sanitary landfills and other waste disposal sites to protect groundwater supplies from other contaminants as well as radioactivity. The new Fermilab model calculates the production of the radionuclides of concern in the same manner. However, instead of using the total production, the average concentration at saturation (i.e., with infinite irradiation time) in water near the vicinity of the beam absorber or target is calculated. The concentration after migration is, then, calculated by using up-to-date modeling techniques to calculate the reduction in the concentration due to dilution, diffusion, and radioactive decay. At the point of concern, usually the location of an aquifer suitable for consumption, the concentrations calculated are then substituted into Eq. (5.54) in order to determine if a shielding design is adequate. The new Fermilab model has some strong advantages over its predecessor. It calculates concentrations directly and also calculates them at saturation, rather than on the basis of annual production. In view of the fact that radionuclides migrate rather slowly in glacial till, the latter may be far more realistic.

A report by the Superconducting Super Collider Central Design Group (Ja87) attempts to estimate the dilution for a shallow uncased well in an aquifer a distance $r$ from a beam loss point also in the aquifer. The loss point is assumed to be within the drawdown zone of the well. This model utilizes an elegant method developed by J. D. Jackson for a simple geology that involves a single uniform strata of earth above some level of impervious stratum. Fig. 5.14 taken from (Ja87) shows the situation described by this model. In this model, a given well is modeled by the profile of depth of water $h(r)$ at distance $r$ from the well. $h(r)$ is determined by the depth of a test well at radius $r$ from the well under consideration. The well is assumed to supply a volume $Q$ of water per day. Conservation of water is the hallmark of this model. The flux of water is determined by the gradient relation,

$$S_r = k \frac{dh(r)}{dr} \quad \text{(5.55)}$$

where $S_r$ is the inward flux at radius $r$ and $k$ is a constant with dimensions of volume per unit time per unit area and is characteristic of the soil.
Conservation of water yields the steady-state equation:

\[ Q = 2\pi r h(r) S = 2\pi k r h \frac{dh}{dr} = \pi k \frac{d(h^2)}{d(\ln r)}. \]  

(5.56)

The quantity \( 2\pi r h \frac{dh}{dr} \) corresponds to the rate of change of volume of the cylindrical shell of height \( h \) ("the head") with respect to \( r \).

This equation has the solution:

\[ Q \ln(r/r_o) = \pi k [h^2(r) - h_o^2]. \]  

(5.57)

where \( r_o \) is the radius of the well and \( h_o \) is the height of water above the impervious stratum at the well. If \( H \) is the depth of the impervious layer below the water table, the radius of influence \( R \) of the well can be defined by the relation:

\[ \ln \left(\frac{R}{r_o}\right) = \frac{\pi k [H^2 - h_o^2]}{Q}. \]  

(5.58)

---

**Fig. 5.14** Hydrological model of a shallow well in proximity to an accelerator tunnel where a beam loss occurs. The radioactive region is represented in cross section by the shaded rectangle on the right. \( H \), represents the elevation of the water table above the impervious stratum. [Reproduced from (Ja87).]
Chapter 5  Induced Radioactivity at Accelerators

However, the detailed solution is not necessary.

Now, suppose that there is a well a distance r away from the region of deposition of radioactivity near an accelerator. We also assume that the activation zone lies below the water table and that the deposition region lies within the radius of influence of the well. This assumption leads to higher concentrations than would be obtained if the activation zone were totally, or partially, above the water table.

The amount of activity drawn into the well is determined by the rate of pumping Q and the necessary total flow through a cylinder of radius r and height h(r) as we have seen. Let \( \Delta V \) be the volume of soil yielding Q gallons of water.

The cylindrical shell providing this amount of water will be of radial thickness \( \Delta r \), where \( \Delta V = 2\pi rh(r)\Delta r \). The fraction \( F \) of the volume of activity included in this shell can be said to be given by:

\[
F = \frac{\Delta r}{t} = \frac{2\pi rh(r)\Delta r}{2\pi rh} = \frac{\Delta V}{2\pi rh} \tag{5.59}
\]

provided that \( \Delta r < t \).

If the activated region contains activity, \( A \) (either total activity or that of a particular radionuclide of interest), the corresponding specific activity, \( a \), in water drawn from the well is thus given by:

\[
a = \frac{fA}{Q} = \frac{fA}{p\Delta V} = \left( \frac{\Delta V}{2\pi rh} \right) f \left( \frac{1}{\Delta V} \right) A = \left( \frac{1}{2\pi rhD} \right) f A \tag{5.60}
\]

where \( f = D/h \) is the fraction of the total height of the cylindrical shell occupied by the activated region and \( p \) is the porosity of the soil. The pumping volume \( Q \) is implicit in \( f \). Porosity values vary considerably but in general are in the range,

\[
0.2 < p < 0.35. \tag{5.61}
\]

Thus, this formula may be used to obtain an estimate of the specific activity as a function of distance from the well, although it is perhaps not too useful for applications to beam losses far from the well. By definition, \( f \leq 1 \) and the lower value of porosity can be used to obtain upper limit estimates of the concentration. It must be emphasized that this model depends upon uniformity of water conduction by the strata. The presence of "cracks", of course, can provide much more rapid movement that is not well-described by this simple model.
Chapter 5  Induced Radioactivity at Accelerators

References


(Bo72)  T. B. Borak, M. Awschalom, W. Fairman, F. Iwami, and J. Sedlet, "The underground migration of radionuclides produced in soil near high energy proton accelerators", Health Physics 23 (1972) 679-887.


(H69)  M. Höfert, "Radiation hazard of induced activity in air as produced by high energy accelerators", in Proceedings of the Second International Conference on Accelerator Radiation Dosimetry and Experience, (Stanford, CA 1969).


Chapter 5  Induced Radioactivity at Accelerators


Chapter 5  Induced Radioactivity at Accelerators-Problems

1. A copper beam stop has been bombarded with high energy hadrons for 30 days and exhibits a dose rate of 100 mrem/hr at 1 meter away 1 day after the beam is turned off. Maintenance work needs to be scheduled in the vicinity within the next 6 months. Using both Gollon's Rule #3 (as derived by Sullivan and Overton) and the Barbier Danger parameter curves, predict the cooling curve and determine when the dose rate is less than a 20 mrem/hr maintenance work criteria. Make a table of dose rate versus cooling time in days for both methods. How well do the two methods agree? (Hint: Use initial value of the dose rate to scale "D" values.)

2. A 100 GeV beam (10^{12} protons/second) strikes the center of a large solid iron cylinder 30 cm in radius for 30 days. Use the FLUKA star density curves from the Chapter 3 (Fig 3.15) and the "\omega" factors calculated by Gollon to estimate the residual dose rate after 1 day cooldown at contact with the side of the cylinder in the "hottest" spot. Using Gollon's third rule, how long must the repair crew wait to service this time in a contact radiation field of absorbed dose rate < 10 rad/hr?

3. A copper target is bombarded with high energy protons such that 10 stars per incident proton are produced. If the incident beam is 10^{11} p/s, what is the specific activity (average) of \(^{54}\text{Mn}\) that is produced after two years of operation? \(^{54}\text{Mn}\) has a high energy spallation production cross section of about 20 mb in Cu. The target is a cylinder, 10 cm radius by 15 cm long. The half-life of \(^{54}\text{Mn}\) is 312 days. Express the answer in both Bq/cm\(^3\) and Ci/cm\(^3\). (Hint: This problem is best if the calculation is done at saturation and then corrected for the noninfinite irradiation time. Also, one needs to use the inelastic cross section given, for example, in Chapter 3.)

4. A 20 m long air gap has a beam of 10^{12} p/s of high energy protons passing through it. First, calculate the production rate of \(^{11}\text{C}\) in the gap at equilibrium if one approximates air in the gap by nitrogen and assumes \(\sigma(\text{\(^{11}\text{C}\))} = 10\) mb. Assume that there are no significant losses of beam by interaction after checking to see that this assumption is, in fact, true. Table 1.2 should contain helpful information.
Chapter 5 Induced Radioactivity at Accelerators-Problems

a) If the air gap is in a 10 X 10 X 20 meter$^3$ enclosure with no ventilation, calculate the equilibrium concentration of $^{11}$C in the room (in units of $\mu$Ci/m$^3$) assuming extremely rapid mixing (i.e., no time allowed for decay while mixing occurs) of the enclosed air. Compare the concentration with the derived air concentration values in Table 5.5 and calculate, using simple scaling, the dose equivalent to a worker who spends full time in this room. (This is a purely hypothetical scenario due to the much larger hazards due to such an intense direct beam!)

b) Calculate the concentration if two (2) air changes/hr are provided.

c) Assume the exhaust of the ventilation described in part "b" is through a 10 cm radius stack 25 m tall. Calculate the air speed in the stock, and the the emission rate Ci/s. Then using Cember's version of Sutton's equation for tall stacks to estimate the concentration directly downwind at ground level, and hence the dose equivalent 1 km away with moderately stable meteorological conditions and an average wind speed of 10 km/hr.

d) Perform the same calculation requested in "c" using the more general version of Sutton's equation appropriate to short stacks and assume the stack height to be 3 meters. All other conditions of the problems are the same as in "c".

5. In soil conditions similar to those at Fermilab, a volume of soil around a beam dump approximately 10 m wide X 10 m high 20 m long is the scene of a star production rate (averaged over the year) of 0.02 star/proton at a beam intensity of $10^{12}$ protons/sec.

a) Calculate the annual production of $^3$H ($t_{1/2}$ = 12.3 years), the saturated activity (in Bq & Ci), and the average saturated specific activity in the above volume's water (assume 10% water content by volume).

b) Use the older "Fermilab Model" to calculate the concentration at the nearest well. Assume the activation region (beam loss point) is 50 m above the aquifer and the usual migration velocities.

c) "Conservatively" apply the "Jackson Model" to estimate the concentration at a well 100 meters distant from the center of the activation region.
Chapter 6  Topics in Radiation Protection Instrumentation at Accelerators

I. Introduction

The purpose of this discussion is to summarize instruments and dosimeters currently used in the environment of particle accelerators to measure and characterize the radiation fields. In particular, the emphasis here is on instrumentation that addresses those aspects of accelerator radiation fields that pose special problems. Thomas and Stevenson (Th88) and Swanson and Thomas (Sw90) also discuss these matters. Cember (Ce69) is a comprehensive health physics textbook that covers the basics of radiation measurement instrumentation quite well. The book by Knoll (Kn79) is a well-written and reasonably up-to-date treatise on this subject. It should be noted here that virtually all particle detection techniques that have been devised by physicists have, to some degree, been employed in radiation measurements at accelerators. For example, the particle yields discussed previously are to a large degree a "product" of the basic scientific research program for which the accelerators have been built. The "burden" is on the radiation protection practitioner to be able to astutely determine which data, among the flood of results which pours out of the physics experiments, has immediate practical application to radiation protection!

Cember (Ce69) has given a good summary of counting statistics which bears repeating here. Radioactive decays are randomly occurring events having a sampling distribution which is correctly described by the binomial distribution given by the expansion of the binomial:

\[(p + q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2!}p^{n-2}q^2 + \frac{n(n-1)(n-2)}{3!}p^{n-3}q^3 + \ldots \]  

(6.1)

where \(p\) is the mean probability for occurrence of an event, \(q\) is the mean probability of non-occurrence of the event and thus \(p + q = 1\), and \(n\) is the number of chances of occurrence. The probability of exactly \(n\) events occurring is given by the first term, the probability of \(n - 1\) events is given by the second term, etc. For example, in the throwing of a die, the probability of throwing a "one" is \(1/6\) and the probability of throwing a "one" 3 times in a row (\(n = 3\)) is:

\[P = (1/6)^3 = 1/216.\]  

(6.2)

In three throws, the probabilities of throwing 2 "ones", 1 "one" and no "ones" are given by the 2nd, 3rd, and 4th terms of the expansion.

This distribution becomes equivalent to the normal distribution (or Gaussian distribution) given by the following, when \(n\) has an approximate value of at least 30:

\[p(n) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\left( \frac{n - \bar{n}}{2\sigma^2} \right)^2 \right], \]  

(6.3)

where \(p(n)\) is the probability of finding exactly \(n\), \(\bar{n}\) is the mean value, and \(\sigma\) is the standard deviation.

For rare (highly improbable) events (typically radioactive decays or particle interactions fall into this category), the binomial distribution approaches the Poisson distribution. In this distribution, the probability of obtaining \(n\) events if the mean value is \(\bar{n}\), is given by:

\[p(n) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}. \]  

(6.4)
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For example consider the decay of 0.001 μCi of activity. For this, \( n = 37 \) decays/sec. The probability of exactly observing this number of events in any one second is:

\[
p(37) = \frac{(37)^{37} e^{-37}}{37!},
\]

(6.5)

where one can apply Sterling's approximation

\[
n! \approx (2\pi n)^{1/2} \left(\frac{n}{e}\right)^n
\]

(6.6)

to evaluate the factorial (and get considerable cancellation). Thus \( p(37) = 0.066 \). As in the case of the normal distribution, 68% of the events would lie within one standard deviation of the mean, 96% of the events would lie within 2 standard deviations of the mean. For the Poisson distribution,

\[
\sigma = \sqrt{n}.
\]

(6.7)

The relative error, \( \sigma/n \), is thus \( \frac{\sqrt{n}}{n} \).

Often, when dealing with instrumentation, counting rates are involved. For these the following holds:

\[
r \pm \sigma_r = \frac{n}{t} \pm \frac{\sqrt{n}}{t}
\]

(6.8)

where \( r \) is the counting rate per unit time, \( \sigma_r \) is its standard deviation, and \( t \) is the counting time during which the rate is measured. (The quantity \( t \), for example, could even be the integration time constant of some instrument.)

It follows that

\[
\sigma_r = \frac{\sqrt{n}}{t} = \sqrt{\frac{n}{t^2}} = \sqrt{\frac{r}{t}}.
\]

(6.9)

Usually, backgrounds are present and must be dealt with. The standard deviation of the net counting rate is given as

\[
\sigma_n = \sqrt{\sigma_g^2 + \sigma_{bg}^2} = \sqrt{\frac{r_g}{t_g} + \frac{r_{bg}}{t_{bg}}},
\]

(6.10)

where the subscripts \( g \) refer to the measurement of the gross counting rate while the subscripts \( bg \) refer to the measurement of the background counting rate.

In general, the common statistical tests are valid for Poisson statistics.
Another quantity that sometimes becomes important is the resolving time of an instrument. This is the time that the detector, following an event, is incapable of measuring a second event. It can be measured by exposure to two different sources of radiation [(the "two-source method" of (Ce69)]. A certain detector has a measured background rate of $R_{bg}$ and responds to first source alone with a rate $R_1$ and to the second source alone with a rate $R_2$ (both $R_1$ and $R_2$ include the background). When exposed to the two sources simultaneously, the measured rate is $R_{12}$. The resolving time, $\tau$, is given by

$$\tau = \frac{R_1 + R_2 - R_{12} - R_{bg}}{R_{12}^2 - R_1^2 - R_2^2}. \quad (6.11)$$

In many situations, it is often easier to determine $\tau$ from the physical properties of the detection mechanism or from the electronic time constants related to the resolving time in the measurement circuitry. When the observed counting rate of a sample is $R_0$, then the "true" counting rate, $R$, that would have been observed with a "perfect" instrument having a resolving time of zero is given by

$$R = \frac{R_0}{1 - R_0 \tau}. \quad (6.12)$$

(Kn79) has a very detailed discussion of count rate considerations and the optimization of the counting statistics. He also presents a discussion of paralyzable versus non-paralyzable dead time corrections.
Chapter 6  Topics in Radiation Protection Instrumentation at Accelerators

II. Special Considerations for Accelerator Environments

There are a number of features of accelerator radiation fields which merit attention in choosing instrumentation or measurement techniques. The most important of these are briefly summarized here.

Large Range of Flux Densities, Dose Equivalent Rates, etc.

The magnitudes of the quantities to be measured encountered at accelerators may range from fractional mrem/year (environmental monitoring and studies) to the very large (up to megard) values of absorbed dose of concern in radiation damage situations. [It is customary to quantify radiation fields in terms of absorbed dose at levels above those encountered in personnel protection (= 1 rem).]

Possible Large Instantaneous Values of Flux Densities of Flux Densities, Dose Equivalent Rates, etc.

Certain accelerators (e.g., linacs, rapid cycle synchrotrons, and "single-turn" extracted beams from synchrotrons) can have very low average flux densities, etc. but have extremely high instantaneous rates. Such circumstances arise at accelerators at high intensities or in situations where the "duty factor" (the fraction of the time the beam is actually present because of accelerator characteristics) of a high intensity radiation field is small. Thus, the dead time considerations described above must be taken into account in special ways or the measured results can be found to be misleadingly low. Some instruments can be completely paralyzed by high instantaneous rates. In those cases, the effect of dead time on the instantaneous counting rate that is present is the relevant parameter.

Large Dynamic Range of Neutron Radiation Fields

At any given accelerator capable of producing neutrons, the properties of nuclear interactions make it highly probable that neutrons will be present at all energies from thermal ($<E_n> = 0.025$ eV) up to the energy of the beam. As we will see below and in the references cited, the methods of detection of neutrons vary considerably over this energy domain. Thus the choice of instrumentation is crucial to the success of the measurement. For no other particle-type is the energy range so large and so diverse with respect to applicable detection techniques as it is for neutrons.

Presence of Mixed Radiation Fields

At accelerators, one has to consider that any given radiation field external to shielding is likely to be comprised of a mixture of photons, neutrons, and (at high energies and forward angles) muons. (Inside of shielding, these particles will often be accompanied by a multitude of others.) Also, virtually all neutron fields contain at least some photon component due, at least, to the capture of thermal neutrons in $(n,\gamma)$ processes. Also, muon fields, at least those near ion accelerators, contain some neutron component. Thus the choice of instrumentation is somewhat dependent upon what particles are present in addition to the one being measured. In certain situations, the radiation field component that is not of interest can actually mask the one of concern.
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Directional Sensitivity

Certain instruments intrinsically exhibit directional sensitivity. This feature can be either beneficial or harmful, depending upon the situation. In all instances, it must be understood. It can lead to underestimates in radiation fields where all particles are not monodirectional. Directional sensitivity can actually be useful in certain circumstances to "find" sources of unwanted radiation.

Sensitivity to Features of the Accelerator Environment Other than Ionizing Radiation

While the focus of this discussion is on ionizing radiation, other features must be taken into account. The most prominent of these is the presence of radiofrequency radiation (RF) at some locations that can perturb instruments acting as "antennas". Environmental features such as temperature and humidity can also be important.
Chapter 6  Topics in Radiation Protection Instrumentation at Accelerators

III. Standard Instruments and Dosimeters

This section will review instruments and dosimeters that are generally available from commercial sources.

Ionization Chambers

A basic type of instrument used at accelerators to measure absorbed dose rates is the ion chamber. Such devices are used at high energy accelerators extensively. They rely on the collection of charge liberated by particles passing through a gas. Some advanced concept detectors now employ liquids for the ionization medium. A fortunate result of atomic physics is that the energy dissipation per ion pair, $W$, is nearly a constant over a number of gases as exhibited by Table 6.1 taken from (Kn79).

Table 6.1 Values of the energy deposition per ion pair (the $W$-value) for different gases*. [Reproduced from (Kn79).]

<table>
<thead>
<tr>
<th>Gas</th>
<th>Fast Electrons</th>
<th>Alphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>27.0</td>
<td>25.9</td>
</tr>
<tr>
<td>He</td>
<td>32.5</td>
<td>31.7</td>
</tr>
<tr>
<td>H₂</td>
<td>38.0</td>
<td>37.0</td>
</tr>
<tr>
<td>N₂</td>
<td>35.8</td>
<td>36.0</td>
</tr>
<tr>
<td>Air</td>
<td>35.0</td>
<td>35.2</td>
</tr>
<tr>
<td>O₂</td>
<td>32.2</td>
<td>32.2</td>
</tr>
<tr>
<td>CH₄</td>
<td>30.2</td>
<td>29.0</td>
</tr>
</tbody>
</table>


Thus, if a given charged particle liberates a certain amount of energy, $\varepsilon$, in the chamber, an electrical charge, $Q$, will be released according to:

$$Q \text{ (Coulombs)} = \frac{1.6 \times 10^{-13} \varepsilon \text{ (MeV)}}{W \text{ (eV/ion pair)}}.$$  \hspace{1cm} (6.13)

$Q$ can be collected by electrodes held at some voltage $V$. The collected charge generates a small change in $V$, $\Delta V$ (volts), in accord with the relation,

$$\Delta V = \frac{\Delta Q}{C}.$$  \hspace{1cm} (6.14)

where $C$ is the capacitance of the total circuit (including that of the chamber) in units of Farads. From (Kn79), for typical chambers, $C$ is of the order of $10^{-10}$ Farads. Knoll (Kn79) gives many details of the processes that determine the size and form of the electrical signals that can be generated in a measurement. Such chambers can be operated either in a "DC" or "ratemeter" mode, or in a mode in which the charge is integrated over some time period with the total charge collected, then "digitized" into pulses that represent some increment of "dose". In the "ion chamber" mode of operation, the applied voltage is sufficiently small so that gas multiplication (charge amplification) does not occur.
In the most simple-minded approach, one might believe that for measurements in photon fields one could fill such a chamber with gases that "mimic" tissue (so called "tissue equivalent" gases, or even hydrocarbons, for most purposes) and, with suitable calibration, convert the charge collected into absorbed dose. However, since ion chamber gases are in general much less dense than tissue, one must also "capture" the energy of the secondary electrons which, in the region of a few MeV, have ranges of several meters. It is thus necessary to use "compensation" techniques in which the solid material of the walls is chosen because of properties that "match" those of the gas. This condition can be readily achieved by use of any material with atomic number close to that of the contained gas, to sufficient accuracy for most practical purposes. Thus, aluminum and especially plastics, for example, are reasonably "equivalent" to tissue and air. Such walls must be of thickness to establish "electronic equilibrium". In this condition, the flux of secondary electrons leaving the inner surface of the wall is independent of the thickness. Table 6.2 taken from (Kn79) gives the wall thicknesses needed to establish electronic equilibrium for photons.

<table>
<thead>
<tr>
<th>Photon Energy (MeV)</th>
<th>Thickness* (g cm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0042</td>
</tr>
<tr>
<td>0.1</td>
<td>0.014</td>
</tr>
<tr>
<td>0.2</td>
<td>0.044</td>
</tr>
<tr>
<td>0.5</td>
<td>0.17</td>
</tr>
<tr>
<td>1</td>
<td>0.43</td>
</tr>
<tr>
<td>2</td>
<td>0.96</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>10</td>
<td>4.9</td>
</tr>
</tbody>
</table>


The thicknesses quoted are based on the range of electrons in water. The values will be substantially correct for tissue-equivalent ionization chamber walls and also for air. Half of the above thickness will give an ionization current within a few percent of its equilibrium value.

Finally, the measurement of absorbed dose is accomplished by application of the Bragg-Gray principle, which states that the absorbed dose \( D_m \) in a given material can be deduced (with suitable unit conversions) from the ionization produced in a small gas-filled cavity within that material as follows:

\[
D_m = WS_m P,
\]

where \( W \) is the average energy loss per ion pair, \( S_m \) is the ratio of mass stopping power (energy loss per unit density, e.g., MeV/g cm⁻²) of the material of interest relative to the chamber gas, and \( P \) is the number of ion pairs formed. For \( D_m \) to be in grays (J/kg), \( W \) must be expressed in Joules per ion pair and \( P \) in ion pairs per kg.
For radiation fields at accelerators containing neutrons, or mixtures of neutrons with muons and photons, one is commonly able to measure with an ideal ion chamber the absorbed dose, $D$, and determine the dose equivalent, $H$ by using the average quality factor, $Q$ as follows (where $Q$ is determined separately):

$$H = QD.$$ (6.16)

Ion chambers with so-called tissue equivalent walls have been used in this manner at many accelerators. The major limitation is that the value of $Q$ has to be determined by some other means. Aswchalom, described the use of such instruments at Fermilab (Aw72). These chambers (which with modifications, are still in use) are filled with suitable gases and have tissue equivalent plastic walls. They have a net volume of about 1.6 liters. Current versions of these instruments have chambers produced commercially and are made of 4 mm thick walls of phenolic. They are filled with propane gas at atmospheric pressure and contain an electrometer encased in a sealed container. Typically, such chambers are calibrated using photons and can have a typical "quality factor" built in to the electronics. Such chambers are available either as line-powered fixed monitors or as hand-held survey instruments.

The use of such instruments at accelerators must be done with the assurance that the instrument will respond correctly to the radiation field present. Neutron radiation fields are generally considered to be the most difficult in which to do this successfully. Höffert and Raffnsee of CERN have made measurements of the response of various instruments, including tissue equivalent ion chambers (Hö80). They were able to test such chambers, along with others (see discussion below), in neutron radiation fields having neutron energies ranging from thermal to 280 MeV. The neutron fields originated from reactor and radioactive sources except that at 280 MeV, a neutron beam from the 600 MeV CERN Synchrocyclotron was used. Table 6.3 adapted from (Hö80) shows the results:

**Table 6.3 Absorbed dose response and measurement errors for tissue equivalent ion chambers as a function of neutron energy.** [Adapted from (Hö80)]

<table>
<thead>
<tr>
<th>$E_n$ (MeV)</th>
<th>Absorbed Dose Response ($10^{-5}$ C Gy$^{-1}$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>thermal</td>
<td>0.446</td>
<td>9.8</td>
</tr>
<tr>
<td>0.0245</td>
<td>0.404</td>
<td>12.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.622</td>
<td>6.1</td>
</tr>
<tr>
<td>0.25</td>
<td>0.806</td>
<td>7.1</td>
</tr>
<tr>
<td>0.57</td>
<td>0.885</td>
<td>5.4</td>
</tr>
<tr>
<td>1.0</td>
<td>0.885</td>
<td>5.4</td>
</tr>
<tr>
<td>2.5</td>
<td>0.993</td>
<td>6.1</td>
</tr>
<tr>
<td>5.0</td>
<td>1.179</td>
<td>5.2</td>
</tr>
<tr>
<td>15.5</td>
<td>1.370</td>
<td>5.2</td>
</tr>
<tr>
<td>19.0</td>
<td>1.664</td>
<td>12.1</td>
</tr>
<tr>
<td>280.0</td>
<td>0.389</td>
<td>10.1</td>
</tr>
</tbody>
</table>

As seen, the performance is reasonably independent of energy in the energy region that typically dominates the dose equivalent (= up to about 5 MeV).

Freeman and Krueger (Fr84) have tested several ion chamber type instruments currently used at Fermilab in both photon and neutron radiation fields from radioactive sources. Both hand-held and fixed monitors were included in this study. $\gamma$-ray sources ($^{137}$Cs) were used to
provide the photons while a $^{238}$PuBe neutron source was used for the neutron measurements. [The PuBe source has neutrons spanning the region from $E_n < 0.25$ MeV to $E_n = 11$ MeV with an average energy of about 4.1 MeV.] Several different detectors were tested and are briefly described in Table 6.4 taken from (Fr84). The results of the measurement are given in Tables 6.5 and 6.6 taken from (Fr84). The measurements were made both indoors and outdoors to be able to understand the effects of room scattering.

Simple tests that have been conducted at Fermilab indicate that absorbed dose measured in muon fields is adequately understood using the γ-ray calibration of the instruments. These tests have involved comparison with direct measurements of the muon fluence using counter-telescope techniques, and typically are in agreement within about 10 per cent for the Fermilab-built instruments described previously. This is to be expected since muons at high energies behave as "minimum ionizing particles" whose loss of energy in matter proceeds, to first order, exactly as does that of electrons.

Practical problems encountered with such ion chambers are mostly those due to radiofrequency interference, pulsed radiation charged fields, and environmental factors such as temperature extremes and humidity. Cossairt and Elwyn (Co87) determined that air-filled pocket ion chambers of the type that are commonly issued to personnel to allow real-time monitoring of exposure to γ-rays, performed very well in muon radiation fields (measuring absorbed doses to within about ± 15%). This is probably due to the fact that the ratio of muon stopping power in tissue to that in air for energies between 1 and 800 GeV is 1.07 ± 0.05 (St83).

Table 6.4 Descriptions of ionization chamber used at Fermilab. The instruments designated "new" were produced after 1980 while those designated "old" were produced earlier. [Reproduced from (Fr84).]

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old Chipmunk</td>
<td>A high-pressure gas-filled ionization chamber designed by Fermilab and built by LND, Inc. with 4 mm thick walls of tissue-equivalent plastic. The fill gas is 150 PSIG of ethane. The chamber is enclosed in a protective box which contains a sensitive electrometer and associated electronics to measure the current output and convert it to dose-equivalent rate. Switch-selectable Quality Factors of 1, 2.5 or 5 are available.</td>
</tr>
<tr>
<td>New Chipmunk</td>
<td>Similar to Old Chipmunk except for use of a phenolic-lined ionization chamber, filled with propane gas at atmospheric pressure and an electrometer encased in a sealed container.</td>
</tr>
<tr>
<td>Old Scarecrow</td>
<td>A high-pressure ionization chamber with bare (stainless steel) walls filled with 150 PSIG of ethane gas. Similar to electrometer for Old Chipmunk but with a fixed Quality Factor of 4 and capability to measure dose rates 100 times higher (up to 10 Rem/hr).</td>
</tr>
<tr>
<td>New Scarecrow</td>
<td>Same electronics as Old Scarecrow, but with phenolic-lined chamber and electrometer enclosure identical to the ones in the New Chipmunk.</td>
</tr>
<tr>
<td>TEIR</td>
<td>Tissue Equivalent Integrating Ratemeter. A commercially available Tissue Equivalent ion chamber 'like that supplied with the REM, Inc. model 112, but interfaced to Fermilab-designed electronics.' The ion-chamber is filled with 18 PSIG ethane gas and has a nearly parallel plate geometry.</td>
</tr>
<tr>
<td>HPI 1010</td>
<td>Commercially available survey meter consisting of a gas-filled multiplying ion chamber and associated electronics. The chamber is filled with 100 mm Hg of TE gas inside a TE plastic chamber.</td>
</tr>
</tbody>
</table>
Chapter 6  Topics in Radiation Protection Instrumentation at Accelerators

Table 6.5 γ-ray test results for ionization chambers used at Fermilab. The instruments designated "new" were produced after 1980 while those designated "old" were produced earlier. [Reproduced from (Fr84).]

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Serial No.</th>
<th>Outdoor (mrad/hr) Gross</th>
<th>Indoor (mrad/hr) Gross</th>
<th>Ratio Indoor Outdoor</th>
<th>Calibration Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old Chipmunk</td>
<td>90</td>
<td>.122</td>
<td>.533</td>
<td>.411</td>
<td>.126</td>
</tr>
<tr>
<td>New Chipmunk</td>
<td>173</td>
<td>.148</td>
<td>.549</td>
<td>.401</td>
<td>.167</td>
</tr>
<tr>
<td>Old Scarecrow</td>
<td>28</td>
<td>25.47</td>
<td>25.88</td>
<td>.41</td>
<td>25.38</td>
</tr>
<tr>
<td>New Scarecrow</td>
<td>33</td>
<td>24.68</td>
<td>25.09</td>
<td>.41</td>
<td>24.71</td>
</tr>
<tr>
<td>TEIR</td>
<td>3</td>
<td>.035</td>
<td>.386</td>
<td>.351</td>
<td>.025</td>
</tr>
<tr>
<td>HPI 1010</td>
<td>2</td>
<td>.0107</td>
<td>.372</td>
<td>.361</td>
<td>.016</td>
</tr>
</tbody>
</table>

Table 6.6 Neutron test results for ionization chambers used at Fermilab. The instruments designated "new" were produced after 1980 while those designated "old" were produced earlier. [Reproduced from (Fr84).]

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Serial No.</th>
<th>Outdoor (mrad/hr) Gross</th>
<th>Indoor (mrad/hr) Gross</th>
<th>Ratio Indoor Outdoor</th>
<th>Calibration Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old Chipmunk</td>
<td>90</td>
<td>.122</td>
<td>.967</td>
<td>.765</td>
<td>(.585)</td>
</tr>
<tr>
<td>New Chipmunk</td>
<td>173</td>
<td>.148</td>
<td>.868</td>
<td>.668</td>
<td>(.488)</td>
</tr>
<tr>
<td>Old Scarecrow</td>
<td>28</td>
<td>25.47</td>
<td>26.24</td>
<td>.70</td>
<td>(.52)</td>
</tr>
<tr>
<td>New Scarecrow</td>
<td>33</td>
<td>24.68</td>
<td>25.43</td>
<td>.68</td>
<td>(.50)</td>
</tr>
<tr>
<td>TEIR</td>
<td>3</td>
<td>.035</td>
<td>.822</td>
<td>.834</td>
<td>(.654)</td>
</tr>
<tr>
<td>HPI 1010</td>
<td>2</td>
<td>.0107</td>
<td>.501</td>
<td>.505</td>
<td>(.325)</td>
</tr>
</tbody>
</table>

*Numbers not in parentheses result from applying the calibration correction factors in Table 6.5
*Numbers in parentheses include subtraction of 0.18 mrad/hr γ-contribution.
Geiger-Mueller Detectors

These instruments, among the oldest developed for the detection of radiation, are in conspicuous use at particle accelerators primarily with respect to detection and measurement of induced activation and removable induced activity (contamination). In some instances such instruments can be used to identify prompt radiation fields. They are very rugged and remarkably insensitive to environmental effects such as temperature and humidity. However, the typical dead time of 100 µsec renders them to be generally useless in fields having high instantaneous rates.

Thermoluminescent Dosimeters (TLDs) [Mostly from (Sw90) and (Kn79)]

These dosimeters are an attractive alternative to photographic film particularly to monitor personnel exposures in β and γ radiation fields. They have also been found to be useful in measuring neutron radiation fields when used as a pair of ⁶LiF and ⁷LiF TLDs "chips" in the same dosimeter. Such use exploits the fact that the ⁶Li(n,α)³H reaction has a large thermal neutron cross section of 940 barns while the ⁷Li(n,γ)⁸Li reaction cross section is only 0.037 barns for thermal neutrons. Since both ⁶Li and ⁷Li have comparable efficiencies for γ or muon radiation, measurement of the response of the two detectors can, then, be used to determine the dose equivalent due to thermal neutrons in the presence of photons or muons (or of fast neutrons if moderation, see below, is supplied).

TLDs operate on the principal that some of the radiation liberated by the ionizing particle is "trapped" in band gaps in the crystal lattice. The process is well-described in (Kn79). In particular, ionization elevates electrons from the valence to the conduction band where they are then captured by a "trapping center". At room temperatures, there is only a small probability per unit time that such "trapped" electrons will escape back to the conduction band from the valence band. Thus exposure to radiation continuously populates the traps. ("Holes" are similarly trapped in the valence band.) When readout of the dose is desired, the crystal is heated and this thermally excites the electrons and holes out of the traps. This process is accompanied by the emission of light that can, then, be measured as a so-called "glow curve". As discussed in (Kn79), a number of other materials can function as TLDs; notably CaSO₄:Mn, CaF₂, and CaF₂:Mn. These materials have properties that can be optimized for different applications. The latter is particularly useful for environmental monitoring purposes. The large numbers of trapped electrons and holes per unit of dose permits sensitivity to absorbed doses as small as 2 X 10⁻³ rads. LiF "fades" less than most of the other materials at room temperature and its average atomic number is very close to that of tissue, so it is particularly useful for personnel dosimetry.

TLDs can give valid results for fields as high as 100 rads. Higher doses can be measured under certain conditions if one takes care to use crystals calibrated in the high fields since linearity of the response breaks down in the high dose region. These devices become superlinear. Also, TLDs are not particularly susceptible to dose rate problems.

Nuclear Track Emulsions [Mostly from (Sw90).]

For many years, thin (= 25 micron) emulsions (NTA) have been used for personal dosimetry in fast neutron fields. The technique is based upon detection of tracks left by proton recoils in the film. The energy range for which these dosimeters are effective is from roughly 0.5 to 25 MeV because below that range, the tracks are too short to be read out while above it there are too few tracks because the (n,p) cross section (elastic scattering, mostly) decreases with energy. However, this energy range is the one that results in significant neutron dose equivalents at accelerators. The singular important problem with NTA is that the latent image fades and leads to underestimates of the dose equivalent. (The fading time can be a short as 2 weeks!) "Heroic"
efforts to keep out the moisture, and experience in dry climates give some indication that his problem can be overcome.

M. Höfert (Hö84) has given a good summary of experience with this dosimeter at accelerators. The dose equivalent range from about 10 mrem to few hundred mrem is that for which this dosimeter can be expected to perform acceptably. Any technique based upon track formation should not be dependent upon dose rate effects.

**Track Etch Dosimeters** [Mostly from (Sw90).]

In these detectors, the passage of a charged particle through a dielectric material will result in a trail of damaged molecules in the material. These tracks can be made visible upon etching in a strong acid or base solution. The tracks will be etched at a faster rate than the undamaged portions of the material. As with nuclear emulsions, there is a minimum detectable track length that sets a threshold of about 0.5 MeV on the neutron detection. Such detectors have been reviewed extensively by Griffith and Tommasino (Ge83). Mica, Lexan, and other materials are suitable for this purpose and electronic methods of readout are becoming available.

**CR-39 Dosimeters** [Mostly from (Sw90).]

This material is a serious candidate for replacing NTA as a film dosimeter; it is also a "track detector". It is a casting resin that is transparent (it was developed for use in eyeglass lenses) and is thermoset (rather than thermoplastic). It is the most sensitive of the track detectors and registers recoil protons up to 15 MeV and down to about 0.1 MeV. It is read out either chemically or electrochemically. The lower limit of detection appears to be improved over NTA and Track-Etch (Lexan). There are about $7 \times 10^3$ tracks cm$^{-2}$ rem$^{-1}$, which appears to be adequate. The sensitivity may be as much as a factor of two lower in high energy spectra. Fading appears to be insignificant. However, natural radon gas can contribute to background readings and the angle of incidence is important. Greenhouse, et al. have experimented with these dosimeters in an accelerator environment with "mixed" results (Gr87). However, the general conclusion of practitioners is that this material is promising.

**Bubble Detectors** [Mostly from (Sw90).]

The bubble damage polymer detector is a relatively new dosimeter that is similar to a bubble chamber in that a liquid whose normal boiling point is below room temperature is kept under pressure. When the pressure is released bubbles form along the path of a charged particle that has traversed it. To enhance the effect, superheated droplets of a volatile liquid are dispersed in a gelatinous medium. There are two types of these detectors that have been developed; one type by R. Apfel (Ap79) and the other type by H. Ing (In84). The polymer or gel is supplied in a clear vial. When a neutron interacts with it, a bubble is created that expands to optically visible dimensions and can thus be counted. There is no angular dependence but temperature effects may be a problem. The Ing detector presently exhibits constant response over the range $15 < T < 35$ °C. The material can be tailored to match a chosen neutron energy threshold which can be as low as 10 keV or less. Indeed, sets have been prepared with arbitrary thresholds of 0.010, 0.100, 0.500, 1, 3, and 10 MeV. The sensitivity can be adjusted over the range of 1-30 bubbles per mrem in a volume of 4 cm$^3$ and the physical mechanism is not readily sensitive to dose rate effects. Disadvantages include a high unit cost, and the fact that once the vial is opened it is only good for a few weeks of dose integration. The materials are presently being tested at accelerator laboratories. These detectors could not be expected to give accurate results in high dose rates.
One can see that no single commercial instrument "solves all problems" simultaneously, especially for neutron fields. The practitioner is encouraged to utilize a variety of instruments, including some of the special techniques below to fully understand the radiation fields.

IV. Specialized Detectors

Thermal Neutron Detectors

Although thermal neutrons are not the major source of neutron dose equivalent at particle accelerators, they are of considerable importance in accelerator radiation protection because of the ability to moderate the fast neutrons (as we shall see below). Furthermore, because some of the most prominent thermal neutron detectors rely upon radioactivation (by neutron capture) as the detection mechanism, they have the advantage that the response is entirely independent of dose rate effects and hence free of deadtime effects. An excellent discussion, summarized here, on thermal neutron detectors is given in (Kn79).

At the outset, there are some general features concerning thermal neutrons that need to be recalled (Ce69). The kinetic energies of thermal neutrons have the familiar relationship as a function of temperature, given by the Maxwell-Boltzmann distribution:

\[
f(E) = \frac{2\pi}{(\pi k T)^{3/2}} E^{1/2} \exp\left[-\frac{E}{kT}\right]
\]

where \( f(E) \) is the fraction of neutrons (or gas molecules) of energy \( E \) per unit energy interval, \( k = 1.38 \times 10^{-16} \) erg\(^o\)K or \( 8.62 \times 10^{-15} \) eV\(^o\)K (Boltzmann constant) and \( T \) is the absolute temperature of the gas (\(^o\)K). The most probable energy, \( E_{mp} \), is given by

\[
E_{mp} = kT
\]

while the average energy at any given temperature, \( \langle E \rangle \), is

\[
\langle E \rangle = (3/2)kT.
\]

At room temperature, \( T = 293 \) \(^o\)K, so that the most probable energy is 0.025 eV. The average velocity, \( \langle v \rangle \), at \( T = 293 \) \(^o\)K (since thermal neutrons are decidedly nonrelativistic!) is given by

\[
\frac{1}{2}m\langle v \rangle^2 = kT, \quad \langle v \rangle = 2200 \text{ m/sec.}
\]

As the neutron energy increases above the thermal value (up to about 1 keV), unless there are "resonances" present in the cross section, the absorption cross section, \( \sigma \), has been found to be approximately described by,

\[
\sigma \propto \frac{1}{\sqrt{E}} \propto \frac{1}{v}.
\]

which is known as the "1/v law". Thus, one can scale from the tabulated "thermal" cross section, \( \sigma_{th} \), as follows (within the range of validity of the 1/v law):

\[
\sigma(E) = \sigma_{th}\sqrt{\frac{E_0}{E}}.
\]
Several different nuclear reactions which are initiated by thermal neutrons are used as the basis of detectors. They all involve particular target nuclei and thus the detector materials sometimes depend upon isotopically separated materials to enhance the effectiveness.

**Boron-10**

The $^{10}\text{B}(n, \alpha)^7\text{Li}$ reaction is exothermic ($Q_V = 2.792$ MeV) and leads either to the ground state of $^7\text{Li}$ or its first excited state (0.482 MeV). The latter occurs about 94% of the time when thermal neutrons are incident. Thus, the reaction imparts about 2.31 MeV (for the dominant transition to the excited state) to the reaction products. (This energy is much larger in energy than is that of the incoming thermal neutron!). Since energy and momenta must be conserved, for the dominant excited state branch, the energy of the alpha particle, $E(\alpha)$, is 1.47 MeV and $E(^7\text{Li}) = 0.84$ MeV. This is because the following must hold:

$$E(^7\text{Li}) + E(\alpha) = 2.31$$  \hspace{1cm} (6.23)

(energy conservation for the excited state branch, neglecting the kinetic energy of the thermal neutron), and

$$[2m(^7\text{Li})E(^7\text{Li})]^{1/2} = [2m(\alpha)E(\alpha)]^{1/2}.$$  \hspace{1cm} (6.24)

(momentum conservation; the two products are oppositely directed to conserve momentum, if one discounts the very small momentum of the thermal neutron and recalls that nonrelativistically, $p^2 = 2mE$)

The excited state subsequently decays by emission of a photon. For this reaction, $\sigma_{th} = 3837$ barns and the natural abundance of $^{10}\text{B}$ is 20% (the only other stable isotope is $^{11}\text{B}$) (Se81). The large natural abundance of the crucial isotope makes this reaction very favorable for thermal neutron detection. In addition, material enriched in $^{10}\text{B}$ is readily available at present. Also the reaction products (and their deposited energies), being of short range, are contained in "reasonable" detector geometries. Figure 6.1 taken from (Kn79) gives the cross sections as a function of neutron energy for several of the thermal capture reactions described here. Note that the Boron-10 reaction has a rather featureless cross section and obeys the $1/v$ law quite well even up to approximately $3 \times 10^5$ eV.

The capture reaction on $^{10}\text{B}$ has been used principally in the form of BF$_3$ gas in proportional tubes. Proportional counters are somewhat similar in concept to ionization chambers except that electric fields of sufficient strength to exceed the threshold for liberating secondary electrons are applied. In typical gases at one atmosphere, this is of the order $10^6$ volts/meter. Under proper conditions, the number of electrons generated in this process can be kept proportional to the energy loss but the number of electrons released (and hence the size of the signal) can be "amplified" by a "gain" of many thousands. In proportional chambers, the region in which these secondary electrons are released is kept small compared to the chamber volume. If the voltage is raised beyond these conditions, then proportionality is lost and the counter enters the Geiger-Mueller mode. (Kn79) contains a detailed exposition on proportional chambers and the gas multiplication process. BF$_3$ is the best of the boron-containing gases as a proportional counter gas because of its "good" properties as a counter gas and also because of the high concentration of boron in the gas molecule.
Typical BF₃ tubes operate at 2000 to 3000 volts potential with gas gains of 100-500. An enriched (96%) BF₃ tube can have an absolute detection efficiency of 91% at 0.025 eV dropping to 3.8% at 100 eV for neutrons incident upon it. Alternatives with somewhat better gas properties (and cleaner signals) have been achieved by using boron-lined chambers with other gases.
Lithium-6
The reaction of interest is $^6\text{Li}(n, \alpha)^3\text{H}$. This reaction has a Q-value of 4.78 MeV and leads only to the ground state of $^3\text{H}$. As discussed in connection with the $^{10}\text{B}(n, \alpha)^7\text{Li}$ reaction, conservation of energy and momentum can be shown to yield the result that

$$E(\alpha) = 2.05 \text{ MeV} \quad \text{and} \quad E(3\text{H}) = 2.73 \text{ MeV}.$$ 

For incident thermal neutrons, $\sigma_{th} = 940$ barns. The natural isotopic abundance of $^6\text{Li}$ is about 7.5%. Fig. 6.1 plots the cross section as a function of neutron kinetic energy. The cross section exhibit a significant resonance at about $3 \times 10^5$ eV. The apparent disadvantage of the "small" thermal cross section is offset by the higher Q-value and resultant larger signals.

Concerning gas counters, no equivalent to the convenience of BF$_3$ gas has been found. Instead, $^6\text{Li}$ has been successfully added to scintillators. With the addition of a small amount (< 0.1% of the total atoms) of europium to LiI [LiI(Eu)], the light output is as much as 35% of NaI(Tl). Such scintillators have a decay time of approximately 0.3 μs. Of course, $^6\text{LiF}$ is also in prominent use as a TLD. (The TLD can be used in high dose rates, provided "instantaneous" readout is not required.)

Helium-3
This element, gaseous at room temperature, is used through the reaction $^3\text{He}(n, p)^3\text{H}$. The Q-value is 0.765 MeV so that, as for the other reactions, $E(p) = 0.574$ MeV and $E(3\text{H}) = 0.191$ MeV for incident thermal neutrons. For this reaction, $\sigma_{th} = 5327$ barns. Although this isotope of helium can be used directly as a detector gas, it has the disadvantages that the natural abundance is only 0.000138% (rendering enriched $^3\text{He}$ to be extremely costly), and that some of the energy can escape the sensitive volume of the detector because of the relatively long range of the proton. Again, the cross section as a function of energy is shown on the previous page. As seen, the cross section is quite "well-behaved". $^3\text{He}$ is a reasonable gas for proportional chambers; however no compounds are available since it is a noble gas. In sufficient purity it will work as an acceptable proportional gas. Because a proton is the reaction product instead of the short range $\alpha$-particle, "wall effects" (i.e., effects in which some energy escapes the counting gas volume) may be somewhat more severe than for BF$_3$. However, these tubes can be operated at much higher pressures than can BF$_3$ and can thus have enhanced detection efficiency compared to the former.

Cadmium-113
The discussion would be incomplete without discussing cadmium. This element, averaged over its naturally present isotopes, has a value of $\sigma_{th} = 2450$ barns. More spectacularly, the reaction $^{113}\text{Cd}(n, \gamma)^{114}\text{Cd}$ has a value of $\sigma_{th} = 19910$ barns. Thus, even without using enriched material, the thermal neutron cross section is large. This element is not used directly in the detector medium, as a general rule. Rather, it is used to shield other detectors from thermal neutrons because in the enriched ($^{113}\text{Cd}$ is 12.2% of natural abundance) form, its large cross section has the effect of essentially eliminating all neutrons < 0.4 eV. Hence, one can do measurements with and without the Cd inside of some moderator and have a very clear understanding of the thermal component.
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Silver

M. Awschalom was able to use thermal neutron capture on silver as a basis of a moderated detector (Aw72). As it occurs in nature, silver has two stable isotopes which both capture thermal neutrons via the \( (n, \gamma) \) process; \( ^{107}\text{Ag} \) (51.8\%, \( \sigma_{\text{th}} = 40 \text{ barns} \)) and \( ^{109}\text{Ag} \) (48.2\%, \( \sigma_{\text{th}} = 93.5 \text{ barns} \)). The average value of \( \sigma_{\text{th}} = 63.6 \text{ barns} \). While the cross sections are not as large as those of some of the other reactions discussed, the material is readily available and enrichment is not needed. The detector which utilized these capture reactions was a moderated one in which the output of a Geiger-Mueller tube wrapped with silver that sensed the capture \( \gamma \)-rays was compared with an identical tube wrapped with tin (average mass number = 118.7). Tin has an average value of \( \sigma_{\text{th}} = 0.63 \text{ barns} \) and is thus comparatively "dead" to thermal neutrons. The tin-wrapped tube was, then, used to subtract background due to muons, photons, etc.

Neutron-Induced Fission Reactions

\( ^{233}\text{U}, ^{235}\text{U}, \) and \( ^{239}\text{Pu} \) all have relatively large fission cross sections at low neutron energies. The Q-values are very large (approximately 200 MeV) so that huge output pulses are possible. The cross sections are shown in Fig. 6.2 taken from (Kn79). The fission processes continue up into the MeV region and beyond. Since the spontaneous decay mode of these radionuclides is by \( \alpha \)-particle emissions, \( \alpha \)-particles will always contribute to the signal. Major disadvantages with the use of these materials is that, because of their "sensitivity" as nuclear weapons materials, stringent regulatory provisions apply to their procurement and use. Instruments based upon these capture reactions have not been extensively used at particle accelerators.
Fig. 6.2  Fission cross sections of some common target nuclides used in fission chambers. Part (a) includes the slow neutron region where the cross sections shown are relatively large. The fast neutron region is shown in (b). Chambers with $^{237}$Np or $^{238}$U are sensitive only to fast neutrons. [Reproduced from (Kn79).]
Moderated Neutron Detectors

As seen, many neutron reactions tend to have much smaller cross sections in the MeV region than they have in the "thermal" region. Historically, it was observed that surrounding a thermal neutron detector with hydrogenous materials enhance detection rates exhibited by a "bare" thermal neutron detector placed in the same radiation field. The reason this occurs with hydrogenous materials is because in nonrelativistic elastic scattering, the most likely interaction between fast neutrons and low-atomic-numbered absorbers, the fraction of the incident energy, \( E_0 \), that can be transferred to the target nucleus after a collision in which the target nucleus recoils at angle \( \theta \), is determined by conservation of momentum and energy to be given by,

\[
\frac{\Delta E}{E_0} = \frac{4M}{(1 + M)^2 \cos^2 \theta},
\]

where \( M \) is the mass of the target nucleus in units where 1 is the mass of the neutron. The head-on collision case (\( \theta = 0 \)) represents the maximum energy that can be transferred and has its maximum value (1) when \( M = 1 \) (hydrogen). Even for as light a nucleus as \( ^{12}\text{C} \), the quantity \( \Delta E/E_0 \) is only 0.28.

If this were the only factor present, one would expect detection efficiency to improve with the thickness of the moderator. However as the moderator thickness increases, the probability that a given neutron will actually ever reach the detector decreases. Fig. 6.3 taken from (Kn79) illustrates these tradeoffs. In general, the optimum thickness will, for moderators such as polyethylene, range from a few centimeters for keV neutrons to several tens of centimeters for MeV neutrons. Furthermore, for any given thickness, the overall counting efficiency as a function of energy will tend to show a peak at some energy determined by the thickness.

Spherical Moderators, Bonner Spheres, and Related Detectors

Bramblett, Ewing and Bonner employed spherical moderators to obtain low resolution neutron spectra (Br60). In this technique moderating spheres of different diameters surrounding a thermal neutron detector of some type are placed in a given radiation field. The normalized relative (or absolute) responses are, then, indicative of the neutron energy spectra. As one might expect, the determination of the efficiency of each sphere as a function of energy is a rather complicated matter, and such response functions have been calculated, using techniques like Monte-Carlo, by a number of authors over the years since this method, the Bonner sphere technique, was invented. Recent calculations of the response function for spheres comprising the "standard" set have been made by Hertel and Davidson (He85). The response functions are dependent upon detector size as well as upon moderator thickness and density (typically 0.95 g/cm\(^3\) for polyethylene). Their results are given in Figs. 6.4 and 6.5 for cylindrical LiI(Eu) detectors of lengths equal to their diameters. The diameters are specified in the figure captions. Note that the largest sphere has a diameter of 45.72 cm (18 inches!) and weighs approximately 48 kg! Most of the efficiency calculations have been made for \(^6\text{LiI}(\text{Eu})\) scintillators, but also can be used for \(^6\text{LiF}\) TLD dosimeters. As one can see, the larger detector readily gives a higher efficiency response at the higher energies as intuitively expected from the enhanced detector volume. Awschalom and Sanna (Aw85) have obtained similar results. There are other sets of response functions extant. Experimental verifications of the details of these response functions are rather rare because of the difficulty of the measurements. Kosako, et. al. (Ko85) have successfully verified some of the important response functions using a neutron time-of-flight technique in the especially difficult keV energy region of neutron energy.
Fig. 6.3 Schematic representation of neutron histories in moderated detectors. The small thermal neutron detector at the center is shown surrounded by two different thicknesses of moderator material. Histories labeled 1 represent incident fast neutrons which are successfully moderated and detected. Those labeled 2 are partially or fully moderated, but escape without reaching the detector. History 3 represents those neutrons that are parasitically captured by the moderator. Larger moderators will tend to enhance process 3 while reducing process 2. [Reproduced from (Kn79).]
Fig. 6.4  The calculated 171-neutron group responses for the bare 4 mm LiI detector and for the same detector inside 5.08, 7.62, 12.7, 20.32, 25.4, 30.48, 38.1, and 45.72 cm diameter spheres.
[reproduced from (He85).]
Fig. 6.5  The calculated 171-neutron group responses for the bare 12.7 mm Li\textsubscript{I} detector and for the same detector inside 5.08, 7.62, 12.7, 20.32, 25.4, 30.48, 38.1, and 45.72 cm diameter spheres. [reproduced from (Ha85).]
A Bonner sphere determination of the neutron spectrum is comprised of a set of measurements of
the responses for the different spheres of radius \( r \), \( C_r \), where \( r \) has the discrete values based on
the available set. Such responses will be given, ideally, by,

\[
C_r = \int_0^\infty \frac{dN}{dE} R_r(E)dE,
\]

(6.26)

where \( dN/dE \) is the differential neutron flux density (the neutron spectrum) and \( R_r(E) \) is the
energy-dependent response function for the sphere of radius \( r \). One measures \( C_r \) and knows
\( R_r(E) \) with the objective of determining \( dN/dE \) by "unfolding" the spectrum. In practice, one
works with a discrete approximation to the integral;

\[
C_r = \sum_i \frac{dN}{dE_i} R_r(E_i)\Delta E_i,
\]

(6.27)

where the index, \( i \), labels each member of the set of "energy groups" used. The unfolding
procedure is a difficult mathematical problem that, unfortunately, suffers from being
underdetermined and ill-conditioned mathematically. (One has as many as 31 or more
"unknowns" corresponding to 31 energy groups, with typically only 8 or 9 measurements to
determine the response!) A variety of numerical techniques have been developed to do the
unfolding.

Prominent codes in use at accelerators include BUNKI (L084), LOUHI (R080), and SWIFT
(O'Br81). The first uses an interactive recursion method and the second uses a least squares
fitting procedure with user-controlled constraints. One essentially starts with an "educated
guess" at the spectrum and iterates to fit the responses. As we have seen, a \( 1/E \) spectrum is a
good starting point for an accelerator spectrum. SWIFT is different; it is a Monte-Carlo program
that makes no \textit{a priori} assumptions on the spectrum and can thus provide a "reality check" on
results using the other two. It has the disadvantage in that it is known to sometimes produce
nonphysical peaks in the unfolded spectrum. In general, the codes agree best with each other for
those properties that are determined by integrating over the spectrum such as the average quality
factor, total fluence, and total absorbed dose and dose equivalent. Typical spectra obtained from
such unfolding procedures have been reported at a number of laboratories. Fermilab results have
been summarized in (C088) and are, in general, similar to those obtained at other laboratories.
(See Chapter 4 for discussion of results of Fermilab neutron measurements.)

It is sometimes important to verify the "reasonableness" of the unfolded spectrum. Comparisons
can be made with known spectra from radioactive sources such PuBe or AmBe and such
comparisons have been made in, for example, (Co88). Sometimes, the normalized responses, \( C_r \),
themselves can be used to check the reasonableness of the unfolded spectrum. For example, in
the labyrinth measurement [(Co85b) and (Co88)] and in the iron leakage measurements at
Fermilab [(Co88) and (El86)], such plots were made. These are shown in Fig. 6.6 taken from
(Co85b) and Fig. 6.7 taken from (El86).

The labyrinth responses are compared with the sphere responses for a pure thermal neutron
spectrum. The enhanced responses for the intermediate-sized spheres indicates the somewhat
"harder" unfolded neutron spectrum than was observed. For the iron leakage spectrum, one can
see evidence for the "softening" of the spectrum after the concrete was added. (See Chapter 4
for discussion of the dramatic spectral changes after the addition of concrete.) Other
"verifications", of course can be obtained using entirely independent measurement techniques.
Fig. 6.6 Normalized response from the detector as a function of spherical moderator diameter. The solid circles are the measurements within the second leg of the labyrinth shown in Fig. 4.10. The open circles represent calculated results assuming a purely thermal spectrum while the crosses are the results for Spectrum I shown in Fig. 3.26 unfolded using the program SWIFT. The solid and dashed curves are drawn to guide the eye. The inset shows a typical gated spectrum of the pulse heights in the $^{6}$Li(Eu) phosphor detector described in the text. [Reproduced from (Co85b)].

Fig. 6.7 Normalized detector response as a function of spherical moderator diameter for the situation resulting Spectra E and F of Fig. 3.25. The open circles are the measurements before, and the X's are the measurements after the placement of the additional concrete shielding. [Reproduced from (El86)].
In the use of $^6$LiI(Eu) scintillators in such detectors in mixed fields, there are situations in which the signals from photons and/or muons can overwhelm the neutron signal. Awschalom and Coulson (Aw73) have developed a technique in which the $^6$LiI(Eu) is surrounded by plastic scintillator. A schematic diagram of the electronic readout circuitry, a schematic of the phoswich detector, and a typical pulse height spectrum obtained by use of this detector in a long exposure to environmental neutrons are given in Figs. 6.8, 6.9, and 6.10, respectively, all taken from (Aw73). The same detector was used to produce the pulse-height spectrum shown in the inset in Fig. 6.6.

In this technique, the fast discriminator is set to respond to the $2-3$ nsec decay time of the plastic scintillation signal while the other discriminator is set to respond to the $1.4 \mu$sec decay time of the crystal. Selecting the slow counts not accompanied by fast counts clearly gives superior discrimination against non-neutron events from environmental radiation to which both the crystal and the plastic scintillator are sensitive.

In performing Bonner sphere measurements in neutron fields that are suspected of being nonuniform in space, it may be necessary to measure $C_r$ over the set of spheres individually because arranging them in an array may result not only in undesired "cross-talk" between the moderators but also in the need to make corrections for the non-uniformities of the radiation field.

Since accelerator neutron fields are often quite similar to each other, it was noticed that the choice of a single moderator size might well offer the opportunity to construct a "rem-meter" that would use a given sphere response function particularly well matched to energy dependence of the fluence-to-dose equivalent conversion factor. The standard implementation of this is in the development of the Andersson-Braun detector (An62) which uses a BF$_3$ detector. The use of such counters is reviewed in (Th88). At present, an ion chamber version of this instrument is used at CERN. Generally, the 25.4 cm (10 inch) diameter polyethylene sphere has been selected because its response curve provides the best match to the curve of fluence-to-dose equivalent. Höfert and Raffnsøe (Hö80) have measured the dose equivalent response of such an instrument as a function of neutron energy. Their results are displayed in Table 6.7.

<table>
<thead>
<tr>
<th>$E_n$ (MeV)</th>
<th>Dose Equivalent Response ($10^{-5}$ C Sv$^{-1}$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>thermal</td>
<td>0.349</td>
<td>10.0</td>
</tr>
<tr>
<td>0.0245</td>
<td>3.209</td>
<td>12.1</td>
</tr>
<tr>
<td>0.1</td>
<td>1.335</td>
<td>6.8</td>
</tr>
<tr>
<td>0.25</td>
<td>1.082</td>
<td>6.1</td>
</tr>
<tr>
<td>0.57</td>
<td>0.923</td>
<td>5.2</td>
</tr>
<tr>
<td>1.0</td>
<td>0.845</td>
<td>5.2</td>
</tr>
<tr>
<td>2.5</td>
<td>0.784</td>
<td>6.1</td>
</tr>
<tr>
<td>5.0</td>
<td>0.653</td>
<td>5.2</td>
</tr>
<tr>
<td>15.5</td>
<td>0.348</td>
<td>5.2</td>
</tr>
<tr>
<td>19.0</td>
<td>0.445</td>
<td>12.2</td>
</tr>
<tr>
<td>280.0</td>
<td>0.157</td>
<td>10.1</td>
</tr>
</tbody>
</table>
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Fig. 6.10  Upper curve is spectrum of all slow pulses. Lower curve is spectrum of slow pulses not accompanied by fast pulses, e.g., "neutrons." [Reproduced from (Aw73).]

Fig. 6.8  Electronics block diagram. [Reproduced from (Aw73).]

Fig. 6.9  Cross section of 8 mm X 8 mm NaI scintillator. [Reproduced from (Aw73).]
These results are discussed further in (Th85). Generally, commercial versions of this instrument operate in the proportional counter mode. This renders them suspect in accelerator fields with high instantaneous dose rates that arise because of the small "duty factor" due to pulsed beams. A similar detector has been developed by Hankins and employed $^6$LiI(Eu) as the detector (Ha62). Hankins obtained the response shown in Fig. 6.11 compared with the "Inverse of the Radiation Protection Guide (RPG) curve". [This figure is also found in (Kn79).] In the keV region, comparisons are difficult and there is some evidence that the detector overresponds considerably. However, the "match" was verified at thermal neutron energies. An alternative detector of this type has been developed by Leake (Le68). In this detector a $^3$He proportional counter is used in a 20.8 diameter sphere to reduce background due to photons along with a cadmium filter against thermal neutrons. It is claimed that this detector is effective in photon fields as high as 20 R/h. There are concerns that above 10 MeV this type of instrument seriously underestimates neutron dose equivalent rates.

It is not necessary, for radiation protection purposes, that a "spherical" moderator be an exact sphere. Awschalom (Aw72) demonstrated that an octagon of revolution (a "pseudosphere") having volume equivalent to that of a 25.4 cm diameter sphere had a response indistinguishable from that of the 25.4 cm sphere as a function of polar angle and that the response of a cylinder of equal volume was not far different. This feature was investigated because cylinders and pseudospheres are cheaper to produce than spheres.

![Graph of Sensitivity of a 25.4 cm (10 in.) diameter moderating sphere surrounding a 4 mm x 4 mm LiI scintillator. Also shown is the relative dose per neutron labeled as "Inverse of RPG curve".](image)

Fig. 6.11 Sensitivity of a 25.4 cm (10 in.) diameter moderating sphere surrounding a 4 mm x 4 mm LiI scintillator. Also shown is the relative dose per neutron labeled as "Inverse of RPG curve". [Reproduced from (Kn79 as adapted from (Ha62).]
Another type of moderated neutron detector that has been used extensively is the long counter. The idea is to adjust the configuration of moderators around some thermal neutron detector in such a manner as to assure that the detection efficiency plotted as a function of neutron energy is a straight line. It has been found over the years that the best detector is a cylinder of moderating materials surrounding a thermal neutron detector (also cylindrical) on the axis. Since a cylindrical detector is desired, the BF$_3$ proportional counter is the most popular. One end of the cylinder "views" the neutron source for best results. Hanson and McKibben (Ha47) were the pioneers of the technique.

An improved version, which has rather widespread use, is that developed by J. DePangher and L. I. Nichols (De66). Figure 6.12 taken from (De66) shows the layout of this detector. The length and diameter are both approximately 41 cm and the mass is about 45 kg. The neutrons are to be incident on the "front" face.

Perhaps the best calibration data on this device is that of Slaughter and Rueppel (Sl77). They used filtered beams from a reactor (2 keV) as well as monoenergetic neutron beams from (p, n) and (d, n) reactions at accelerators to cover the energy range from 10 keV to 19 MeV. The sensitivity data resulted in an average of about 3.5 counts/(n cm$^{-2}$) over this large dynamic range as indicated in Fig. 6.13 taken from (Sl77). [Fig. 6.13 also shows data from other workers as discussed in (Sl77)].

This detector has been used to conduct studies of skyshine at Fermilab [(Co85a) and (El86)]. The large peak in the pulse-height spectrum of the BF$_3$ tube from thermal neutron capture (Q-value = 2.79 MeV) renders the detector essentially dead, with the application of a suitable discriminator, to all other radiations.

Knoll (Kn79) summarizes results with modified long counters that have achieved better uniformity and sensitivity over more restricted energy domains.
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Fig. 6.12 Sketch of DePangher Long Counter. This version contained a built-in PuBe source. [Reproduced from De66.]

Fig. 6.13 Sensitivity [counts/(n cm\(^{-2}\))] as a function of neutron energy. [Adapted from (S177) and references cited therein.]
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Activation and Threshold Detectors

As we have seen, certain nuclear reactions have sharp thresholds which can be used to determine portions of a hadron spectrum that exceed it since the "leveling off" of the cross sections is generally "well-behaved". Of these, the production of $^{11}$C is one of the best known and has the cross sections shown in Fig. 6.14 taken from (Sw90). In addition to information on reaction thresholds provided in Chapter 4, where referral was made to threshold techniques, Table 6.8 taken from (Sw90) summarizes some of the useful reactions. [(Pa73) contains a large list of other reactions that might have useful thresholds.]

Fig. 6.14  Excitation functions for the reactions $^{12}$C $\rightarrow$ $^{11}$C induced by neutrons, pions, and protons. The arithmetic mean of the positive and negative pions cross-sections is shown as the pion curve. [Reproduced from (Sw90) as adapted (St84).]

Table 6.8 Types of activation threshold detectors.  [Reproduced from Sw90).

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Sample type</th>
<th>Threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{31}$S-$^{32}$P</td>
<td>Sulfur powder or pellets</td>
<td>3</td>
</tr>
<tr>
<td>$^{27}$Al-$^{27}$Mg</td>
<td>Aluminum disks or pellets</td>
<td>3</td>
</tr>
<tr>
<td>$^{24}$Al-$^{24}$Na</td>
<td>Aluminum disks or pellets</td>
<td>6</td>
</tr>
<tr>
<td>$^{24}$Al-$^{22}$Na</td>
<td>Aluminum disks or pellets</td>
<td>35</td>
</tr>
<tr>
<td>$^{19}$F-$^{19}$F</td>
<td>Teflon cylinders</td>
<td>12</td>
</tr>
<tr>
<td>$^{13}$C-$^{11}$C</td>
<td>Polyethylene cylinders or plastic cylinders</td>
<td>20</td>
</tr>
<tr>
<td>$^{12}$C-$^{7}$Be</td>
<td>Polyethylene cylinders or plastic cylinders</td>
<td>35</td>
</tr>
<tr>
<td>Bi fission</td>
<td>Fission chamber</td>
<td>50</td>
</tr>
</tbody>
</table>
The $^{12}\text{C} \rightarrow ^{11}\text{C}$ producing reactions are of special interest because of the fact that plastic scintillators can themselves become activated by hadrons (especially neutrons and protons) exceeding 20 MeV. This technique was first developed at the Lawrence Radiation Laboratory by McCaslin (McC60). Stevenson (St84) has determined that a value of 28 fSv m$^2$ is an appropriate multiplicative factor for the conversion of the measured fluence of neutrons with $E_n > 20$ MeV (neutrons m$^{-2}$) to the dose equivalent due to those energetic neutrons. This assumes a "typical" accelerator spectrum in side shields of earth or concrete where neutrons clearly dominate. Such measurements can be useful to determine the contribution of the high energy ($E_n > 20$ MeV) neutrons to the total neutron dose equivalent.

Moritz (Mo89) has found that the use of NE102A scintillators activated by the $^{12}\text{C}(n, 2n)^{11}\text{C}$ can be included as an "additional detector" in a Bonner sphere measurement in order to extend the energy range. Moritz, following Stevenson, used an average cross section of 22 mb for the $^{12}\text{C}(n, 2n)^{11}\text{C}$ reaction. NE102A has a carbon content of $4.92 \times 10^{22}$ atoms/gram and a density of 1.032 (Kn79). Moritz used a cylindrical detector 5 cm in diameter by 5 cm long and achieved an efficiency of 93% in detecting the 0.511 annihilation $\gamma$-rays produced as a result of the $^{11}\text{C}$ decay. In effect, the addition of this reaction reduced the degeneracy of the spectrum unfolding process using the code LOUHI (see Chapter 3).

Pertinent information, including some practical detector sizes for commonly used threshold reactions, are given in Table 6.9. Table 6.10 taken from (Sw90) gives a more extensive list of possible threshold reactions. The Hg $\rightarrow ^{149}\text{Tb}$ reaction is a suitable monitor for very high energy particles and is sometimes used as a beam calibrator. However, it has been found by S. Baker [(Ba84) and (Ba91)] that there are three reactions involving copper targets which are more useful for this purpose because they have longer half-lives than the 4 hours of $^{149}\text{Tb}$. These cross sections have been measured for energies from 30 to 800 GeV and are included in Table 6.9.
## Table 6.9 Important characteristics of various activation-detector techniques

<table>
<thead>
<tr>
<th>Detector</th>
<th>Reaction Range</th>
<th>Energy (MeV)</th>
<th>Half-Life</th>
<th>Typical Detector Size</th>
<th>Cross Sections Peak (mb)</th>
<th>High Energy (mb)</th>
<th>Particle Detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>sulfur</td>
<td>$^{32}\text{S}(n,p)^{32}\text{P}$</td>
<td>$\geq3$</td>
<td>14.3 d</td>
<td>2.54 cm diam, 4 g disk</td>
<td>500$^a$</td>
<td>10$^a$</td>
<td>$\beta^+$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$^{27}\text{Al}(n,\alpha)^{24}\text{Na}$</td>
<td>$\geq6$</td>
<td>15 h</td>
<td>16.9 to 6600 g</td>
<td>11$^b$</td>
<td>9$^b$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$^{27}\text{Al}(n,2p4n)^{22}\text{Na}$</td>
<td>$\geq25$</td>
<td>2.6 $\gamma$</td>
<td>16.9 g</td>
<td>30$^b$</td>
<td>10$^b$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Plastic scintillator</td>
<td>$^{12}\text{C}(n,2n)^{11}\text{C}$</td>
<td>$\geq20$</td>
<td>20.4 min</td>
<td>13 to 2700 g</td>
<td>90$^b$</td>
<td>30$^b$</td>
<td>$\beta^+,\gamma$</td>
</tr>
<tr>
<td>Plastic scintillator</td>
<td>$^{12}\text{C}(n,\text{spall})^{7}\text{Be}$</td>
<td>$\geq30$</td>
<td>53 d</td>
<td>16.9 g (2.54 cm high)</td>
<td>18$^b$</td>
<td>10$^b$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Mercury</td>
<td>$^{198}\text{Hg}(n,\text{spall})^{149}\text{Tb}$</td>
<td>$\geq600$</td>
<td>4.1 h</td>
<td>up to 500 g</td>
<td>2$^a$</td>
<td>1$^a$</td>
<td>$\alpha,\gamma$</td>
</tr>
<tr>
<td>Gold foils</td>
<td>$^{197}\text{Au}(n, \text{spall})^{149}\text{Tb}$</td>
<td>$\geq600$</td>
<td>4.1 h</td>
<td>2.54 cm diam, 0.5 g</td>
<td>1.6$^b$</td>
<td>0.7$^b$</td>
<td>$\alpha,\gamma$</td>
</tr>
<tr>
<td>Copper foils</td>
<td>$\text{Cu}(p, \text{spall})^{24}\text{Na}$</td>
<td>$\geq600$</td>
<td>14.7 h</td>
<td>5.6 cm diam, 580 g</td>
<td>4$^c$</td>
<td>3.9$^c$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Copper foils</td>
<td>$\text{Cu}(p, \text{spall})^{52}\text{Mn}$</td>
<td>$\geq70$</td>
<td>5.7 d</td>
<td>5.6 cm diam, 580 g</td>
<td>5$^c$</td>
<td>4.6$^c$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Copper foils</td>
<td>$\text{Cu}(p, \text{spall})^{54}\text{Mn}$</td>
<td>$\geq80$</td>
<td>310 d</td>
<td>5.6 cm diam, 580 g</td>
<td>11$^c$</td>
<td>11$^c$</td>
<td>$\gamma$</td>
</tr>
</tbody>
</table>

$^a$Swanson and Thomas (Sw90)
$^b$Barbier (Ba69)
$^c$Baker, et al (Ba91) and (Ba84).
Table 6.10 Activation reactions possibly suitable for threshold detectors. [Reproduced from (Th88) as adapted from references cited therein.]

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Half-life of product</th>
<th>Energy of gamma ray (MeV)</th>
<th>Approximate threshold energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{3}$Li(n,α) $^{7}$H</td>
<td>12.3 a</td>
<td>0.019 (β)</td>
<td>3.8</td>
</tr>
<tr>
<td>$^{14}$C(n,α) $^{14}$C</td>
<td>20.3 min</td>
<td>0.51</td>
<td>20</td>
</tr>
<tr>
<td>$^{12}$C(n,spall) $^{12}$Be</td>
<td>53.6 d</td>
<td>0.48</td>
<td>30</td>
</tr>
<tr>
<td>$^{15}$F(n,α) $^{15}$F</td>
<td>109.7 min</td>
<td>0.51</td>
<td>13.1</td>
</tr>
<tr>
<td>$^{24}$Mg(n,p) $^{24}$Na</td>
<td>15.0 h</td>
<td>1.37, 2.75</td>
<td>7.5</td>
</tr>
<tr>
<td>$^{27}$Al(n,p) $^{27}$Mg</td>
<td>9.5 min</td>
<td>0.84, 1.01</td>
<td>3.8</td>
</tr>
<tr>
<td>$^{27}$Al(n,α) $^{27}$Na</td>
<td>15.0 h</td>
<td>1.37, 2.75</td>
<td>4.9</td>
</tr>
<tr>
<td>$^{27}$Al(n,spall) $^{27}$Na</td>
<td>262 a</td>
<td>0.51, 1.28</td>
<td>25</td>
</tr>
<tr>
<td>$^{32}$S(n,p) $^{32}$P</td>
<td>14.3 d</td>
<td>1.71 (β)</td>
<td>3.3</td>
</tr>
<tr>
<td>$^{40}$Ti(n,p) $^{40}$Sc</td>
<td>83.9 d</td>
<td>0.89, 1.12</td>
<td>2.9</td>
</tr>
<tr>
<td>$^{41}$Ti(n,p) $^{41}$Sc</td>
<td>3.43 d</td>
<td>0.16</td>
<td>2.4</td>
</tr>
<tr>
<td>$^{41}$Ti(n,p) $^{41}$Sc</td>
<td>1.83 d</td>
<td>0.98, 1.13</td>
<td>7.1</td>
</tr>
<tr>
<td>$^{52}$Cr(n,α) $^{52}$Cr</td>
<td>41.9 min</td>
<td>0.15</td>
<td>13.5</td>
</tr>
<tr>
<td>$^{52}$Cr(n,α) $^{52}$Cr</td>
<td>27.7 d</td>
<td>0.32</td>
<td>12.4</td>
</tr>
<tr>
<td>$^{59}$Fe(n,p) $^{59}$Mn</td>
<td>503 d</td>
<td>0.84</td>
<td>2.2</td>
</tr>
<tr>
<td>$^{59}$Fe(n,α) $^{59}$Fe</td>
<td>8.53 min</td>
<td>0.38</td>
<td>13.9</td>
</tr>
<tr>
<td>$^{59}$Fe(n,p) $^{59}$Mn</td>
<td>2.58 h</td>
<td>0.85</td>
<td>5.0</td>
</tr>
<tr>
<td>$^{59}$Ni(n,p) $^{59}$Co</td>
<td>71.3 d</td>
<td>0.51, 0.81</td>
<td>1.3</td>
</tr>
<tr>
<td>$^{59}$Ni(n,α) $^{59}$Ni</td>
<td>36.0 h</td>
<td>0.51, 1.37</td>
<td>12.6</td>
</tr>
<tr>
<td>$^{59}$Co(n,α) $^{59}$Mn</td>
<td>2.58 h</td>
<td>0.85</td>
<td>5.2</td>
</tr>
<tr>
<td>$^{59}$Co(n,α) $^{59}$Mn</td>
<td>71.3 d</td>
<td>0.51, 0.81</td>
<td>10.3</td>
</tr>
<tr>
<td>$^{60}$Cu(n,α) $^{60}$Cu</td>
<td>9.76 min</td>
<td>0.51, 1.17</td>
<td>11.3</td>
</tr>
<tr>
<td>$^{64}$Zn(n,p) $^{64}$Cu</td>
<td>12.8 h</td>
<td>0.51, 1.35</td>
<td>2.0</td>
</tr>
<tr>
<td>$^{64}$Cu(n,p) $^{64}$Ni</td>
<td>2.56 h</td>
<td>1.12</td>
<td>3.2</td>
</tr>
<tr>
<td>$^{64}$Cu(n,α) $^{64}$Cu</td>
<td>12.7 h</td>
<td>1.35, 0.51, 1.35</td>
<td>10.3</td>
</tr>
<tr>
<td>$^{60}$Zr(n,α) $^{60}$Zr</td>
<td>78 h</td>
<td>0.91</td>
<td>12</td>
</tr>
<tr>
<td>$^{60}$Nb(n,α) $^{60}$Nb</td>
<td>13.6 a</td>
<td>0.019, 0.017</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Half-life of product</th>
<th>Energy of gamma ray (MeV)</th>
<th>Approximate threshold energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{90}$Nb(n,α) $^{90}$Nb</td>
<td>1.2 a</td>
<td>0.019 (β)</td>
<td>3.8</td>
</tr>
<tr>
<td>$^{160}$Rb(n,α) $^{160}$Rb</td>
<td>20.3 min</td>
<td>0.51</td>
<td>20</td>
</tr>
<tr>
<td>$^{113}$In(n,α) $^{113}$In</td>
<td>4.5 h</td>
<td>0.34</td>
<td>1.5</td>
</tr>
<tr>
<td>$^{177}$Ir(n,α) $^{177}$Ir</td>
<td>12.8 d</td>
<td>0.39, 0.67</td>
<td>9.3</td>
</tr>
<tr>
<td>$^{197}$Au(n,α) $^{197}$Au</td>
<td>6.2 d</td>
<td>0.36</td>
<td>8.6</td>
</tr>
<tr>
<td>$^{197}$Au(n,α) $^{197}$Au</td>
<td>39.5 h</td>
<td>0.33</td>
<td>24</td>
</tr>
<tr>
<td>$^{197}$Au(n,spall) $^{197}$Au</td>
<td>4.1 h</td>
<td>0.17</td>
<td>600</td>
</tr>
<tr>
<td>$^{197}$Au(n,spall) $^{197}$Au</td>
<td>4.1 h</td>
<td>0.17</td>
<td>600</td>
</tr>
<tr>
<td>$^{200}$Bi(n,α) $^{200}$Bi</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{231}$Pa(n,α) $^{231}$Pa</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{233}$Th(n,α) $^{233}$Th</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{235}$U(n,α) $^{235}$U</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{235}$U(n,α) $^{235}$U</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fission Counters

The fission reactions described above have been exploited as neutron (or hadron) detectors at accelerators. In addition, other elements, not normally thought of as "fissionable", exhibit significant fission cross sections. Figure 6.15 taken from (Pa73) shows the excitation functions of some of these materials. Fission of $^{209}$Bi is especially interesting since this reaction has a threshold of about 50 MeV and also exhibits strong evidence that the neutron and proton-induced fission cross sections are approximately equal. Bismuth has been employed in ionization chambers where the large energy deposited by the fission fragments gives a clear "signature" of this process. Like the use of $^{12}$C, it can provide further information about high energy neutrons and resolve ambiguities in the unfolding of spectra from Bonner sphere data. McCaslin, et.al. has summarized results obtained at the Lawrence Berkeley Laboratory (McC68).

![Fission cross sections of natural uranium, natural thorium, bismuth, gold, and tantalum as a function of neutron or photon energy. (Reproduced from (Pa73).)]
Proton Recoil Counters

Knoll (Kn79) describes a variety of techniques for detecting neutrons based upon measuring the energy of recoil particles. The $^3\text{He}(n,p)^3\text{H}$ reaction has a reasonable cross section even into the MeV region but suffers from competition with $(n, d)$ processes and elastic scattering. Elastic scattering of neutrons in which the energy of the recoil particle is measured and correlated with the neutron energy has received a great deal of attention. The most obvious recoil particle to measure is the proton because hydrogenous detector materials (e.g., plastic scintillator) are readily available and also because the proton can receive the most energy in the recoil process. Detector designers have been able to exploit the fact that scattering from hydrogen in the region $E_n < 10$ MeV is isotropic in the center of mass frame. In (Kn79) it is shown that the probability, $P(E_r)$, of creating a recoil with energy $E_r$ is also independent of angle in the laboratory frame within this energy domain. Thus the recoil energy is only a function of the incident neutron energy. However, complexities enter the picture because in scintillators, carbon is present along with the hydrogen and can contribute recoil protons. Furthermore, the magnitude of the cross sections is a function of neutron energy as is the efficiency of neutron detection in the scintillator. These effects, along with finite pulse height resolution, can lead to the need to resort to unfolding techniques, discussed in detail in (Kn79), in which the pulse height, indicative of the energy of the recoil proton, is correlated with the average neutron energy which could produce such a pulse. The technique has exhibited some promise in measuring the energy spectra of neutron radiation fields. The best recent summary is that of Griffith and Thorngate (Gr85) who were able to determine neutron energy spectra in the 2-20 MeV region.

TEPCs and LET Spectrometry

In mixed field dosimetry, a promising technique, now reaching commercial potential is that of the tissue-equivalent proportional chamber (TEPC) sometimes referred to as the "Rossi counter" after its inventor, H. Rossi (Ro55). These have been described by Brackenbush, et al in (Br78). In this chamber, tissue equivalent walls are employed to apply the Bragg-Gray principle. In such chambers, the pressure is maintained at low values, only a few torr (a few hundred pascals) so that the energy deposited is kept small. Thus, the energy so deposited will be equal to the linear energy transfer of the particle multiplied by the path length. At these low pressures, the gas-filled cavity has the same mass stopping power as a sphere of tissue of diameter about 1 μm—hence an "equivalent diameter of 1 μm". In principle, determining the absorbed dose from events in such chambers is a straightforward unit conversion from a measured pulse height spectrum (calibrated in energy) to absorbed dose (in tissue) irrespective of the radiation field:

$$D(\text{rad}) = \frac{(1.602 \times 10^{-8})C}{\rho V} \sum_{h} (hN(h))$$

where the summation is over channels corresponding to the radiation type of interest (see below), $V$ is the sensitive volume (cm$^3$), $\rho$ is the density (g/cm$^3$) and $C$ converts the channel number to energy in MeV while $h$ is the channel number and $N(h)$ is the number of counts in channel number $h$.

In such chambers, the transition between photon and neutron induced events occurs at a pulse height of about 15 keV/μm. It is possible to determine the quality factor, $Q$, from a single TEPC measurement. Under the conditions stated above, one can unfold from the pulse height spectrum the distribution of absorbed dose as a function of LET, $D(L)$, using a complicated formula derived by Rossi (Ro68). [The formula is complicated by the fact that one must average over mean chord lengths in the chamber.] Such a distribution is used to calculate quality factor, and
hence the dose equivalent. The advent of microcomputers has now made such instruments feasible as portable instruments. Fig. 6.16 taken from (Br78) shows a typical pulse height spectrum for such an instrument. In higher energy fields, dose distributions due to other particles with the same characteristic shapes but larger pulse sizes appear as the $^2$H, $^3$H, $^3$He, $^4$He and even $^7$Li drop points. This obviously will add complexities to the unfolding procedures in the determination of LET spectra. A more recent discussion of the application of this technique is given by Vasilik et. al. (Va85).

Recombination Chambers

An adaptation of the ion chamber that has shown considerable potential for usefulness as a "mixed field" dose equivalent meter is based on the exploitation of recombination phenomena in such chambers. As charged particles interact in such a chamber the gas is ionized. The ions left behind in this process will be collected by the electrodes except to the extent that they recombine. Such "columnar recombination" will depend upon the distance between the ions as well as upon the applied voltage (which sets the speed at which the ions migrate to the electrodes). Thus, for a given voltage, a chamber should exhibit more severe recombination for the radiations having high LET (e.g. neutrons, heavy ions, etc.) than for those having low LET (electrons, photons, and muons). The initial work on this subject was done by Zielczynski (Zi63). Later, Baarli and Sullivan (Ba65) further refined the topic. It turns out that the current, $i$ (or charge if integrated over time), measured in a given radiation field, is related to the applied voltage $V$ by the following approximate expression:

$$i = kV^n.$$  \hspace{1cm} (6.29)
The power, $n$, is approximately proportional to the quality factor $Q$. The relationship is shown in Fig. 6.17 taken from (Sw90). Using a different chamber, workers at Fermilab were able to obtain very similar curves over a more limited range of $Q$ ($2 < Q < 7$) using mixed fields of $\gamma$-rays and PuBe neutrons (Co84). The relationship between $Q$ and $n$ determined in (Co84) for a particular chamber used in this manner was

$$n = 0.00762 + 0.16Q.$$  \hfill (6.30)

Fig. 6.17 Response of a high-pressure parallel-plate recombination chamber as a function of quality factor $Q$. The curve shows the response predicted from theory. The experimental points were reported by Baarli and Sullivan (Ba65). [Reproduced from (Sa).]

At Fermilab, the normalized response (usually using some adjacent ion chamber) is measured as a function of time for the special chamber provided for the purpose over the voltage range $20 \leq V \leq 1200$ volts. The method of least squares is then applied to determine $n$ by taking advantage of the fact that the above relationship can be rewritten as

$$\ln (i) = \ln(k) + n \ln (V).$$  \hfill (6.31)

In typical situations, this log-log fit is of moderately good quality. The quality factor, $Q$, then, can be determined directly from $n$ using the above "calibration". Data taken in the process of investigation of the iron leakage spectrum described by Elwyn and Cossairt (El86) where the spectrum was "softened" after the addition of concrete to the bare iron shield are shown in Fig. 6.18 taken from (El86). The spectra in which these measurements were made are Spectra E and F shown in Fig. 3.25. Fig. 6.19 taken from (Co87) shows the response measured in a field known to be dominated by muons ($Q = 1$).

These measurements have been used to check the quality factors obtained in the unfolding of Bonner sphere data. Table 6.12 taken from (Co88) illustrates the good agreement between these entirely different techniques.
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Fig. 6.19  Recombination chamber response as a function of chamber potential in a radiation field nearly purely due to muons. [Reproduced from (Co87).]

Table 6.11 Average Quality Factors. The spectra are those of Figs. 3.24, 3.25, and 3.26. [Reproduced from (Co88).]

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>Unfolding</th>
<th>Recombination</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1.4 ± 0.2*</td>
<td>1.1 ± 0.3*</td>
</tr>
<tr>
<td>E</td>
<td>5.4 ± 0.2</td>
<td>6.0 ± 0.6</td>
</tr>
<tr>
<td>F</td>
<td>2.5 ± 0.3</td>
<td>3.0 ± 0.3</td>
</tr>
<tr>
<td>I</td>
<td>3.1 ± 0.7</td>
<td>3.4 ± 0.1</td>
</tr>
</tbody>
</table>

*Mixed field, includes muon component

Fig. 6.18  Recombination chamber response functions measured both before (top) and after (bottom) the placement of additional shielding [see Spectra E and F of Fig. 3.25]. Values of the quality factor Q are obtained from the fitted N-values according to Eq. (6.30). [Reproduced from (El86).]
Early work was done by Zel'chinskij and Zharnovetskij (Ze67) in which they proposed using two chambers placed in the radiation field of interest, one operated at a low voltage and other at a high voltage. The differences in currents read out by the two chambers would then be proportional to the dose equivalent rate. It turns out that measuring differences in small ion chamber currents found in practical chambers is difficult due to the small currents and connector leakage problems.

Höfert and Raffnsøe (Hö80) have measured the dose equivalent response of such an instrument as a function of neutron energy and obtained the results in Table 6.12.

<table>
<thead>
<tr>
<th>$E_n$ (MeV)</th>
<th>Dose Equivalent Response ($10^{-5}$ C Sv$^{-1}$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>thermal</td>
<td>0.830</td>
<td>10.0</td>
</tr>
<tr>
<td>0.0245</td>
<td>2.579</td>
<td>12.1</td>
</tr>
<tr>
<td>0.1</td>
<td>1.451</td>
<td>6.2</td>
</tr>
<tr>
<td>0.25</td>
<td>1.585</td>
<td>6.1</td>
</tr>
<tr>
<td>0.57</td>
<td>1.215</td>
<td>5.2</td>
</tr>
<tr>
<td>1.0</td>
<td>1.215</td>
<td>5.2</td>
</tr>
<tr>
<td>2.5</td>
<td>1.112</td>
<td>6.1</td>
</tr>
<tr>
<td>5.0</td>
<td>0.840</td>
<td>5.2</td>
</tr>
<tr>
<td>15.5</td>
<td>0.728</td>
<td>5.2</td>
</tr>
<tr>
<td>19.0</td>
<td>0.998</td>
<td>12.1</td>
</tr>
<tr>
<td>280.0</td>
<td>0.782</td>
<td>10.1</td>
</tr>
</tbody>
</table>

These responses turned out to have the smallest dependence on energy of any of the instruments tested by Höfert and Raffnsøe and the results for which were reported in (Hö80).

Counter Telescopes

Since the fluence-to-dose equivalent conversion factor for muons varies so little over a wide range [as discussed in Chapter 1 and determined by Stevenson, (St83)], scintillation telescopes provide an attractive method for assessing pure muon fields. At suitable distances and at forward angles, muons will dominate the radiation fields and the results is that little or no discrimination against other particles is necessary.

At Fermilab, a pair of 20.32 cm square by 0.635 cm thick plastic scintillators has been constructed (Co83). The separation distance between theses "paddles" provides moderate directional sensitivity when a coincidence is required between the two plates in a relatively parallel beam of muons. A 2.54 cm thick aluminum plate is employed in the gap between the plates to reduce false coincidences due to &delta;-rays (recoil electrons) from the collisions occurring in the first plate. These plates are mounted in a four-wheel drive vehicle (called the Mobile Environmental Radiation Laboratory-MERL) and are supported by an on-board gasoline-powered generator. A microwave telemetry system provides gating pulses and proton beam intensity information so that normalized beam-on and beam-off (background) measurements can be taken simultaneously. The paddles were chosen to provide sufficient sensitivity to obtain statistical errors at the 20% level in remote locations receiving annual dose equivalents in the fractional mrem range in a scan lasting an hour or two. In such a scan, the detectors are moved across a region of elevated muon flux density. In these detectors, a muon beam perpendicular to...
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the detectors yields $1.7 \times 10^5$ counts per minute per mrem/hour. The normal singles background due to cosmic rays is approximately 400 counts per minute.

Smaller, more portable systems can be useful in conducting muon surveys. Fermilab has such a system, called a "muon finder", consisting of a pair of small plastic scintillators mounted in a compact package which is battery powered and can be carried by one (athletic!) person. It is read out by scalers and can record both singles and coincidence rates. The ratio of the two can be used to "find" unknown muon sources; hence the name of the detector. Also, the separation distance can be adjusted to enhance, or limit, the directional sensitivity.

The parameters of this system are given in Table 6.13.

<table>
<thead>
<tr>
<th>Table 6.13 Parameters of &quot;muon finder&quot; used at Fermilab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillator diameter</td>
</tr>
<tr>
<td>Scintillator thickness</td>
</tr>
<tr>
<td>Scintillator area</td>
</tr>
<tr>
<td>Scintillator spacing</td>
</tr>
<tr>
<td>Half-angle cone of sensitivity</td>
</tr>
<tr>
<td>Dose calibration factor (muons $\perp$ detectors)</td>
</tr>
<tr>
<td>Dose rate cal. factor (muons $\perp$ detectors)</td>
</tr>
</tbody>
</table>

Of course, the use of such scintillators, especially in the "singles" mode, in mixed fields of muons and neutrons requires that one must be aware of the fact that the plastic scintillators have nonzero detection efficiency for the neutrons. Vylet (Vy91) has used the values of total cross sections to calculate the neutron detection efficiency of the detectors described above for neutrons over a range of energies. The results are given in Fig. 6.20 taken from (Vy91). In this figure, effects due to successive collisions as well as those due to the first collisions ("1st collision with H") are given. The total efficiencies at the upper end of the energy region measured were an efficiency of 0.058 for the MERL paddles and 0.0235 for the muon finders.

![Figure 6.20 Calculated neutron efficiencies as a function of neutron energy of scintillation counters used in the "singles" mode at Fermilab. (Reproduced from (Vy91).)
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References


(Aw72) M. Awschalom, "Bonner spheres and tissue equivalent chambers for extensive radiation area monitoring around a 1/2 TeV proton synchrotron", in Proceedings of IAEA Symposium on Neutron Monitoring for Radiation Protection Purposes (Vienna, 1972, vol 1, p. 297).


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(Fr84) W. S. Freeman and F. P. Krueger, "Neutron calibration tests of Fermilab radiation detectors", Fermilab Radiation Physics Note No. 48 (1984).


(Kr68) A. J. Elwyn and J. D. Cossairt, "A study of neutron leakage through an Fe shield at an accelerator", Health Physics 51 (1986) 723-735.

(Fr84) W. S. Freeman and F. P. Krueger, "Neutron calibration tests of Fermilab radiation detectors", Fermilab Radiation Physics Note No. 48 (1984).


(Hö00) M. Höffert and Ch. Raffnsee, "Measurement of absolute absorbed dose and dose equivalent response for instruments used around high-energy proton accelerators", Nucl. Instr. and Meth. 176 (1985) 443-448.


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(Ro55) H. H. Rossi and W. Rosenzweig, "A device for the measurement of dose as a function of specific ionization", Radiology 64 (1955) 404-411.


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Chapter 6  Topics in Radiation Protection Instrumentation at Accelerators -
Problems

Note: The problems in this chapter will require a number of references to the previous chapters.

1. A cylindrical ion chamber is 5 cm in radius and 20 cm long. It is filled with methane (CH₄) at 1 atmosphere absolute pressure. It is bombarded by a uniform flux density of high energy (minimum-ionizing) muons incident perpendicular to one of the ends. One can safely make the assumption that the passage of the muons through the entire length of the chamber represents insignificant degradation of the muon energy or direction. The dose equivalent rate in the radiation field is 0.1 mrem/hour.

   a) Calculate the electric current that will be drawn from this chamber which represents the "signal" to be measured and correlated with the dose equivalent rate. One needs to use Table 1.2 values of (dE/dx)_{min} and to obtain the density of CH₄.

   b) If the charge liberated in the chamber is collected (i.e., integrated electronically) for 1 second and the chamber and circuit represent a capacitance of 10⁻¹⁰ Farads, calculate the size of the signal pulse in volts if one neglects any "pulse-shaping" of the readout electronics.

2. Consider the detector based on the 25.4 cm moderating sphere whose response curve is displayed in Fig. 6.11.

   a) Calculate the approximate absolute detection efficiency for neutrons. This is to be done for the 2 < E_n < 8 MeV energy domain and the sharp peaks in the detector response curve are to be ignored (i.e., averaged out). In this problem, 100 % efficiency is defined to be 1 count generated for every neutron which strikes the sphere. Assume the incident neutrons to be monodirectionally aimed at the detector and originate from a "point" source" despite the fact that this is not quite true.

   b) Since the LiI detector only responds to thermal neutrons, calculate the efficiency with which the moderator transforms fast neutrons incident upon it into thermal neutrons present at the LiI. For this calculation, neglect any "dopants" in the LiI, assume that the Li is "natural" lithium with respect to isotopic abundance and use the fact that the atomic weight of iodine is 127. The density of LiI is 3.5 g/cm³. Assume that the detector is 100% efficient in detecting thermal neutron captures within its volume.
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3. A BF$_3$ proportional chamber is used in a DePangher long counter. This detector, when 
placed in a certain neutron field known to be dominated by neutrons of approximately 5 
MeV kinetic energy, generates counts due to neutrons at the rate of 1 count/minute. 
The detector sensitivity is that represented in Fig. 6.13. The counter operates at one 
atmosphere absolute pressure, the atomic weight of boron is 10.8 while the atomic weight 
of fluorine is 19. At STP the density of BF$_3$ is 2.99 grams/liter.

a) What is the dose equivalent rate of this radiation field?

b) If the radiation field persists full time, is this detector sufficiently sensitive to detect a 
dose rate of 10 mrem/year?

c) In this radiation field, high energy minimum ionizing muons pass through this detector, 
including the proportional counter. The largest muon signals in the proportional counter 
will obviously result when the muons pass lengthwise through the tube. If the tube is 40 
cm long, what will be the size of the largest muon-induced signal relative to the neutron-
induced signal? Is it likely that a simple discriminator circuit can be used to eliminate the 
muon-induced signals. It is quite permissible to "guess" the value of $(dE/dx)_{\text{min}}$ by 
roughly interpolating among the values tabulated in Table 1.20.

4. One needs to understand the sensitivity of the technique of using the $^{12}$C($n$, 2$n$)$^{11}$C 
reaction in plastic scintillator to measure dose equivalent rate external to thick concrete or 
earth shielding near a high energy accelerator. The detector discussed in the text used by 
Moritz has a sensitive volume of approximately 100 cm$^3$ (a 5 cm diameter by 5 cm long 
cylinder). The NE102A scintillator, from (Kn79), has a density of 1.032 g/cm$^3$. This 
detector is nearly 100 % efficient at sensing the 0.511 MeV annihilation photons 
produced in the course of the $^{11}$C decay.

a) This detector is irradiated in a particular radiation field external to such accelerator 
shielding. The irradiation, which is steady in time, is of sufficient length in time to result in 
saturation of the production of $^{11}$C in the scintillator. After the beam is turned off, the 
detector counts at a rate of 10 counts per minute (including appropriate decay-correction to 
the instant of beam shutdown). Calculate the flux density of neutrons with $E_n > 20$ MeV 
during the irradiation and use the result along with Stevenson's conclusion concerning the 
conversion from the flux density of neutrons with $E_n > 20$ MeV to dose equivalent to 
determine the dose equivalent rate.
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Problems

b) Assuming this count rate is the smallest that can be reliably detected, how much smaller in
volume can the detector be for it to barely be sensitive to a dose equivalent rate of 2
mrem/hour?
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Chapter 7 Accelerator Radiation Protection Program Elements

I. Introduction

This chapter summarizes, in outline form, the author's view of the elements needed for a radiation protection program to succeed at an accelerator facility and also gives a short synopsis of some of the important "regulatory" considerations.

II. Accelerator radiation protection program elements

A. Establishment of technical design and siting criteria to address radiation protection concerns

1. off-site dose considerations:
   - direct radiation
   - skyshine
   - air activation
   - water activation
   - groundwater/hydrogeology
   - storage and transportation considerations
   - radiation buffer zones

2. regulatory considerations
   - state/local regulations on groundwater protection
   - state/local regulations and requirements for radiation sources
   - environmental permits for radiological emissions
   - environmental permits for discharges
   - environmental monitoring requirements

B. Quantification of radiation source terms

1. primary source terms
   - particle type (protons, heavy ions, electrons)
   - beam power
   - beam energy
   - utilization factors

2. secondary radiation sources
   - hadrons-through shielding, secondary beams
   - electrons-primary beams at electron beams, secondary beams elsewhere
   - photons-from electron beams and activated components
   - muons (at high energies)

3. incidental radiation sources
   - RF sources (e.g., klystrons, RF cavities)
   - electrostatic septa
   - radiography devices
   - calibration sources
Chapter 7  Accelerator Radiation Protection Program Elements

C. Specification of shielding design criteria

- usually some fraction of regulatory limits
- must consider "worst case" (so-called "accident") conditions
- must address normal losses of beam
- should consider dose to personnel maintaining the accelerator
- calculations should be verified by measurement to extent possible

D. Provision for accelerator access control-interlocks and warning devices

Such systems are of paramount importance. Accidental exposure to the accelerated beam can be the most significant radiation hazard, especially at high energies.

1. functional features of the access control system

- emergency off ("scram") switches
- emergency exit provisions
- warning lights/alarms/signs
- search and secure provisions
- "controlled access"
- accommodation to experimental program needs-flexibility
- beam containment devices
- fences, gates, and locks
- hierarchy of access precautions based upon potential dose rates
- exclusion areas should be both locked and interlocked

2. design features of the access control system

- use "fail-safe" radiation damage resistant components
  
  {area should be rendered safe in the event that a component fails or power is lost}
- "redundancy" is the general rule
  
  in circuitry
  
  in methods of disabling beam
- protection of cabling and any function switches
  
  from damage (including radiation damage)
  
  from tampering
- use solid state devices in such systems with "caution"
  
  questions about software reliability
  
  need for self-checking features (more tricky than simple "loop" circuits with relays)-see Ref. 2
- system must be designed to be tested
  
  written testing procedures must be followed
  
  all inputs to be tested
  
  periodic testing must be done

2. Radiation alarm system

- measure radiation fields in occupied areas and set off alarms/turn off beams if preset limits are exceeded (may be related to the safety envelope)
- long-term logging of radiation levels to provide occupational and environmental radiation "documentation"
- instrumentation should be calibrated and regularly verified to be functional
visible/audible alarms should be used in some situations

E. Provision for the control of radioactivation and contamination (removable radioactivity)

1. Control of residual activation in beamline enclosures
   - surveys upon entry after operations
   - marking/labeling of activated components
   - checking of personnel and equipment leaving enclosures for "contamination" ("removable" radioactivity)
   - entry control/restrictions

2. Control of activated materials outside of beamline enclosures
   - labeling system
   - exclusion for areas frequented by the general public/nonradiation workers
   - exclusion from lunchrooms, etc.
   - procedures/criteria for checking materials being shipped for radioactivity
   - work procedures for activities which might generate removable radioactivity (e.g., welding, grinding, fine wires, dust)
   - liquids containing tritium (e.g., cooling water, pump oil, etc.)
   - targets (sometimes made with hazardous materials; e.g., lithium, beryllium)

F. Provision for the proper management of radioactive waste
   - identification of radioactive waste
   - characterization of radioactive waste
   - avoidance of waste types for which disposal is difficult/impossible
   - waste minimization
   - proper shipment and disposal

G. Program for the control of radioactive sources
   - high intensity calibration sources
   - sources mounted on experimental apparatus
   - labeling/inventory considerations
   - periodic wipes
   - procedures for
     - issuing sources
     - monitoring the inventory and proper usage of sources
     - transporting sources
     - collecting sources no longer needed

H. Program for meeting transportation requirements for radioactive materials and sources
   - U. S. Dept. of Transportation regulations (Code of Federal Regulations Chapter 49-denoted 49 CFR)
   - procedures for shipments to/from universities and commercial entities
   - air transport
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I. ALARA program
   must keep exposures "as low as reasonably achievable"
   usually have ALARA committee
   review jobs, plan jobs based on estimated exposures
   should be embedded in job planning process where radiation exposures are possible
   involves job supervision where significant exposures are probable

J. Decontamination and decommissioning program-planning for the demise of the accelerator
   documentation of activated components, especially structures, groundwater, and soil
   documentation of contaminated components, structures, etc.
   accurate drawings of all structures and components associated with the accelerator
   documentation of the operational history

K. Development and maintenance of a safety envelope for accelerator operations
   the set of physical and administrative conditions that define the bounding conditions for safe operation at an accelerator facility

L. Audit program
   must arrange for objective assessments of program performance
   DOE requires complete radiation protection assessment every 3 years
   audits are needed to establish that requirements are being met

M. Training program
   qualification of radiation workers
   radiological control technicians
   professional radiation protection personnel
   training of experimenters
   addressing the needs of visitors

N. Configuration control program
   maintenance of drawings of shielding and components
   prevention of unauthorized removal of protective features
   
   shielding
   beam containment devices
   interlocked detectors
   alarms
   beam intensity measurement devices
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O. Dosimetry program

must meet designated standards, either National Voluntary Laboratory Accreditation Program (non-DOE) or DOE Laboratory Accreditation Program (DOELAP) (DOE facilities)

dosimeters must be worn, collected on time, read out, and actions must be taken to correct unnecessarily high exposures

dosimetry records must be retained essentially forever

P. Provision for record keeping program

design calculations
dosimetry results
job planning records
log books of accelerator and beamline operation
survey records
release of radioactive material into the environment
prompt environmental radiation exposures
access control system test results
decontamination and decommissioning program
all written procedures
training records
configuration control program
ALARA analysis
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III. Summary of regulatory requirements

In the United States the regulation of the manufacture, distribution and operation of particle accelerators in a manner that does not jeopardize public health and safety is a complex matter shared by several government agencies.

A. International and national advisory bodies have a role chiefly in developing standards and recommendations applicable to general radiation protection (not specific to accelerators) for adoption by individual nations. These standards include:

- recommended dose limits and their technical basis in radiobiology and epidemiology
- standards on measuring/calculating external exposure
- standards on measuring/calculating internal exposure
- standards on instrumentation
- program recommendations

These bodies are:

1. The International Atomic Energy Agency (IAEA)-In addition to regulatory advice, IAEA has issued three reports on radiation safety at electron linacs, neutron generators, and proton accelerators.

2. The International Commission on Radiological Protection (ICRP)-In addition to regulatory advice, ICRP has issued Publication 51 which contains particle fluence to dose equivalent conversion coefficients that are most useful at accelerators.

3. The International Commission on Radiation Units and Measurements (ICRU)-In addition to regulatory advice, ICRP has issued Report 28, "Basis Aspects of High-Energy Particle Interactions and Radiation Dosimetry" which presents a good summary of the subject.

4. The National Council on Radiation Protection and Measurements (NCRP)-independently charted by the U. S. Congress. This body has issued a number of reports of interest in accelerator radiation protection.

5. The American National Standards Institute (ANSI)-has standards on instrument calibration requirements.


7. The Institute of Electrical and electronic Engineers (IEEE)-hosts symposia on instrumentation and issues publications of great utility (nuclear science section).
8. Professional organizations promote this field:

The Health Physics Society
The American Academy of Health Physics
The American Association of Physicist in Medicine

B. Promulgation of Regulations in the United States pertinent to radiation protection programs at particle accelerators

1. **U. S. Environmental Protection Agency (USEPA)** has responsibility for developing guidance on radiological protection for all Federal agencies—usually, but not always, based on ICRP and NCRP recommendations. Promulgation requires presidential approval. The last modifications were approved by President Reagan. "Ancestors" to this system date back to the Eisenhower administration.

USEPA also has considerable direct regulatory authority with enforcement authority which affects accelerators (which may be delegated to the state "environmental" organizations):

a. EPA has issued regulations (40 CFR) on airborne radioactivity emissions (Clean Air Act) from DOE-owned facilities (10 mrem/year dose equivalent to offsite personnel).

b. EPA has issued regulations (40 CFR) on concentrations of radioactivity in drinking water used by community water systems (Safe Drinking Water Act).

2. **The U. S. Department of Transportation (DOT)** issues regulations on transportation of shipments of radioactive materials (49 CFR). In some states these regulations are enforced by state police forces.

3 **U. S. Department of Energy** has regulatory authority over the accelerators it owns and exercises some authority over institutions to which it issues grants.

a. has issued, and continues to issue many "Orders" on radiological protection, noteworthy are:

5480.11-"Radiation Protection for Occupational Workers"

sets dose limit of 5 rem per year to occupational workers
sets dose limit of 100 mrem per year to members of the public
requires notification of off-site dose > 10 mrem/year
many other program requirements

This Order is largely replaced, effectively, in Regulation 10 CFR 835 which became effective in December 1993.
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5400.5- "Radiation Protection of the Public and the Environment"

sets environmental release limits
institutes USEPA requirements
sets forth environmental monitoring program

This Order will be largely replaced by Regulation 10 CFR 834, when the latter is issued.

Many other orders on virtually every facet of radiation protection program

5480.25- "Safety of Accelerator Facilities" has been issued. This Order extensively regulates the entirety of safety at DOE-owned accelerators.

DOE Notice N 5480.6 "DOE Radiological Control Manual" places stringent requirements on all radiological control programs at DOE facilities.

b. As of this writing, May 1995, the Department of Energy was extensively reevaluating its directive system so that many changes can be expected in the near term future.

3. 36 states have regulations concerning accelerators (may or may not apply to federally owned accelerators)

uniformity is encouraged by the Conference of Radiation Control Program Directors which has issued recommendations

often the requirements are patterned after U. S. Nuclear Regulatory Requirements (USNRC) specified in 10 CFR

usually state regulations are incorporated into other state programs for controls of radioactive materials in accord with "agreement state" licensing status with USNRC (USNRC does not explicitly regulate accelerators)

4. Some local governments (cities, counties) etc., may have additional requirements. New York City is a well-known example.
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Bibliography


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